CBSE Test Paper 05

Chapter 9 Some Applications of Trigonometry

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1. A river is 60 m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to the foot of the tree, on the other bank, is 30^{0}. The height of the tree is **(1)**
   a. $30\sqrt{3}$ m
   b. $10\sqrt{3}$ m
   c. $20\sqrt{3}$ m
   d. $60\sqrt{3}$ m
2. If the altitude of the sun is 60°, the height of a tower which casts a shadow of length 90 m is **(1)**
   a. 60 m
   b. $90\sqrt{3}$ m
   c. 90 m
   d. $60\sqrt{3}$ m
3. If the shadow of a tower is 30 m long when the sun's elevation is 30°. The length of the shadow, when the sun's elevation is 60° is **(1)**
   a. 10 m
   b. 30 m
   c. $10\sqrt{3}$ m
   d. 20 m
4. Two men are on opposite sides of a tower. They observe the angles of elevation of the top of the tower as 60° and 45° respectively. If the height of the tower is 60m, then the distance between them is **(1)**
   a. $20(3-\sqrt{3})m$
   b. $20(\sqrt{3}-3)m$
   c. None of these
   d. $20(\sqrt{3}+3)m$
5. A plane is observed to be approaching the airport. It is at a distance of 12 km from the point of observation and makes an angle of elevation of 30° there at. Its height above the ground is **(1)**

a. 10 km
b. 12 km
c. 6 km
d. none of these

6. A ladder, leaning against a wall, makes an angle of 60^o with the horizontal.If the foot of the ladder is 2.5 m away from the wall, then find the length of the ladder. **(1)**

7. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^{0}. Find the length of the string assuming that there is no slack in the string. **(1)**

8. A kite is flying at a height of 30 m from the ground. The length of the string from kite to the ground is 60 m. Assuming that there is no slack in the string, find the angle of elevation of the kite at the ground. **(1)**

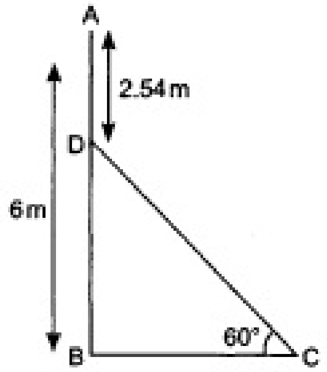
9. If a pole 6m high throws shadow of $2\sqrt{3}$ m, then find the angle of elevation of the sun. **(1)**

10. The length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the sun. **(1)**

11. The angle of elevation of the top of a building from the foot of the tower is 30^{0} and the angle of elevation of the top of the tower from the foot of the building is 45^{0}. If the tower is 30 m high, find the height of the building. **(2)**

12. A person standing on the bank of a river, observes that the angle of elevation of the top of the tree standing on the opposite bank is 60^{0}. When he retreats 20 m from the bank, he finds the angle of elevation to be 30^{0}. Find the height of the tree and the breadth of the river. **(2)**

13. In fig. AB is 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point of pole. If AD = 2.54 m, find the length of the ladder, (use $\sqrt{3}$ = 1.73) **(2)**



14. A person observed the angle of elevation of the top of a tower is 30^0. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60^0. Find the height of the tower. **(3)**

15. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30^0. If the height of the first pole is 24 m, find the height of the hsecond pole.[Use $\sqrt{3}$ = 1.732] **(3)**

16. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and height of the hill. **(3)**

17. The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree, when seen from the top of the second tree is 45°. If the height of the second tree is 80 m, find the height of the first tree. **(3)**

18. The elevation of a tower at a station A due north of is α and at a station B due west of A is β. prove that the height of the tower is $\frac{AB \sin\alpha \sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}$. **(4)**

19. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60^0, then find the height of the flagstaff. [Use $\sqrt{3}$ = 1.73]. **(4)**

20. From an aeroplane vertically above a straight horizontal plane, the angles of depression of two consecutive kilometer stones on the opposite sides of the aeroplane are found to be α and β . Show that the height of the aeroplane is $\frac{\tan\alpha \tan\beta}{\tan\alpha + \tan\beta}$. **(4)**

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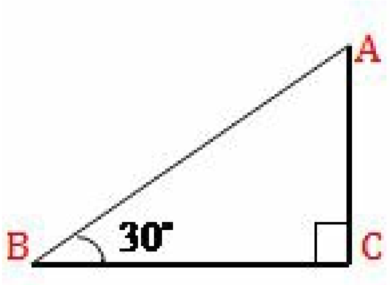
Solution

1. c. $20\sqrt{3}$ m

   Explanation: Let BC = 60 m be the width of the river and angle of elevation = 30°

   To find: Height of the tree AC

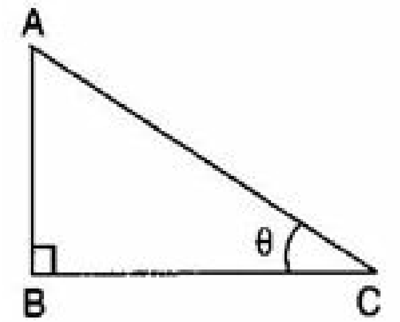
   $\therefore \tan 30^{\circ} = \frac{\mathrm{AC}}{\mathrm{BC}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{\mathrm{AC}}{60}$

   $\Rightarrow$ AC = $\frac{60}{\sqrt{3}}$

   = $\frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}\mathrm{m}$

   Therefore, the height of the tree is $20\sqrt{3}\mathrm{m}$.

2. b. $90\sqrt{3}$ m

   Explanation: 

   Let Height of the tower = AB = h meters,

   Length of the shadow = BC = 90 m

   And angle of elevation $\theta = 60^{\circ}$

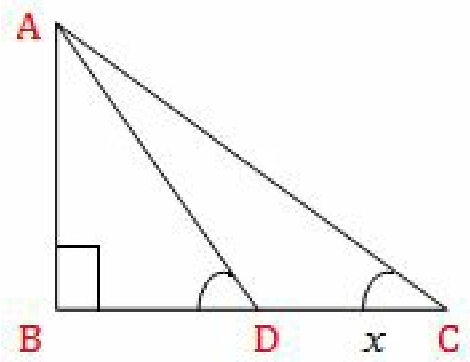
   $\therefore \tan 60^{\circ} = \frac{\mathrm{AB}}{\mathrm{BC}}$

   $\Rightarrow \sqrt{3} = \frac{h}{90}$

   $\Rightarrow h = 90\sqrt{3}$ meters

3. a. 10 m

Explanation:



Let the height of the tower be h

$\therefore \tan 30^\circ = \frac{h}{30}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30}$

$\Rightarrow h = \frac{30}{\sqrt{3}}$

Again $\tan 60^\circ = \frac{h}{x}$

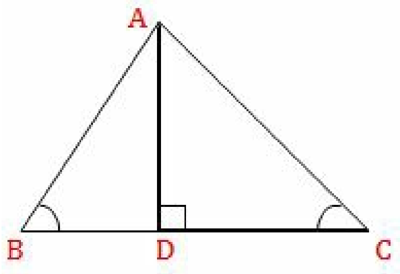
$\Rightarrow \sqrt{3} = \frac{30}{\sqrt{3} \times x}$

$\Rightarrow x = 10 \text{ m}$

Therefore, the length of the shadow is 10 m long.

4. d. $20(\sqrt{3}+3)m$

Explanation:



Let the height of the tower = AD = 60 m and angles of elevation of the top of the tower of two men are 60° and 45° respectively.

To find: Distance between two men = BC

In triangle ABD,

$\tan 60^\circ = \frac{60}{\text{BD}} \Rightarrow \sqrt{3} = \frac{60}{\text{BD}}$

$\Rightarrow \text{BD} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ = $20\sqrt{3}$ m

In triangle ADC,

$\tan 45^\circ = \frac{60}{\text{DC}}$

$\Rightarrow 1 = \frac{60}{\text{DC}}$

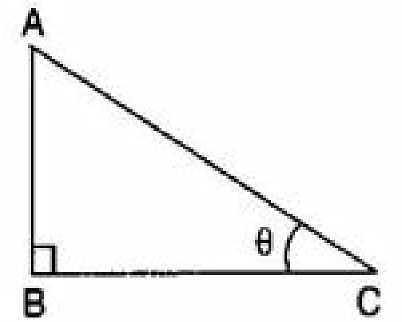
$\Rightarrow \mathrm{DC} = 60 \text{ m}$

$\therefore$ BC = BD + DC = $20\sqrt{3} + 60$

= $20\left(\sqrt{3} + 3\right)$ m

5. c. 6 km

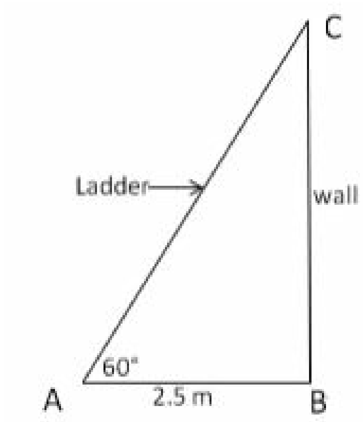
Explanation:



Let the height of the flying plane be AB = h meters, distance from the poisnt of observation AC = 12 m and angle of elevation $\theta = 30^\circ$

$\therefore \sin 30^\circ = \frac{\mathrm{AB}}{\mathrm{AC}} \Rightarrow \frac{1}{2} = \frac{h}{12} \Rightarrow$ h = 6 meters

6.



Let AC be the ladder of length $h\ m$ and BC be the wall.

Then, AB = 2.5 m and $\angle CAB = 60^o$

In right-angled $\triangle ABC$,

$\sec 60^o = \frac{H}{B} = \frac{AC}{AB}$

$\sec 60^o = \frac{h}{2.5}$

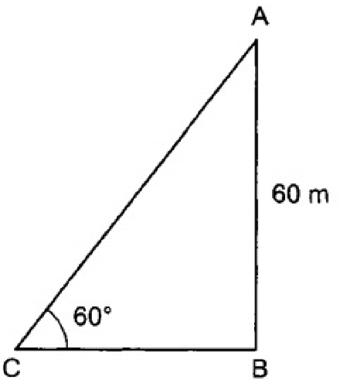
$\Rightarrow 2 = \frac{h}{2.5}\ [\because\ Sec\ 60^o =\ \ 2]$

$\Rightarrow h = 5\ m$

Therefore, find the length of the ladder = 5 m

7. Let A be the kite and CA be the string attached to the kite such that its one end is tied to a point C on the ground. The inclination of the string CA with the ground is 60°.

In $\triangle ABC$, we are given that $\angle C = 60^{\circ}$ and perpendicular AB = 60 m and we have to find hypotenuse AC. So, we use the trigonometric ratio involving perpendicular and hypotenuse.



In $\triangle ABC$, we have

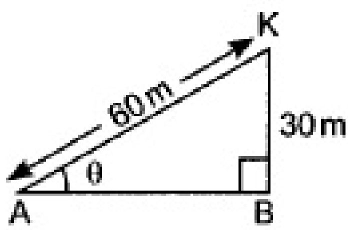
$\sin C = \frac{AB}{AC}$

$\Rightarrow \quad \sin 60^{\circ} = \frac{AB}{AC}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{AC}$

$\Rightarrow \quad AC = \frac{120}{\sqrt{3}} = 40\sqrt{3}\text{m}.$

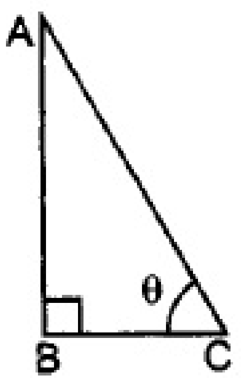
Hence, the length of the string is $40\sqrt{3}\text{m}$.

8. 

Let kite is at point K,then AK = length of string

$\therefore$ In right $\triangle$ABK, $\frac{\mathbf{BK}}{\text{AK}} = \sin\theta$

$= \sin\theta = \frac{30}{60} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^{\circ}\text{m}$

9. 

given,

height of the pole = 6m

Let AB is pole and BC is its shadow

$\therefore AB = 6m, BC = 2\sqrt{3}$ m

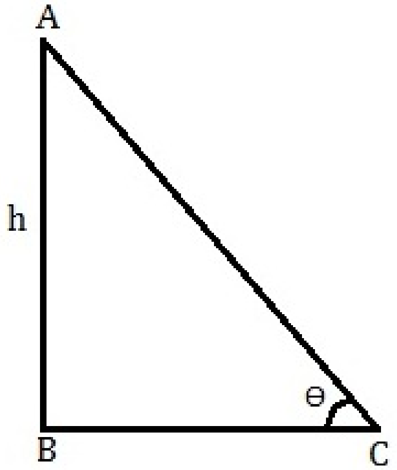
In right ABC, $\frac{AB}{BC} = \tan\theta$

$\Rightarrow \tan\theta = \frac{6}{2\sqrt{3}}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60^\circ$

10. According to the question, the length of shadow of the tower on the plane ground is $\sqrt{3}$ times the height of the tower.



Let θ be the angle of elevation.

BC = $\sqrt{3}$ $\times$ AB

$\Rightarrow \frac{BC}{AB}$ = $\sqrt{3}$

$\Rightarrow \frac{BC}{h}$ = cotθ

$\Rightarrow$cotθ = $\frac{BC}{h}$ = $\sqrt{3}$

$\Rightarrow$ cot $\theta = cot 30^o$

$\Rightarrow \theta$ = 30^o

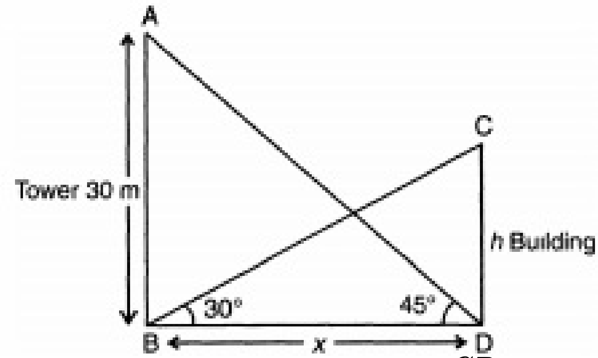
11. Let the height of the building be CD = h m. and distance between tower and building be, BD = x m.

and height of the tower be AB = 30m

In $\triangle ABD, \tan 45^\circ = \frac{AB}{BD}$

$\Rightarrow \quad 1 = \frac{30}{x}$

$\Rightarrow$ x = 30 .....(i)



Now in $\triangle BDC$, $\tan 30^\circ = \frac{CD}{BD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \quad \sqrt{3}h = x \Rightarrow h = \frac{x}{\sqrt{3}}$ ....(ii)

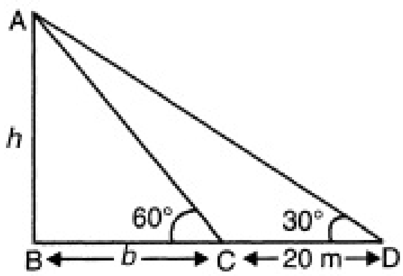
From (i) and (ii), $h = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

Rationalising Numerator and Denominator above we get

$= 10\sqrt{3}\text{m}$

$\therefore$ Height of the building ,h = $10\sqrt{3}m$

12. Let the height be 'h' m and breadth of river be 'b' m.



In $\triangle$ABC,

$\frac{h}{b} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow \quad h = \sqrt{3}b$

In $\triangle$ABD,

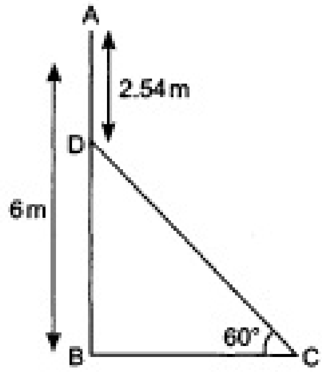
$\frac{h}{b+20} = tan 30^\circ = \frac{1}{\sqrt{3}}$

$h = \frac{b+20}{\sqrt{3}}$

b + 20 = h$\sqrt{3}$ = 3b

b = 10 m

And h = 10 $\times$ $\sqrt{3}$ = 17.3m

13. 

$BD = AB - AD = 6 - 2.54 = 3.46$

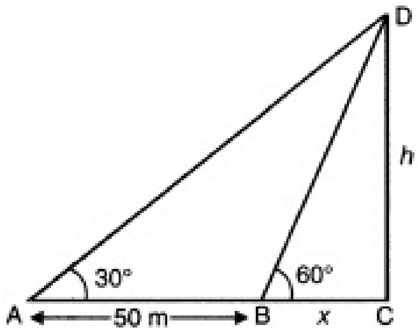
In right $\triangle$DBC

$\frac{BD}{CD} = \sin 60^\circ$

$\frac{3.46}{CD} = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{3.46 \times 2}{\sqrt{3}} = \mathrm{CD}$

$\Rightarrow \frac{3.46 \times 2}{1.73} = \mathrm{CD} \Rightarrow \mathrm{CD} = 4\mathrm{m}$

14. 

Let height of the tower be DC = h m and BC = x m AC = (50 + x) m

In $\triangle$DBC, $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow \quad h = \sqrt{3}x$ ..(i)

In $\triangle$DAC, $\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$,

$\Rightarrow \quad \sqrt{3}h = x + 50$ ...(ii)

Substituting the value of h from (i) in (ii), we get

3x = x + 50

or, 3x - x = 50

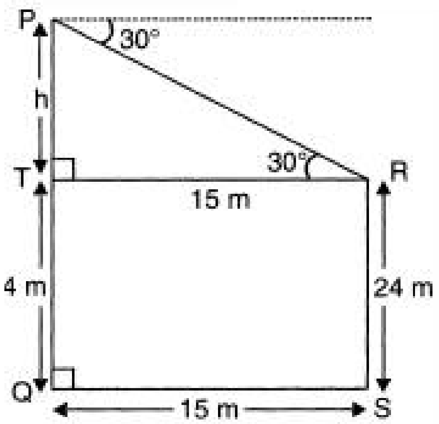
$\Rightarrow$ 2x = 50

$\Rightarrow$ x = 25 m

$h = 25\sqrt{3} = 25 \times 1 \cdot 732\mathrm{m}$

= 43.3 m

Hence, Height of tower = 43.3 m.

15. 

In $\triangle$ PTR,

$\tan 30^\circ = \frac{PT}{TR}$

$\frac{1}{\sqrt{3}} = \frac{h}{15}$

$h = \frac{15}{\sqrt{3}}$

$= 5\sqrt{3}$

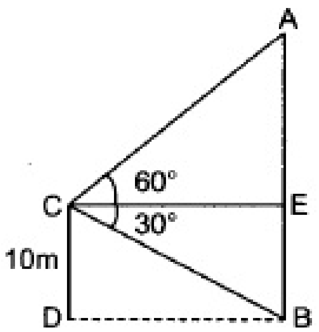
= 5 $\times$ 1.732 = 8.66

PQ = PT + TQ

= 8.66 + 24

= 32.66 m

Height of the second pole is 32.66 m.

16. 

Let CD is deck of ship and AB is hill. $CD = BE = 10m$

In $\triangle$BEC, $\frac{CE}{BE} = \cot 30^\circ$

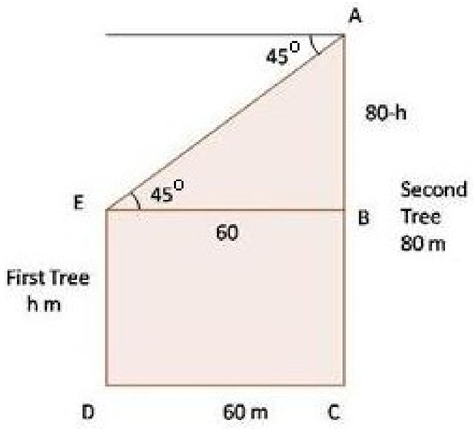
$\Rightarrow$ CE = BE.cot 30° $\Rightarrow$ $\text{CE} = 10 \times \sqrt{3}\text{m} = 17.3\text{m}$

In $\triangle$AEC, $\frac{\text{AE}}{\text{CE}}$ = tan 60°

$\Rightarrow$ AE = CE.$\sqrt{3}$ m $\Rightarrow$ $\text{AE} = 10 \times \sqrt{3} \times \sqrt{3}\text{m} = 30\text{m}$

$\therefore$ Height of hill $AB = AE + EB = (30 + 10)m = 40m$

Distance of hill $CE = 17.3m$

17. 

Let height of first tree = h m

Height of second tree = 80 m

$\therefore$ AB = (80 - h)m

Distance between both trees = 60 m

In ΔABE

$\tan 45^\circ = \frac{AB}{BE}$

$\Rightarrow 1 = \frac{80-h}{60}$

$\Rightarrow$ 60 = 80 - h

$\Rightarrow h = 80 - 60 = 20m$

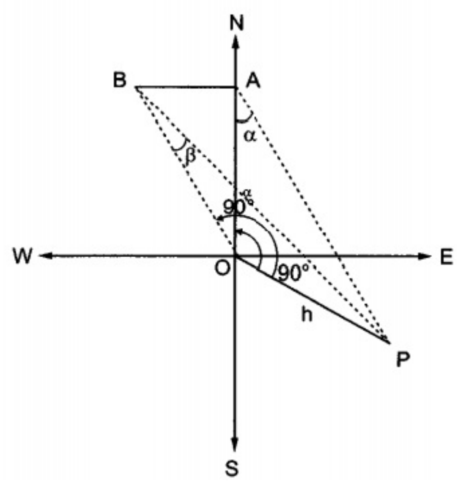
18. Let OP be the tower and let A be a point due north of the tower OP and let B be the point due west of A. Such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let h be the height of the tower.

In right-angled triangle OAP and OBP, we have

$\tan \alpha = \frac{h}{OA}$ and $\tan\beta = \frac{h}{OB}$

$\Rightarrow OA = h \cot \alpha$ and $OB = h \cot \beta$

In $\triangle OAB$, we have



OB ^{2} = OA ^{2} + AB

$\Rightarrow$ AB^2 = OB^2 - OA ^{2}

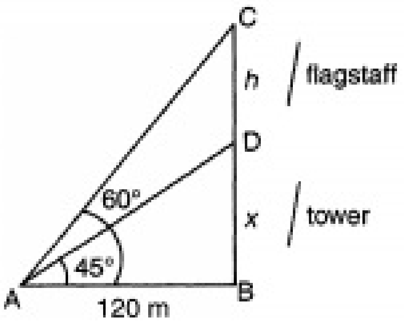
$$\Rightarrow AB^2 = h^2\left[\cot^2\beta - \cot^2\alpha\right]$$
$$\Rightarrow AB^2 = h^2\left[\left(cosec^2\beta - 1\right) - \left(cosec^2\alpha - 1\right)\right]$$
$$\Rightarrow AB^2 = h^2\left(cosec^2\beta - cosec^2\alpha\right)$$
$$\Rightarrow AB^2 = h^2\left(\frac{\sin^2\alpha - \sin^2\beta}{\sin^2\alpha\sin^2\beta}\right)$$
$$\Rightarrow h = \frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}$$

19. Height of flagstaff = CD = h m

Height of tower = BD = x m

$\angle DAB = 45^\circ, \angle CAB = 60^\circ$

AB = 120 m



$\triangle$ABD is right angled triangle

tan 45^{o} = 1

$$\frac{x}{AB} = 1$$

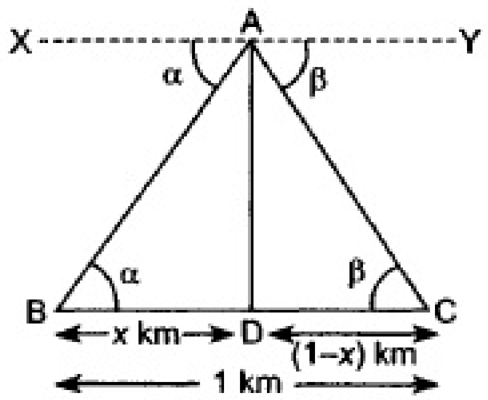
x = AB = 120 m

$\triangle$ ACB is right angled triangle

$$\frac{h+x}{120} = \tan 60^\circ = \sqrt{3}$$
$$h + 120 = 120\sqrt{3}$$
$$h = 120\sqrt{3} - 120$$
$$h = 120(\sqrt{3} - 1)$$
$$h = 120(1.73 - 1)$$
$$h = 120 \times 0.73$$

h = 87.6 m

Hence, height of the flagstaff = 87.6 m.

20. 

Let us suppose that aeroplane is at A

Suppose B and C are two consecutive kilometre stones such that

$\angle\mathbf{XAB} = \alpha$ and $\angle\mathrm{YAC} = \beta$

$\therefore \angle ABD = \alpha$ and $\angle ACD = \beta$

Let us suppose that BD = x km.

AD is height of aeroplane.

In $\triangle$ADB, $\frac{AD}{BD} = \tan\alpha$

$\Rightarrow \quad \frac{\mathrm{AD}}{x} = \tan\alpha$

$\Rightarrow x = \frac{\mathrm{AD}}{\tan\alpha}$

In $\triangle$ADC, $\frac{\mathrm{AD}}{\mathrm{DC}} = \tan\beta$

$\Rightarrow \quad \frac{\mathrm{AD}}{1-x} = \tan\beta$

$\Rightarrow \frac{\mathrm{AD}}{1-\frac{\mathrm{AD}}{\tan\alpha}} = \tan\beta$

$\Rightarrow \frac{\mathrm{AD}\tan\alpha}{\tan\alpha-\mathrm{AD}} = \tan\beta$

$\Rightarrow AD\tan\alpha = \tan\alpha.\tan\beta - AD\tan\beta$

$\Rightarrow AD\tan\alpha + AD\tan\beta = \tan\alpha.\tan\beta$

$\Rightarrow \mathbf{AD} = \frac{\tan\alpha\cdot\tan\beta}{\tan\alpha+\tan\beta}$

Hence proved.