CBSE Class 12 - Mathematics
Sample Paper 11

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. The system of linear equations ax+ by= 0, cx + dy = 0 has a non-trival solution if

   a. ad – bc = 0

   b. ad – bc < 0

   c. ad –bc = 0.

   d. ac + bd = 0

2. If ω is non real cube root of unity, then $\begin{vmatrix} 2 & 2\omega & -\omega^2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$ is equal to

a. –1

b. 0

c. None of these

d. 1

3. If $\sqrt{x}+\sqrt{y}=\sqrt{a},$ then $\left(\frac{d^2y}{dx^2}\right)_{x=a}$ is equal to

a. $\frac{1}{2a}$

b. a

c. None of these

d. $\frac{1}{a}$

4. A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by

a. $P\left(X=r\right)=C_r^n p^r q^{n-r-2}$

b. $P\left(X=r\right)=C_r^n p^r q^{n-r}$

c. $P\left(X=r\right)=C_r^n p^{2r} q^{n-r}$

d. $P\left(X=r\right)=C_{r-2}^n p^r q^{n-r}$

5. If E and F are independent, then which one of the following is right?

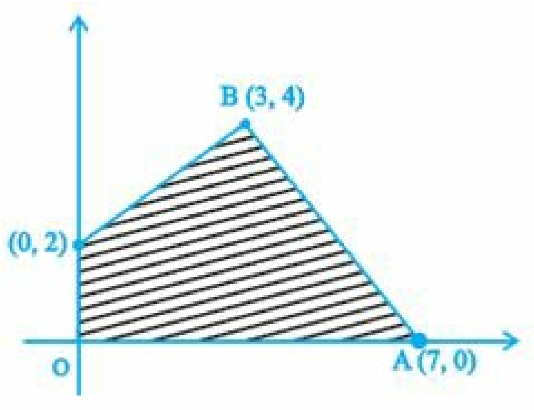
a. P (E|F) = P (E’), P (F) ≠ 0E’ is complement of E

b. P (E|F) = P (E), P (F) ≠ 0

c. P (E|F) = P (F), P (F) ≠ 0

d. P (E|F) =P (E’∪ F)E’ is complement of E

6. Feasible region (shaded) for a LPP is shown in Figure. Maximize Z = 5x + 7y.



a. 45

b. 49

c. 47

d. 43

7. If and x + y + z = xyz, then a value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is

a. π

b. $\frac{\pi}{2}$

c. None of these

d. $\frac{3\pi}{2}$

8. $\frac{d}{dx}(\int f(x)dx)$ is equal to

a. $\frac{[(f(x)]^2}{2}$

b. $\frac{f(x)}{2}$

c. f(x)

d. x

9. The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by

a. $\sin\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

b. $\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|.$

c. $\tan\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

d. $\cot\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

10. Unit vectors along the axes OX, OY and OZ are denoted by

a. $\bar{i},\ \hat{j}\ and\ \hat{k}$

b. $\hat{i},\ \ddot{j} and\ \hat{k}$

c. $\hat{i},\ \hat{j}\ and\ \hat{k}$

d. $\hat{i},\ \ddot{j} and\ \hat{k}$

11. Fill in the blanks:

The set of second elements of all ordered pairs in R, i.e. {y : (x, y) ∈ R} is called the ________ of relation R.

12. Fill in the blanks:

The derivative of sin x w.r.t. cos x is ________.

13. Fill in the blanks:

________ matrix is both symmetric and skew-symmetric matrix.

14. Fill in the blanks:

A plane passes through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4). The equation of plane is ________.

OR

Fill in the blanks:

The cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$

15. Fill in the blanks:

If vectors $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then its area is given by ________.

OR

Fill in the blanks:

If $\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2$ + $\left|\overrightarrow{a} . \overrightarrow{b}\right|^2$ = 144 and $\left|\overrightarrow{a}\right|$ = 4, then $\left|\overrightarrow{b}\right|$ is equal to ________.

16. Find |adj A|, if A= $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$.

17. Evaluate $\int\limits_0^1 xe^x dx$

OR

Evaluate $\int_1^2 \frac{x^3-1}{x^2} dx.$

18. Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx.$

19. Find the interval in which the function f(x) = cot^{-1}x + x is increasing.

20. Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves xy = C cos x.

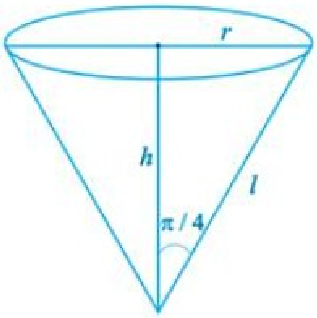
Section B

21. Find the principal values of cosec^{-1}(2) and cosec^{-1} $\left(-\frac{2}{\sqrt{3}}\right)$.

OR

Find fog and gof, if f(x) = x + 1, g(x) = e^x.

22. Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2cm^2/$ sec in the surface area, through a tiny hole at the vertex of the bottom. When the slant height of cone is 4 cm, find the rate of decrease of the slant height of water.



23. Find all points of discontinuity of f, where $f(x) = \begin{cases} \frac{\sin x}{x}, & if\ x < 0 \\ x+1, & if\ x \geqslant 0 \end{cases}$.

24. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + \hat{2}j + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is $\perp$ to $\vec{c}$ is then find the value of λ.

OR

Find $\vec{a} \cdot (\vec{b} \times \vec{c})$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

25. Find the Cartesian equation of the plane $\vec{r}.(4\hat{i} - 2\hat{j} - 5\hat{k}) = 45$

26. Three events A, B and C have probabilities $\frac{2}{5}, \frac{1}{3}, \frac{1}{2}$ respectively. Given that $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$ find the value of P(C|B) and $P(A' \cap C')$.

Section C

27. If f : $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow R$ and g : [-1,1] $\rightarrow$ R be defined as f(x) = tan x and g(x) = $\sqrt{1-x^2}$ respectively. Describe fog and gof.

28. If cosy = x cos (a + y) prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

OR

For what value of k, is the function defined by f(x) =

$\begin{cases} k\left(x^2+2\right), & \text{if } x \leq 0 \\ 3x+1, & \text{if } x>0 \end{cases}$ continuous at x = 0? Also, find whether the function is continuous at x = 1.

29. Find the general solution: $\left(1+x^2\right) \frac{dy}{dx}+2xy=\frac{1}{1+x^2} ; y=0$ when x = 1

30. $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right) dx$

31. Ten coins are tossed. What is the probability of getting at least 8 heads?

OR

Two cards are drawn simultaneously (without replacement) from a well-shuffled deck of 52 cards. Find the mean and variance of number of red cards.

32. In order to supplement daily diet, a person wishes to take some X and some wishes Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligram per tablet) are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of X and Y is Rs 2 and Re 1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

Section D

33. $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$,

Prove $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

OR

Using properties of determinants, show that ΔABC is isosceles, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix}=0$$

34. Find the area of circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

35. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

OR

Find the equation, of the tangent line to the curve $y = x^2 – 2x + 7$ which is

a. Parallels to the line $2x – y + 9 = 0$
b. Perpendicular to the line $5y – 15x = 13$

36. Show that the straight lines whose direction cosines are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angles.

Solution
Section A

1. (a) ad – bc = 0

   Explanation:

   The given system of equations has a non – trivial solution if :
   $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0 \Rightarrow ad - bc = 0.$
2. (b) 0

   Explanation:

   Expanding along R_3

   $= 1(2\omega + \omega^2) + 1(2 + \omega^2) = (2 + 2\omega + 2\omega^2 = 2(1 + \omega + \omega^2) = 2(0) = 0$
3. (a) $\frac{1}{2a}$

   Explanation:

   $\sqrt{x} + \sqrt{y} = \sqrt{a} \ldots\ldots..(1)$

   $\Rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

   $\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \ldots\ldots.(2)$

   $\Rightarrow \frac{d^2y}{dx^2} = -\frac{\sqrt{x}\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} - \sqrt{y}\frac{1}{2}x^{-\frac{1}{2}}}{x}$

   $= \frac{-\left(\frac{\sqrt{x}}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right) - \frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$

   $= \frac{\sqrt{x}+\sqrt{y}}{2x\sqrt{x}} = \frac{\sqrt{a}}{2x\sqrt{x}.} = \frac{\sqrt{a}}{2a\sqrt{a}} = \frac{1}{2a}$
4. (b) $P\left(X = r\right) = C_r^n p^r q^{n-r}$

   Explanation:

A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by :
$P(X=r)=C_r^n p^r q^{n-r}$.

5. (b) P (E|F) = P (E), P (F) ≠ 0

Explanation:

If E and F are independent, then P (E|F) = P (E), P (F) ≠ 0 .

6. (d) 43

Explanation:

Corner points	Z = 5x + 7y
O(0,0)	0
B (3,4)	43
A(7,0)	35
C(0,2)	14

Hence the maximum value is 43

7. (a) π

Explanation:

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$$

$$\Rightarrow \tan^{-1}\left[\frac{x+y}{1-xy}\right] + \tan^{-1}z$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x+y}{1-xy}+z}{1-\left(\frac{x+y}{1-xy}\right)z}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x+y+z-xyz}{1-xy}}{\frac{1-xy-xz-yz}{1-xy}}\right]$$

$$= \tan^{-1}\left[\frac{xyz-xyz}{1-xy-xz-yz}\right] \quad x+y+z = xyz$$

$$= \tan^{-1}(0) = \pi$$

8. (c) f(x)

Explanation:

$\frac{d}{dx}(\int f(x)dx) = f(x)$

9. (b) $\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

Explanation:

By definition , The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by :

$\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$.

10. (c) $\hat{i},\ \hat{j}\ and\ \hat{k}$

Explanation:

$\hat{i},\ \hat{j}\ and\ \hat{k}$ represents the unit vectors along the co ordinate axes i.e. OX ,OY and OZ respectively.

11. range

12. -cot x

13. Null

14. $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

OR

x + y - z = 2

15. $\frac{1}{2}|\vec{a} \times \vec{b}|$

OR

3

16. Given A = $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

Clearly, |A| = $\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}$ = 15 - 14 =1

We know that, if A is a non-singular matrix of order n |adj (A) | = $|A|^{n-1}$.

$\therefore$|adj(A) |= $|A|^{2-1}$ $\Rightarrow$ |adj(A) |= $(1)^{2-1}$ = 1

17. $\int\limits_0^1 x\, e^x\, dx = xe^x - \int 1.e^x dx$

= $xe^x - e^x + c$

$\int_0^1 xe^x dx = [xe^x - e^x]_0^1$

= $(1.e^1 - e^1) - (0.e^0 - e^0)$

= 0 - (0 - 1) $[\because e^0 = 1]$

= 1

OR

According to the question , $I = \int_1^2 \frac{x^3-1}{x^2} dx$

$= \int_1^2 \left(x - \frac{1}{x^2}\right) dx$

$= \left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2$

$= \left(\frac{(2)^2}{2} + \frac{1}{(2)}\right) - \left(\frac{(1)^2}{2} + \frac{1}{(1)}\right)$

$= \left(2 + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right)$

$= 1$

18. According to the question , $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

Let , $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad \frac{1}{\sqrt{x}}dx = 2dt$

$I = \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

$= 2\int \cos t dt$

$= 2\sin t + C$

$= 2\sin\sqrt{x} + C[\,put\ t = \sqrt{x}]$

19. Since, f(x)= $\cot^{-1}$ x+x

    On differentiating w.r.t x,we get

    f'(x)=$-\frac{1}{1+x^2} + 1 = \frac{x^2}{1+x^2} \geq 0$, since for -ve values of x , the expression becomes positive since we have x^2.

    Also when x = 0 , the value is 0. And the positive values of x gives values greater than zero. Since the derivative of f(x) is non negative, f(x) is increasing function for all x $\in (-\infty, \infty)$.

20. Given Equation of family of curves is xy = C cos x. ... (i)

    On differentiating both sides w.r.t. x, we get

    $1 \cdot y + x\frac{dy}{dx} = C(-\sin x)$

    $\Rightarrow \quad y + x\frac{dy}{dx} = -\left(\frac{xy}{\cos x}\right)\sin x$ [from Eq. (i)]

    $\therefore \quad y + x\frac{dy}{dx} + xy\ \tan x = 0$

Section B

21. For $x \in (-\infty, -1] \cup [1, \infty)$, $\operatorname{cosec}^{-1}x$ is an angle $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$ such that $cosec\ \theta$ = x.

    $\therefore$ $\operatorname{cosec}^{-1}(2) = \left(\text{ An angle } \theta \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right] \text{ such that } cosec\ \theta = 2\right) = \frac{\pi}{6}$

    and $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

    $= \left(\text{ An angle } \theta \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right] \text{ such that } cosec\ \theta = -\frac{2}{\sqrt{3}}\right) = -\frac{\pi}{3}$

OR

f(x) = x + 1 and g(x) = e^x

Range of f = R ⊂ Domain of g = R ⇒ gof exist

Range of g = (0, ∞) ⊂ Domain of f = R ⇒ fog exist

Now,

gof (x) = g(f(x)) = g(x + 1) = e^{x+1}

And

fog(x) = f(g(x)) = f(e^x) = e^x + 1

22. If s represents the surface area, then
$\frac{ds}{dt} = 2cm^2/\sec$
Also, on using trigonometric ratios, radius of cone can be taken as
$r = l\sin\frac{\pi}{4}$
$s = \pi rl = \pi l.\sin\frac{\pi}{4}l = \frac{\pi}{\sqrt{2}}l^2$
Therefore, $\frac{ds}{dt} = \frac{2\pi}{\sqrt{2}}l.\frac{dl}{dt} = \sqrt{2}\pi l.\frac{dl}{dt}$
$\frac{dl}{dt} = \frac{1}{\sqrt{2}\pi.l}\cdot\frac{ds}{dt}$
when $l = 4cm, \frac{dl}{dt} = \frac{1}{\sqrt{2}\pi.4}.2 = \frac{1}{2\sqrt{2}\pi} = \frac{\sqrt{2}}{4\pi}cm/s$
Thus, rate of decrease of slant height of water is $\frac{\sqrt{2}}{4\pi}cm/\sec$

23. Given: $f(x) = \begin{cases} \frac{\sin x}{x}, \ if\ x < 0 \\ x + 1, \ if\, x \geqslant 0 \end{cases}$

At x = 0, L.H.L. $= \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0}\frac{\sin(x)}{x} = 1$

R.H.L. $= \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0}(x+1) = 0 + 1 = 1$

f(0) = 1

∴ f is continuous at x = 0.

When x < 0, then f(x) is the ratio of two functions x and sin x which are both continuous, therefore $\frac{\sin x}{x}$ is also continuous.

When $x > 0, f(x) = x + 1$ is a polynomial, then f is continuous.

Therefore, f is continuous at any point.

24. $\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$

$=(2-\lambda)\,\hat{i}+(2+2\lambda)\,\hat{j}+(3+\lambda)\,\hat{k}$

$\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=\ 0\ \left[\because \vec{a}+\lambda\vec{b}\perp\vec{c}\right]$

$\left[(2-\lambda)\,\hat{i}+(2+2\lambda)\,\hat{j}+(3+\lambda)\,\hat{k}\right].\left(3\hat{i}+\hat{j}\right)=0$

$3\,(2-\lambda)+(2+2\lambda)=0$

$-\lambda=-8$

$\lambda=8$

OR

Given, $\vec{a}=\ 2\hat{i}+\hat{j}+3\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$and $\vec{c}=3\,\hat{\imath}+\hat{j}+2\hat{k}$

Now, $(\vec{b}\times\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 1\\ 3 & 1 & 2\end{vmatrix}$

$\Rightarrow(\vec{b}\times\vec{c})=\hat{i}(4-1)-\hat{j}(-2-3)+\hat{k}(-1-6)$

$\Rightarrow(\vec{b}\times\vec{c})=3\hat{i}+5\hat{j}-7\hat{k}$

Now,

$\vec{a}\cdot(\vec{b}\times\vec{c})=(2\hat{i}+\hat{j}+3\hat{k})(3\hat{i}+5\hat{j}-7\hat{k})$
$\vec{a}\cdot(\vec{b}\times\vec{c})=(2\cdot 3+1\cdot 5+3\cdot -7)$ = 6 + 5 -21 = 11 - 21 = -10

25. Let $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\hat{k}$, then,

$(x\hat{i}+y\hat{j}+z\hat{k}).(4\hat{i}-2\hat{j}-5\hat{k})$

$\Rightarrow 4x-2y-5z=45$

This is the catesian equation of the required plane.

26. Here, $P(A)=\frac{2}{5}, P(B)=\frac{1}{3}, P(C)=\frac{1}{2}\cdot P(A\cap C)=\frac{1}{5}$ and $P(B\cap C)=\frac{1}{4}$

$P(C/B)=\frac{P(B\cap C)}{P(B)}=\frac{\frac{1}{4}}{\frac{1}{3}}=\frac{3}{4}$

and $P(A'\cap C')=1-P(A\cup C)=1-[P(A)+P(C)-P(A\cap C)]$

$=1-\left[\frac{2}{5}+\frac{1}{2}-\frac{1}{5}\right]=1-\left[\frac{4+5-2}{10}\right]=1-\frac{7}{10}=\frac{3}{10}$

Section C

27. $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ and g : [-1, 1] $\to$ R defined as f (x) = tan x and g(x) = $\sqrt{1-x^2}$

Range of f : Let y = f(x) $\Rightarrow$ y = tan x

$\Rightarrow$ x = $\tan^{-1}$ y

Since $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right), y\in(-\infty,\infty)$

$\therefore$ Range of f $\subset$ Domain of g = [-1, 1]

$\therefore$ gof exists.

By similar argument, fog exists.

Now,

fog(x) = f(g(x)) = $f\left(\sqrt{1-x^2}\right)$

fog(x) = tan $\sqrt{1-x^2}$

Again

gof(x) = g(f(x)) = g(tan x)

gof (x) = $\sqrt{1-\tan^2 x}$

28. $\cos y=x.\cos(a+y)$ (given)

$x=\frac{\cos y}{\cos(a+y)}$

$\frac{dx}{dy}=\frac{\cos(a+y)(-\sin y)-\cos y.(-\sin(a+y))}{\cos^2(a+y)}$

$\frac{dx}{dy}=\frac{\cos y.\sin(a+y)-\sin y\cos(a+y)}{\cos^2(a+y)}$

$\frac{dx}{dy}=\frac{\sin(a+y-y)}{\cos^2(a+y)}$

$\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$

OR

Let f(x) = $\begin{cases} k\left(x^2+2\right), \text{ if } x \leq 0 \\ 3x+1, \text{ if } x > 0 \end{cases}$ is continuous at x = 0.

Then, LHL = RHL = f(0).......(i)

Here, LHL = $\lim\limits_{x \rightarrow 0^-} f(x) = \lim\limits_{x \rightarrow 0^-} k\left(x^2+2\right)$

$= \lim\limits_{h \rightarrow 0} k\left[(0-h)^2+2\right]$

$= \lim\limits_{h \rightarrow 0} k\left(h^2+2\right)$

$\Rightarrow$ LHL = 2k

and RHL = $\lim\limits_{x \rightarrow 0^+} f(x) = \lim\limits_{x \rightarrow 0^+}(3x+1)$

$= \lim\limits_{h \rightarrow 0}[3(0+h)+1]$

$= \lim\limits_{h \rightarrow 0}(3h+1)$

$\Rightarrow$ RHL = 1

From Eq. (i), we have

LHL=RHL

$\Rightarrow$2k = 1

$\therefore \quad k = \frac{1}{2}$

Now, let us check the continuity of the given function f(x) at x = 1.

Consider, $\lim\limits_{x \rightarrow 1} f(x) = \lim\limits_{x \rightarrow 1}(3x+1)$

= 4 = f(1)

Therefore, f(x) is continuous at x = 1.

29. Given: Differential equation $\left(1+x^2\right) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}; y = 0$ when x = 1

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{1}{\left(1+x^2\right)^2}$

Comparing with $\frac{dy}{dx} + Py = Q$,

we have $P = \frac{2x}{1+x^2}$ and $Q = \frac{1}{\left(1+x^2\right)^2}$.

$\therefore \int P dx = \int \frac{2x}{1+x^2} dx = \log\left(1+x^2\right)$

$\Rightarrow I.F = e^{\int P dx} = e^{\log\left(1+x^2\right)} = 1+x^2$

Solution is $y\,(I.F) = \int Q\,(I.F) dx + c$

$\Rightarrow y\left(1+x^2\right)=\int \frac{1}{\left(1+x^2\right)^2}\left(1+x^2\right) d x+c$

$\Rightarrow y\left(1+x^2\right)=\int \frac{1}{\left(1+x^2\right)} d x=\tan ^{-1} x+c$ ...(i)

Now putting y = 0, x = 1 in (i),we get, $0=\tan ^{-1} 1+c \Rightarrow 0=\frac{\pi}{4}+c \Rightarrow c=-\frac{\pi}{4}$
Putting the value of c in eq. (i), $\Rightarrow y\left(1+x^2\right)=\tan ^{-1} x-\frac{\pi}{4}$

30. $\int e^x\left(\tan ^{-1} x+\frac{1}{1+x^2}\right) d x \; f(x)=\tan ^{-1} x \; f^{\prime}(x)=\frac{1}{1+x^2}$
$\int e^x\left(\tan ^{-1} x+\frac{1}{1+x^2}\right) d x=e^x \tan ^{-1} x+c$
$\left[\because \int e^x\left[f(x)+f^{\prime}(x)\right] d x=e^x f(x)+C\right]$

31. In this case, we have to find out the probability of getting at least 8 heads. Let X is the random variable for getting a head.
Here, $n=10, r \geqslant 8$,
i.e., r = 8, 9, 10, $p=\frac{1}{2}, q=\frac{1}{2}$
we know that, P(X = r) $={ }^n C_r p^r q^{n-r}$
$\therefore$ P(X = r) = P(r = 8) + P(r = 9) + P(r = 10)
$={ }^{10} C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{10-8}+{ }^{10} C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{10-9}+{ }^{10} C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$
$=\frac{10!}{8!2!}\left(\frac{1}{2}\right)^{10}+\frac{10!}{9!1!}\left(\frac{1}{2}\right)^{10}+\frac{10!}{0!10!}\left(\frac{1}{2}\right)^{10}$
$=\left(\frac{1}{2}\right)^{10}\left[\frac{10 \times 9}{2}+10+1\right]=\left(\frac{1}{2}\right)^{10} \cdot 56=\frac{1}{2^7 \cdot 2^3} \cdot 56=\frac{7}{128}$

OR

Firstly, find the probability distribution table of number of red cards, then, using this table, find the mean and variance.
Let X be the number of red cards. Then, X can take values 0, 1 and 2.
Now, P (X = 0) = P (having no red card)
$=\frac{{ }^{26} C_2}{{ }^{52} C_2}=\frac{26 \times 25 /(2 \times 1)}{52 \times 51 /(2 \times 1)}$
$=\frac{1}{2} \times \frac{25}{51}=\frac{25}{102}$
P (X = 1) = P (having one red card)
$=\frac{{ }^{26} C_1 \times{ }^{26} C_1}{{ }^{52} C_2}$
$=\frac{26 \times 26 \times 2}{52 \times 51}=\frac{26}{51}$
P (X = 2) = P (having two red cards)
$=\frac{{ }^{26} C_2}{{ }^{52} C_2}=\frac{26 \times 25 / 2 \times 1}{52 \times 51 / 2 \times 1}=\frac{25}{102}$

$\therefore$ The probability distribution of number of red cards is given below

X	0	1	2
P(X)	$\frac{25}{102}$	$\frac{26}{51}$	$\frac{25}{102}$

Now, we know that, mean=$\sum XP(X)$
and variance =$\sum X^2P(X)-[\sum XP(X)]^2$

X	**P(X)**	**X. P (X)**	**X^2 P(X)**
0	$\frac{25}{102}$	0	0
1	$\frac{26}{51}$	$\frac{26}{51}$	$\frac{26}{51}$
2	$\frac{25}{102}$	$\frac{25}{51}$	$\frac{50}{51}$

$\therefore \quad \text{Mean } =\sum X\cdot P(X)$
$=0+\frac{26}{51}+\frac{25}{51}=\frac{51}{51}=1$
$=\sum X^2P(X)-[\sum XP(X)]^2$
$=\frac{76}{51}-1=\frac{76-51}{51}=\frac{25}{51}$

32. Let the person takes x units of tablet X and y unit of tablet Y.

So, from the given information, we have

$6x+2y\geqslant 18\Rightarrow 3x+y\geqslant 9\ldots(i)$

$3x+3y\geqslant 21\Rightarrow x+y\geqslant 7\ldots(ii)$

And $2x+4y\geqslant 16\Rightarrow x+2y\geqslant 8\ \ldots(iii)$

Also, we know that here, $x\geqslant 0, y\geqslant 0\ldots(iv)$

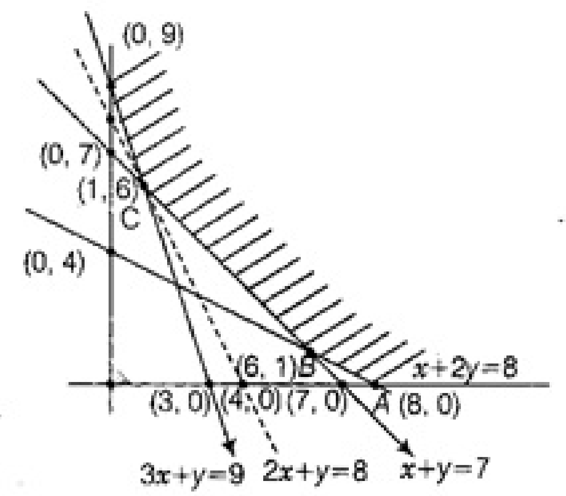
The price of each tablet of X and Y is Rs 2 and Rs 1, respectively.

So, the corresponding LPP is minimise Z = 2x + y subject to

$3x+y\geqslant 9, x+y\geqslant 7x, x+y\geqslant 7, x+2y\geqslant 8, x\geqslant 0, y\geqslant 0$

From the shaded graph, we see that for the shown unbounded region, we have coordinates of corner points A, B, C and D as (8,0),(6,1),(1,6) and (0,9) respectively.

[On solving x + 2y = 8 and x + y = 7 we get x = 6, y = 1 and on solving 3x + y = 9 and x + y = 7, we get x = 1, y = 6]



Corner Points	Values of Z = 2x + y
(8, 0)	16
(6, 1)	13
(1, 6)	8 (minimum)
(0, 9)	9

Thus, we see that 8 is the minimum value of Z at the corner point (1, 6). Here we see that the feasible region is unbounded. Therefore, 8 may or may not be the minimum value of Z. To decide this issue, we graph the inequality

2x + y < 8 ...(v)

And check whether the resulting open half has points in common with feasible region or not. If it has common point, then 8 will not be the minimum value of Z, otherwise 8 will be the minimum value of Z.

Thus, from the graph it is clear that, it has no common point.

Therefore, Z = 2x + y has 8 as minimum value subject to the given constraint.

Hence, the person should take 1 unit of X tablet and 6 unit of Y tablets to satisfy the given requirements and at the minimum cost of Rs 8.

Section D

33. Put $\tan \frac{\alpha}{2} = t$

$$A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$
$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$
$$\text{L.H.S.} = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$
$$= (I - A) \begin{bmatrix} \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \frac{2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{-2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{t.2t}{1+t^2} & \frac{-2t}{1+t^2} + t\left(\frac{1-t^2}{1+t^2}\right) \\ -t\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} & -t\left(\frac{-2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t^3-t}{1+t^2} \\ \frac{t^3+t}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

L.H.S = R.H.S

Hence proved

OR

Given , $\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix}$ = 0

On applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B-\cos A & (\cos\ C\ -\ \cos\ A) \\ \left(\cos^2 A+\cos A\right) & \left(\cos^2 B-\cos^2 A\right)+(\cos B-\cos A) & (\cos^2 C\ -\ \cos^2 A)+(\cos\ C\ -\ \cos A) \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & (\cos B-\cos A) & (\cos\ C\ -\ \cos\ A) \\ \left(\cos^2 A+\cos A\right) & (\cos B-\cos A)(\cos B+\cos A+1) & (\cos C\ -\ \cos A)+(\cos\ C\ -\ \cos A+1) \end{vmatrix}=0$$

Therefore,on taking common (cos B - cos A) from C_2 and (cos C - cos A) from C_3, we get

= (cos B - cos A)(cos C - cos A)

$$\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A+\cos A & 1+\cos A+\cos B & 1+\cos A+\cos C \end{vmatrix}=0$$

Therefore,on expanding along R_1, we get

= (cos B - cos A)(cos C - cos A)[1 + cos A + cos C - 1 - cos A - cos B] = 0

$\Rightarrow$(cos B - cos A)(cos C - cos A)(cos C - cos B) = 0

$\Rightarrow$cos B = cos A or cos C = cos A

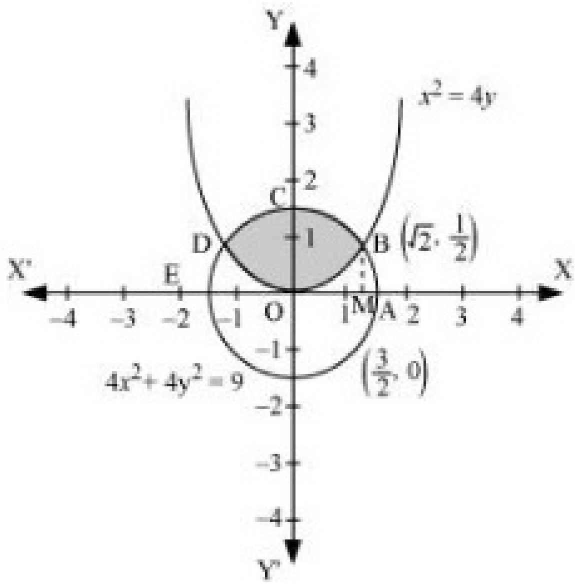
or, cos C = cos B

$\Rightarrow \quad \angle B=\angle A$

or, $\angle C=\angle A$

or, $\angle C=\angle B$

Therefore, ΔABC is an isosceles triangle.

34. 

Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as

B$\left(\sqrt{2}, \frac{1}{2}\right)$ and D$\left(-\sqrt{2}, \frac{1}{2}\right)$

It can be observed that the required area is symmetrical about y-axis.

$\therefore$ Area OBCDO = 2 $\times$ Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are ($\sqrt{2}$, 0)

Therefore, Area OBCO = Area OMBCO - Area OMBO

= $\int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \frac{x^2}{4} dx$

= $\frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx$

= $\frac{1}{4}\left[x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$

= $\frac{1}{4}\left[\sqrt{2}\sqrt{9-8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}(\sqrt{2})^3$

= $\frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$

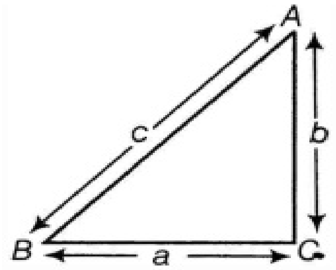
= $\frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$

= $\frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$

Therefore, the required area OBCDO = $2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$ =

$\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]$ sq. units.

35. Let a and b be the sides of right-angled triangle and c be the hypotenuse.



From ΔABC, we have

$c^2 = a^2 + b^2$

Area of ΔABC, (A) $= \frac{1}{2} a \cdot b = \frac{1}{2} a \sqrt{c^2 - a^2}$

On differentiating both sides w.r.t. a, we get

$$\frac{dA}{da} = \frac{1}{2} \cdot 1 \cdot \sqrt{c^2 - a^2} + \frac{1}{2} \cdot a \cdot \frac{1}{2} \cdot \frac{(-2a)}{\sqrt{c^2 - a^2}}$$

$$= \frac{1}{2}\left(\sqrt{c^2 - a^2} - \frac{a^2}{\sqrt{c^2 - a^2}}\right)$$

For maxima or minima, put $\frac{dA}{da} = 0$

$$\Rightarrow \frac{1}{2}\left(\sqrt{c^2 - a^2} - \frac{a^2}{\sqrt{c^2 - a^2}}\right) = 0$$

$\Rightarrow c^2 - a^2 - a^2 = 0$

$\Rightarrow c^2 = 2a^2$

Now, $\frac{d^2 A}{da^2} = \frac{1}{2}\left[\frac{-a}{\sqrt{c^2 - a^2}} - \frac{a^3}{(c^2 - a^2)^{3/2}}\right]$

$$= -\frac{1}{2} a \left[\frac{c^2 - a^2 + a^2}{(c^2 - a^2)^{3/2}}\right]$$

$$= -\frac{1}{2} \frac{c^2 a}{(c^2 - a^2)^{3/2}} < 0$$

Therefore, Area of triangle ABC is maximum and

b = $\sqrt{c^2 - a^2} = \sqrt{2a^2 - a^2}$ = a

Hence, the triangle is isosceles.

OR

Let (x, y) be the point

a. $y = x^2 - 2x + 7$ ....(1)

$$\frac{dy}{dx} = 2x - 2$$

$$\left.\frac{dy}{dx}\right]_{x_1, y_1} = 2x_1 - 2$$

Given curve is parallel to the line 2x - y + 9 = 0

Slope of this line = 2

According To Question,

$2x_1 - 2 = 2$

$x_1 = 2$

$y_1 = x_1^2 - 2x_1 + 7$ [from (1)]

$y_1 = 4 - 4 + 7$

= 7

Equation of tangent

$y - y_1 = \frac{dy}{dx}(x - x_1)$

y - 7 = 2x - 4

2x - y + 3 =0

b. 5y - 15x = 13

Slope of Line= $\frac{-(-15)}{5} = 3$

Now given curve is perpendicular to the line 5y - 15x = 13

Therefore,

$m_1 \times m_2 = -1$

$(2x_1 - 2) \times 3 = -1$

$x_1 = \frac{5}{6}$

Put x_1 in equation (1)

$y_1 = \left(\frac{5}{6}\right)^2 - 2\left(\frac{5}{6}\right) + 7$

$= \frac{25}{36} - \frac{10}{6} + 7$

$= \frac{25-60+7\times36}{36}$

$= \frac{-35+252}{36}$

$= \frac{217}{36}$

Equation of tangent

$y - y_1 = \frac{dy}{dx}(x - x_1)$

$y - \frac{217}{36} = \frac{-1}{3}\left(x - \frac{5}{6}\right)$

$\frac{36y-217}{36} = \frac{-1}{3}\left(\frac{6x-5}{6}\right)$

$\frac{36y-217}{36} = -6x + 5$

36y - 217 = -12x + 10

12x + 36y - 277 = 0

36. We have, 2l + 2m - n = 0 .... (i)

And mn + nl + lm = 0 .... (ii)

Eliminating m from the both equations, we get

$m = \frac{n-2l}{2}$ [from Eq. (i)]

$\Rightarrow \left(\frac{n-2l}{2}\right) n + nl + l\left(\frac{n-2l}{2}\right) = 0$

$\Rightarrow \frac{n^2 - 2nl + 2nl + nl - 2l^2}{2} = 0$

$\Rightarrow n^2 + nl - 2l^2 = 0$

$\Rightarrow n^2 + 2nl - nl - 2l^2 = 0$

$\Rightarrow (n + 2l)(n - l) = 0$

$\Rightarrow n = -2l$ and $n = l$

$\therefore m = \frac{-2l-2l}{2}, m = \frac{l-2l}{2}$

$\Rightarrow m = -2l, m = \frac{-l}{2}$

Thus, the direction ratios of two lines are proportional to l, -2l, -2 and $l, \frac{-l}{2}, l$.

$\Rightarrow$ 1, -2, -2 and $1, \frac{-1}{2}, 1$

$\Rightarrow$ 1, -2, -2 and 2, -1, 2

Also, the vectors parallel to these lines are $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ respectively.

$\therefore \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{(i - 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 2\hat{k})}{3 \cdot 3}$

$= \frac{2+2-4}{9} = 0$

$\therefore \theta = \frac{\pi}{2} \left[\because \cos\frac{\pi}{2} = 0\right]$