MATRICES

MATHEMATICS

may not teach us how to add love or minus hate. But it gives us every reason to hope that every problem has a solution.

Definition of Matrices

A system of mn number arranged in the form of an ordered set of m rows, each row consisting of an ordered set of n numbers, is called an $m \times n$ (to be read as m by n) matrix.

The given matrix is,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

a_{11} → 1 Row, 1 column

$m \times n$ → m: no. of rows, n: no. of columns

For example :-

$$\text{Let } A = \begin{array}{c} \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 6 \end{bmatrix}_{2 \times 3} \begin{array}{l} \to \text{Ist row } (R_1) \\ \to \text{2nd row } (R_2) \end{array} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \\ \downarrow & \downarrow & \downarrow \\ \text{Ist col.} & \text{2nd} & \text{3rd Column} \end{array} \end{array}$$

Note:

There are four different symbols [ ], ( ), || ||, { }.
for inclosing the numbers of array.

⇒ We shall apply mostly the first two symbols.

Equality of Matrices:

Two matrices A and B are said to be equal if and only if:

(i) A and B are of the same type i.e. their order are equal

(ii) All the elements of A are the same as the corresponding elements of B.

<u>OR</u>

Two matrices $A = [a_{ij}]$, $B = [b_{ij}]$ of the same type are said to be equal if $a_{ij} = b_{ij} \ \forall \ i,j$.

Note ÷ There are four different symbols [ ], ( ), || ||, { } for inclosing the numbers of the arrays.

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SPECIAL TYPES OF MATRICES

Row Matrix :

An $m \times n$ matrix is called a row matrix if $m = 1$.

OR

A matrix is said to be a row matrix if it has one row.

For example : Let $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1 \times 3}$.

Column matrix :

An $m \times n$ matrix is called a column matrix if $n = 1$

OR

A matrix is said to be a column matrix if it has only 1 column.

For example : Let $A = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}_{3 \times 1} \rightarrow$ one column

or let $A = \begin{bmatrix} 27 \\ 3 \\ 8 \\ 11 \end{bmatrix}_{4 \times 1}$

one column

Square matrix

An $m \times n$ matrix is called a square matrix if $m = n$

OR

A matrix is said to be square matrix if it has same no. of rows and columns.

For example: Let $A = \begin{bmatrix} 2 & 1 \\ 9 & 8 \end{bmatrix}_{2\times 2}$

is a square matrix or sometimes called 2-rowed matrix.

Rectangular matrix

A matrix which is not a square matrix, is called a rectangular matrix. OR

An $m \times n$ matrix is called a rectangular matrix if $m \neq n$

For example: Let $A = \begin{bmatrix} 1 & 7 & 4 \\ 5 & 6 & 9 \end{bmatrix}_{2\times 3}$

Imp. Example

Let $A = [2]_{1\times 1}$, which is a Row matrix, column matrix and square matrix.

Zero matrix

A matrix each of whose elements is zero is called a zero matrix and is denoted by O. Zero matrix is also called Null matrix.

For example ÷ Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a zero matrix.

Diagonal matrix

A square matrix with all its non-diagonal elements as zero is called a diagonal matrix.

Therefore if $A = [a_{ij}]$ is a diagonal matrix, then $a_{ij} = 0 \ \forall \ i \neq j$.

For example ÷ Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ is a diagonal matrix.

Note ÷ A Diagonal matrix

Let. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3 \times 3}$

a_{ij} $(i<j)$ non-diagonal enteries

a_{ij} $(i>j)$ non-diagonal enteries

a_{ij} $(i=j)$ Diagonal enteries

Scalar matrix

A diagonal matrix all of whose diagonal elements are equal is called a scalar matrix.

for example ÷ $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

is a scalar matrix.

Identity matrix

A scalar matrix all of whose diagonal elements are equal to unity i.e. 1 is called a unit matrix. It is denoted by I_n, if it is of order n. Unit matrix is also called an Identity matrix.

Note ÷ Sum of diagonal elements of a square matrix is called trace of A

Triangular matrix

If every element above or below the diagonal is zero, the matrix is said to be a triangular matrix.

OR

A matrix $A = [a_{ij}]$ is called a triangular matrix if $a_{ij} = 0$, whenever $i > j$ or $i < j$.

Note ⇒ Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

a_{ij} $(i<j)$

a_{ij} $(i>j)$

$a_{ij} = (i=j)$

⇒ Upper triangular matrix

If $a_{ij} = 0$ when $i > j$, the matrix is called Upper triangular matrix.

for example ÷ Let $A = \begin{bmatrix} 1 & 2 & 7 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix}$

is an upper triangular matrix.

⇒ Lower triangular matrix

If $a_{ij} = 0$ when $i < j$ the matrix is called lower triangular matrix.

For e.g. :- $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 3 & 4 \end{bmatrix}$ is lower triangular matrix.

Ques. Construct a 3×4 matrix, whose elements are given by : $a_{ij} = \frac{1}{2}|-3i + j|$.

Sol. The given matrix is.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$

$\therefore \; a_{11} = \frac{1}{2}|-3(1)+1| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$

$a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$.

$a_{13} = \frac{1}{2}|-3(1)+3| = \frac{1}{2}|0| = 0$

$a_{14} = \frac{1}{2}|-3(1)+4| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$.

$a_{21} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5|$

$= \frac{5}{2}$.

$$a_{22} = \frac{1}{2}|-3(2)+2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$

$$a_{23} = \frac{1}{2}|-3(2)+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}.$$

$$a_{24} = \frac{1}{2}|-3(2)+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1.$$

$$a_{31} = \frac{1}{2}|-3(3)+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{\cancel{8}^{4}}{\cancel{2}} = 4$$

$$a_{32} = \frac{1}{2}|-3(3)+2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}.$$

$$a_{33} = \frac{1}{2}|-3(3)+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{34} = \frac{1}{2}|-3(3)+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}.$$

$$\therefore A = \begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}_{3\times 4}$$

Sol.

Ques: Construct a 2×3 matrix whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{(i+2j)^2}{2}$

(iii) $a_{ij} = \frac{1}{2}|i-3j|$.

(i) Sol. The given matrix is,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3}.$$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = \frac{4}{2} = 2.$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{13} = \frac{(1+3)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8.$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}.$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8.$$

$$a_{23} = \frac{(2+3)^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}.$$

$$\therefore A = \begin{bmatrix} 2 & 9/2 & 8 \\ 9/2 & 8 & 25/2 \end{bmatrix}_{2\times3}$$

(ii) Sol. The given matrix is,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3}.$$

$$\therefore a_{11} = \frac{(1+2(1))^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}.$$

$$a_{12} = \frac{(1+2(2))^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}.$$

$$a_{13} = \frac{(1+2(3))^2}{2} = \frac{(7)^2}{2} = \frac{49}{2}.$$

$$a_{21} = \frac{(2+2(1))^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8.$$

$$a_{22} = \frac{(2+2(2))^2}{2} = \frac{(6)^2}{2} = \frac{36}{2} = 18.$$

$$a_{23} = \frac{(2+2(3))^2}{2} = \frac{(8)^2}{2} = \frac{64}{2} = 32.$$

$$\therefore A = \begin{bmatrix} 9/2 & 25/2 & 49/2 \\ 8 & 18 & 32 \end{bmatrix}_{2\times 3}$$

(iii) Sol. The given matrix is,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}.$$

$$a_{11} = \frac{1}{2}|1-3(1)| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1.$$

$$a_{12} = \frac{1}{2}|1-3(2)| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}.$$

$$a_{13} = \frac{1}{2}|1-3(3)| = \frac{1}{2}|-8| = \frac{8}{2} = 4.$$

$$a_{21} = \frac{1}{2}|2-3(1)| = \frac{1}{2}|-1| = \frac{1}{2}.$$

$$a_{22} = \frac{1}{2}|2-3(2)| = \frac{1}{2}|2-6| = \frac{1}{2}|-4| = 2.$$

$$a_{23} = \frac{1}{2}|2-3(3)| = \frac{1}{2}|2-9| = \frac{1}{2}|-7| = \frac{7}{2}.$$

$$\therefore A = \begin{bmatrix} 1 & 5/2 & 4 \\ 1/2 & 2 & 7/2 \end{bmatrix}_{2\times3}$$

Ques: If a matrix has 18 elements what are the possible order, but if it has 5 elements

Sol: When number of elements in a matrix are 18.

$\therefore$ possible orders are 1×18, 2×9, 9×2, 3×6, 6×3, 18×1.

But if no. of elements $=5$.

Then, possible orders are 1×5, 5×1.

Ques: (a) Construct a 2×3 matrix whose elements are given by $a_{ij} = i+2j$.

(b) Construct a 2×4 matrix $A=(a_{ij})$ whose elements are given by $a_{ij} = -2i-j$.

Sol: (a) The given matrix is.

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3}$$

$$a_{11} = 1+2(1) = 1+2 = 3$$

$$a_{12} = 1+2(2) = 1+4 = 5$$

$$a_{13} = 1+2(3) = 1+6 = 7$$

$$a_{21} = 2+2(1) = 2+2 = 4$$

$a_{22} = 2+2(2) = 2+4 = 6$

$a_{23} = 2+2(3) = 2+6 = 8$

∴ Required matrix is,

$$A = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \end{bmatrix}_{2\times 3}$$

(b) Sol. The given matrix is,

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}_{2\times 4}$

$a_{11} = -2(1)-1 = -2-1 = -3$

$a_{12} = -2(1)-2 = -2-2 = -4$

$a_{13} = -2(1)-3 = -2-3 = -5$

$a_{14} = -2(1)-4 = -2-4 = -6$

$a_{21} = -2(2)-1 = -4-1 = -5$

$a_{22} = -2(2)-2 = -4-2 = -6$

$a_{23} = -2(2)-3 = -4-3 = -7$

$a_{24} = -2(2)-4 = -4-4 = -8$

∴ Required matrix is,

$$A = \begin{bmatrix} -3 & -4 & -5 & -6 \\ -5 & -6 & -7 & -8 \end{bmatrix}_{2\times 4}$$

Ques: Construct a 3×4 matrix whose elements are

(1) $a_{ij} = i+j$ (2) $a_{ij} = i-j$

(3) $a_{ij} = i \times j$ (4) $a_{ij} = \frac{i}{j}$

Sol(1) The given matrix is,

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$

We are given that, $\underline{\underline{a_{ij} = i+j}}$

$\therefore a_{11} = 1+1 = 2$

$a_{12} = 1+2 = 3$

$a_{13} = 1+3 = 4$

$a_{14} = 1+4 = 5$

$a_{21} = 2+1 = 3$

$a_{22} = 2+2 = 4$

$a_{23} = 2+3 = 5$

$a_{24} = 2+4 = 6$

$a_{31} = 3+1 = 4$

$a_{32} = 3+2 = 5$

$a_{33} = 3+3 = 6$

$a_{34} = 3+4 = 7$

∴ Required matrix is,

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}_{3\times4}$$

Sol.(2) The given matrix is.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$

We are given that, $a_{ij} = i-j$

$\therefore\ a_{11} = 1-1 = 0$

$a_{12} = 1-2 = -1$

$a_{13} = 1-3 = -2$

$a_{14} = 1-4 = -3$

$a_{21} = 2-1 = 1$

$a_{22} = 2-2 = 0$

$a_{23} = 2-3 = -1$

$a_{24} = 2-4 = -2$

$a_{31} = 3-1 = 2$

$a_{32} = 3-2 = 1$

$a_{33} = 3-3 = 0$

$a_{34} = 3-4 = -1$

∴ Required matrix is,

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}_{3\times 4}$$

Sol (3) The given matrix is

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}$

We are given that, $a_{ij} = i \times j$

$a_{11} = 1 \times 1 = 1$

$a_{12} = 1 \times 2 = 2$

$a_{13} = 1 \times 3 = 3$

$a_{14} = 1 \times 4 = 4$

$a_{21} = 2 \times 1 = 2$

$a_{22} = 2 \times 2 = 4$

$a_{23} = 2 \times 3 = 6$

$a_{24} = 2 \times 4 = 8$

$a_{31} = 3 \times 1 = 3$

$a_{32} = 3 \times 2 = 6$

$a_{33} = 3 \times 3 = 9$

$a_{34} = 3 \times 4 = 12$.

∴ required matrix is,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}_{3\times 4}$$

Sol (4) The given matrix is,

Let $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}$$

We are given that, $a_{ij} = \frac{i}{j}$.

∴ $a_{11} = 1/1 = 1$

$a_{12} = 1/2 = 1/2$

$a_{13} = 1/3$

$a_{14} = 1/4$

$a_{21} = 2/1 = 2$

$a_{22} = \frac{2}{2} = 1$

$a_{23} = 2/3$

$a_{24} = \frac{2}{4} = \frac{1}{2}$

$a_{31} = \frac{3}{1} = 3$

$a_{32} = \frac{3}{2}$

$$a_{33} = \frac{3}{3} = 1$$

$$a_{34} = \frac{3}{4}$$

$\therefore$ required matrix is,

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 2 & 1 & 2/3 & 1/2 \\ 3 & 3/2 & 1 & 3/4 \end{bmatrix}_{3\times 4}$$

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Ques: Find the value of x and y.

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Sol: We are given that

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

$\therefore$ by equality of matrices, we have

$$3x+y = 1 \longrightarrow (1)$$

$$-y = 2 \longrightarrow (2)$$

$$2y-x = -5 \longrightarrow (3)$$

$\therefore$ from (2), $\boxed{y=-2}$

$\therefore$ ① $\Rightarrow$ $3x - 2 = 1$

$\Rightarrow$ $3x = 1 + 2$

$\Rightarrow$ $\boxed{x = 1}$

$\therefore$ $x = 1$ and $y = -2$

Ques: If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of x and y.

Sol. We are given that,

$$\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$$

$\therefore$ by equality of matrices, we have,

$x = 3$ ——— ①

$x - y = 1$ ——— ②

$2x + y = 8$ ——— ③

$\therefore$ from ①, $x = 3$

$\therefore$ ② $\Rightarrow$ $3 - y = 1$

$\Rightarrow$ $y = 2$

$\therefore$ $x = 3$, $y = 2$.

Ques: If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, then find x and y.

Sol: We are given,

$$\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$$

$\therefore$ by equality of matrices, we have,

$$y+2x = 7 \longrightarrow (1)$$
$$-x = -2 \longrightarrow (2)$$

from (2), $\boxed{x = 2}$

$\therefore$ (1) $\Rightarrow y + 2(2) = 7$

$\Rightarrow y + 4 = 7$

$\Rightarrow \boxed{y = 3}$

$\therefore \underline{\underline{x = 2}}, \underline{\underline{y = 3}}$

Ques: Write the no. of all the possible matrices of order 2×2 with each entry 1, 2 or 3.

Sol: Let $A = [a_{ij}]_{2\times 2}$

$$\therefore A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

$\therefore a_{ij} = 1, 2$ or $3 \ \forall\, i \leq 2, j \leq 2$.

$\therefore$ total no. of possible matrices

$= 3 \times 3 \times 3 \times 3$

$= 3^4 = \underline{\underline{81}}$ Ans.

Ques: If $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$, find x and y.

Sol: We are given that,

$$\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$$

$\therefore$ by equality of two matrices,

$$3y - x = 5 \longrightarrow ①$$

$$-2x = -2 \longrightarrow ②$$

$\therefore$ from ②, $x = 1$

$\therefore$ ① $\Rightarrow 3y - 1 = 5$

$\Rightarrow y = \frac{6}{3} = 2$

$\Rightarrow y = 2$

$\therefore \underline{\underline{x = 1}}, \underline{\underline{y = 2}}$

Ques: If $\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$ then find the value of x & y.

Sol: We are given that,

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$

$\therefore$ by equality of matrices, we have

$$x+y = 3 \longrightarrow (1)$$

$$3y = 6 \longrightarrow (2)$$

$\therefore$ from (2), $y = \frac{6}{3} \Rightarrow y=2$

$\therefore$ (1) $\Rightarrow x+2=3$

$\Rightarrow x=1$

$\therefore$ $\underline{\underline{x=1}}$, $\underline{\underline{y=2}}$.

Ques: If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find the value of a and b.

Sol: We are given,

$$\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}.$$

$\therefore$ by equality of two matrices,

$$a+b = 6 \longrightarrow (1)$$

$$ab = 8 \longrightarrow (2)$$

$\therefore$ (2) $\Rightarrow b = \frac{8}{a} \longrightarrow (3)$

Putting $b = \frac{8}{a}$ in ①, we get,

$$a + \frac{8}{a} = 6$$

$$\therefore a^2 + 8 = 6a$$

$$\therefore a^2 - 6a + 8 = 0$$

$$\therefore a(a-4) - 2(a-4) = 0$$

$$\therefore (a-2)(a-4) = 0$$

$$\therefore a = 2, 4$$

When $a=2$,

then ③ $\Rightarrow b = \frac{8}{2}$

$$\Rightarrow \boxed{b=4}$$

$\therefore$ $a=2$, $b=4$

When $a=4$,

③ $\Rightarrow b = \frac{8}{4} = 2$

$\therefore$ $a=4$, $b=2$

Addition of Matrices

If $A = [a_{ij}]$, $B = [b_{ij}]$ be two matrices of the same type $m \times n$, then their sum $A+B$ is defined as the matrix $A+B = [a_{ij} + b_{ij}]$.

Note:-
Two matrices which can be added are said to be conformable for addition.

Ques: If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ then find $A+B$.

Sol: The given matrices are,

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$\therefore A+B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$\therefore A+B = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}_{2\times 2}$$

Subtraction of Matrices

If $A = [a_{ij}]$, $B = [b_{ij}]$ are two matrices of the same type, then difference is defined as the matrix $C = [c_{ij}]$ where $c_{ij} = a_{ij} - b_{ij}$.

Ques: If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find $A-B$.

Sol: The given matrices are,

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}.$$

$$\therefore A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}_{2\times 2}$$

SCALAR MULTIPLICATIVE

If each element of a matrix A is multiplied by a scalar k (we call k a scalar to distinguish it from $[k]$ which is 1×1 matrix), then the resulting matrix $kA = Ak$ is called the scalar multiple of A by k.

For example : If $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$, find $3A$.

$$\therefore A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \Rightarrow 3A = \begin{bmatrix} 3 & -6 \\ 3 & 9 \end{bmatrix}$$

NEGATIVE OF A MATRIX

Negative of a matrix A i.e. $-A$ is the matrix obtained from A by multiplying each of its elements by -1.

For example : If $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, find $-3A$.

We have, $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

$$\therefore -3A = \begin{bmatrix} -6 & -9 \\ 3 & -12 \end{bmatrix}$$

Imp. Note :

1.) $A + B = B + A$ (addition is commutative)

2.) $A + (B + C) = (A + B) + C$ (addition is associative)

{Only if A, B and C are three matrices of the same order.}

Note : If A and B are two matrices of the same order and k an l are numbers then

1.) $k(A + B) = kA + kB$

2.) $(k + l)A = kA + lA$

Ques: If $A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 0 & 6 \\ -2 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 & 2 \\ 6 & -1 & 4 \\ 5 & 3 & -4 \end{bmatrix}$

then find $2A - 3B$.

Sol. The given matrices are,

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 0 & 6 \\ -2 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 1 & 2 \\ 6 & -1 & 4 \\ 5 & 3 & -4 \end{bmatrix}$$

$$\therefore 2A-3B = 2\begin{bmatrix} 2 & 3 & -4 \\ 1 & 0 & 6 \\ -2 & 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 5 & 1 & 2 \\ 6 & -1 & 4 \\ 5 & 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & -8 \\ 2 & 0 & 12 \\ -4 & 2 & 10 \end{bmatrix} - \begin{bmatrix} 15 & 3 & 6 \\ 18 & -3 & 12 \\ 15 & 9 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 3 & -14 \\ -16 & 3 & 0 \\ -19 & -7 & 22 \end{bmatrix}_{3\times 3}$$

Ques: If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$, then find a matrix X such that $3A-2B+4X=0$, where $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Sol We have,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Now, $3A-2B+4X=0$

$$\Rightarrow 4X = -3A+2B$$

$$\Rightarrow 4X = -3\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\Rightarrow 4X = \begin{bmatrix} -6 & -9 \\ -12 & -15 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$\Rightarrow 4X = \begin{bmatrix} 0 & -1 \\ -2 & -3 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{4}\begin{bmatrix} 0 & -1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{3}{4} \end{bmatrix}$$

Ques: If $2X+3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X+2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$, find X and Y.

Sol: We have,

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \longrightarrow (1)$$

$$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \longrightarrow (2)$$

$$\therefore 3 \times (1) \Rightarrow 6X + 9Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} \longrightarrow (3)$$

$$\therefore 2 \times (2) \Rightarrow 6X + 4Y = \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix} \longrightarrow (4)$$

$\therefore (3) - (4) \Rightarrow$

$$5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$$

$$\therefore \; 5Y = \begin{bmatrix} 6+4 & 9-4 \\ 12-2 & 0+10 \end{bmatrix}$$

$$\therefore \; Y = \frac{1}{5}\begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\therefore \; Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}_{2\times 2}$$

$$\therefore \; (1) \Rightarrow \; 2X + 3\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \; 2X + \begin{bmatrix} 6 & 3 \\ 6 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \; 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ 6 & 6 \end{bmatrix}$$

$$\Rightarrow \; X = \frac{1}{2}\begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$$

$$\Rightarrow \; X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Sol.

Ques: Given $A = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$, then find $A+B$.

Sol. The given matrices are,

$$A = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$$

$$\therefore A + B = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$$

$$\therefore A + B = \begin{bmatrix} 2+\sqrt{3} & 1+\sqrt{5} & 0 \\ 0 & 6 & 1/2 \end{bmatrix}$$

Ques: Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

Sol: $= \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

Sol: $= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}_{2\times 2}$$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

Sol: $= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 11 & 0 \\ 6 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}_{3\times3}$

(iv) $\begin{bmatrix} \cos^2x & \sin^2x \\ \sin^2x & \cos^2x \end{bmatrix} + \begin{bmatrix} \sin^2x & \cos^2x \\ \cos^2x & \sin^2x \end{bmatrix}$

Sol: $= \begin{bmatrix} \cos^2x + \sin^2x & \sin^2x + \cos^2x \\ \sin^2x + \cos^2x & \cos^2x + \sin^2x \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2\times2}$

Ques: If $A = \begin{bmatrix} 2/3 & 1 & 5/3 \\ 1/3 & 2/3 & 4/3 \\ 7/3 & 2 & 2/3 \end{bmatrix}$ and

$B = \begin{bmatrix} 2/5 & 3/5 & 1 \\ 1/5 & 2/5 & 4/5 \\ 7/5 & 6/5 & 2/5 \end{bmatrix}$ then compute $3A - 5B$.

Sol: $3A-5B = 3\begin{bmatrix} 2/3 & 1 & 5/3 \\ 1/3 & 2/3 & 4/3 \\ 7/3 & 2 & 2/3 \end{bmatrix} - 5\begin{bmatrix} 2/5 & 3/5 & 1 \\ 1/5 & 2/5 & 4/5 \\ 7/5 & 6/5 & 2/5 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$

Ques: If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, find the matrix C such that $A+B+C$ is a zero matrix.

Sol. The given matrices are,

$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

Since, $A+B+C = O$

$\Rightarrow C = -A-B$

$\Rightarrow C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

$$\Rightarrow C = \begin{bmatrix} -1 & 3 & -2 \\ -2 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}_{2\times 3}$$

Ques: If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then, find the matrix X such that $2A + 3X = 5B$

Sol: The given matrices are,

$$A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

Now, $2A + 3X = 5B$

$$\Rightarrow 3X = 5B - 2A$$

$$= 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{3}\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & -10/3 \\ 4 & 14/3 \\ -31/3 & -7/3 \end{bmatrix}_{3\times 2}$$

Ques: Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Sol: The given matrices are,

$$Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \longrightarrow (1)$$

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \longrightarrow (2)$$

$\therefore$ (2) $-$ (1) $\Rightarrow$

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}_{2\times 2}$$

Ques: Find X and Y if $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$, $X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

Sol: The given equations are,

$$X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \longrightarrow (1)$$

$$X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \longrightarrow (2)$$

$\therefore$ (1)+(2) $\Rightarrow$

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\therefore 2X = \begin{bmatrix} 10 & 0 \\ 2 & 3 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Putting X in (1), we get,

$$\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{Ans.}$$

Ques: Find X and Y if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

Sol: We have,

$$2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \longrightarrow (1)$$

$$X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \longrightarrow (2)$$

$\therefore 2 \times (2) \Rightarrow$

$$2X + 4Y = \begin{bmatrix} 6 & 4 & 10 \\ -4 & 2 & -14 \end{bmatrix} \longrightarrow (3)$$

$\therefore (3) - (1) \Rightarrow$

$$5Y = \begin{bmatrix} 6 & 4 & 10 \\ -4 & 2 & -14 \end{bmatrix} - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$\therefore 5Y = \begin{bmatrix} 0 & 10 & 10 \\ 0 & 0 & -15 \end{bmatrix}.$$

$$\therefore Y = \frac{1}{5}\begin{bmatrix} 0 & 10 & 10 \\ 0 & 0 & -15 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

∴ putting Y in ①, we get,

$$2X - \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$\therefore 2X = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\begin{bmatrix} 6 & -4 & 2 \\ -4 & 2 & -2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$ Sol:

Ques: If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then which of the sums $A+B$, $B+C$, $C+D$ and $B+D$ is defined?

Sol: The given matrices are,

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and $D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$

Type of matrix A is 2×2.

Type of matrix B is 2×3.

Type of matrix C is 2×1

Type of matrix D is 2×3.

$\therefore$ Sums A+B, B+C and C+D is not defined because of different types of matrices.

Sum B+D is defined because of same type of B and D matrices.

$$\therefore \; B + D = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$$

$$\therefore \; B + D = \begin{bmatrix} 5 & 9 & 10 \\ 9 & 10 & 10 \end{bmatrix}_{2 \times 3}$$

Ques: find the values of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}.$$

Sol: We have,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}.$$

$$\therefore \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} 4+a & 6+a+b \\ -1+c+d & 2d+3 \end{bmatrix}$$

∴ by equality of matrices,

$3a = 4+a \longrightarrow (1)$

$3b = 6+a+b \longrightarrow (2)$

$3c = -1+c+d \longrightarrow (3)$

$3d = 2d+3 \longrightarrow (4)$

$\therefore (1) \Rightarrow 2a = 4$

$\Rightarrow \boxed{a=2}$

$\therefore (4) \Rightarrow \boxed{d=3}$

Now, putting value of a in (2), we get.

$\therefore 3b = 6+2+b$

$\therefore 2b = 8$

$\boxed{\therefore b=4}$

Putting $d=3$ in (3), we get.

$\therefore 3c = -1+c+3$

$\therefore 2c = 2$

$\therefore \boxed{c=1}$

$\therefore a=2, b=4, c=1$ and $d=3$

PRODUCT OF MATRICES

Let $A = [a_{ij}]$, $B = [b_{ij}]$ be two matrices of $m \times n$ and $n \times l$ types respectively i.e. the number of columns of A is the same as the number of rows of B.

A → Pree factor

B → Post factor.

⇒ AB is possible only if no. of column in A is equal to the no. of rows in B.

Ques: (i) Find AB and BA. Is it equal if,

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 2 & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 6 & 4 \end{bmatrix}.$$

Sol. The given matrices are,

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 2 & 8 \end{bmatrix}_{2\times3}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 6 & 4 \end{bmatrix}_{3\times2}$$

$$\therefore AB = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 2 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 6 & 4 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 2+12+36 & 4+8+24 \\ 3+6+48 & 6+4+32 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 50 & 36 \\ 57 & 42 \end{bmatrix}_{2\times2}$$

$$\therefore BA = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 6 & 4 \end{bmatrix}_{3\times2} \begin{bmatrix} 2 & 4 & 6 \\ 3 & 2 & 8 \end{bmatrix}_{2\times3}$$

$$\therefore BA = \begin{bmatrix} 2+6 & 4+4 & 6+16 \\ 6+6 & 12+4 & 18+16 \\ 12+12 & 24+8 & 36+32 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 8 & 8 & 22 \\ 12 & 16 & 34 \\ 24 & 32 & 68 \end{bmatrix}_{3\times3}$$

Thus, $AB \neq BA$.

Ques: Write a single matrix.

$$\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 2 & 4 \\ -7 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

Sol: We have,

$$\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 2 & 4 \\ -7 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2-4+30 \\ 0+8+24 \\ -7+20+0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & -2 & 3 \end{bmatrix}_{1\times 3} \begin{bmatrix} 28 \\ 32 \\ 13 \end{bmatrix}_{3\times 1}$$

$$\therefore A = \begin{bmatrix} 28-64+39 \end{bmatrix} = \begin{bmatrix} 3 \end{bmatrix}_{1\times 1}$$. Sol.

Ques: Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$. Find AB and BA.

Sol: We have,

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}.$$

$$\therefore AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

Now, $BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$\therefore BA = \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix}$

$\therefore BA = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}_{2\times 2}$ Sol.

Note $\Rightarrow$ zero matrix as the product of two non-zero matrix.

Sol. $\Rightarrow$ Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$.

clearly $A \neq O$ also $B \neq O$

$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore BA = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$\therefore AB = BA = 0.$

Ques: Find X such that $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

Sol: We have,

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0+4+0 \\ 0+0+x \\ 0+0+2x \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ x \\ 2x \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 4+2x+2x \end{bmatrix} = 0$$

$$\therefore 4+4x = 0$$

$$\therefore x = \frac{-4}{4} \Rightarrow x = -1$$

Ques: Find X such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$

Sol: We are given that,

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1-4+9 \\ 4-10+18 \\ 3-4+15 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix} = 0$$

$$\therefore [6 + 12x + 14] = 0$$

$$\therefore 12x + 20 = 0$$

$$\therefore x = -\frac{20}{12} \Rightarrow x = -\frac{5}{3}$$

Ques: Find $A^2 - 5A + 6I$. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Sol: We are given,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}_{3\times 3}$$

$$\therefore A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2-0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}_{3\times 3}$$

Ques: If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, verify that $A^2 - 5A + 7I = 0$

Sol: The given matrix is,

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}.$$

$$\therefore A^2 - 5A + 7I = 0$$

$$\therefore \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\therefore 0 = 0$$

Hence, it is verified.

Ques: If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then show that $A^2 - 7A + 10I = 0$

Sol: We are given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

$$\therefore A^2 = \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix}_{3\times 3}$$

$$\therefore A^2 - 7A + 10I = 0$$

$$\therefore \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 27 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$ Hence, proved

Ques: If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$.

Sol: The given matrix is,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Now, $f(x) = x^2 - 2x - 3$

$$\therefore f(A) = A^2 - 2A - 3I$$

$$\therefore A^2 = A \cdot A$$

$$\therefore A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$\therefore f(A) = A^2 - 2A - 3I$$

$\therefore f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\therefore f(A) = \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$

$\therefore f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Hence, proved.

Ques 7 If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ and $f(x) = x^2 - 4x + 2$ then find $f(A)$.

Sol. The given matrix is.

$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

Now, $f(x) = x^2 - 4x + 2$

$\therefore f(A) = A^2 - 4A + 2I$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 4-1 & 2+3 \\ -2-3 & -1+9 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -5 & 8 \end{bmatrix}$

Now, $f(A) = A^2 - 4A + 2I$

$$\therefore f(A) = \begin{bmatrix} 3 & 5 \\ -5 & 8 \end{bmatrix} - 4\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore f(A) = \begin{bmatrix} 3 & 5 \\ -5 & 8 \end{bmatrix} - \begin{bmatrix} 8 & 4 \\ -4 & 12 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore f(A) = \begin{bmatrix} 3-8+2 & 5-4+0 \\ -5+4+0 & 8-12+2 \end{bmatrix}$$

$$\therefore f(A) = \begin{bmatrix} -7 & 1 \\ -1 & -6 \end{bmatrix}_{2\times2}$$

Ques: Simplyfy $\cos\theta \begin{bmatrix} \cos\theta \cdot & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Sol: We have,

$$= \cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ -\cos\theta\sin\theta + \cos\theta\sin\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

Ques: If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$, then find x.

Sol: We have,

$$\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 2x & 3 \end{bmatrix} . \begin{bmatrix} x+6 \\ -3x+0 \end{bmatrix} = 0$$

$$\therefore \left[2x(x+6) + 3(-3x)\right] = 0$$

$$\therefore 2x^2 + 12x - 9x = 0$$

$$\therefore 2x^2 + 3x = 0.$$

$$\therefore x[2x+3] = 0$$

$$\Rightarrow x = 0, \quad x = -\frac{3}{2}$$

Ques: If $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$, then find the value of x and y.

Sol: We have,

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3x-4y \\ x+2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

$\therefore$ by equality of matrices,

$$3x - 4y = 3 \longrightarrow (1)$$
$$x + 2y = 11 \longrightarrow (2)$$

Solving (1) and (2), we get

$$\frac{x}{-44-6} = \frac{y}{3-33} = \frac{-1}{6+4}$$

$$\begin{Bmatrix} 2 & 3 & 1 & 2 \\ -4 & 3 & 3 & -4 \\ 2 & 11 & 1 & 2 \end{Bmatrix}$$

$$\Rightarrow x = \frac{50}{10}, \quad y = \frac{30}{10}$$

$$\Rightarrow \boxed{x=5}, \quad \boxed{y=3}$$

Ques: Give example to show that $AB = O$, even if $A \neq O$, $B \neq O$.

Sol: Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$.

Clearly $A \neq O$, $B \neq O$.

$$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = O$$

$\therefore AB = O$ even if $A \neq O$, $B \neq O$.

Ques: Give example to show that $AB = 0$, but $BA \neq 0$.

Sol Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = 0$$

$$\text{Now } BA = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0+0 & 0+0 \\ 2+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$$

$\therefore AB = 0$ but $BA \neq 0$.

Ques: Give example to show that $AB \neq 0$ but $BA = 0$.

Sol Let $A = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 2+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = 0$$

$\therefore AB \neq 0$, but $BA = 0$.

Ques: If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

find k so that $A^2 = KA - 2I$.

Sol: The given matrices are,

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now, $A^2 = KA - 2I$.

$$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore K\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\therefore K\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow K = 1$$

★★ **Ques:** Prove the following by P.M.I:

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then $A^n \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

for any integer n.

Sol: The given matrix is,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

To Prove: $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} \longrightarrow (1)$

Proof: For $n=1$

$$A^1 = \begin{bmatrix} 1+2\cdot 1 & -4\cdot 1 \\ 1 & 1-2\cdot 1 \end{bmatrix}$$

$\therefore A^1 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, which is true.

$\therefore$ result is true for $n=1$.

Assume that result ① is true for $n=k$.

$$\therefore A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \longrightarrow ②$$

$$\therefore A^{k+1} = A^k \cdot A$$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad \{\because \text{ of } ②\}$$

$$= \begin{bmatrix} 3(1+2k)-4k(1) & -4(1+2k)-1(-4k) \\ 3k+(1)(1-2k) & -4k-(1)(1-2k) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4-4k \\ k+1 & -2k-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$$

$\therefore$ result is true for $n=k+1$.

Hence, by P.M.I. the result is true for all positive integer n.

Ques: Prove the following by P.M.I.,

If $A = \begin{bmatrix} 11 & -25 \\ 4 & -9 \end{bmatrix}$ then $A^n = \begin{bmatrix} 1+10n & -25n \\ 4n & 1-10n \end{bmatrix}$,

where n is +ve Integer.

Sol: The given matrix is,

$$A = \begin{bmatrix} 11 & -25 \\ 4 & -9 \end{bmatrix}$$

To prove: $A^n = \begin{bmatrix} 1+10n & -25n \\ 4n & 1-10n \end{bmatrix} \longrightarrow (1)$

For $n=1$:-

$$A^1 = \begin{bmatrix} 1+10\cdot1 & -25\cdot1 \\ 4\cdot1 & 1-10\cdot1 \end{bmatrix}$$

$$\therefore A^1 = \begin{bmatrix} 11 & -25 \\ 4 & -9 \end{bmatrix}, \text{ which is true.}$$

$\therefore$ result is true for $n=1$

Assume that result is true for $n=k$.

$$\therefore A^K = \begin{bmatrix} 1+10k & -25K \\ 4K & 1-10K \end{bmatrix} \longrightarrow (2)$$

$$\therefore A^{K+1} = A^K . A$$

$$= \begin{bmatrix} 1+10K & -25K \\ 4K & 1-10K \end{bmatrix} \begin{bmatrix} 11 & -25 \\ 4 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 11(1+10K)-(25K)\cdot 4 & -25(1+10K)-25K(-9) \\ 4K\cdot 11+4(1-10K) & -25(4K)-9(1-10K) \end{bmatrix}$$

$$= \begin{bmatrix} 11+110K-100K & -25-25K \\ 44K+4-40K & -9-10K \end{bmatrix}$$

$$= \begin{bmatrix} 11+10K & -25-25K \\ 4+4K & -9-10K \end{bmatrix}$$

$$A^{K+1} = \begin{bmatrix} 1+10(K+1) & -25(K+1) \\ 4(K+1) & 1-10(K+1) \end{bmatrix}$$

∴ result is true for $n = K+1$.

Hence, by P.M.I., result is true for all +ve integer n.

Ques: Let $A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$. Show by P.M.I. that $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$ for every +ve integer n.

Sol: The given matrix is,

$$A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}.$$

<u>To prove</u>: $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$ ——— (1)

For $n=1$

$$A^1 = \begin{bmatrix} 1-2\cdot1 & -4\cdot1 \\ 1 & 1+2\cdot1 \end{bmatrix}$$

$\therefore A^1 = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$ which is true

$\therefore$ result is true for $n=1$.

Assume that result is true for $n=k$.

$$\therefore A^K = \begin{bmatrix} 1-2k & -4k \\ k & 1+2k \end{bmatrix} \longrightarrow (2)$$

$\therefore A^{k+1} = A^K \cdot A$

$$= \begin{bmatrix} 1-2k & -4k \\ k & 1+2k \end{bmatrix} \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix} \quad \{\because \text{of } (2)\}$$

$$= \begin{bmatrix} -1+2k-4k & -4+8k-12k \\ -k+1+2k & -4k+3+6k \end{bmatrix}$$

$$= \begin{bmatrix} -2k-1 & -4k-4 \\ k+1 & 2k+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2(k+1) & -4(k+1) \\ k+1 & 1+2(k+1) \end{bmatrix}$$

$\therefore$ result is true for $n=k+1$.

Hence, by P.M.I. result is true ~~of~~ +ve Integer n.

Ques: If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then show that $A^n \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ for every +ve integer n.

Sol: We are given matrix,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

T.P. :→ $$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix} \longrightarrow (1)$$

For $n=1$

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix}$$

$\therefore$ $A^1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ which is true.

$\therefore$ result is true for $n=1$.

Assume the result is true for $n=k$.

$$\therefore A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$\therefore A^{k+1} = A^k \cdot A$$

$$= \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \{\because \text{of } (2)\}$$

$$= \begin{bmatrix} 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

$$= \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix}$$

$\therefore$ result is true for $n = k+1$.

Hence, by P.M.I. result is true for all +ve integer n.

Ques: Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, show that by P.M.I that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, for all +ve integer n.

Sol: We are given,

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

T.P. : $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

For $n=1$

$$A^1 = \begin{bmatrix} \cos 1\cdot\theta & \sin 1\cdot\theta \\ -\sin 1\cdot\theta & \cos 1\cdot\theta \end{bmatrix}$$

$$A^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$, which is true

$\therefore$ result is true for $n=1$

Assume that result is true for $n=k$.

$$\therefore A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix} \longrightarrow (2)$$

$\therefore A^{k+1} = A^k \cdot A$

$$\therefore A^{k+1} = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta \cos\theta - \sin k\theta \sin\theta & \cos k\theta \sin\theta + \sin k\theta \cos\theta \\ -\sin k\theta \cos\theta - \sin\theta \cos k\theta & -\sin k\theta \sin\theta + \cos k\theta \cdot \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k\theta+\theta) & \sin(k\theta+\theta) \\ -\sin(k\theta+\theta) & \cos(k\theta+\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

$\therefore$ result is true for $n = k+1$

Hence, by P.M.I. result is true for all +ve integer n.

TRANSPOSE OF A MATRIX

The matrix obtained from a given matrix A by interchanging its rows and columns, is called the Transpose of a matrix
It is generally denoted by A' or A^t or A^T

For example Let $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 6 \end{bmatrix}_{2\times3}$

$$\therefore \quad A' = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 3 & 6 \end{bmatrix}_{3\times2}$$

$$\therefore \quad (A')' = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 6 \end{bmatrix}.$$

Important Note :-

1.) $(A+B)' = A' + B'$

2.) $(A-B)' = A' - B'$

3.) $(\underset{\text{scalar}}{K}A)' = KA'$

4.) $(A')' = A$

REVERSAL LAW

If A and B are any two matrices conformable for multiplication, then,

$$(AB)' = B'A' \quad \text{or} \quad (ABC)' = C'B'A'$$

$\underline{\underline{A}}$ $\underline{\underline{B}}$

Pre factor — Post factor

i.e. the transpose of the product of two matrices is the product of the transposes taken in the reverse order.

Ques: If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$, then verify that $(AB)' = B'A'$.

Sol: The given matrices are,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}.$$

$$\therefore AB = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}.$$

$$\therefore AB = \begin{bmatrix} 4+4 & 3+2 \\ 12-8 & 9-4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 4 & 5 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix} \longrightarrow (1)$$

Also, $B'A' = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 4+4 & 12-8 \\ 3+2 & 9-4 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix} \longrightarrow (2)$$

From (1) and (2), $(AB)' = B'A'$.

Ques: If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find $A + A'$.

Sol: The given matrix is,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$$

Ques: If $A = \begin{bmatrix} -1 & 3 & 0 \\ -7 & 2 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 0 \\ 0 & 3 \\ 1 & -8 \end{bmatrix}$

Verify that $(AB)' = B'A'$

Sol: The given matrices are,

$$A = \begin{bmatrix} -1 & 3 & 0 \\ -7 & 2 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} -5 & 0 \\ 0 & 3 \\ 1 & -8 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} -1 & 3 & 0 \\ -7 & 2 & 8 \end{bmatrix} \begin{bmatrix} -5 & 0 \\ 0 & 3 \\ 1 & -8 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 5+0+0 & 0+9-0 \\ 35+0+8 & 0+6-64 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 5 & 9 \\ 43 & -58 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 5 & 43 \\ 9 & -58 \end{bmatrix} \longrightarrow (1)$$

Now, $$B' = \begin{bmatrix} -5 & 0 & 1 \\ 0 & 3 & -8 \end{bmatrix}$$

$$A' = \begin{bmatrix} -1 & -7 \\ 3 & 2 \\ 0 & 8 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -5 & 0 & 1 \\ 0 & 3 & -8 \end{bmatrix} \begin{bmatrix} -1 & -7 \\ 3 & 2 \\ 0 & 8 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 5+0+0 & 35+0+8 \\ 0+9+0 & 0+6-64 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 5 & 43 \\ 9 & -58 \end{bmatrix} \longrightarrow (2)$$

$\therefore$ from (1) and (2),

$$(AB)' = B'A'$$

Ques: If $A = \begin{bmatrix} -1 & 0 & 2 \\ 4 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 0 & 4 \end{bmatrix}$,

then verify that $(AB)' = B'A'$.

Sol: The given matrices are,

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 4 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 0 & 4 \end{bmatrix}.$$

$$\therefore AB = \begin{bmatrix} -1 & 0 & 2 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 0-0+0 & -2+0+8 \\ 0-0+0 & 8+0+12 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 0 & 6 \\ 0 & 20 \end{bmatrix}$$

$\therefore (AB)' = \begin{bmatrix} 0 & 0 \\ 6 & 20 \end{bmatrix} \longrightarrow ①$

$\therefore B' = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix}, \; A' = \begin{bmatrix} -1 & 4 \\ 0 & 0 \\ 2 & 3 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & 0 \\ 2 & 3 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} 0-0+0 & 0-0+0 \\ -2+0+8 & 8+0+12 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} 0 & 0 \\ 6 & 20 \end{bmatrix}_{2\times 2} \longrightarrow ②$

from ① and ②, $\underline{\underline{(AB)'}} = \underline{\underline{B'A'}}$

Ques: If $A = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & 5 \end{bmatrix}$, then verify that $(AB)' = A'B'$.

Sol: The given matrices are,

$A = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & 5 \end{bmatrix}$.

$\therefore AB = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 4 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 4 & 5 \\ 6 & 12 & 15 \\ 12 & 24 & 30 \end{bmatrix}$

$$\therefore (AB)' = \begin{bmatrix} 2 & 6 & 12 \\ 4 & 12 & 24 \\ 5 & 15 & 30 \end{bmatrix}_{3\times3} \longrightarrow (1)$$

Now, $B' = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$ & $A' = \begin{bmatrix} 1 & 3 & 6 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 2 & 6 & 12 \\ 4 & 12 & 24 \\ 5 & 15 & 30 \end{bmatrix} \longrightarrow (2)$$

From (1) and (2),

$$(AB)' = B'A'$$

Ques: For the following matrices A and B. Verify that $(AB)' = B'A'$, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$.

Sol: The given matrices are,

$$A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 2 & 3 \\ 4 & -8 & -12 \\ -3 & 6 & 9 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 3 & -12 & 9 \end{bmatrix} \longrightarrow (1)$$

$$\therefore B' = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 3 & -12 & 9 \end{bmatrix} \longrightarrow (2)$$

$$\therefore \underline{\underline{(AB)' = B'A'}} \quad \{\because \text{ of } (1) \text{ and } (2)\}.$$

Symmetric Matrix

Any square matrix $A = [a_{ij}]_{m \times n}$ is said to be symmetric if $a_{ij} = a_{ji} \ \forall \ i,j$.

<u>Example</u> :- Let $A = \begin{bmatrix} 1 & 3 & -6 \\ 3 & 4 & 7 \\ -6 & 7 & 6 \end{bmatrix}$ is symmetric matrix.

Note For symmetric matrix,

$$\boxed{A' = A}$$

Let the given matrix be,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}_{3 \times 3}$$

For symmetric matrix, $A = A'$

i.e. $a_{12} = a_{21}$

$a_{13} = a_{31}$

$a_{23} = a_{32}$

Skew - Symmetric Matrices

Any square matrix $[a_{ij}]_{m\times n}$ is said to be skew- symmetric matrix if $\boxed{a_{ij} = -a_{ji}}$ $\forall\ i,j$.

Note÷ In a skew- symmetric matrix all the diagonal elements are zero

Proof We know that in skew symmetric matrix A,

Let, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

$$a_{11} = -a_{11} \Rightarrow 2a_{11} = 0 \Rightarrow a_{11} = 0$$

$$a_{22} = -a_{22} \Rightarrow 2a_{22} = 0 \Rightarrow a_{22} = 0$$

$$a_{33} = -a_{33} \Rightarrow 2a_{33} = 0 \Rightarrow a_{33} = 0$$

$$\left(\begin{aligned} a_{21} &= -a_{12} \\ a_{31} &= -a_{13} \\ a_{32} &= -a_{23} \end{aligned}\right) \left\{\text{Only for skew- symmetric matrix}\right\}$$

Art → Show that the elements on the main diagonal of a skew-symmetric matrix are all zero.

Sol ÷→ Let the matrix $A = [a_{ij}]_{m \times n}$ be a skew symmetric matrix.

$\therefore \ a_{ij} = -a_{ji} \quad \forall \ i, j.$

For diagonal elements, put $i = j$

$\therefore \ a_{ii} = -a_{ii} \quad \forall \ i$

$\Rightarrow 2a_{ii} = 0 \quad \forall \ i$

$\Rightarrow a_{ii} = 0 \quad \forall \ i$

Hence, all diagonal elements of a skew-symmetric matric are zero.

Ques:) If matrix A is symmetric as well as skew symmetric then show that $A = 0$.

Sol:) The given matrix A is symmetric as well as skew-symmetric.

$\therefore \ A' = A \longrightarrow ① \ \{\because A \text{ is symmetric}\}$

and $A' = -A \longrightarrow ② \ \{\because A \text{ is skew-symm}\}$

From (I) and (II),

$$A = -A$$

$$\therefore 2A = 0$$

$$\Rightarrow \boxed{A = 0}$$ Hence, proved.

Art → A and B are symmetric matrices. Show that $AB + BA$ is symmetric and $AB - BA$ is skew symmetric matrices.

Sol. → The given matrices A and B are symm- matrices.

$$\left.\begin{aligned} \therefore A' &= A \\ \text{and } B' &= B \end{aligned}\right\} \longrightarrow (1)$$

To prove : 1.) $(AB + BA)$ is symmetric matrix.

2.) $(AB - BA)$ is skew symmetric matrix.

Proof : 1.) $(AB + BA)' = (AB)' + (BA)'$

$$\therefore (AB + BA)' = B'A' + A'B'$$

$$\therefore (AB + BA)' = BA + AB \quad \{\because \text{ of } (1)\}$$

$$\therefore (AB + BA)' = (AB + BA)$$

$\therefore (AB + BA)'$ is a symmetric matrix.

2.) $(AB-BA)' = (AB)' - (BA)'$

$(AB-BA)' = B'A' - A'B'$

{$\because$ of reversal law}

$(AB-BA)' = BA - AB$ {$\because$ of ①}

$(AB-BA)' = -(AB-BA)$

$\therefore$ $(AB-BA)$ is a skew-symmetric matrix.

Art.) Show that the matrix $B'AB$ is symm. or skew-symmetric according as A is symmetric or skew symmetric matrix.

Sol.) 1.) Let the matrix A is symmetric matrix.

$\therefore A' = A \longrightarrow ①$

T.P. $\rightarrow$ $B'AB$ is symmetric.

$\therefore (B'AB)' = B'A'(B')'$ {$\because$ of reversal law}

$\therefore (B'AB)' = B'AB$

$\therefore$ $B'AB$ is symmetric matrix.

2.) Let the matrix A be skew-symmetric matrix.

$\therefore A' = -A \longrightarrow ②$

T.P. → $B'AB$ is skew-symmetric matrix.

$\therefore (B'AB)' = B'A'(B')'$ {$\because$ of reversal law}

$\therefore (B'AB)' = B'(-A)B$ {$\because$ of ②}

$\therefore (B'AB)' = -(B'AB)$

$\therefore B'AB$ is skew-symmetric matrix.

Theorem Any square matrix can be expressed as the sum of a symmetric and skew-symmetric matrix in one and only one method **(unique way).**

Sol Let A be any square matrix.

$$\therefore A = \frac{1}{2}(2A)$$

$$\therefore A = \frac{1}{2}(A + A)$$

$$\therefore A = \frac{1}{2}\left[(A+A') + (A-A')\right]$$

$$\therefore A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$\therefore A = P + Q \longrightarrow ①$$

where, $P=\frac{1}{2}(A+A')$, $Q=\frac{1}{2}(A-A')$.

Now, $P' = \left[\frac{1}{2}(A+A')\right]'$

$\therefore P' = \frac{1}{2}(A+A')'$

$\therefore P' = \frac{1}{2}\left(A'+(A')'\right)$

$\therefore P' = \frac{1}{2}(A'+A) \quad \{\because (A')'=A\}$

$\therefore P' = \frac{1}{2}(A+A')$

$\Rightarrow P' = P$

$\therefore$ P is a symmetric matrix.

Now, $Q' = \left[\frac{1}{2}(A-A')\right]'$

$\therefore Q' = \frac{1}{2}(A-A')'$

$\therefore Q' = \frac{1}{2}\left[A'-(A')'\right]$

$\therefore Q' = \frac{1}{2}(A'-A)$

$\therefore Q' = -\frac{1}{2}(A-A') \Rightarrow Q' = -Q$

$\therefore$ Q is a skew-symmetric matrix.

Uniqueness

Let A can be express in another way.

Let $A = R + S \longrightarrow (1)'$

Where R is symmetric while S is skew-symmetric matrix.

$$\therefore A' = (R+S)'$$

$$\therefore A' = R' + S'$$

$$\therefore A' = R - S \longrightarrow (2)$$

$$\{\because R' = R,\ S' = -S\}$$

Adding (1) and (2), we get.

$$A + A' = 2R$$

$$\therefore 2R = A + A'$$

$$\therefore R = \frac{1}{2}(A + A') = P$$

$\therefore$ (1) − (2) $\Rightarrow$

$$A - A' = 2S$$

$$\therefore S = \frac{1}{2}(A - A') = Q$$

$\therefore R = P$ and $S = Q$

$\therefore$ the representation (1)' is same as representation (1).

Note:

$\Rightarrow$ Sum of two symmetric matrices is always symmetric.

$\Rightarrow$ Subtraction of two symmetric matrices is always symmetric

Ques: If A and B are the symmetric matrices of the same order than show that AB is symmetric if and only if A and B commute i.e. $AB = BA$.

Sol: We are given A and B are symmetric matrices.

$$\left.\begin{aligned} \therefore\ A' &= A \\ \text{and } B' &= B \end{aligned}\right\} \longrightarrow (1)$$

Assume that AB is symmetric matrix.

T.P. $\rightarrow$ $AB = BA$

Since AB is symmetric matrix.

$\therefore\ (AB)' = AB$

$\therefore\ B'A' = AB$

$\therefore\ BA = AB$

$\therefore\ AB = BA$

Now, assume that $AB = BA$

T.P. : AB is symmetric matrix.

$\therefore\ (AB)' = B'A'$ {$\because$ of reversal law}

$\therefore\ (AB)' = BA$

$\therefore\ (AB)' = AB$ {$\because AB = BA$}

$\therefore$ AB is symmetric matrix.

Ques: Express $\begin{bmatrix} 2 & 5 & -1 \\ 3 & 1 & 5 \\ 7 & 6 & 9 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix ?

Sol: Let the given matrix be,

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 1 & 5 \\ 7 & 6 & 9 \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} 2 & 3 & 7 \\ 5 & 1 & 6 \\ -1 & 5 & 9 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 1 & 5 \\ 7 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 7 \\ 5 & 1 & 6 \\ -1 & 5 & 9 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 4 & 8 & 6 \\ 8 & 2 & 11 \\ 6 & 11 & 18 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 4 & 8 & 6 \\ 8 & 2 & 11 \\ 6 & 11 & 18 \end{bmatrix}$$ which is a symmetric matrix.

$$\therefore \frac{1}{2}(A+A') = \begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & 11/2 \\ 3 & 11/2 & 9 \end{bmatrix}$$ which is a symmetric matrix.

Now, $$A-A' = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 1 & 5 \\ 7 & 6 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 7 \\ 5 & 1 & 6 \\ -1 & 5 & 9 \end{bmatrix}$$

$$\therefore A-A' = \begin{bmatrix} 0 & 2 & -8 \\ -2 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 2 & -8 \\ -2 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A') = \begin{bmatrix} 0 & 1 & -4 \\ -1 & 0 & -1/2 \\ -4 & 1/2 & 0 \end{bmatrix}$$

which is a skew-symmetric matrix.

$$\therefore A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$\therefore \begin{bmatrix} 2 & 5 & -1 \\ 3 & 1 & 5 \\ 7 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 4 & 1 & 11/2 \\ 3 & 11/2 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -4 \\ -1 & 0 & -1/2 \\ 4 & 1/2 & 0 \end{bmatrix}$$

$\therefore$ given matrix has been expended as the sum of a symmetric and a skew-symmetric matrix.

Ques: Express the matrix $A=\begin{bmatrix}-2 & 3 & 1\\ 1 & 3 & 2\\ 5 & -4 & 5\end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

Sol: The given matrix is,

$$A=\begin{bmatrix}-2 & 3 & 1\\ 1 & 3 & 2\\ 5 & -4 & 5\end{bmatrix}$$

$$\therefore\ A'=\begin{bmatrix}-2 & 1 & 5\\ 3 & 3 & -4\\ 1 & 2 & 5\end{bmatrix}$$

$$\therefore\ A+A'=\begin{bmatrix}-2 & 3 & 1\\ 1 & 3 & 2\\ 5 & -4 & 5\end{bmatrix}+\begin{bmatrix}-2 & 1 & 5\\ 3 & 3 & -4\\ 1 & 2 & 5\end{bmatrix}$$

$$\therefore\ A+A'=\begin{bmatrix}-4 & 4 & 6\\ 4 & 6 & -2\\ 6 & -2 & 10\end{bmatrix}$$

$$\therefore\ \frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}-4 & 4 & 6\\ 4 & 6 & -2\\ 6 & -2 & 10\end{bmatrix}=\begin{bmatrix}-2 & 2 & 3\\ 2 & 3 & -1\\ 3 & -1 & 5\end{bmatrix}$$

which is a symmetric matrix.

Now, $A-A' = \begin{bmatrix} -2 & 3 & 1 \\ 1 & 3 & 2 \\ 5 & -4 & 5 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 5 \\ 3 & 3 & -4 \\ 1 & 2 & 5 \end{bmatrix}$

$\therefore A - A' = \begin{bmatrix} 0 & 2 & -4 \\ -2 & 0 & 6 \\ 4 & -6 & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 2 & -4 \\ -2 & 0 & 6 \\ 4 & -6 & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A-A') = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$ which is a skew-symm-matrix.

$\therefore A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

$\therefore \begin{bmatrix} -2 & 3 & 1 \\ 1 & 3 & 2 \\ 5 & -4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 2 & 3 \\ 2 & 3 & -1 \\ 3 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

$\therefore$ given matrix has been expended as the sum of a symmetric and skew-symmetric matrix.

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Quet: Express the matrix $A = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

Sol: The given matrix is,

$$A = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix}$$

$$\therefore \quad A' = \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 0 \\ 6 & 1 & 2 \end{bmatrix}$$

$$\therefore \quad A + A' = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 0 \\ 6 & 1 & 2 \end{bmatrix}$$

$$\therefore \quad A + A' = \begin{bmatrix} 8 & 4 & 8 \\ 4 & 0 & 1 \\ 8 & 1 & 4 \end{bmatrix}$$

$$\therefore \quad \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 8 & 4 & 8 \\ 4 & 0 & 1 \\ 8 & 1 & 4 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A') = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 0 & \frac{1}{2} \\ 4 & \frac{1}{2} & 2 \end{bmatrix}$$

which is a symmetric matrix.

Now, $A-A' = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 0 \\ 6 & 1 & 2 \end{bmatrix}$

$$\therefore A-A' = \begin{bmatrix} 0 & 6 & 4 \\ -6 & 0 & 1 \\ -4 & -1 & 0 \end{bmatrix}_{3\times 3}$$

$$\therefore \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 6 & 4 \\ -6 & 0 & 1 \\ -4 & -1 & 0 \end{bmatrix}$$

$\therefore \frac{1}{2}(A-A') = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & \frac{1}{2} \\ -2 & -\frac{1}{2} & 0 \end{bmatrix}$ which is a skew-symm. matrix.

$$\therefore A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$\therefore \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 0 & 1/2 \\ 4 & 1/2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 1/2 \\ -2 & -1/2 & 0 \end{bmatrix}$$

$\therefore$ given matrix has been expended as the sum of a symmetric and skew-symmetric matrix.

Ques: Express the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

Sol: The given matrix is,

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$\therefore A+A' = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A') = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}$$

which is a symmetric matrix.

Now, $(A-A') = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$

$$\therefore A-A' = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$\therefore \frac{1}{2}(A-A') = \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$ which is a skew-symmetric matrix

$$\therefore A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$\therefore \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$$

$\therefore$ the given matrix has been expended as the sum of a symmetric and skew-symmetric matrix.

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