The Triangle and its Properties

- A triangle is a simple closed curve made up of three line segments.

  It has three vertices, three sides and three angles.
- Triangles can be classified on the basis of their sides as:

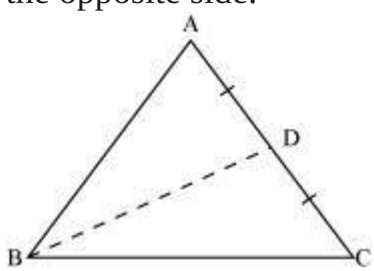
1. Scalene – No side of the triangle is equal
2. Isosceles – Exactly two sides of the triangle are equal
3. Equilateral – All the sides of the triangle are equal

- On the basis of angles, triangles can be classified as:

1. Acute-angled – All the angles of the triangle are less than 90°
2. Obtuse-angled – Any one of the angles of the triangle is greater than 90°
3. Right-angled – Any one of the angles of the triangle is 90°

- Median of a triangle

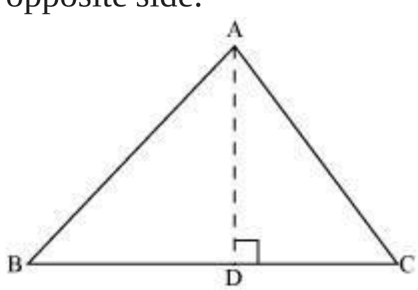
  A median is a line segment joining the vertex of a triangle to the mid-point of the opposite side.



In the given ΔABC, if AD = DC, then BD is the median of ΔABC with respect to the side AC.
A triangle has three medians, one for each side.

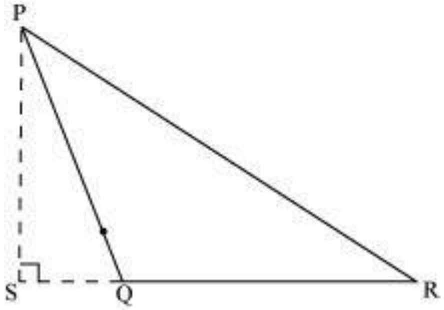
- Altitude of a triangle

  An altitude is the perpendicular drawn from the vertex of a triangle to its opposite side.



In the given figure, AD is the altitude of ΔABC with respect to side BC.

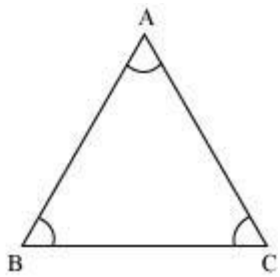
A triangle has three altitudes, one from each vertex.
The altitude of a triangle may or may not lie inside the triangle.
For example, for ΔPQR, its altitude lies outside it.



- **Angle sum property of triangles:**

  The sum of all the three interior angles of a triangle is 180°.

  ∴ ∠A + ∠B + ∠C = 180°



Example:

If the measures of the angles of a triangle are in the ratio 2: 4: 6, then find all the angles of the triangle.

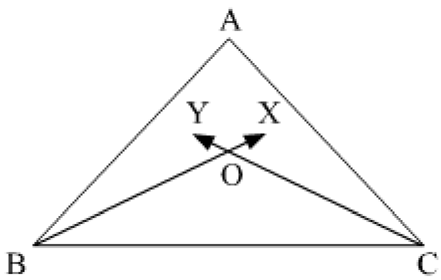
Solution:

Ratio of the measures of angles = 2: 4: 6
Therefore, let the angles of the triangle measure 2*x*, 4*x*, and 6*x*.
Now, $2x + 4x + 6x = 180°$ {By angle sum property of triangles}
$\Rightarrow 12x = 180°$
$\Rightarrow x = 15°$
Thus, the angles of the triangle are
$2x = 2 \times 15° = 30°$,
$4x = 3 \times 15° = 60°$
$6x = 6 \times 15° = 90°$.
The measure of one of angle is 90°.

- **Facts deduced from angle sum property of triangles:**

  There can be no triangle with two right angles or two obtuse angles.
  There can be no triangle with all angles less than or greater than 60°.

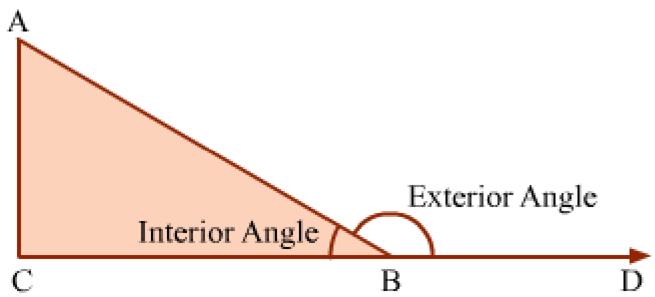
- **Relation between the vertex angle and the angles made by the bisectors of the remaining angles:**



In ΔABC, BX and CY are bisectors of ∠B and ∠C respectively. Also, O is the point of intersection of BX and CY.

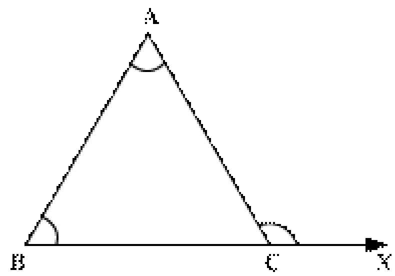
Therefore, $\angle BOC = 90° + \frac{1}{2}\angle A$.

- The angle formed by a side of a triangle with an extended adjacent side is called an **exterior angle of the triangle**.



It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

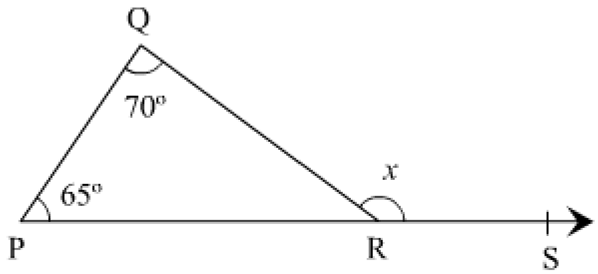


∠ACX = ∠BAC + ∠ABC

This property is known as exterior angle property of a triangle.

Example:

Find the value of *x* in the following figure.



Solution:
∠QRS is an exterior angle of ΔPQR. It is thus equal to the sum of its interior opposite angles.
∴∠QRS = ∠QPR + ∠PQR
⇒ x = 65° + 70° = 135°
Thus, the value of x is 135°.

- Two exterior angles can be drawn at each vertex of triangle. The two angles thus drawn have an equal measure and are equal to the sum of the two opposite interior angles.

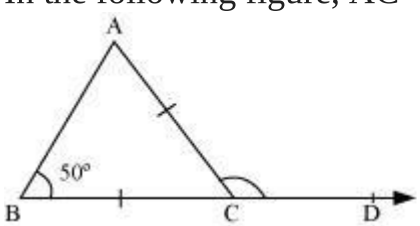
- Equilateral triangle

  In an equilateral triangle, (1) all sides are equal and (2) each angle is of measure 60°.

- Isosceles triangle

  In an isosceles triangle, (1) two sides are of equal length and (2) the base angles opposite to the equal sides are equal.

  Example:
  In the following figure, AC = BC. Find m∠ACD.



  Solution: In ΔABC, AC = BC
  ∴ ∠CBA = ∠CAB
  ∴ ∠ACD = ∠CAB + ∠CBA = 2 × 50° = 100°

- In a triangle, the sum of the lengths of any two sides is greater than the length of the third side.
- The difference between the lengths of any two sides of a triangle is less than the length of the third side.

  Example:

Is it possible to construct a triangle with sides having lengths as 6 cm, 5 cm and 12 cm?

Solution:

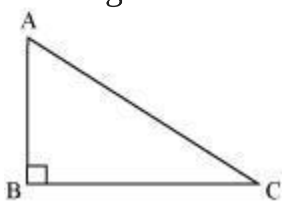
In the given case, 12 cm – 5 cm = 7 cm > 6 cm

Here, the difference between two sides is greater than the third side.

Hence, a triangle with the given lengths cannot be constructed.

- Right-angled triangle

A triangle with one of the angles as 90°, is called a right-angled triangle.

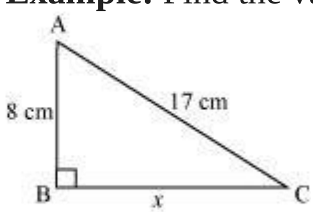


ΔABC is a right-angled triangle with $\angle$B = 90°

- The side opposite to the right angle is called its hypotenuse. The other two sides are called the legs of the right-angled triangle, i.e., in the given figure, AC is the hypotenuse and AB, BC are the legs.
- In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides, i.e., $AC^2 = AB^2 + BC^2$.

This property is called Pythagoras Theorem.

Example: Find the value of *x* in the following figure.



Solution: ΔABC is a right-angled triangle with $\angle$B = 90°.

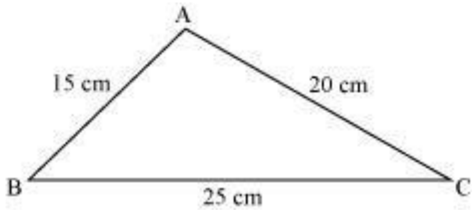
$AC^2 = AB^2 + BC^2$ [By Pythagoras Theorem]

$\Rightarrow (17 \text{ cm})^2 = (8 \text{ cm})^2 + x^2$

$\Rightarrow x = \sqrt{289 - 64}\text{cm} = 15\text{cm}$

- In a triangle, if Pythagoras property holds, then it is a right-angled triangle.

Example:



Prove that ΔABC is right-angled.

Solution: In ΔABC,

AB = 15 cm ⇒ AB^2 = 225 cm^2

AC = 20 cm ⇒ AC^2 = 400 cm^2

BC = 25 cm ⇒ BC^2 = 625 cm^2

$BC^2 = AB^2 + AC^2$

Therefore, Pythagoras property holds in ΔABC. Thus, it is a right-angled triangle.