9. CENTRE OF MASS

Mass Moment : $\vec{M} = m\vec{r}$

CENTRE OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2 + \dots\dots + m_n r_n}{m_1 + m_2 + \dots\dots + m_n} \quad ; \quad r_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} \quad r_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i r_i$$

CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

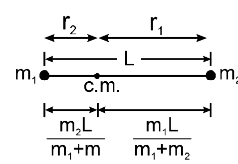
$$x_{cm} = \frac{x\,dm}{dm},\ y_{cm} = \frac{y\,dm}{dm},\ z_{cm} = \frac{z\,dm}{dm}$$

$dm = M$ (mass of the body)

CENTRE OF MASS OF SOME COMMON SYSTEMS

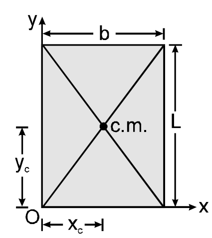
A system of two point masses $m_1 r_1 = m_2 r_2$

The centre of mass lies closer to the heavier mass.



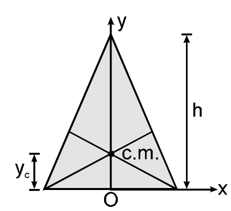
Rectangular plate (By symmetry)

$x_c = \frac{b}{2}$ $\qquad$ $y_c = \frac{L}{2}$



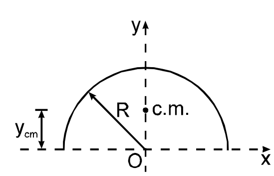
A triangular plate (By qualitative argument)

at the centroid : $y_c = \frac{h}{3}$

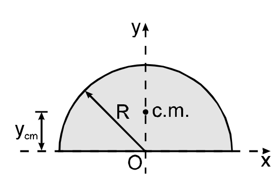


A semi-circular ring

$y_c = \frac{2R}{\pi}$ $\qquad$ $x_c = O$

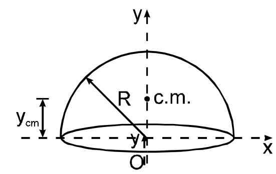


A semi-circular disc



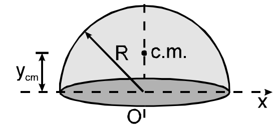
$y_{cm} = \frac{4R}{3\pi}$ $x_{cm} = O$

A hemispherical shell



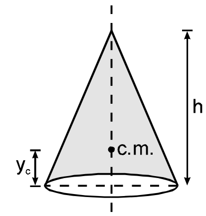
$y_{cm} = \frac{R}{2}$ $x_{cm} = O$

A solid hemisphere



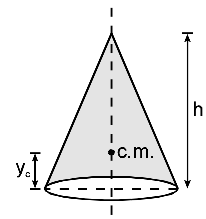
$y_{cm} = \frac{3R}{8}$ $x_{cm} = O$

A circular cone (solid)



$y_{cm} = \frac{h}{4}$

A circular cone (hollow)



$y_{cm} = \frac{h}{3}$

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM :

Velocity of centre of mass of system

$$v_{cm} = \frac{m_1\frac{d\vec{r_1}}{dt} + m_2\frac{d\vec{r_2}}{dt} + m_3\frac{d\vec{r_3}}{dt}.............. + m_n\frac{d\vec{r_n}}{dt}}{M} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3 .... + m_n v_n}{M}$$

$$\vec{P}_{System} = M\vec{v}_{cm}$$

Acceleration of centre of mass of system

$$a_{cm} = \frac{m_1\frac{d\vec{v_1}}{dt} + m_2\frac{d\vec{v_2}}{dt} + m_3\frac{d\vec{v_3}}{dt}.............. + m_n\frac{d\vec{v_n}}{dt}}{M} = \frac{m_1a_1 + m_2a_2 + m_3a_3.......... + m_na_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net External Force} + \text{Net internal Force}}{M} = \frac{\text{Net External Force}}{M}$$

$$F_{ext} = M a_{cm}$$

IMPULSE

Impulse of a force F action on a body is defined as :-

$$J = \int_{t_i}^{t_f} F dt \qquad J = \Delta P \qquad \textbf{(impulse - momentum theorem)}$$

Important points :

1. Gravitational force and spring force are always non-impulsive.

2. An impulsive force can only be balanced by another impulsive force.

COEFFICIENT OF RESTITUTION (e)

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r\, dt}{\int F_d\, dt} = s\,\frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

(a)	$e = 1$	Impulse of Reformation = Impulse of Deformation Velocity of separation = Velocity of approach Kinetic Energy may be conserved Elastic collision.
(b)	$e = 0$	Impulse of Reformation = 0 Velocity of separation = 0 Kinetic Energy is not conserved Perfectly Inelastic collision.
(c)	$0 < e < 1$	Impulse of Reformation < Impulse of Deformation Velocity of separation < Velocity of approach Kinetic Energy is not conserved Inelastic collision.

VARIABLE MASS SYSTEM :

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity v_{rel} (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu |v_{rel}|$.

Thrust Force (F_t)

$$F_t = v_{rel} \frac{dm}{dt}$$

Rocket propulsion :

If gravity is ignored and initial velocity of the rocket u = 0;

$$v = v_r \ln \frac{m_0}{m}\ .$$