CBSE Test Paper 03

CH-11 Conic Sections

1. A and B are two distinct points, Locus of a point P satisfying |PA| + | PB | = 2k, a constant is
   a. nothing can be said as we will not get any specific equation as k is not specified .
   b. the line segment [AB]
   c. an ellipse
   d. a hyperbola
2. The line y = mx + 1 is a tangent to the parabola $y^2 = 4x$ if m =
   a. 2
   b. 1
   c. 3
   d. 4
3. If the length of the major axis of an ellipse is three times the length of its minor axis, its eccentricity is
   a. $\frac{1}{\sqrt{2}}$
   b. $\frac{2\sqrt{2}}{3}$
   c. $\frac{1}{3}$
   d. $\frac{1}{\sqrt{3}}$
4. The equations $x = at^2, y = 4at\ ;\ t \in \ R$ represent
   a. a hyperbola

b. a parabola

c. a circle

d. an ellipse

5. The number of common tangents to the circles $x^2 + y^2 - x = 0, x^2 + y^2 + x = 0$ is

   a. 2

   b. 3

   c. 1

   d. 4

6. Fill in the blanks: The equation of the left-handed parabola is of the form ________.
7. Fill in the blanks: The standard equations of parabolas have focus on one of the coordinate axis, vertex at the origin and the directrix is ________ to the other coordinate axis.
8. Find the centre and radius of the circle. $(x + 5)^2 + (y - 3)^2 = 36$
9. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.
10. Find the equation of hyperbola having Foci (± $3\sqrt{5}$, 0), the latus rectum is of length 8.
11. If the lines 3x - 4y + 4 = 0 and 6x - 8y - 7 = 0 are tangents to a circle, then find the radius of the circle.
12. Find the equation of a circle which touches both the axes and the line 3x - 4y + 8 = 0 and lies in the third quadrant.
13. Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$
14. Find the length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola, where the line segment makes an angle θ to the X-axis.
15. Find the equation of the parabola whose focus is the point (2, 3) and directrix is the line x - 4 y + 3 = 0. Also, find the length of its latus-rectum.

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Solution

1. (a) nothing can be said as we will not get any specific equation as k is not specified.
   Explanation: Since the value of k is not specified, we cannot get a specific equation.
2. (b) 1
   Explanation:

   The contion for a line to be a tangent to the parabola is y = mx + $\frac{a}{m}$

   From the given equation of the parabola we infer that a = 1 and from the equation of the line c = 1

   Therefore m = a/c = 1
3. (b) $\frac{2\sqrt{2}}{3}$ **Explanation:**

   here b/a=1/3

   Hence $\frac{b^2}{a^2}$ = $\frac{1}{9}$, b^2 = $\frac{a^2}{9}$

   Therefore $b^2 = a^{2(}1 - e^2)$

   Substituting the values we get

   e = $\frac{2\sqrt{2}}{3}$
4. (b) a parabola
   Explanation:

   $y = 4at$

   Squaring both sides, we get

   $y^2 = 16a^2t^2$

   Putting the value of at^2 i.e. $at^2 = \frac{y^2}{16a}$ in x = at^2 we get,

$16ax = y^2$

or $y^2 = 4(4a)x$

which is nothing but equation of parabola.

5. (b) 3

   Explanation: The equation of the circles can be written as $(x-\frac{1}{2})^2 + y^2 = \frac{1}{4}$ and $(x+\frac{1}{2})^2 + y^2 = \frac{1}{4}$

   Hence their radii is 1/2 and the centres is (1/2,0) and (-1/2,0)

   Distance between their centres is 1 and sum of the radii is also 1, hence the circles touch externally.

   Therefore no: of tangents that can be drawn is 3

6. $y^2 = -4ax, a > 0$

7. parallel

8. The given equation of circle is
   $(x + 5)^2 + (y - 3)^2 = 36 \Rightarrow (x + 5)^2 + (y - 3)^2 = (16)^2$
   Comparing it with $(x - h)^2 + (y - k)^2 = r^2$ we have
   h = -5, k = 3 and r = 6
   Thus the coordinates of the centre is (-5, 3) and radius is 6.

9. Let the centre of circle in first quadrant be (a, a).
   $\therefore$ Equation of circle is $(x - a)^2 + (y - a)^2 = a^2$
   $\Rightarrow x^2 + a^2 - 2ax + y^2 + a^2 - 2ay = a^2$
   $\Rightarrow x^2 + y^2 - 2ax - 2ay + a^2 = 0$

10. Here foci are $(\pm 3\sqrt{5}, 0)$ which lie on x-axis.
    So the equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
    $\therefore$ foci $(\pm c, 0)$ is $(\pm 3\sqrt{5}, 0) \Rightarrow c = 3\sqrt{5}$

Length of latus rectum $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

We know that $c^2 = a^2 + b^2$

$\therefore (3\sqrt{5})^2 = a^2 + 4a \Rightarrow a^2 + 4a - 45 = 0 \Rightarrow (a + 9)(a - 5) = 0$

$\Rightarrow a = 5$ ($\because a = -9$ is not possible)

Also $a = 5 \Rightarrow b^2 = 4 \times 5 = 20$

Thus required equation of hyperbola is

$\frac{x^2}{25} - \frac{y^2}{20} = 1$

11. Given lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ or $3x - 4y - \frac{7}{2} = 0$ are tangents to the circle.

Also, given lines are parallel.

$\therefore$ Radius = $\frac{1}{2}$ (Distance between two parallel lines)

$= \frac{1}{2} \frac{\left|4+\frac{7}{2}\right|}{\sqrt{3^2+4^2}}$

$= \frac{1}{2} \frac{\left|\frac{15}{2}\right|}{\sqrt{9+16}} = \frac{15}{4\sqrt{25}}$

$= \frac{15}{4\times5} = \frac{3}{4}$

12. Let a be the radius o f the circle and centre of circle will be $(-a, -a)$

Since, the perpendicular distance from centre to the tangent is equal to the radius of the circle.

$\therefore a = \frac{3(-a)-4(-a)+8}{\sqrt{3^2+(-4)^2}}$

$\Rightarrow a = \frac{a+8}{5}$

$\Rightarrow 4a = 8 \Rightarrow a = 2$

$\therefore$ Equation of circle is

$(x + 2)^2 + (y + 2)^2 = 2^2$

$\Rightarrow x^2 + 4x + 4 + y^2 + 4y + 4 = 4$

$\Rightarrow x^2 + y^2 + 4x + 4y + 4 = 0$

13. Here $h = 1$, $k = 1$ and $r = \sqrt{2}$

The equation of circle is

$(x - h)^2 + (y - k)^2 = r^2$

$\therefore (x - 1)^2 + (y - 1)^2 = \left(\sqrt{2}\right)^2$

$\Rightarrow$ $x^2 + 1 - 2x + y^2 + 1 - 2y = 2$

$\Rightarrow$ $x^2 + y^2 - 2x - 2y = 0$

Which is required equation of circle.

14. Let any point be P(h, k) and it satisfy

$y^2 = 4ax$ i.e., $k^2 = 4ah$ ...(i)

Let a line OP makes an angle θ with the X-axis.

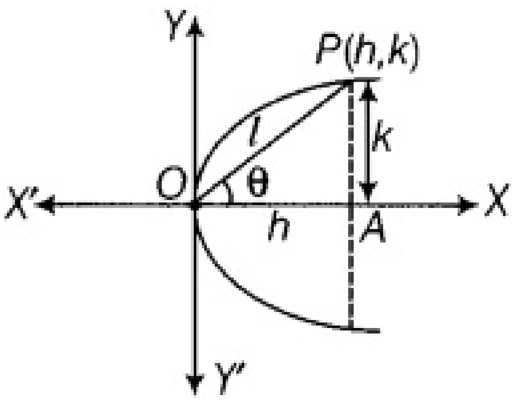
$\therefore$ In ΔOAP,

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{PA}{OP}$

$\Rightarrow \sin\theta = \frac{k}{l}$

$\Rightarrow$ $k = l\sin\theta$

and $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$



$= \frac{OA}{OP}$

$\Rightarrow \cos\theta = \frac{h}{l} \Rightarrow h = l\cos\theta$

Hence, from Eq. (i), we get

$l^2\sin^2\theta = 4a \times l\cos\theta$ [put $k = l\sin\theta$, $h = l\cos\theta$]

$\Rightarrow l = \frac{4a\cos\theta}{\sin^2\theta}$

15. Let P(x, y) be any point on the parabola whose focus is S(2, 3) and the directrix $x - 4y + 3 = 0$. Draw PM perpendicular from P (x, y) on the directrix $x - 4y + 3 = 0$.

Then by definition

SP = PM

$\Rightarrow$ $SP^2 = PM^2$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = \left(\frac{x-4y+3}{\sqrt{1^2+(-4)^2}}\right)^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 9 - 6y = \frac{(x-4y+3)^2}{(\sqrt{17})^2}$

$\Rightarrow x^2 + y^2 - 4x - 6y + 4 + 9 = \frac{(x-4y+3)^2}{17}$

$\Rightarrow 17(x^2 + y^2 - 4x - 6y + 13) = (x - 4y + 3)^2$

$\Rightarrow 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + (-4y)^2 + 9 + 2x(-4y) + 2(-4y) \times 3 + 2 \times 3x$

$\Rightarrow 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$

$\Rightarrow 17x^2 - x^2 + 17y^2 - 16y^2 + 8xy - 68x - 6x - 102y + 24y + 221 - 9 = 0$

$\Rightarrow 16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

Thus is the equation of the required parabola.

Latus Rectum = Length of perpendicular from focus (2,3) on directrix, $x - 4y + 3 = 0$

$= 2\left|\frac{2-12+3}{\sqrt{1+16}}\right|$

$= 2\left|\frac{-7}{\sqrt{17}}\right|$

$= \frac{14}{\sqrt{17}}$