

## Exponents and Powers

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- Very large numbers and very small numbers are difficult to read, understand, and compare. To make this easier, we use exponents by converting many of the large numbers and small numbers into a shorter form.

**For example:** 10,000,000,000,000 can be written as  $(10)^{13}$ .

Here, 10 is called the base and 13 is called the exponent.

- For any non-zero integer  $a$ ,  $a^{-m} = \frac{1}{a^m}$ , where  $m$  is a positive integer.

$a^{-m}$  is called the multiplicative inverse of  $a^m$  and vice-versa.

- Decimal numbers can be written in expanded form using exponents.

For example, the number, 32845.912 can be written in an expanded form as follows:

32845.912

$$\begin{aligned} &= 3 \times 10^4 + 2 \times 10^3 + 8 \times 10^2 + 4 \times 10^1 + 5 \times 1 + 9 \times \frac{1}{10} + 1 \times \frac{1}{10^2} + 2 \times \frac{1}{10^3} \\ &= 3 \times 10^4 + 2 \times 10^3 + 8 \times 10^2 + 4 \times 10^1 + 5 \times 1 + 9 \times 10^{-1} + 1 \times 10^{-2} + 2 \times 10^{-3} \end{aligned}$$

- Laws of exponents** (Here,  $a$  and  $b$  are non-zero integers and  $m$  and  $n$  are integers)

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$
- $(a^m)^n = a^{mn}$
- $a^m \times b^m = (ab)^m$
- $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1 \quad (a \neq 0)$

For example,  $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$  can be simplified using laws of exponents as:

$$\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$$

$$\begin{aligned}
 &= \frac{1^{-2}}{6^{-2}} + \frac{1^{-1}}{7^{-1}} + \frac{1^{-1}}{11^{-1}} & \left( \left( \frac{a}{b} \right)^m &= \frac{a^m}{b^m} \right) \\
 &= \frac{6^2}{1^2} + \frac{7^1}{1^1} + \frac{11^1}{1^1} & \left( a^{-m} &= \frac{1}{a^m} \right) \\
 &= 36 + 7 + 11 \\
 &= 54
 \end{aligned}$$

- **Expressing very small numbers in a simpler way using negative exponents.**

The number 0.00000003812 can be expressed in a simpler way using negative exponents as follows:

$$0.00000003812 = \frac{3812}{100000000000} = \frac{3812}{10^{11}} = \frac{3.812 \times 10^3}{10^{11}} = 3.812 \times 10^{3-11} = 3.812 \times 10^{-8}$$

**Note:** The decimal in 0.00000003812 is moved 8 places to the right.

**Example:** The distance between Earth and Neptune is approximately 4400000000 km. Express this number in the standard form.

$$\begin{aligned}
 \textbf{Solution:} & 4400000000 \\
 &= 44 \times 10^8 \\
 &= 4.4 \times 10^9
 \end{aligned}$$

**Example:** Express the number,  $2.315 \times 10^7$ , in usual form.

$$\begin{aligned}
 \textbf{Solution:} \\
 2.315 \times 10^7 &= 2.315 \times 10000000 = 2315 \times 10000000 \div 1000 = 2315 \times 10000 = 23150000
 \end{aligned}$$