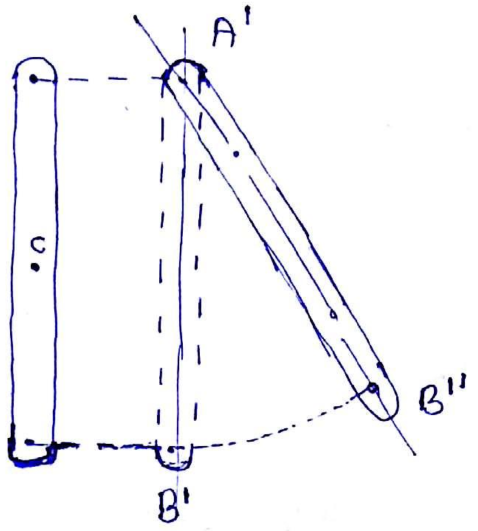
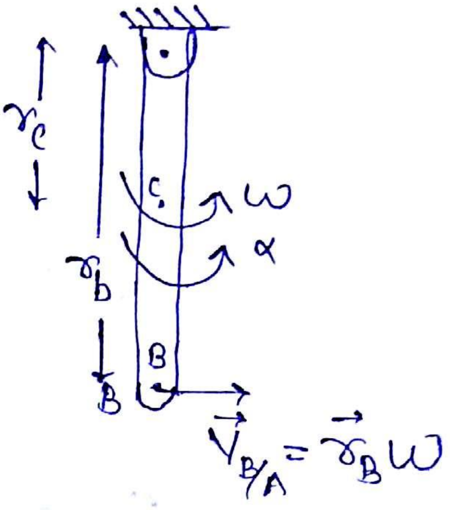
General Motion :—

If a rigid body rotate as well as translates, such a (simultaneous) motion is called General Motion

eg. Motion of a ~~beam~~ wheel, motion of connecting road, motion of ladder.



w.r.t A

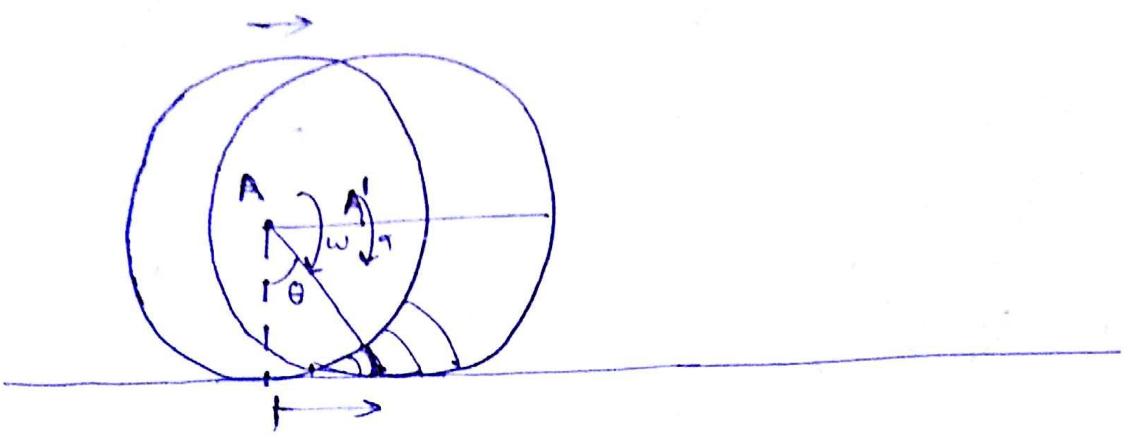


Displacement

$$\vec{S}_B = \vec{S}_{trans.} + \vec{S}_{rotation}$$

$$\left.\begin{array}{l}
\vec{S}_B = \vec{S}_A + \vec{S}_{B/A} \quad [\vec{S}_{B/A} = r_B \vec{\theta}] \\
\vec{S}_c = \vec{S}_A + \vec{S}_{c/A} \quad [\vec{S}_{c/A} = r_c \vec{\theta}] \\
\vec{V}_B = \vec{V}_B + \vec{V}_{B/A} \quad [V_{B/A} = r_B \vec{\omega}] \\
\vec{V}_c = \vec{V}_A + \vec{V}_{c/A} \quad [V_{c/A} = r_c \vec{\omega}] \\
\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \quad [\vec{a}_{B/A} = r_B \vec{\omega}^2 + r_B \vec{\alpha}] \\
\vec{a}_c = \vec{a}_A + \vec{a}_{c/A} \quad [a_{c/A} = r_c \vec{\omega}^2 + r_c \vec{\alpha}]
\end{array}\right\} \text{Charles eqn}$$

Pure Rolling :- {DOF = 1}



$S_A = r\theta$

if $S_A = r\theta$

In 1 complete cycle

$\theta = 2\pi$

$S_A = 2\pi r$

For n cycle

$$\boxed{S_A = n(2\pi r)}$$

if $S_A < r\theta \rightarrow$ slipping

$S_A > r\theta \rightarrow$ skidding

$\Rightarrow$

$$\boxed{\vec{S}_A = r\theta\,\hat{i}}$$

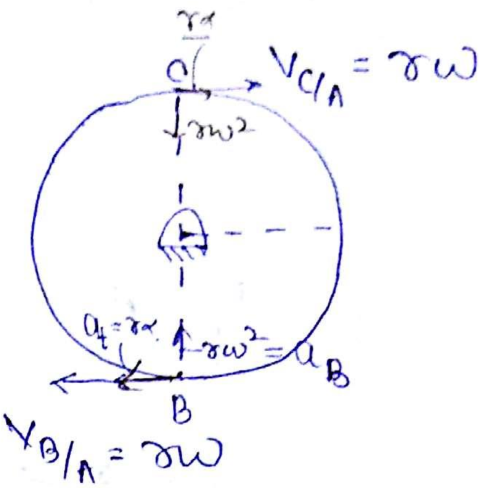
$$\boxed{\vec{V}_A = r\frac{d\theta}{dt}\hat{i} = r\omega\,\hat{i}}$$

$$\boxed{a_A = r\frac{d\omega}{dt}\hat{i} = r\alpha\,\hat{i}}$$

$\rightarrow$ total acc^n

Now

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$



$$\vec{V}_B = r\omega\hat{i} + r\omega(-\hat{i})$$

$$\boxed{\vec{V}_B = 0}$$

$$\vec{V}_C = \vec{V}_A + \vec{V}_{C/A}$$

$$\vec{V}_C = r\omega\hat{i} + r\omega\hat{i}$$

$$\boxed{V_C = 2r\omega\hat{i}}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = r\alpha\hat{i} + r\alpha(-\hat{i}) + \alpha\omega^2(\hat{j})$$

$$\boxed{\vec{a}_B = r\omega^2(\hat{j})}$$

$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A}$$

$$= \underline{r\alpha\hat{i}} + \underline{r\alpha\hat{i}} + r\omega^2(-\hat{j})$$

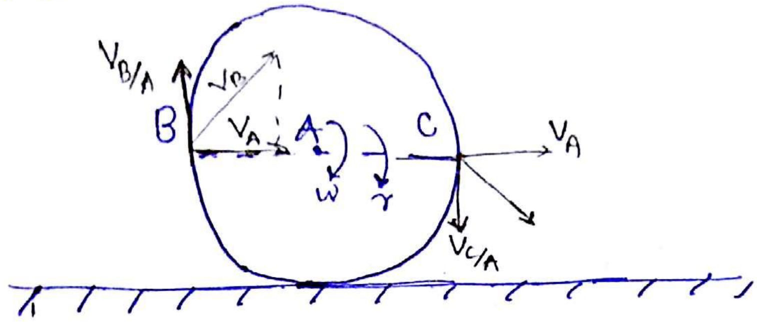
$$\boxed{\vec{a}_C = 2r\alpha\hat{i} + r\omega^2(-\hat{j})}$$

$$\Rightarrow \vec{V}_{B/C} = \vec{V}_B - \vec{V}_C = 0 - 2r\omega^2\hat{i} = \underline{\underline{-2r\omega^2\hat{i}}}$$

★ **At contact point**

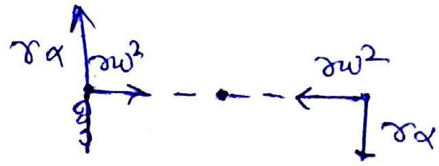
$$\underline{V = 0} \; ; \; a \neq 0$$

Ques A wheel of radius of 2 m rolles freely as shown in fig if $V_A = 6$ m/s & $a_A = 20$ m/s^2 then what are the velocity and acc^n at point B and C



Sol^n

$$V_A = 6 = r\omega$$



$$\omega = \frac{6}{2} = 3 \text{ rad/s}$$

$$a_A = 20 = r\alpha$$

$$\alpha = \frac{20}{2} = 10 \text{ rad/sec}^2$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$\vec{V}_B = r\omega\hat{i} + r\omega\,\hat{j}$$

$$\vec{V}_B = 6\hat{i} + 6\hat{j}$$

$$\vec{V}_C = \vec{V}_A + \vec{V}_{C/A}$$

$$\vec{V}_C = r\omega\hat{i} + r\omega(-\hat{j})$$

$$\vec{V}_C = 6\hat{i} - 6\hat{j}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_B = r\alpha\hat{i} + r\omega^2\hat{i} + r\alpha\hat{j}$$

$$\vec{a}_B = 38\hat{i} + 20\hat{j}$$

$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A}$$

$$\vec{a}_C = r\alpha\hat{i} + r\omega^2(-\hat{i}) + r\alpha(-\hat{j})$$

$$\vec{a}_C = 20\hat{i} + 2\times3^2(-\hat{i}) + 20(-\hat{j})$$

$$\vec{a}_C = 20\hat{i} - 18\hat{i} + 20\hat{j}$$

$$\vec{a}_C = 2\hat{i} - 20\hat{j}$$

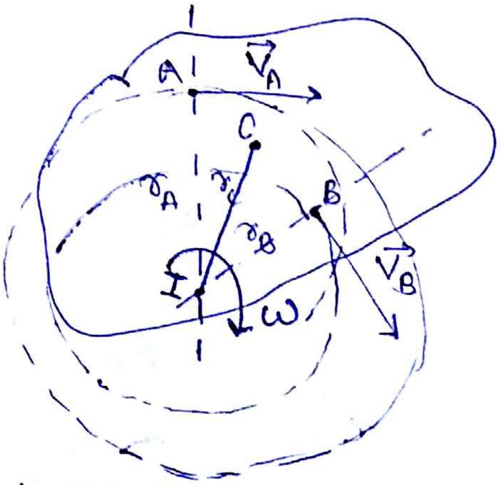
Instantaneous Centre/Axis of Rot^n →

It is a point or line in space about which a body in General motion can be assume as in pure rotation to find velocities

Note:- 'I'- centre has zero velocity but not zero acc^n. ($v = 0$; $a \neq 0$)

locations of I- Centres:-

Case-1



$V_A = r_A \omega$

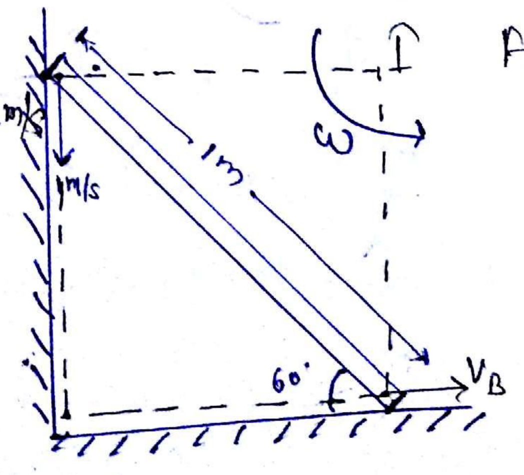
$V_B = r_B \omega$

$V_C = r_C \omega$

$$\boxed{\frac{V_A}{r_A} = \frac{V_B}{r_B} = \frac{V_C}{r_C} \; --}$$

Probo

Q.5.5
Pg.92
rate



Find $V_B = ?$

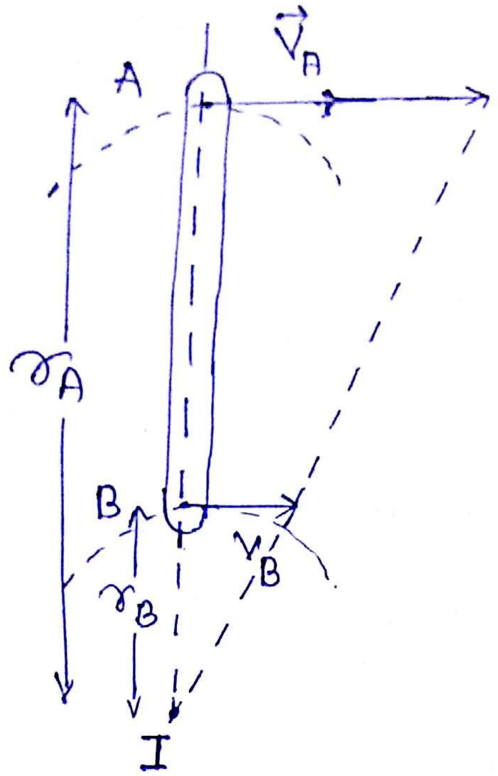
$V_A = r_A \omega$

$1 = \cos 60^\circ \omega$

$\omega = 2$ rad/sec.

$V_B = r_B \omega = 1 \times \sin 60^\circ \times 2 = \sqrt{3}$ m/s

Case-2



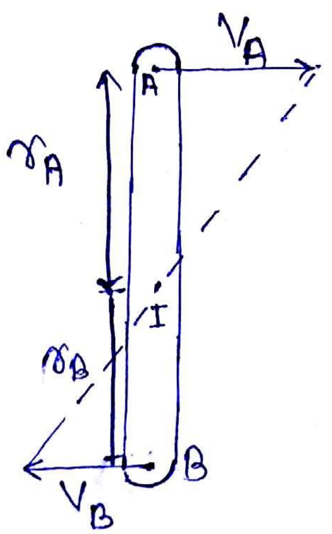
$$V_A = r_A \omega$$

$$V_B = r_B \omega$$

$$\frac{V_A}{V_B} = \frac{r_A}{r_B} \quad -(1)$$

$$AB = r_A - r_B \quad -(2)$$

Case-3



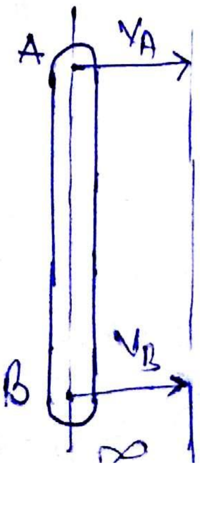
$$V_A = r_A \omega$$

$$V_B = r_B \omega$$

$$\frac{V_A}{V_B} = \frac{r_A}{r_B} \quad (1)$$

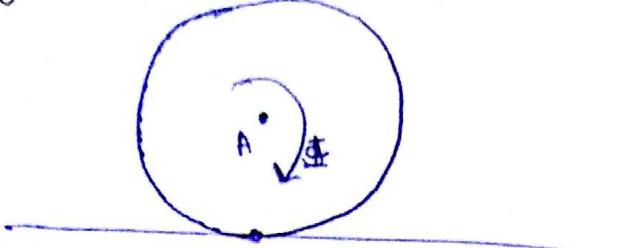
$$r_A + r_B = AB \quad -(2)$$

Case-4



$$V_A = V_B$$

Case-5



$I_o \rightarrow$ Inst. centre

$$(K.E.)_{Rolling} = (K.E.)_{trans.} + (K.E.)_{rota.}$$

$$= \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2$$

$$= \frac{1}{2} m r^2 \omega^2 + \frac{1}{2} I_A \omega^2$$

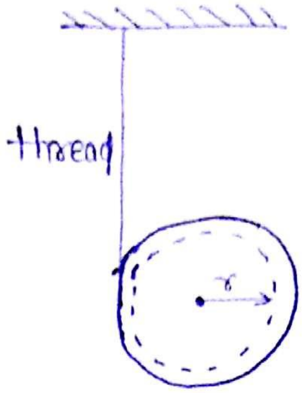
$$(K.E.)_{Rolling} = \frac{1}{2} \left[I_A + m r^2 \right] \omega^2$$

$$\boxed{(K.E.)_{Rolling} = \frac{1}{2} \left[I_{I\text{-}centre} \right] \omega^2}$$

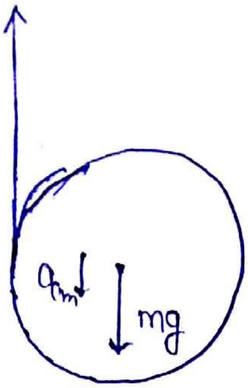
Angular momentum

$$\boxed{L_{I\text{-}cen.} = I_{I\text{-}cen} \, \omega}$$

Question 5.6 & 5.7
Page 92 & 93



Sol^n



$$mg - T = m a_{cm} \quad - 2^{nd} \text{ law}$$

$$mg - T = m r \alpha \quad - (1)$$

$$[\text{Torque}]_{centre} = (I)_{centre} \alpha$$

$$Tr = mk^2 \alpha \quad - (2)$$

$$\alpha = \frac{Tr}{mk^2}$$

from (1) & (2)

$$mg - T = m \cdot r \frac{Tr^2}{mk^2}$$

$$\boxed{T = \frac{mgk^2}{r^2 + k^2}}$$

$$a_{cm} = r\alpha = \frac{Tr^2}{mk^2} = \frac{mgk^2}{r^2+k^2} \times \frac{r^2}{mk^2}$$

$$\boxed{a_{cm} = \frac{gr^2}{r^2+k^2}}$$