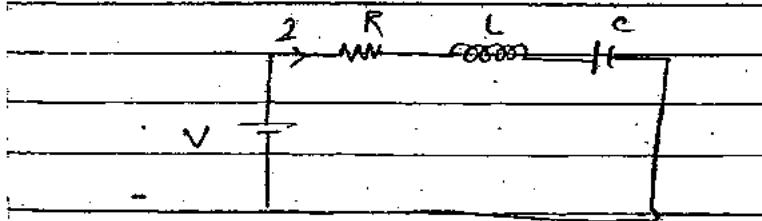


Basically our main aim is to transfer the energy from one point to another and for this we required an interconnection of elements and this interconnection of elements or component is called as an electrical circuit.

1) Difference b/w a circuit and network.

- Circuit always form a closed path but the network may or may not be closed.



Charge: It is the electrical property of the atomic particles of which metal consists and in electrical circuit the most quantity is charge and its unit is coulombs.

$$1e^- = 1.6 \times 10^{-19} \text{ C}$$

Coulomb is a large unit of charge as 1 coulomb of charge is contributed by $6.24 \times 10^{18} e^-$

$$\therefore 1.6 \times 10^{-19} \text{ C} = 1e^-$$

$$\therefore 1\text{C} = \frac{1}{1.6 \times 10^{-19}} e^- = 6.24 \times 10^{18} e^-$$

iv of Conservation of Charge:

It states that charge can neither be created nor be destroyed it can only be transferred from body to another. $(\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t})$ (cont quantity, law).

Def- The flow of electrons at the time rate change of charge is called as current and its unit is ampere.

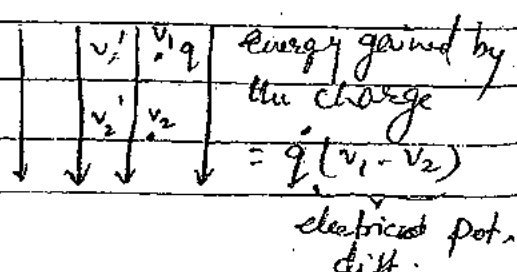
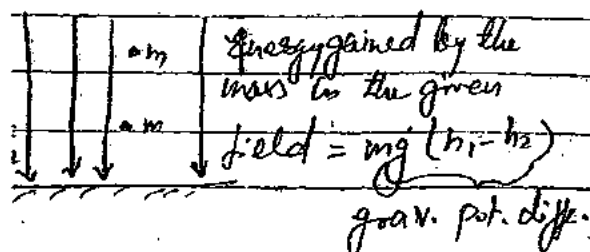
$$i = \frac{dq}{dt} \quad \text{C/s or Ampere}$$

e- The direction of current flow is conventionally taken the direction of the positive charge movement.

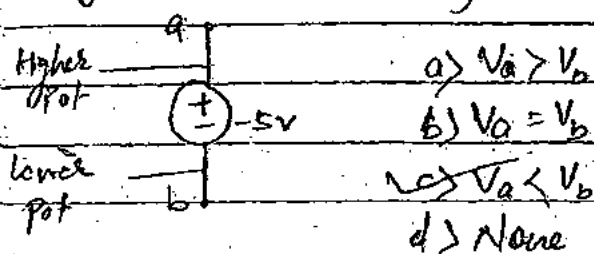
Def- To move the electron from one point to other point in a particular direction an external force is required and in an electrical circuit this external force is provided by the electromotive force. This is given by

$$V = \frac{dW}{dq} \quad \text{J/C or volt.}$$

If we place a positive charge in an electric field it will be accelerated in the direction of the field, it like a mass placed in a gravitational field gets accelerated in the direction of field.



Even if h_1 & h_2 are taken from top of the building rather than from the earth surface, then h_1 & h_2 will be different but the difference b/w them will be the same as before. Similarly, what matters in the electric ckt is that the potential difference b/w the two points. And hence potential at point 1 & 2 can be arbitrarily shifted can be any value by shifting the reference.



$$\text{Higher pot} - \text{lower pot} = -5$$

$$V_a - V_b = -5$$

$$\boxed{V_a = V_b - 5}$$

Power: It is the time rate of change of energy (expending or absorbing) and its unit is watt.

$$P = \frac{dw}{dt} \quad \text{J/s or watts}$$

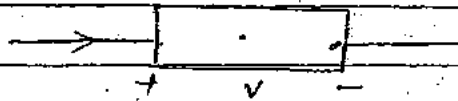
$$= \frac{dw}{dq} \cdot \frac{dq}{dt}$$

$$\boxed{p(t) = v(t) \cdot i(t)}$$

Voltage polarity and current direction are important in determining the sign of the power.

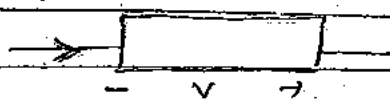
Note - 1. When the current is entering at the +ve terminal then the element is absorbing power.

When the current is leaving from the +ve terminal then the element is delivering power.



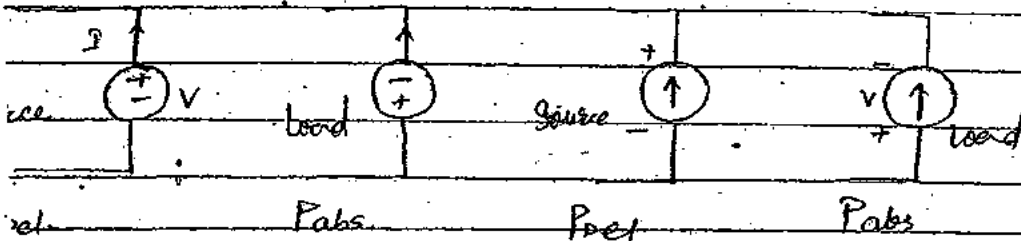
$$P = +VI$$

was absorbed as
was received as
was dissipated



$$P = -VI$$

power delivered



2nd of Conservation of Energy

It states that energy can neither be created nor be destroyed it can only be transformed from one form to another.

For this reason the algebraic sum of the power in circuit at any instant of time must be equal to zero.

$$\text{i.e. } \sum P = 0$$

$$\sum P_{del} = \sum P_{abs}$$

Find the power of the each element in the below circuit.

Energy:

The capacity to do the work is called as energy and its unit Joules.

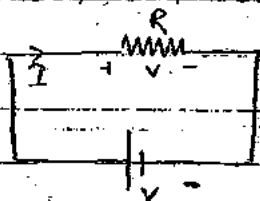
$$\text{Work done } W = \int_0^t P dt \quad \text{W-sec or Joules}$$

CIRCUIT ELEMENTS

An element can be completely characterised by which V-I relationship.

Resistor - If the voltage across the element is linearly proportional to the current through it then that element is called as resistor.

Resistance - Resistance can be described as that property of circuit element which offers opposition to the flow of the current and while doing so it convert electrical energy to the heat energy.



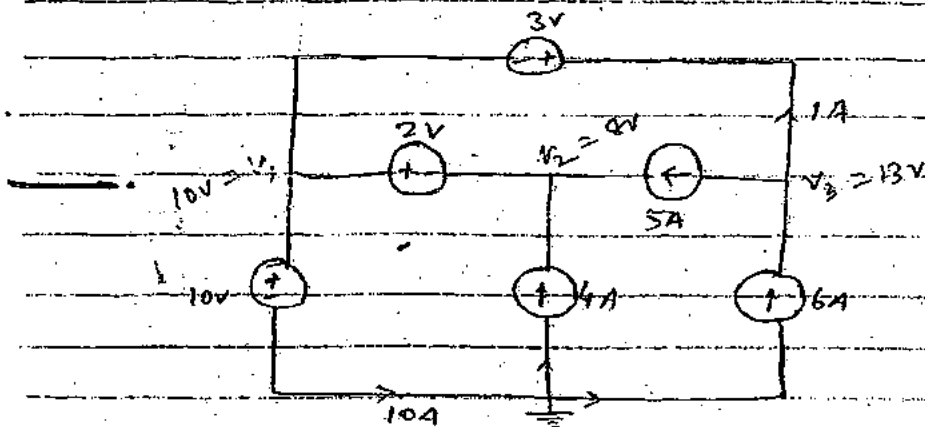
$$P = VI$$
$$= (IR)I$$

$$\boxed{P = I^2 R}$$

$$W = \int_0^t P dt$$
$$= \int_0^t I^2 R dt$$

$$W = I^2 R t$$

$$\therefore \boxed{R = \frac{W}{I^2 t}} \quad \frac{\text{Joule}}{\text{Sec} \cdot \text{A}^2}$$



$$P_{10V} = +10 \times 10$$

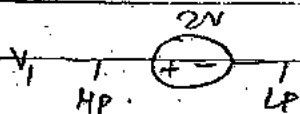
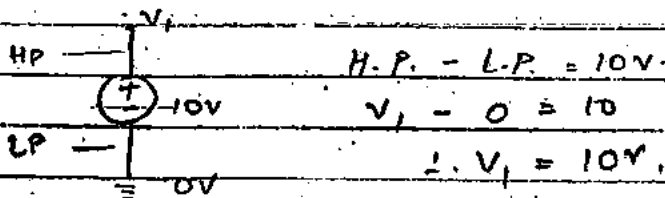
$$= 100W \text{ (abs)}$$

$$P_{2V} = -2 \times 9$$

$$= 18 \text{ (Del)}$$

$$P_{3V} = 3 \times 1$$

$$= 3 \text{ (abs)}$$



$$H.P. - L.P. = 2$$

$$V_1 - V_2 = 2$$

$$\Rightarrow 10 - V_2 = 2$$

$$\therefore V_2 = 8V$$

$$P_{4A} = 32W \text{ (Del)}$$

$$P_{5A} = 25W \text{ (Abs)}$$

$$P_{6A} = 78W \text{ (Del)}$$

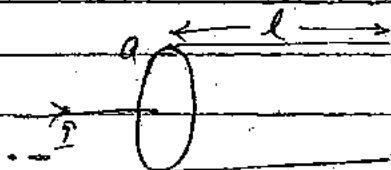
$$\Sigma \text{Del} = \{ \text{Abs} \}$$

$$(32 + 18 + 78) = (100 + 3 + 25)$$

$$128W = 128W$$

Ohm's law:

It states that at constant temperature current density is directly proportional to the electric field intensity.



$J \propto E$
$I = \sigma E$

1st form or point or microscopic form of Ohm's law.

$$\therefore \frac{I}{a} = \frac{1}{\rho} \cdot \frac{V}{l}$$

$$V = I \left(\frac{\rho l}{a} \right)$$

$R = \frac{\rho l}{a}$

$$V = IR$$

$$V \propto I$$

2nd form of Ohm's law

or Circuital form of Ohm's law.

i.e. at constant temperature the potential difference across the element is directly proportional to the current flowing through the element.

$$V = IR$$

$$I = V/R$$

$I = GV$
$I \propto V$

} 3rd form of Ohm's law

$$V = IR$$

$V = R \frac{dq}{dt}$

4th form of Ohm's law.

Experiment shows that the resistance of the metallic conductor varies with the temperature as

$$R_{T_2} = R_{T_1} [1 + \alpha \Delta T]$$

Where $\Delta T = T_2 - T_1$

R_{T_1} = Resistance at T_1

R_{T_2} = Resistance at T_2

α = temp. coefficient of resistance

2.

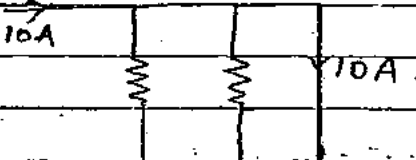
i.e. Ohm's law is valid when the temperature and conductivity of the material are constant.

CONCEPTS OF SHORT CIRCUIT:

$$R = 0$$

Ideally, $R = 0$, $V = 0$, $I = \infty$

Practically



I depends on circuit.

1

The voltage across the terminals of short circuit is always zero regardless of the value of current.

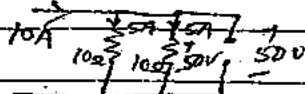
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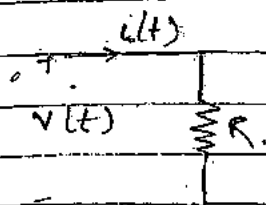
Concepts of Open Circuit:

$$R = \infty$$

ideally, $R = \infty$, $I = 0$, $V = \infty$

Practically,





$$v(t) = i(t) \cdot R \rightarrow \text{Time domain} \quad \left. \begin{array}{l} \text{Non-Sinusoidal} \\ \text{Excitation} \end{array} \right\}$$

$$v(s) = I(s) \cdot R \rightarrow \text{Laplace domain}$$

$$\bar{V} = \bar{I} R \rightarrow \text{Phasor domain} \quad \left. \begin{array}{l} \text{Sinusoidal} \\ \text{Excitation} \end{array} \right\}$$

where $\bar{V} = |V| \angle \theta$
 $\bar{I} = |I| \angle \theta$

2. Inductor:

If the terminal voltage across an element is directly proportional to the derivative of the current with respect to time, then that element is called an inductor.

Inductor is designed to store the energy in its magnetic field.

Any conductor carrying current has inductive properties and may be considered as an inductor but its inductive effect is very weak and to increase its inductive effect the no. of turns are provided.

Faraday's Law of Electromagnetic Induction:

1st - If a flux linking a coil varies as a function of time then an emf is induced b/w its terminals (Statically induced emf like transformer) or,

When a conductor cuts the magnetic lines of force an emf is induced (dynamically induced emf like generator or PMMC instruments)

nd law - Value of induced emf is directly proportional to the rate of change of flux

$$\psi = N\phi$$

$$\phi \propto i$$

$$\therefore \psi \propto i$$

$$v = \frac{d\psi}{dt}$$

$$\boxed{\psi = Li}$$

$$v = N \frac{d\phi}{dt}$$

$$v = \frac{d\psi}{dt} = \frac{d(Li)}{dt}$$

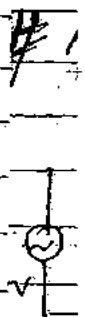
$$= L \frac{di}{dt} + i \frac{dL}{dt}$$

For network,

$$\boxed{v = L \frac{di}{dt}}$$

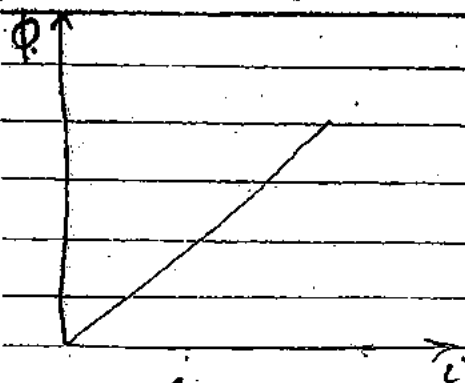
$$\therefore Li = N\phi$$

$$\boxed{L = \frac{N\phi}{i}}$$



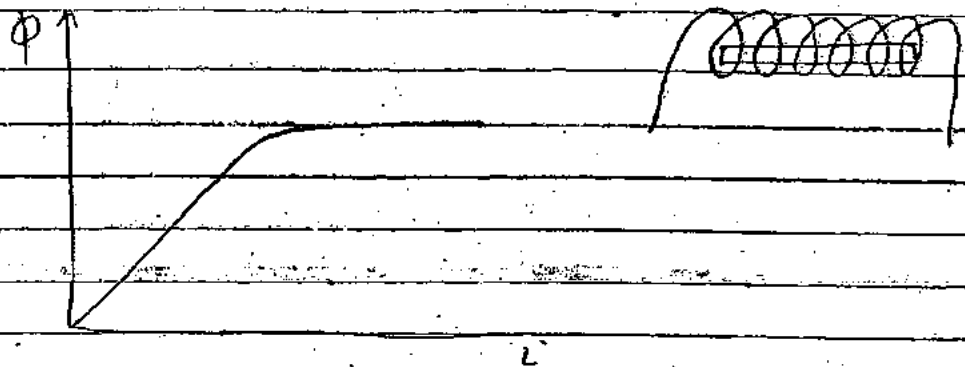
linear Inductor - When inductance of inductor is independent of the current magnitude then the inductor is called as linear inductor.

$$L = N \frac{\phi}{i}$$

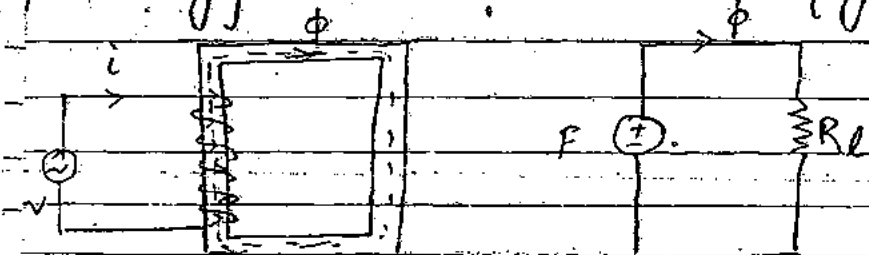


Non-linear Inductor - When inductance of the inductor depends on the current magnitude then the inductor is called as non-linear inductor.

For example - Iron cored inductor.

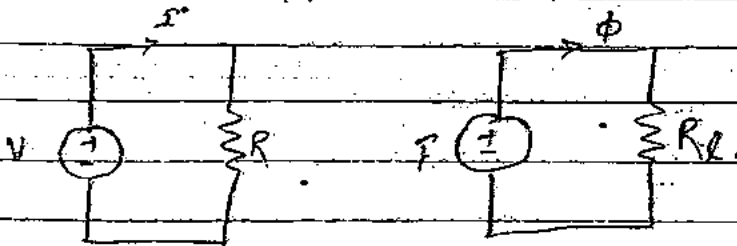


Analogy between Electric & Magnetic Circuit



$$F = \text{mmf}$$

$$F = NI \text{ (Amp-turn)}$$



$$V \rightarrow \text{emf}$$

$$I = \frac{V}{R}$$

$$R = \frac{l}{\mu a}$$

$$F = \text{mmf}$$

$$\Phi = \frac{F}{R_L}$$

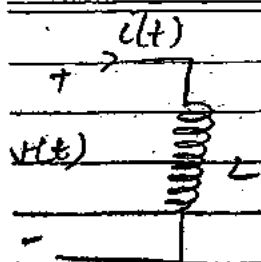
$$R_L = \frac{l}{\mu a}$$

$$\therefore L = \frac{N\Phi}{I}$$

$$= \frac{N}{l} \cdot \frac{F}{R_L}$$

$$= \frac{N}{l} \cdot \frac{N I}{\frac{l}{\mu a}}$$

$$\therefore L = \frac{N^2 \mu a}{l}$$



$$v = L \frac{di}{dt}$$

→ 5th form of Ohm's law

$$i = \frac{1}{L} \int_{-\infty}^{\infty} v dt$$

→ 6th form of Ohm's law

$$i = i(0) + \frac{1}{L} \int_0^t v dt$$

$$W = \int P dt$$

$$= \int v i dt$$

$$= \int L \frac{di}{dt} \cdot i dt$$

$$W = \frac{1}{2} L i^2$$

The power dissipation in an ideal inductor is zero.
(because its internal resistance is zero)

The energy stored in the inductor is in the form of magnetic field (Kinetic energy)

e.g. For the dc source under steady state condition inductor acts as a short circuit.

$$\therefore v = L \frac{di}{dt}$$

dc source $i \rightarrow \text{constant}$

$$\frac{di}{dt} = 0$$

$$\therefore \boxed{v = 0} \text{ (dc source)}$$

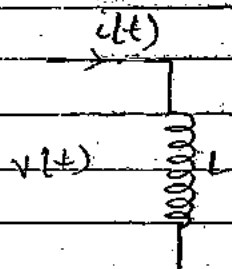
2. Inductor doesn't allow sudden change in current because for sudden change in current infinite voltage is required which is practically not feasible.

$$V = L \frac{di}{dt}$$

$$\frac{di}{dt} \rightarrow \text{finite}$$

$$dt \rightarrow 0$$

$$\therefore \boxed{V = \infty} \text{ not feasible.}$$



$$V(t) = L \frac{di}{dt} \rightarrow \text{Time domain}$$

$$V(s) = sL I(s) \rightarrow \text{Laplace domain}$$

$$\bar{V} = j\omega L \bar{I}$$

$$\bar{V} = |\bar{V}| \angle \theta$$

$$\bar{I} = |\bar{I}| \angle \theta$$

Non
sinusoidal
excitation

Sinusoidal
excitation

→ When the excitation is non-sinusoidal then the given n/w can be analysed either in time domain or in Laplace domain.

→ When the excitation is sinusoidal then the n/w is analysed by using phasors.

of Analysis of circuit in S-domain

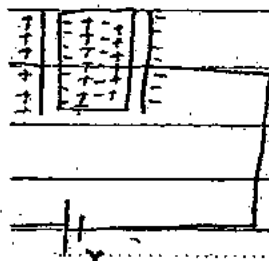
essential or integral equation is transform into equation and hence the calculation becomes

conditions, if any, are taken automatically into

capacitor:

the terminal voltage is proportional to the integral with respect to time. This element is called capacitor.

is a device which store energy in the electric field.



$$\therefore Q \propto v$$

$$Q = CV$$

$$\frac{dq}{dt} = \frac{d}{dt}(CV)$$

$$\frac{dv}{dt} + v \frac{dc}{dt}$$

$$\frac{dv}{dt} \rightarrow \text{1}^{\text{th}} \text{ form of Ohm's law}$$

$$V = \frac{1}{C} \int_{-\infty}^t i dt$$

8th form of ohm's law

Now,

$$V(t) = V(0) + \frac{1}{C} \int_0^t i(t) dt$$

$$P_{\text{dissipation}} = 0$$

$$\therefore W = \int P dt$$

$$= \int v i dt$$

$$W = \int v C \frac{dv}{dt} \cdot dt$$

$$W = \frac{1}{2} C v^2$$

- Power dissipation in an ideal capacitor is equal to zero.
- Capacitor stores energy in the form of electric field (potential energy)

Note: For dc source under steady state conditions capacitor acts as an open circuit.

$$i = C \frac{dv}{dt}$$

dc source $v \rightarrow \text{constant}$

$$\frac{dv}{dt} \rightarrow 0$$

$$\therefore i = 0 \text{ (open circuit)}$$

Capacitor doesn't allow sudden change in voltage

$$i = C \frac{dv}{dt}$$

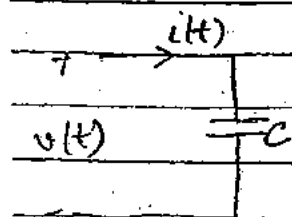
$$dv \rightarrow \text{finite}$$

$$t \rightarrow 0$$

$$\therefore [i = \infty] \text{ (which is not possible)}$$

Linear Capacitor - If the capacitance value is independent of the voltage magnitude then the capacitor is called as linear capacitor.

If the capacitance value is dependent on the voltage magnitude then the capacitor is called non-linear capacitor.
eg - varactor diode.



$$i(t) = C \frac{dv}{dt}$$

Time domain

$$I(s) = CS V(s)$$

Laplace domain

$$\bar{I} = j\omega C \bar{V}$$

Phasor domain

Fig :

Non-ideal, Sin excitation and

is seen

v_i

Peak Res vol by the

SOURCES:

If the terminal voltage is completely independent of current or the current is completely independent of voltage then the element is called as independent source.

The element for which either the voltage or the current depends upon voltage or current elsewhere in the circuit then it is called as dependent source.

Voltage Source

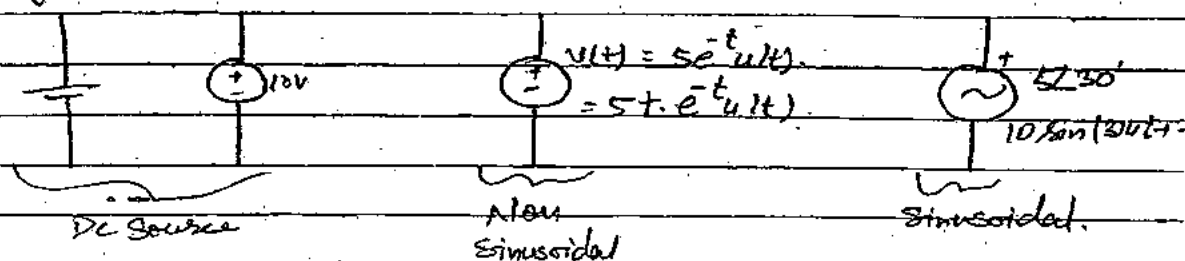
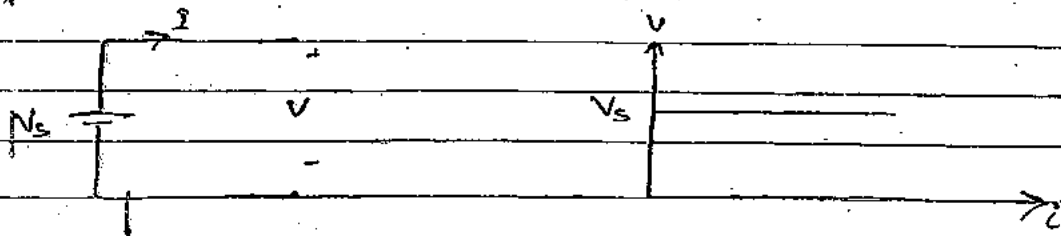
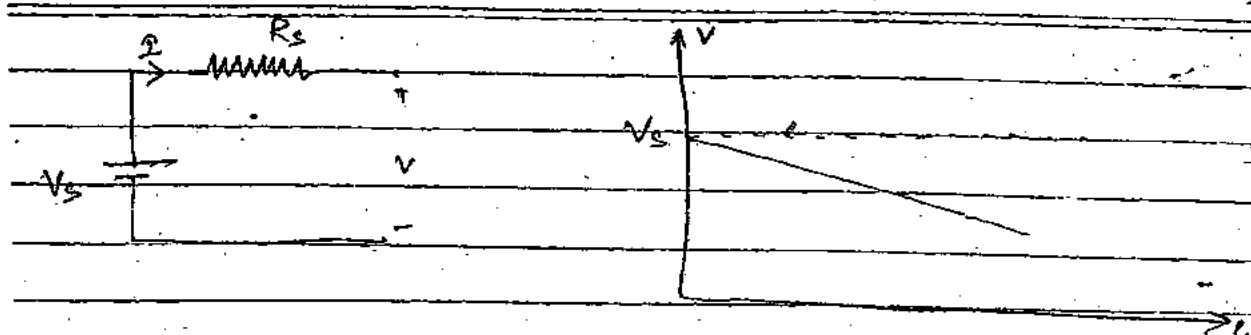


Fig: Independent voltage source representations.

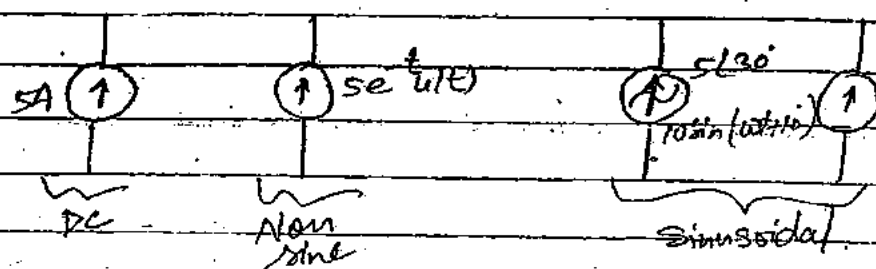
~~ideal~~ ^{ideal} A ideal voltage source has zero internal resistance and it delivers energy at a specified voltage which is independent of the current delivered by the source.



Practical voltage source have some finite internal resistance and it delivers energy at a specified voltage which depends on the current delivered by the source.

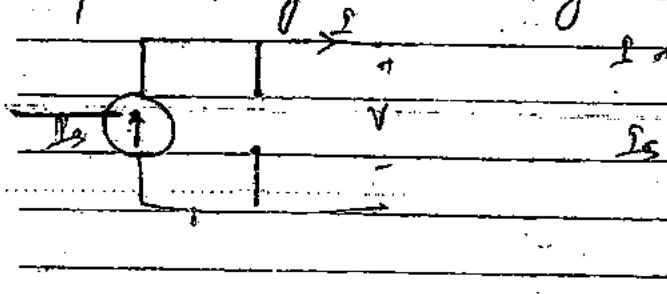


Current Sources:

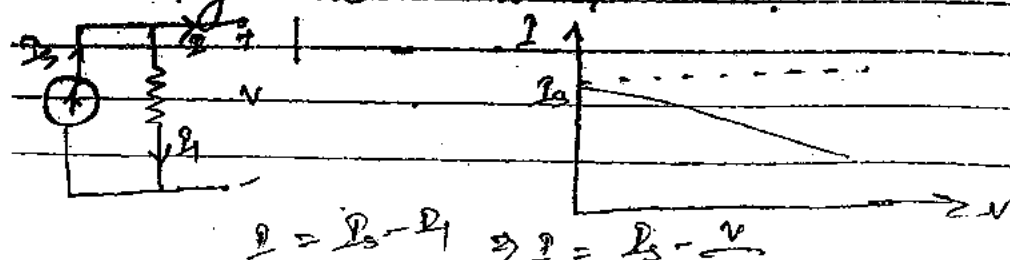


9- Independent Current Source Representation

For ideal current source internal resistance is infinite and it delivers energy at a specified current which is independent of the voltage across the source.



Practical current source has finite internal resistance & delivers energy at a specified current which depends the voltage across the source.



Note - 1. Current source doesn't have zero voltage across its terminal by providing the fixed current it depends entirely on the circuit to which it is connected.

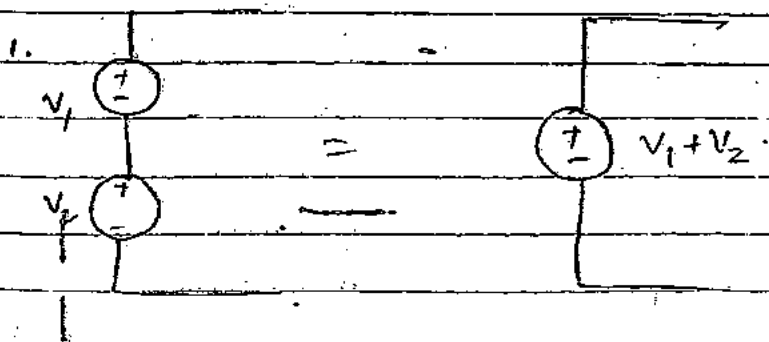
2. In the real time system only independent voltage source exist. To make calculation simple by using source transformation technique current source concept is introduced.

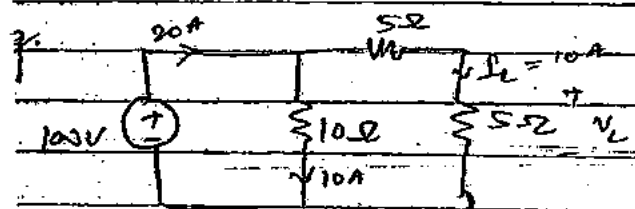
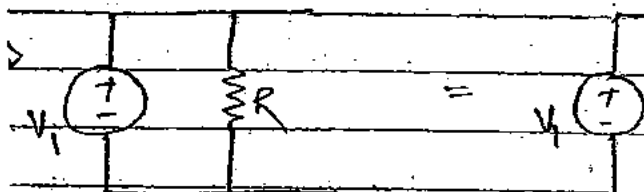
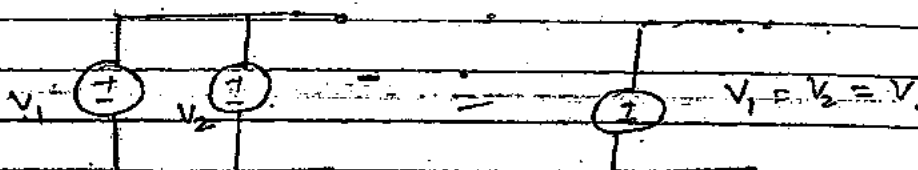
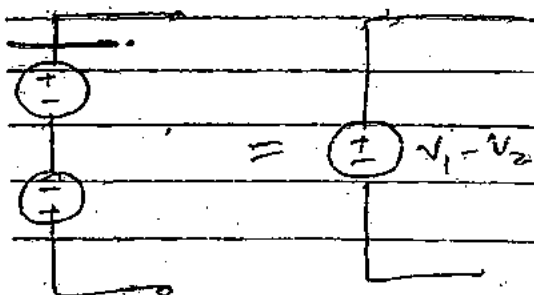
3. While analyzing any electrical network using K.V.L and K.C.L equations the independent & dependent voltage & current sources are handled exactly in the same manner except in the following two cases.

(a.) Analysis of the circuit using superposition theorem

(b.) Analysis of the n/w using Thevenin & Norton theorem

In such cases all the independent voltage & current sources are replaced by their internal resistance and all the dependent sources remain as they are.



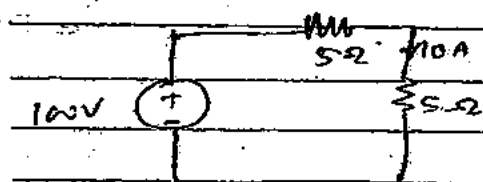


$$P_L = 10A \cdot V_L$$

$$V_L = 50V$$

$$I_S = 20A$$

$$P_S = 2kW$$



$$I_L = 10A$$

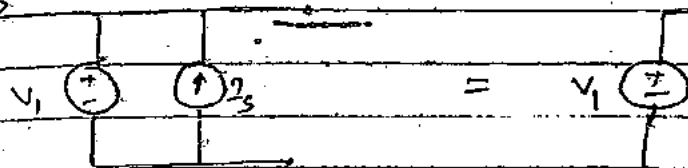
$$V_L = 50V$$

$$I_S = 10A$$

$$(P_S = 1kW)$$

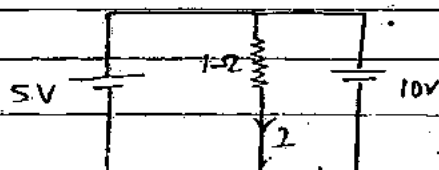
In the above ckt the resistance R can be neglected
 in doing the calculations for the load current & load voltage
 not for source current or source power.

Ex-7



In above circuit the current source can be neglected while doing the calculation for the load current or load voltage but not for the source current or source power.

Q.7 Find the current I in the below given ckt.



a) 5 A

b) 10 A

c) 15 A

✓ None

Ans →

$$5V - (I_1 \cdot 1) = 0$$

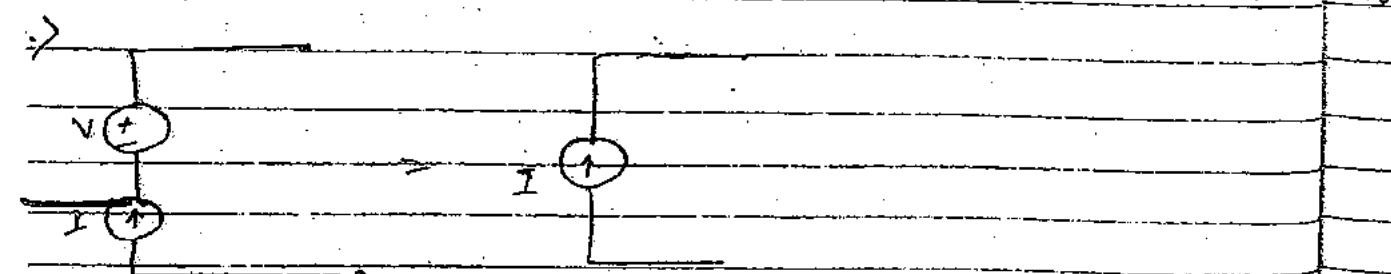
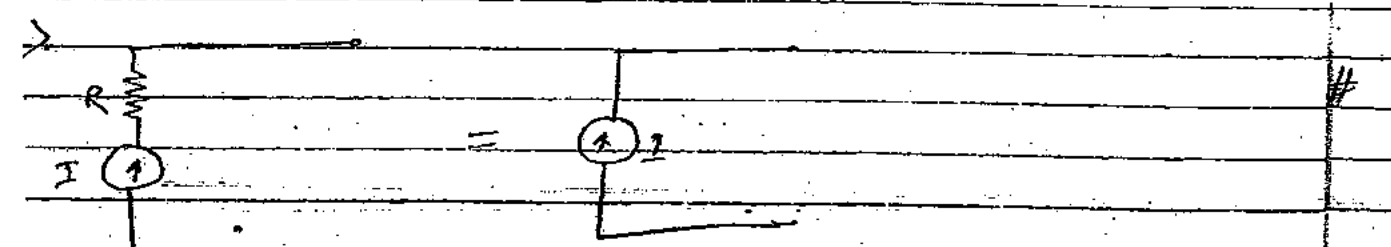
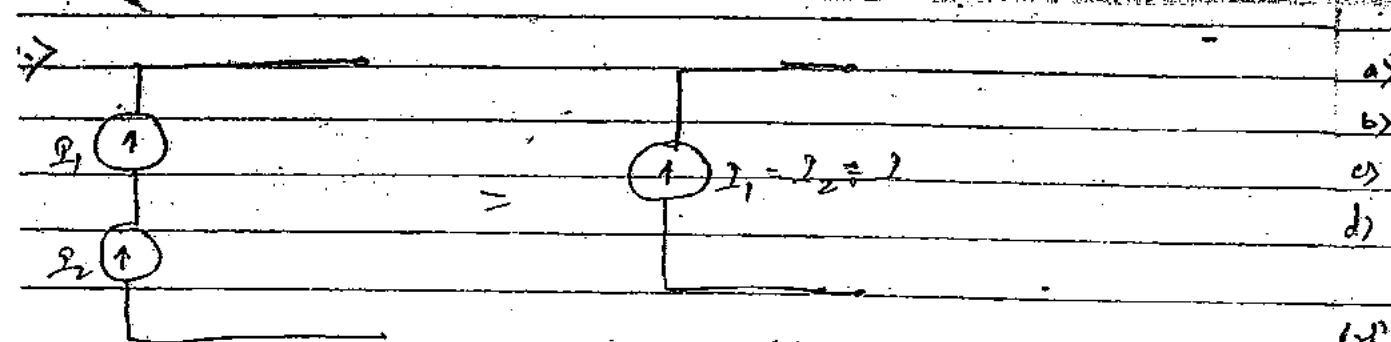
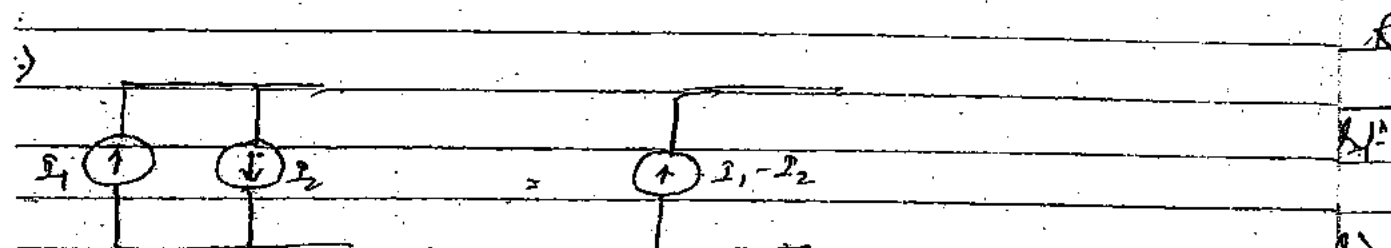
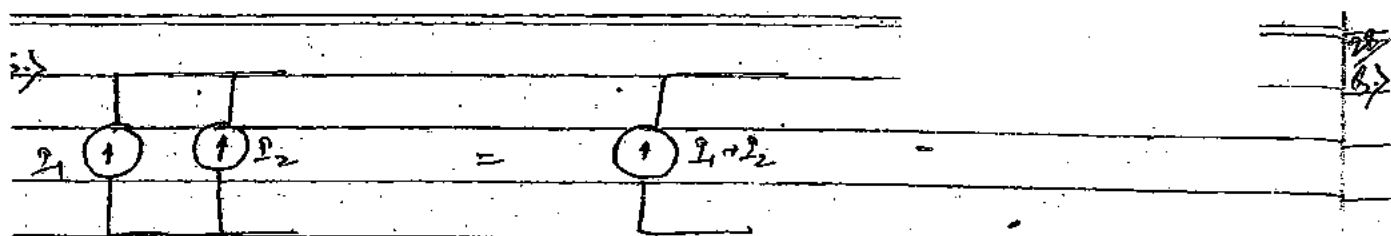
$$10 - (I_2 - I_1) = 0$$

$$5V - I_1 + I_2 = 0$$

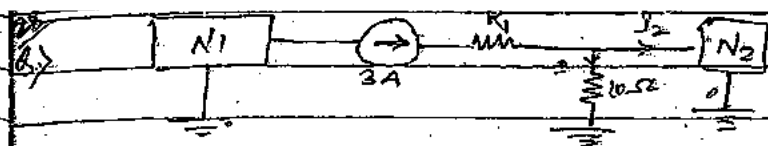
$$10 - I_2 + I_1 = 0$$

$$\begin{cases} I_1 - I_2 = 5 \\ -I_1 + I_2 = 10 \end{cases}$$

We cannot determine the current for this ckt.



the point ③ & ④ resistance & voltage source can be neglected
 like calculating either load current or load voltage but
 while calculating voltage across the source or power of the load



8) In the above ckt the current I_2 is 2A when the value of R_1 is 20Ω . What will be the value of I_2 when R_1 is changed to:

- a) 1A ~~b) 2A~~ c) 3A d) 4A

Ans: b) 2A

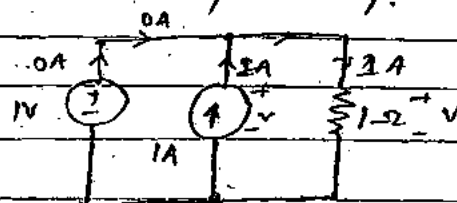
9) What will be the power consumed by the voltage source, current source & resistance respectively.

- a) 1W, 1W, 2W

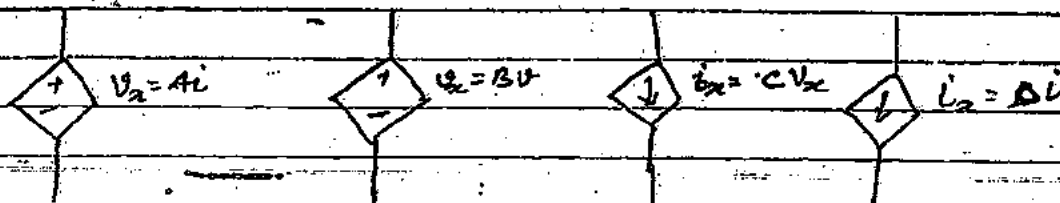
- b) 0W, 1W, 1W

- c) 1W, 0W, 1W

- d) 0W, 0W, 0W



Ans: b) 0W, 1W, 1W



Current Control voltage
Source

$$A = \Omega$$

Voltage control
voltage source

$$B = \text{Unitless}$$

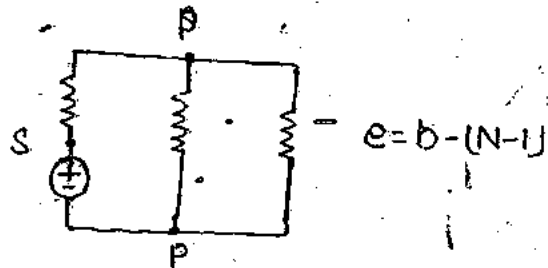
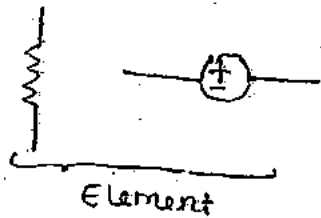
Voltage Control
Current Source

$$C = \Omega$$

Current control
Current Source

$$D = \text{Unitless}$$

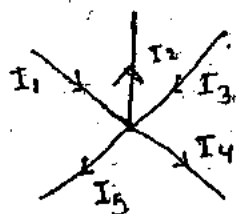
Branch: A branch is a single element or component in the ckt. If several elements in the ckt carrying same current they can also be referred as a single branch.



Node: When two or more than two elements are connected together, their common point is called as Node.

Principle:

KCL:



$$I_1 - I_2 + I_3 - I_4 - I_5 = 0$$

It says that algebraic sum of I entering a node or meeting at a point is zero.

Outgoing current is considered as -ve and incoming current as +ve.

It is based on law of conservation of charge.

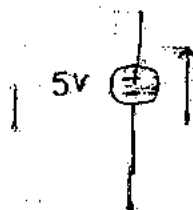
2nd: It says that sum of all the voltages in a closed loop is zero.

It is based on law of conservation of energy.

Standard Notation:

In a loop, rise in potential = -ve

Drop in potential = +ve



Mesh Analysis:

Mesh is a loop which does not contain any inner loop.
It is applicable only for the ^{Planar} n/w . A $Planar\ n/w$ is one that can be drawn with no branches crossing one another.

Procedure:

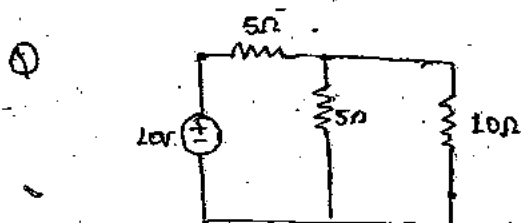
1. Identify total no of Meshes
2. Assign mesh current
3. Develop KVL eqⁿ for each mesh.
4. Solve eqⁿ to find loop current.

Note:-

1. The dirⁿ of mesh current can be taken any dirⁿ (⌚ or ⌚)
But taking clock wise dirⁿ give simple Analysis.

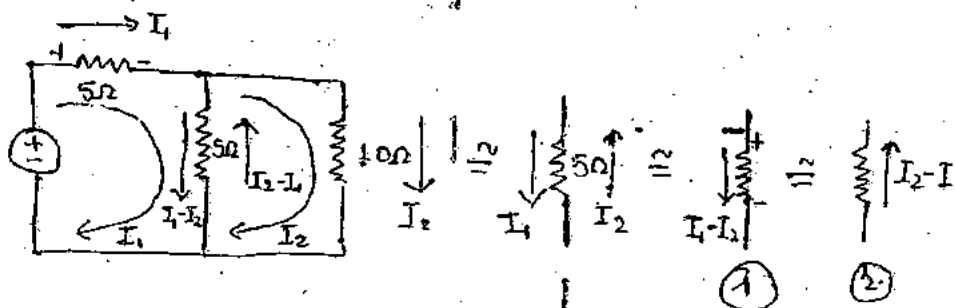
$$\begin{aligned} \text{2. NO of eq}^n &= \text{NO of mesh} \\ &= b - (N - 1) \end{aligned}$$

Where b = total no of branches
 N = total no of nodes.



using mesh analysis find the power loss in 10Ω resistor

Solⁿ:



Loop 1:

$$-10 + 5I_1 + 5(I_1 - I_2) = 0$$

$$10I_1 - 5I_2 = 10$$

$$2I_1 - I_2 = 2 \quad \text{--- (1)}$$

Loop 2:

$$10I_2 + 5(I_2 - I_1) = 0$$

$$-5I_1 + 15I_2 = 0$$

$$-I_1 + 3I_2 = 0 \quad \text{--- (2)}$$

$$(1) + 2 \times (2)$$

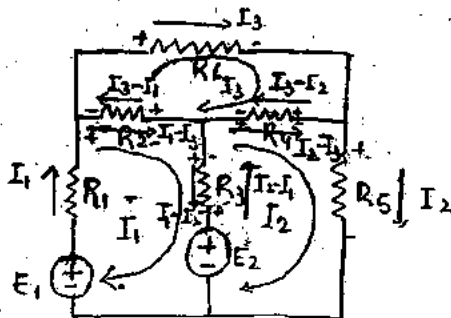
$$5I_2 = 2$$

$$I_2 = 2/5 \text{ A}$$

$$\therefore P_{10} = I_2^2 (10)$$

$$= \frac{4}{25} \times 10 = \frac{40}{25} \text{ W} = \frac{8}{5} \text{ W}$$

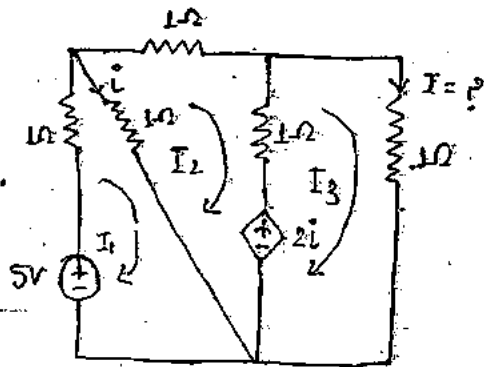
Developed the mesh eqⁿ for the below given n/w?



$$\text{Loop 1: } -E_1 + I_1 R_1 + (I_1 - I_3) R_2 + (I_1 - I_2) R_3 + E_2 = 0$$

$$\text{Loop 2: } I_2 R_5 - E_2 + (I_2 - I_1) R_3 + (I_2 - I_3) R_4 = 0$$

$$\text{Loop 3: } I_3 R_4 + (I_3 - I_2) R_4 + (I_3 - I_1) R_2 = 0$$



Find the value of I by using mesh Analysis

Soln:

$$\textcircled{1} :- -5 + I_1 + (I_1 - I_2)1 = 0$$

$$2I_1 - I_2 = 5 \quad \textcircled{1}$$

$$\textcircled{2} :- -I_2 + (I_2 - I_3) + 2i + (I_2 - I_1) = 0$$

$$-I_1 + 3I_2 - I_3 + 2i = 0 \quad \textcircled{2}$$

$$\textcircled{3} :- I_3 - 2i + I_3 - I_2 = 0$$

$$-I_2 + 2I_3 - 2i = 0 \quad \textcircled{3}$$

$$i = I_1 - I_2 \quad \textcircled{4}$$

$$\textcircled{4} \rightarrow \textcircled{2}$$

$$-I_1 + 3I_2 - I_3 + 2I_1 - 2I_2 = 0$$

$$I_1 + I_2 - I_3 = 0 \quad \textcircled{5}$$

$$\textcircled{4} \rightarrow \textcircled{3}$$

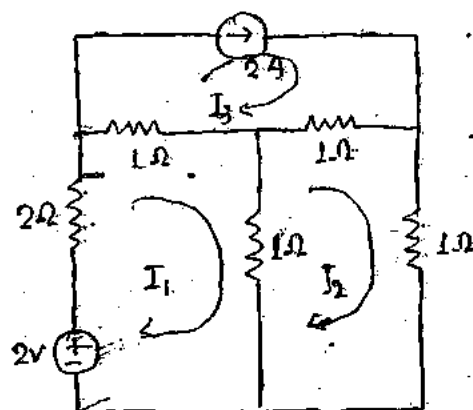
$$-I_2 + 2I_3 - 2I_1 + 2I_2 = 0$$

$$-2I_1 + I_2 + 2I_3 = 0 \quad \textcircled{6}$$

$$\textcircled{1} + \textcircled{6}$$

$$2I_3 = 5$$

$$I_3 = \frac{5}{2} \text{ A}$$



Soln: Loop 1: $-2 + 2I_1 + (I_1 - I_3) + (I_1 - I_2) = 0$

$$4I_1 - I_3 - I_2 = 2 \quad (1)$$

Loop 2: $-I_1 + I_2 + (I_2 - I_1) + (I_2 - I_3) = 0$

$$-I_1 + 3I_2 - I_3 = 0 \quad (2)$$

Loop 3: $I_3 = 2A \quad (3)$

(3) in eqn (1) & (2)

$$4I_1 - I_2 = 4 \quad (4)$$

$$-I_1 + 3I_2 = 2 \quad (5)$$

$$3 \times (4) + (5)$$

$$11I_1 = 14$$

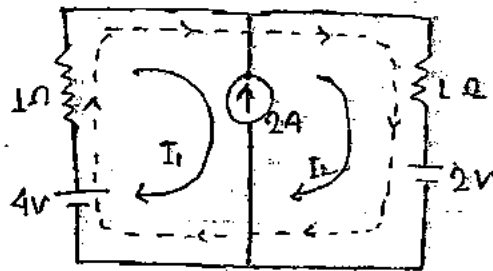
$$I_1 = \frac{14}{11} A$$

$$-\frac{14}{11} + 3I_2 = 2$$

$$I_2 = \frac{12}{11} A$$

Super mesh Analysis (Problems)

Q.



Soln:

$$-4 + I_1 + I_2 + 2 = 0$$

$$I_1 + I_2 = 2 \quad (1)$$

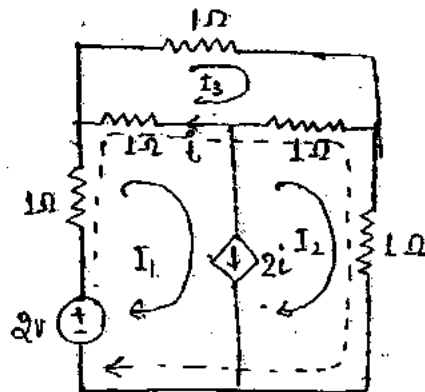
$$-I_1 + I_2 = 2 \quad (2)$$

$$(1) + (2)$$

$$I_2 = 2A$$

$$I_1 = 0$$

Q.



Soln:

$$(3) :- I_3 + (I_3 - I_2) + (I_3 - I_1) = 0$$

$$-I_1 + 3I_3 - I_2 = 0 \quad (1)$$

Supermesh

$$-2 + I_1 + (I_1 - I_3) + (I_2 - I_3) + I_2 = 0$$

$$2I_1 + 2I_2 - 2I_3 = 2 \quad (2)$$

$$I_1 - I_2 = 2i \quad (3)$$

$$i = I_3 - I_1$$

$$I_1 - I_2 = 2(I_3 - I_1)$$

$$I_1 - I_2 = 2I_3 - 2I_1$$

$$3I_1 - I_2 - 2I_3 = 0 \quad (4)$$

$$I_1 = \frac{5}{8} \text{ A} \quad I_2 = \frac{1}{8} \text{ A} \quad , \quad I_3 = \frac{1}{2} \text{ A}$$

Mesh $\rightarrow$ KVL + Ohm's law
 Supermesh $\rightarrow$ KVL + KCL + Ohm's law

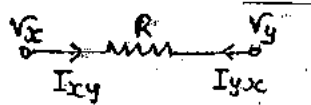
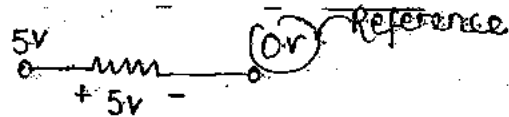
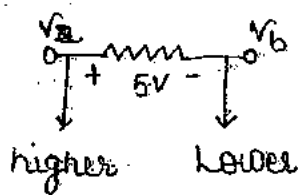
NODAL ANALYSIS

PROCEDURE:

- Identify total no. of nodes.
- Assign the voltage at each node.
- One of the node taken as reference (Datum)
- Reference node potential should be equal to ground node potential.
- Develop KCL eqⁿ for each non reference node.
- ~~By~~ Solving KCL eqⁿ find node voltages.

Note:

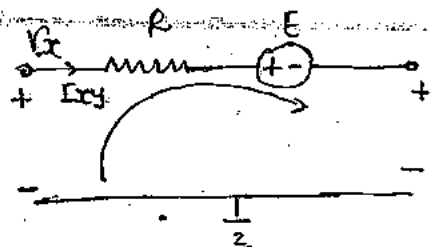
- It can be apply both planar and non planar n/w.
- Total no of eqⁿ $N-1$



$$I_{xy} = \frac{V_x - V_y}{R}$$

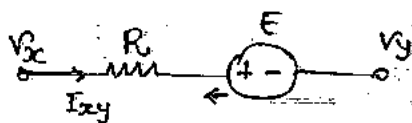
$$I_{yx} = \frac{V_y - V_x}{R}$$

$$I_{xy} = -I_{yx}$$

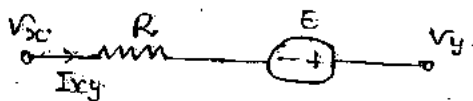


$$-V_x + I_{xy}R + E + V_y = 0$$

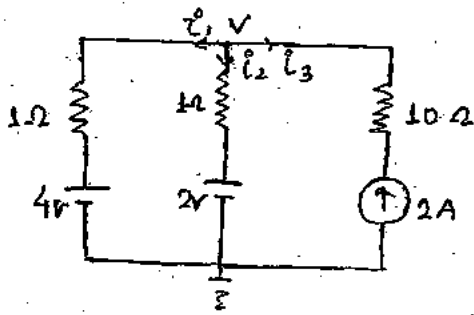
$$I_{xy} = \frac{(V_x - V_y) + E}{R}$$



$$I_{xy} = \frac{(V_x - V_y) - E}{R}$$



$$I_{xy} = \frac{(V_x - V_y) + E}{R}$$

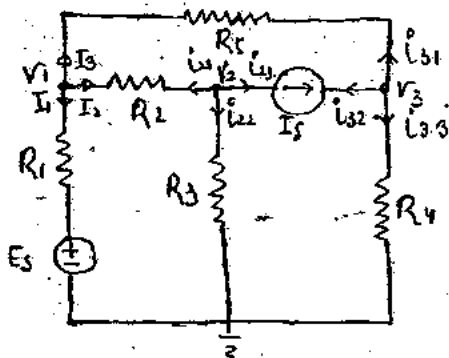


$$i_1 + i_2 + i_3 = 0$$

$$\frac{(V-0)-4}{1} + \frac{(V-0)-2}{1} - 2 = 0$$

$$2V - 4 - 2 - 2 = 0$$

$$V = 4V$$



Node 1:- $i_1 + i_2 + i_3 = 0$

$$\frac{(V_1-0)-E_1}{R_1} + \frac{(V_1-V_2)}{R_2} + \frac{V_1-V_3}{R_5} = 0$$

Node 2:-

$$i_{21} + i_{22} + i_{23} = 0$$

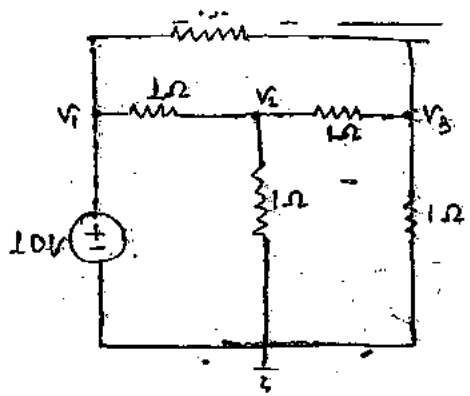
$$\frac{(V_2-0)}{R_3} + \frac{V_2-V_1}{R_2} + I_1 = 0$$

Node 3:-

$$i_{31} + i_{32} + i_{33} = 0$$

$$\frac{V_3-V_1}{R_5} - I_1 + \frac{V_3-0}{R_4} = 0$$

Q.



Node 1: $\frac{v_1 - 10}{1} + \frac{v_1 - v_2}{1} + \frac{v_1 - v_3}{1} = 0$ given $v_1 = 10V$

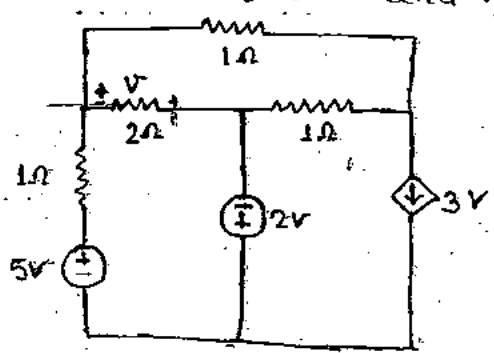
Node 2: $\frac{v_2 - v_1}{1} + \frac{v_2 - 0}{1} + \frac{v_2 - v_3}{1} = 0$ — (1)

Node 3: $\frac{v_3 - v_2}{1} + \frac{v_3 - 0}{1} + \frac{v_3 - v_1}{1} = 0$ — (2)

Solving Eq. (1) and (2)

$v_2 = v_3 = 5V$ and $v_1 = 10V$

Q.



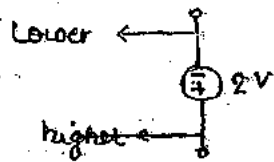
Find the power delivered by 2V source

Soln: Node 1: $\frac{v_1 - 5}{1} + \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{1} = 0$

$= v_1 - 5 + \frac{v_1}{2} - \frac{v_2}{2} + v_1 - v_3 = 0$

$= \frac{5v_1}{2} - \frac{v_2}{2} - v_3 = 10$

$5v_1 - v_2 - 2v_3 = 20$ — (1)



$$0 - V_2 = 2V$$

$$V_2 = -2V \quad (2)$$

$$(2) \rightarrow (1)$$

$$5V_1 - 2V_3 = 8 \quad (3)$$

At node 3 -

$$\frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1} + 3V = 0$$

$$V = V_3 - V_2$$

$$-V_1 - V_2 + 2V_3 + 3V_3 - 3V_1 = 0$$

$$-4V_1 + 2V_3 = 4 \quad (4)$$

Add (3) and (4)

$$V_1 = 12V$$

$$V_3 = 26V$$

Power delivered by 2V source is

$$i_{21} + i_{22} + i_{23} = 0$$

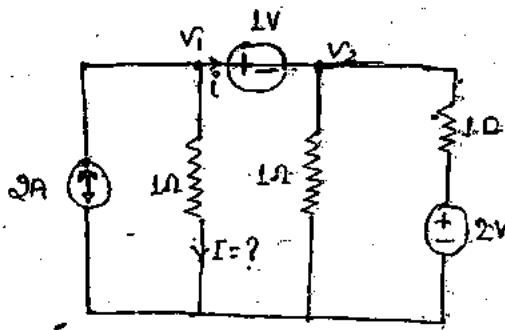
$$i_{21} = -[i_{22} + i_{23}]$$

$$= -\left[\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1}\right]$$

$$= -\left[\frac{-2 - 12}{1} + \frac{-2 - 26}{1}\right] = 35A$$

$$P_{2V} = 35 \times 2 = 70W \text{ Del.}$$

Q.



at node 1:

$$2 + \frac{V_1}{1} + i = 0 \quad (1)$$

at node 2:

$$\frac{V_2}{1} + \frac{V_2 - 2}{1} - i = 0 \quad (2)$$

$$(1) + (2)$$

$$: 2 + \frac{V_1}{1} + \frac{V_2}{1} + \frac{V_2 - 2}{1} = 0 \quad \text{Super node Eqn}$$

$$V_1 + 2V_2 = 0 \quad (3)$$

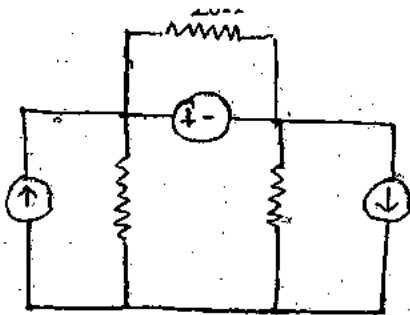
$$V_1 - V_2 = 1V \quad (4)$$

$$(3) - (4)$$

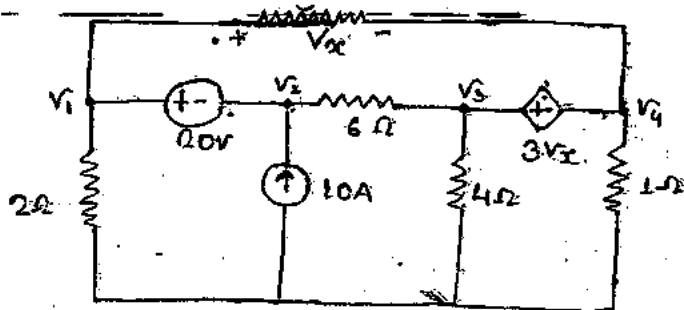
$$V_2 = -\frac{1}{3}V$$

$$V_1 = \frac{2}{3}V$$

$$I = \frac{V_1}{1} = \frac{2}{3}A$$



4.



Find the node voltage v_1 , v_2 , v_3 and v_4 ?

Soln: Super node (1-2)

$$\frac{v_1}{2} + \frac{v_1 - v_4}{3} = 10 + \frac{v_2 - v_3}{6} = 0$$

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \quad (1)$$

Super node (3-4)

$$\frac{v_3 - v_2}{6} + \frac{v_3}{4} + \frac{v_4}{1} + \frac{v_4 - v_1}{3} = 0$$

$$-4v_1 - 2v_2 + 5v_3 + 16v_4 = 0 \quad (2)$$

$$v_1 - v_2 = 20 \quad (3)$$

$$v_3 - v_4 = 3v_x$$

$$v_x = v_1 - v_4$$

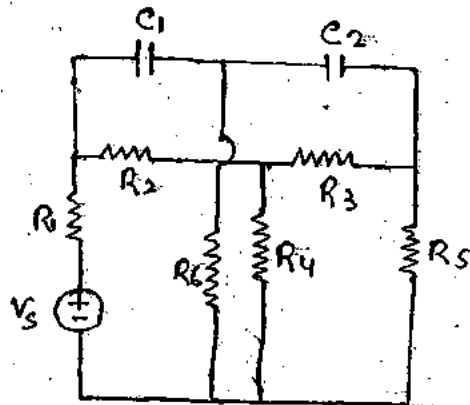
$$\therefore v_3 - v_4 = 3v_1 - 3v_4$$

$$3v_1 - v_3 - 2v_4 = 0 \quad (4)$$

$$(3) \rightarrow (1) \& (2)$$

$$6v_1 - v_3 - 2v_4 = 80 \quad (5)$$

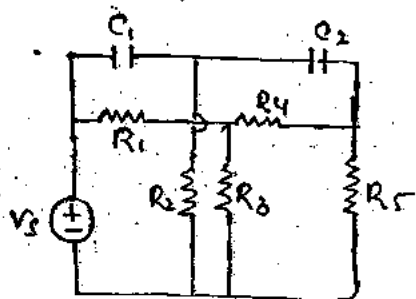
$$6v_1 - 5v_3 - 16v_4 = 40 \quad (6)$$



Find the minⁿ no of eqⁿ required to solve the problem?

$$n = 5 \quad b = 8$$

$$C = N - 1 = 4$$



$$N = 5$$

$$C = N - 1 = 4$$

but Here one voltage source is already given
so only 3 no of Eqⁿ are required to solve
the problem.

Q A n/w consists of principal node and 'b' branches.
If mesh analysis is simpler than nodal analysis
then n is greater than ____.

- (a) $b+1$ (b) $b-1$ (c) $\frac{b}{2}+1$ (d) $\frac{b}{2}-1$

Ans

$$M < e$$

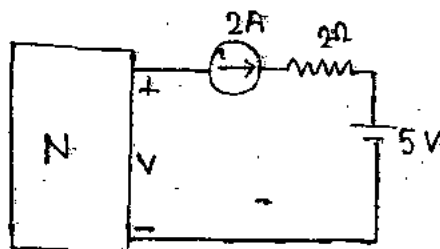
$$b - (N - 1) < N - 1$$

$$b - N + 1 < N - 1$$

$$b + 2 < 2N$$

$$N > \frac{b}{2} + 1$$

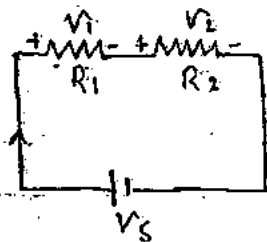
Q.



- (a) 5V (b) 8V (c) 11V (d) None

Ans: The voltage across the current is not given the Eqn. and hence we can find voltage. So the Answer is none (d)

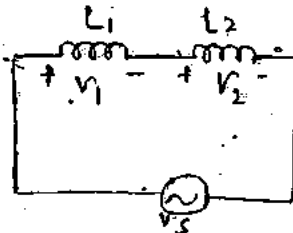
Voltage Division and Series Combination:



$$R_{eq} = R_1 + R_2$$

$$V_1 = \frac{V_S}{R_1 + R_2} \cdot R_1$$

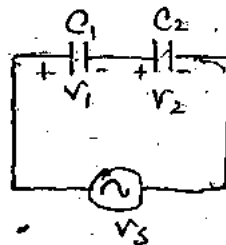
$$V_2 = \frac{V_S}{R_1 + R_2} \cdot R_2$$



$$L_{eq} = L_1 + L_2$$

$$V_1 = \frac{V_S}{L_1 + L_2} \cdot L_1$$

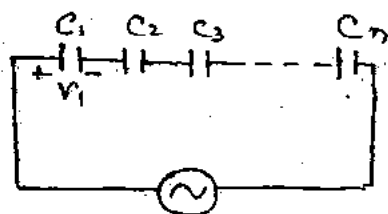
$$V_2 = \frac{V_S}{L_1 + L_2} \cdot L_2$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$V_1 = \frac{V_S}{C_1 + C_2} \cdot C_2$$

$$V_2 = \frac{V_S}{C_1 + C_2} \cdot C_1$$



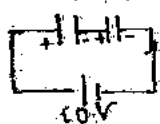
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$V_1 = \frac{V_S \cdot \frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

Note: voltage is equally distributed when $R_1 = R_2$, $L_1 = L_2$, $C_1 = C_2$

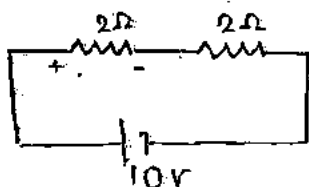
Ex:

$$C_1 = C_2 = 2F$$



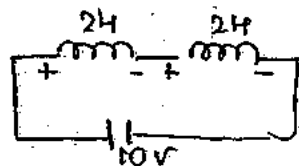
$$V_{C1} = 10 \times \frac{2}{4} = 5V$$

$$V_{C2} = 10 \times \frac{2}{4} = 5V$$



$$V_{R1} = 10 \times \frac{2}{2+2} = 5V$$

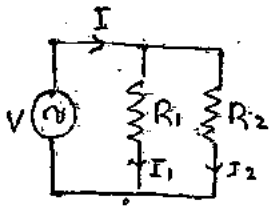
$$V_{R2} = 10 \times \frac{2}{2+2} = 5V$$



$$V_{L1} = 10 \times \frac{2}{4} = 5V$$

$$V_{L2} = 10 \times \frac{2}{4} = 5V$$

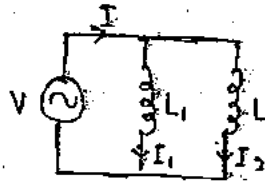
11 combination and current division:



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$I_1 = \frac{I \cdot R_2}{R_1 + R_2}$$

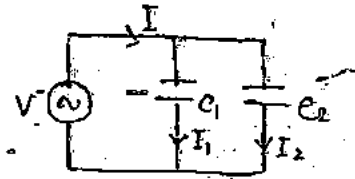
$$I_2 = \frac{I \cdot R_1}{R_1 + R_2}$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$I_1 = \frac{I \cdot L_2}{L_1 + L_2}$$

$$I_2 = \frac{I \cdot L_1}{L_1 + L_2}$$

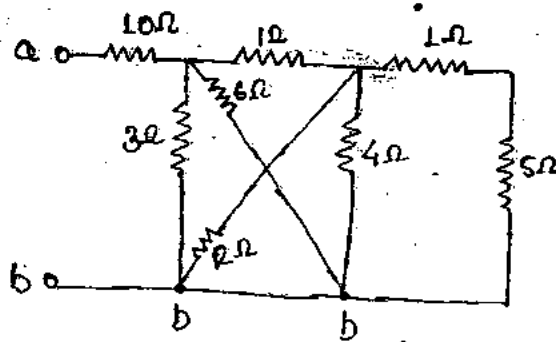


$$C_{eq} = C_1 + C_2$$

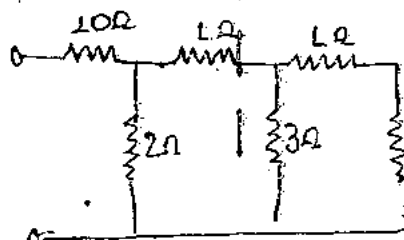
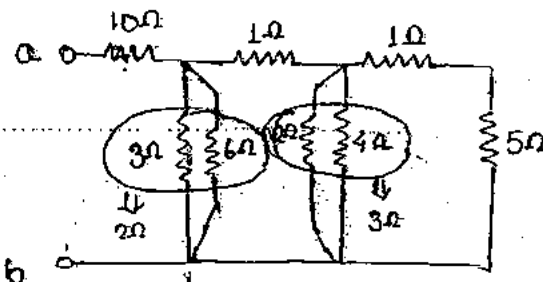
$$I_1 = \frac{I \cdot C_2}{C_1 + C_2}$$

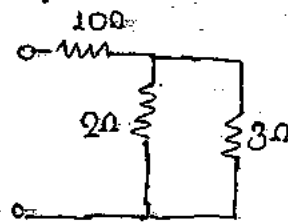
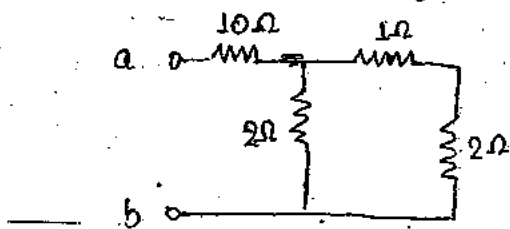
$$I_2 = \frac{I \cdot C_1}{C_1 + C_2}$$

Note: Current is equally distributed when $R_1 = R_2$, $L_1 = L_2$, $C_1 = C_2$
Find the equivalent resistance b/w a & b?



Soln

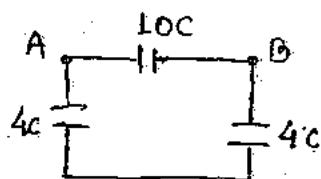
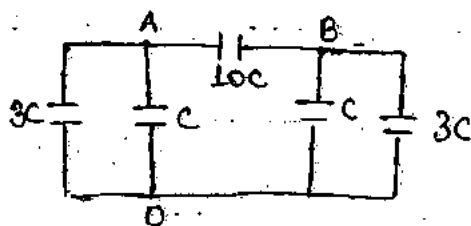
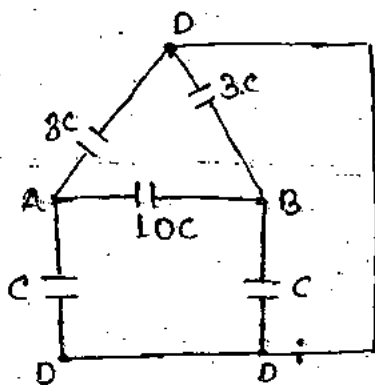




$$R_{ab} = 10 + \frac{2 \times 3}{2+3}$$

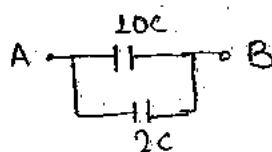
$$= 10 + \frac{6}{5} = 11.2 \Omega$$

Q.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$\frac{1}{C_{eq}} = \frac{C}{n}$$



$$C_{eq} = 10C + 2C = 12C$$

Q A n/w consists of principal node and 'b' branches.

If mesh analysis is simpler than nodal analysis then n is greater than —

- (a) $b+1$ (b) $b-1$ (c) $\frac{b}{2}+1$ (d) $\frac{b}{2}-1$

Ans

$$M < e$$

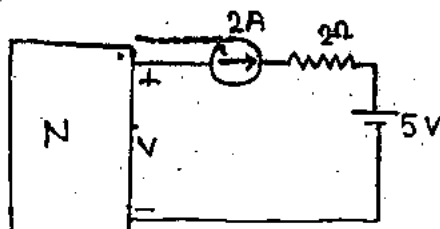
$$b - (N-1) < N-1$$

$$b - N + 1 < N - 1$$

$$b + 2 < 2N$$

$$N > \frac{b}{2} + 1$$

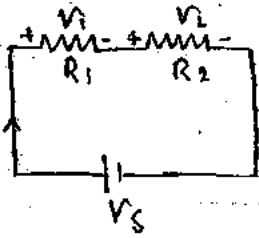
Q.



- (a) 5V (b) 9V (c) 11V (d) None

Soln: The voltage across the current is not given the Eqn. and hence we can find voltage. So the Answer is none (d)

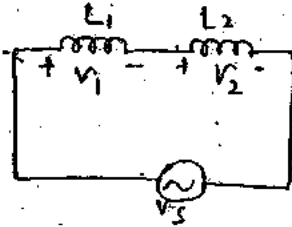
Voltage Division and Series Combination:



$$R_{eq} = R_1 + R_2$$

$$V_1 = \frac{V_S}{R_1 + R_2} \cdot R_1$$

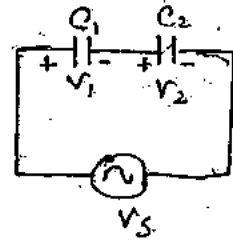
$$V_2 = \frac{V_S}{R_1 + R_2} \cdot R_2$$



$$L_{eq} = L_1 + L_2$$

$$V_1 = \frac{V_S}{L_1 + L_2} \cdot L_1$$

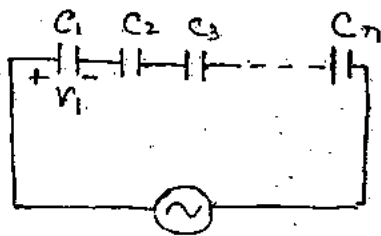
$$V_2 = \frac{V_S}{L_1 + L_2} \cdot L_2$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$V_1 = \frac{V_S}{C_1 + C_2} \cdot C_2$$

$$V_2 = \frac{V_S}{C_1 + C_2} \cdot C_1$$

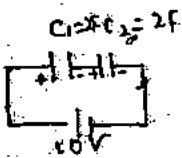


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$V_1 = \frac{V_S \cdot \frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

Note: Voltage is equally distributed when $R_1 = R_2$, $L_1 = L_2$, $C_1 = C_2$

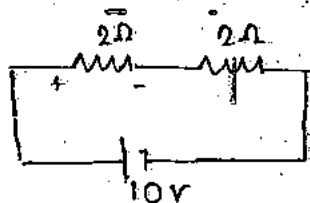
Ex:



$$C_1 = C_2 = 2F$$

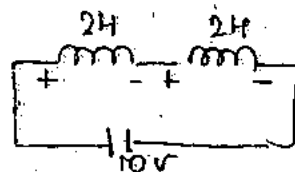
$$V_{C1} = 10 \times \frac{2}{4} = 5V$$

$$V_{C2} = 10 \times \frac{2}{4} = 5V$$



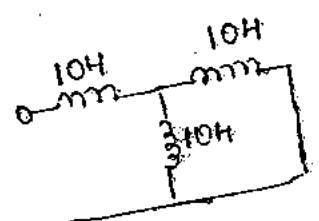
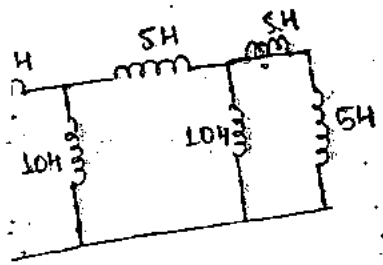
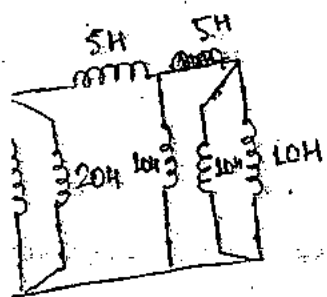
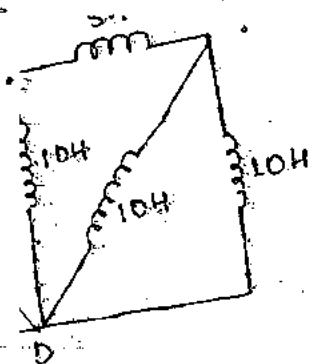
$$V_{R1} = 10 \times \frac{2}{2+2} = 5V$$

$$V_{R2} = 10 \times \frac{2}{2+2} = 5V$$



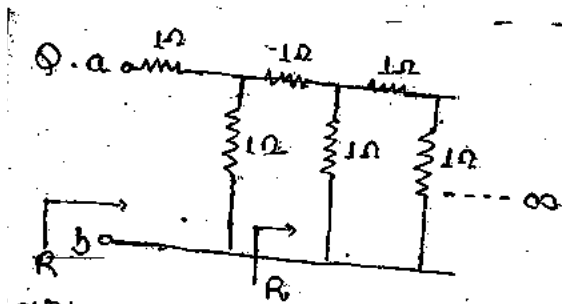
$$V_{L1} = 10 \times \frac{2}{4} = 5V$$

$$V_{L2} = 10 \times \frac{2}{4} = 5V$$

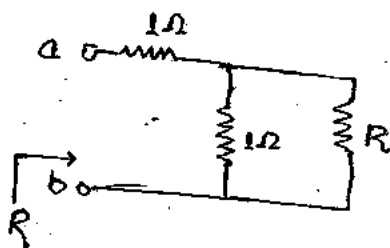


$$= 10H + (10H || 10H)$$

$$= 10H + 5H = 15H \quad \underline{\text{Ans}}$$



Q. a am
 R b
 Method 1:



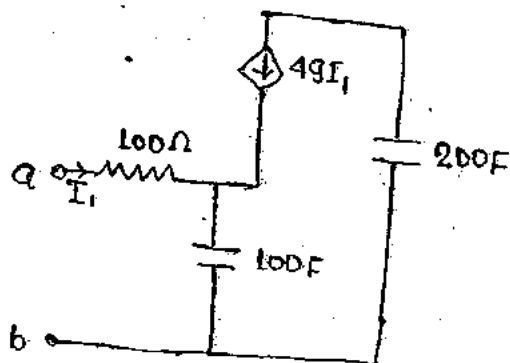
$$R = 1 + \frac{R \cdot 1}{R + 1}$$

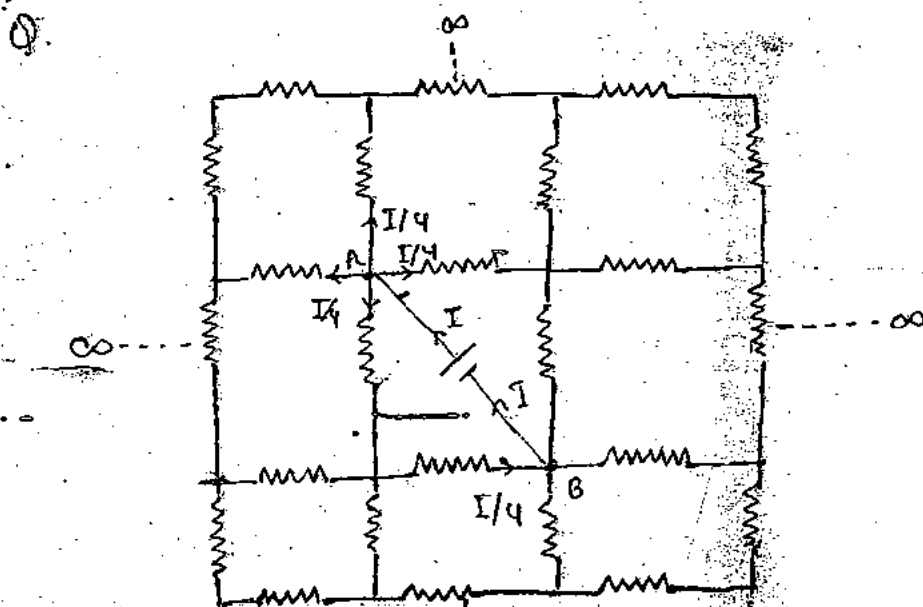
$$\Rightarrow R^2 + R = 2R + 1$$

$$\Rightarrow R^2 - R - 1 = 0$$

$$R = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore R = \frac{1 + \sqrt{5}}{2} \Omega$$





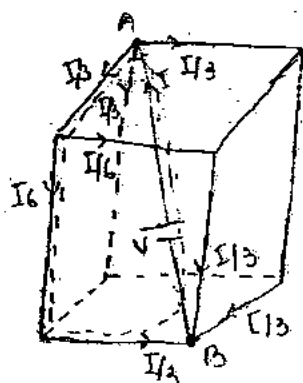
Find the eq. resistance a & b when each resistance is $R \Omega$

Solⁿ: Assume, let battery V connected b/w a and b which draw current I

$$-V + \frac{I}{4}R + \frac{I}{4}R = 0$$

$$V = I \left[\frac{R}{4} + \frac{R}{4} \right]$$

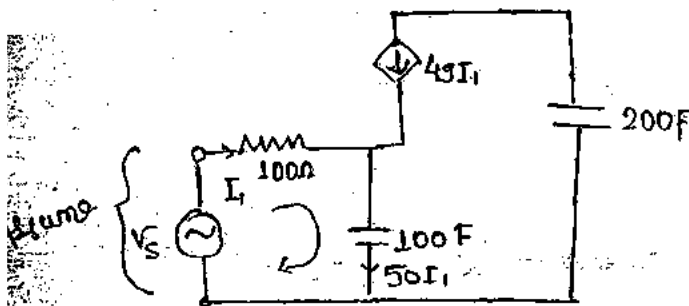
$$\frac{V}{I} = R_{eq} = \frac{R}{2} \quad \text{Ans}$$



$$\frac{V}{I} = Z_{eq} = R + sL + \frac{1}{sC}$$

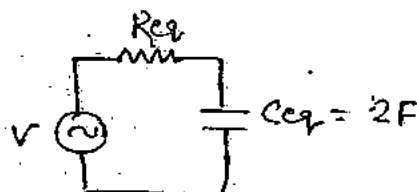
$$= R + j\omega L - \frac{j}{\omega C}$$

$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$



$$\textcircled{1} \quad V_s = I_1(100) + \frac{1}{100} \int 50I_1 dt$$

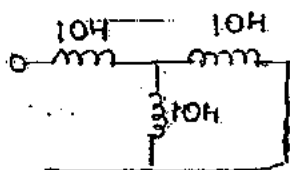
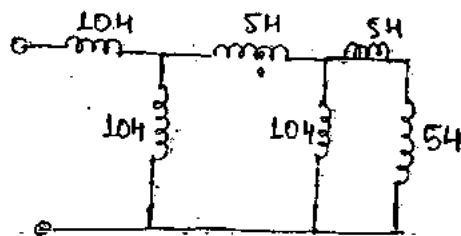
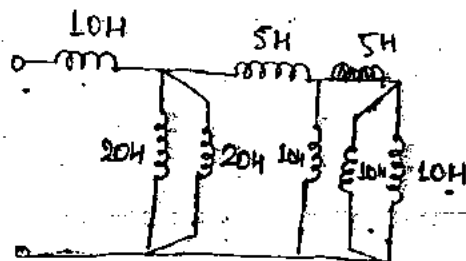
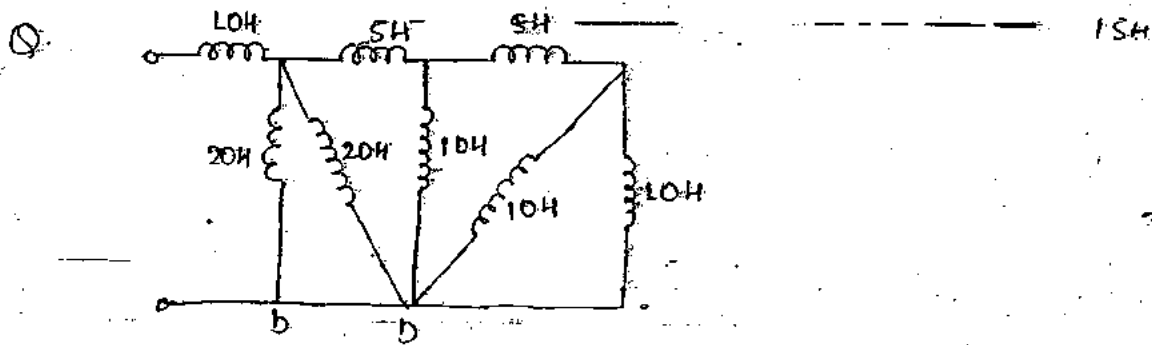
$$= I_1(100) + \frac{1}{2} \int I_1 dt$$



$$\textcircled{2} \quad V(s) = 100 I_1(s) + \left(\frac{50}{s} I_1(s) \right) \cdot \frac{1}{s(100)}$$

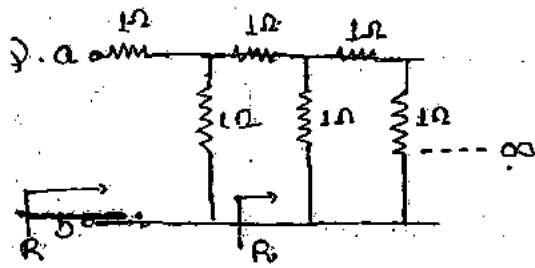
$$\frac{V(s)}{I_1(s)} = 100 + \frac{1}{2s}$$

$$\therefore C_{eq} = 2F$$

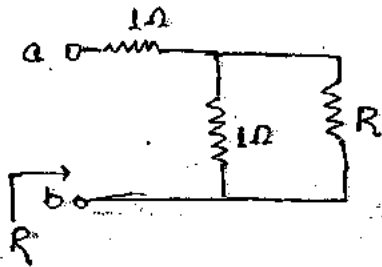


$$= 10H + (10H \parallel 10H)$$

$$= 10H + 5H = 15H \text{ Ans}$$



Q7: Method 1:



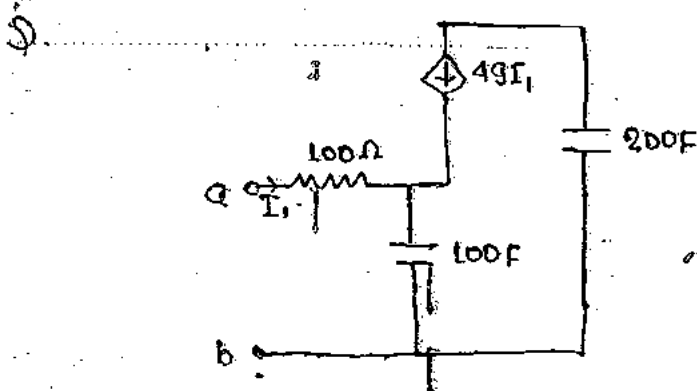
$$R = 1 + \frac{R \cdot 1}{R + 1}$$

$$\Rightarrow R^2 + R = 2R + 1$$

$$\Rightarrow R^2 - R - 1 = 0$$

$$R = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore R = \frac{1 + \sqrt{5}}{2} \Omega$$



$$-V + \frac{I}{3}R + \frac{I}{6}R + \frac{I}{3}R = 0$$

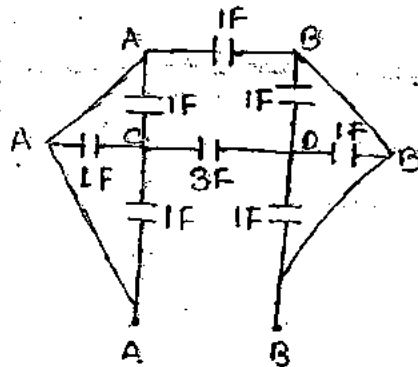
$$\frac{V}{I} = R_{eq} = \left[\frac{R}{3} + \frac{R}{6} + \frac{R}{3} \right]$$

$$R_{eq} = \frac{5}{6}R$$

~ If L present in the ckt $L_{eq} = \frac{5}{6}L$

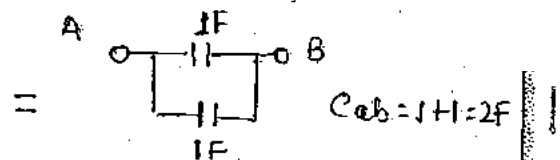
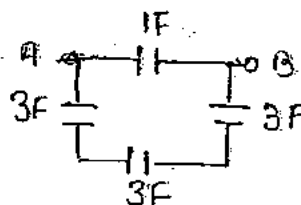
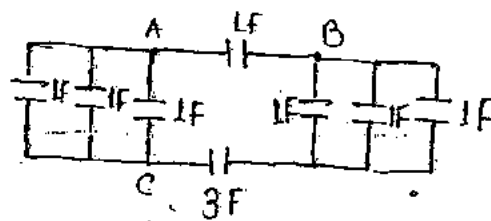
Similarly C present in the ckt $C_{eq} = \frac{6}{5}C$

Q.

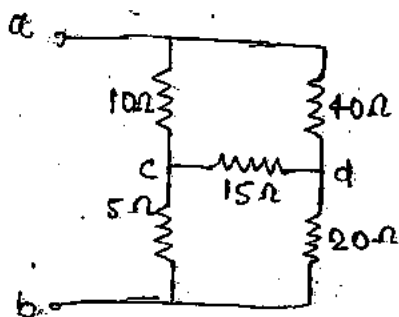


Find the equivalent capacitance b/w ~~A and B~~ A and B

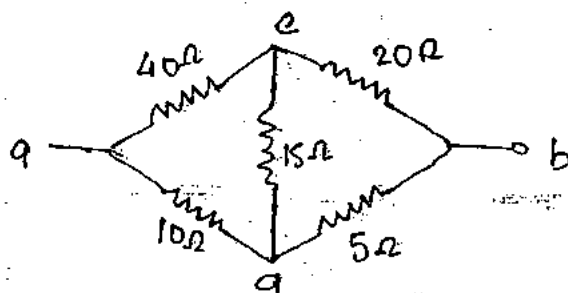
Solⁿ:



Q.

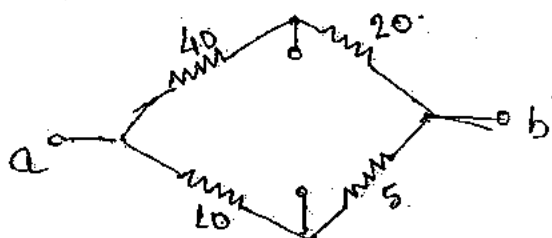


Q1.



$$40 \times 5 = 20 \times 10$$

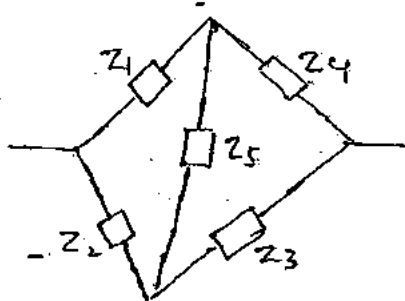
$$200 = 200 \text{ balanced}$$



$$R_{eq} = (40 + 20) \parallel (10 + 5)$$

$$= 18 \Omega$$

100:
 : If the impedances are there in the bridge it should satisfy both angle as well as magnitude



① $Z_1 Z_4 = Z_2 Z_3$

② $|Z_1| |Z_4| = |Z_2| |Z_3|$
 $\angle$

$\angle \theta_1 + \theta_4 = \angle \theta_2 + \theta_3$

If the above bridge is balanced then find the value of Z_4

$Z_1 = 10 \angle 30^\circ$ $Z_2 = 5 \angle -50^\circ$ $Z_3 = 4 \angle -40^\circ$ $Z_4 = ?$

a) $2 \angle 120^\circ$ (b) $2 \angle -120^\circ$ (c) $10 \angle 30^\circ$ (d) None

$Z_1 Z_4 = Z_2 Z_3$

$10 \angle 30^\circ (Z_4) = (5 \angle -50^\circ) (4 \angle -40^\circ)$

$Z_4 = \frac{(5 \angle -50^\circ) (4 \angle -40^\circ)}{10 \angle 30^\circ} = \frac{20 \angle -90^\circ}{10 \angle 30^\circ} = 2 \angle -120^\circ$

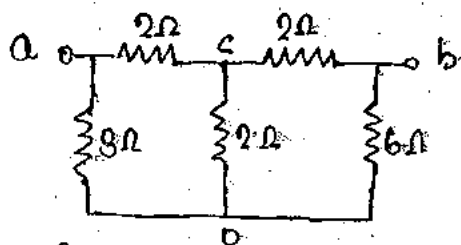
$Z_4 = 2 [\cos(-120^\circ) + j \sin(-120^\circ)]$

$= -1 - j\sqrt{3}$

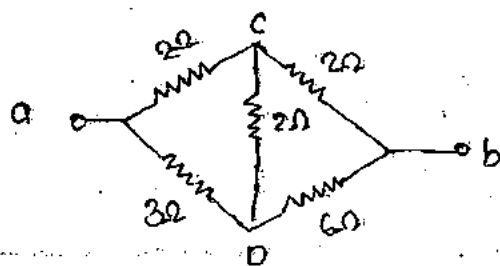
$Z_4 = R + jX_c$

R should never be

2. Find the equivalent resistance of a and b.

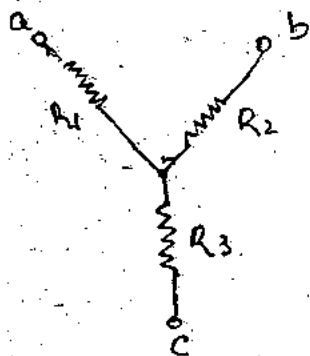


217

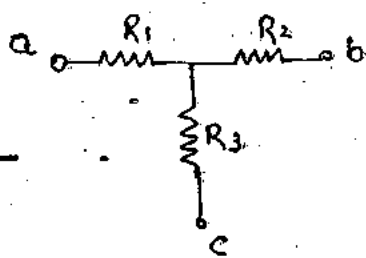


$$12\Omega \times 6\Omega = 3\Omega \times 2\Omega$$

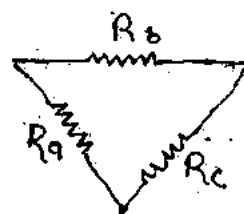
$12\Omega \neq 6\Omega$ (not balanced)



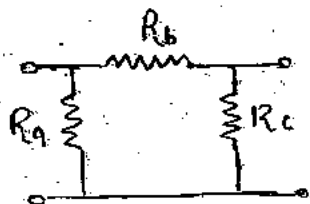
STAR/YTE



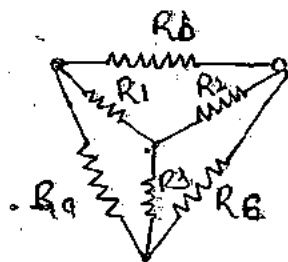
T-N/w



Delta



PI- π /w



Delta to Star:

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

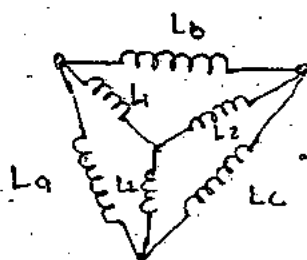
$$R_3 = \frac{R_c R_a}{R_a + R_b + R_c}$$

Star to Delta:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

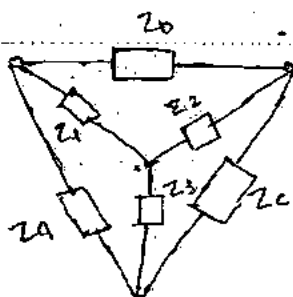
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

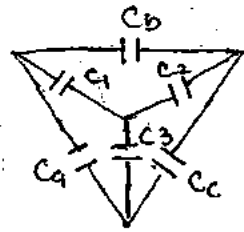


$$L_1 = \frac{L_a L_b}{L_a + L_b + L_c}$$

$$L_a = \frac{L_1 L_2 + L_2 L_3 + L_3 L_1}{L_2}$$

Hints (same as above)





Star to delta

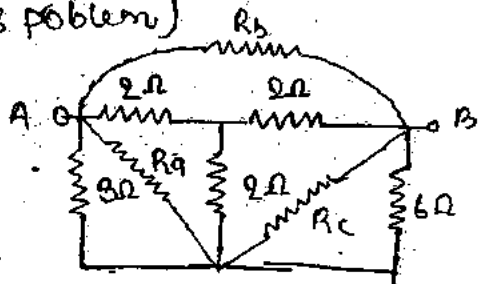
$$\frac{1}{C_1} = \frac{\frac{1}{C_4} \cdot \frac{1}{C_5}}{\frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_3}}$$

$$\frac{1}{C_2} = \frac{\frac{1}{C_3} \cdot \frac{1}{C_5}}{\frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_3}}$$

Star to delta

$$\frac{1}{C_3} = \frac{\frac{1}{C_1} \cdot \frac{1}{C_2} + \frac{1}{C_2} \cdot \frac{1}{C_4} + \frac{1}{C_4} \cdot \frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_4}}$$

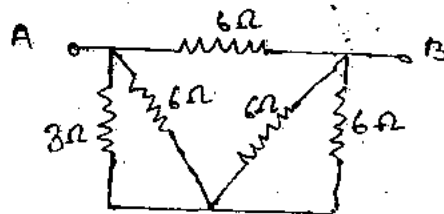
17. (Previous problem)

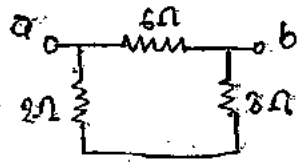


$$R_A = \frac{2 \times 2 + 2 \times 2 + 2 \times 2}{2} = 6\Omega$$

$$R_B = \frac{2 \times 2 + 2 \times 2 + 2 \times 2}{2} = 6\Omega$$

$$R_C = \frac{2 \times 2 + 2 \times 2 + 2 \times 2}{2} = 6\Omega$$





$$\begin{aligned}
 R_{ab} &= 6 \parallel (2+3) \\
 &= 6 \parallel 5 \\
 &= \frac{30}{11} \Omega
 \end{aligned}$$

Note:

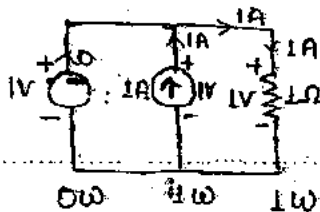
- (1) when resistor of equal value transform from star to delta the resistance value $\uparrow$ es by 3 times. Same for inductance and impedances
- (2) when capacitor's of equal value transform from star to delta capacitance value $\uparrow$ es by 3 times.

Workbook Ques 31. Page: 7

Soln. $R_c = \frac{R_a \cdot R_b}{R_a + R_b + R_c}$

$$= \frac{\frac{R_a}{K} \cdot \frac{R_b}{K}}{\frac{R_a}{K} + \frac{R_b}{K} + \frac{R_c}{K}} = \left(\frac{R_a R_b}{R_a + R_b + R_c} \right) \frac{K}{K}$$

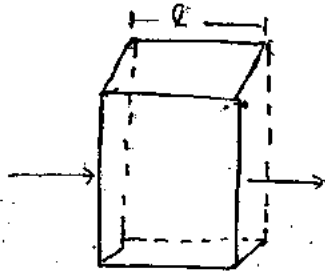
Q.B.
Q.L.



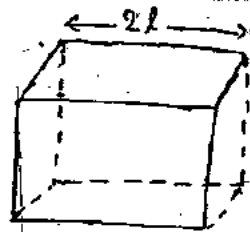
Ans: b.

1. The resistance of cube shape materials any of its two faces is 2Ω . If this material is stretched in one dir by applying linear force to double its original length. Then the resistance b/w the two opposite stress faces will be.

Ans:



$$V = l \times a$$



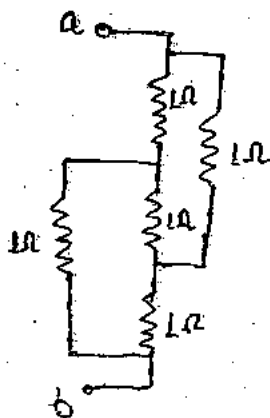
$$V = 2l \times a'$$

$$2l \times a' = l \times a$$

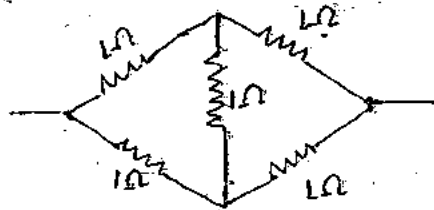
$$a' = a/2$$

$$R_1 = \frac{\rho \cdot l_1}{a_1} = \frac{\rho \cdot l}{a} = 2\Omega$$

$$R_2 = \frac{\rho \cdot l_2}{a_2} = \frac{\rho \cdot (2l)}{a/2} = \left(\frac{\rho l}{a}\right) 4 = 2 \times 4 = 8\Omega$$

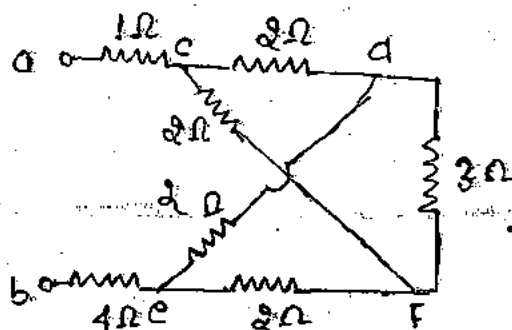


Soln:



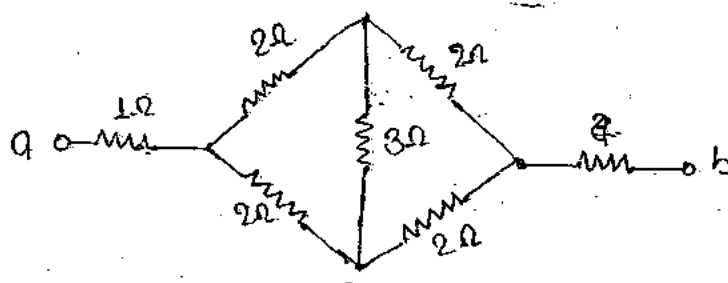
$$2 \parallel 2 = 1 \Omega \text{ Ans}$$

Q.

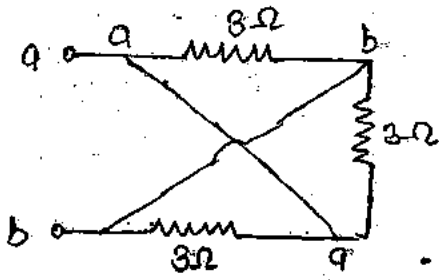


Find equivalent resistance
b/w a & b.

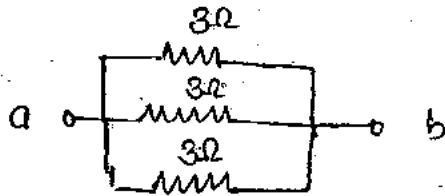
Soln:



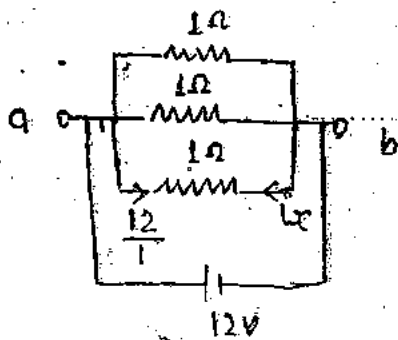
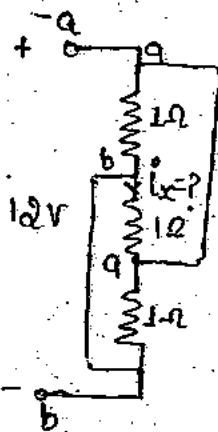
$$R_{eq} = 1 + 4 \parallel 4 + 2 = 7 \Omega$$



n.



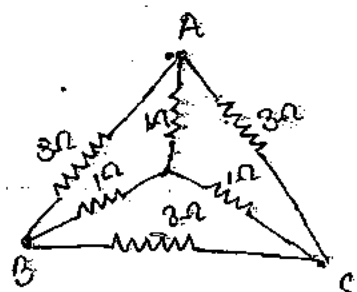
$$R_{eq} = \frac{3}{3} = 1\Omega$$



$$i_x = -12A$$

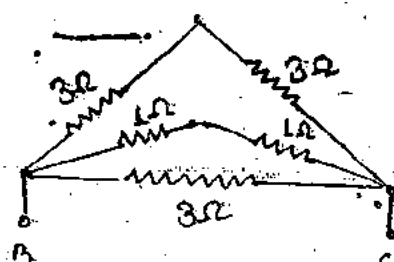
Quiz 18 Page 5

Q.



Find the equivalent Resistance b/w B & C

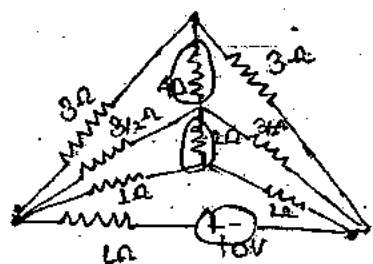
Soln:



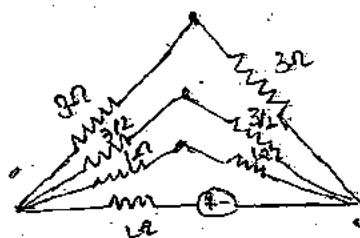
$$R_{eq} = 6 || 2 || 3 :$$

$$= 1\Omega \text{ Ans}$$

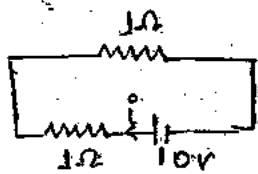
Q.



Soln:



$$6 || 3 || 2 = 1\Omega$$



$$I = \frac{10}{2} = 5A \quad \text{Ans}$$

cls of elements:

Active elements: When an element is capable of delivering energy indefinitely during infinite time and capable of providing power gain then element is called Active element.

Current source, voltage source, transistor, op-amp etc.

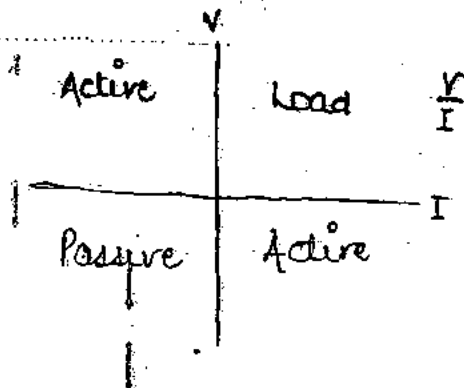
Passive elements: When an element isn't capable of providing energy for infinite time and not capable of providing power gain called passive elements.

Ex: Resistor, capacitor, inductor etc.

ratio of voltage to current at any point of char. curve then element is called Active otherwise passive.

$$P = VI \quad \text{Passive}$$

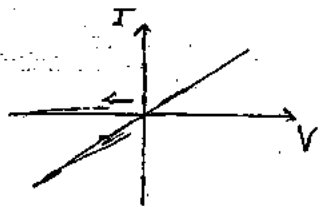
$$P = -VI \quad \text{Delivered}$$



Bidirectional elements: When element properties and char. are independent of dirⁿ of current then element is called bidirectional elements.

Unidirectional element: When element properties and char. depends upon dirⁿ of current then element is called unidirectional element.

- If the char. curve is similar in opposite quadrant then element is bidirectional otherwise unidirectional.

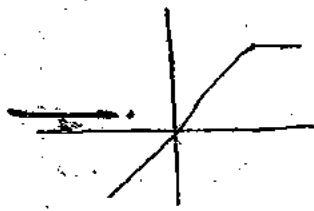


Lump elements: It is considered as a separate element, which is very small in size.

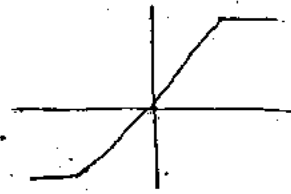
Distributed elements: It is electrically not separable.

Linear & Non linear elements:

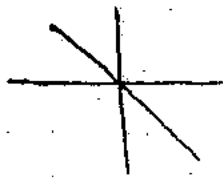
- Linearity is a property of elements describing linearity b/w excitation & response.
- Every linear element must exhibit bidirectional property.



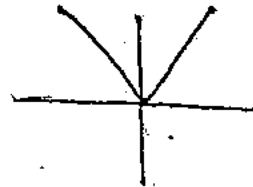
NL
P
Unidirectional



NL
Passive
Bidirectional



Active
Linear
Bidirectional

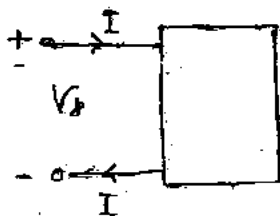


Unidirectional
Active
NL

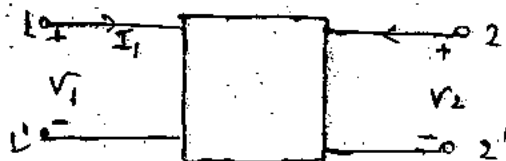
Two port n/w:

A pair of terminal through which a current may enter or leave the n/w is called port.

Two terminal devices such as resistor, inductor, capacitor result in ~~two~~^{one} port n/w



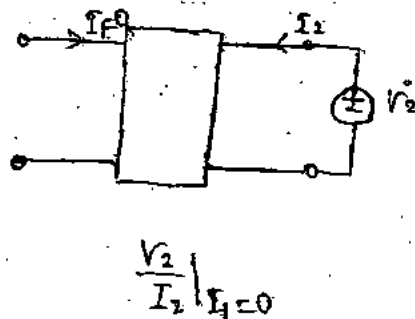
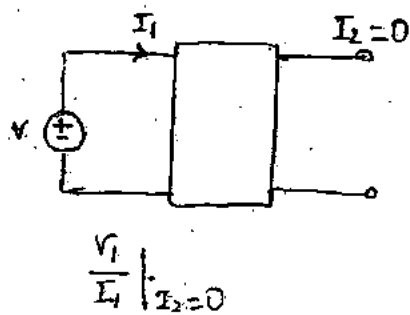
current entering one terminal of port and leaves the other so that net current entering is zero.



- Two port n/w has two separate port we apply i/p — at one port and get o/p from other port.
- current entering to the port in 2-port n/w.
- Here we have four variables V_1, I_1 (For i/p-port) and V_2, I_2 (O/P port)
- Only 2 out of 4-variables V_1, I_1, V_2, I_2 are independent.
- The n/w inside the port is consider as black box which should consist of linear, passive and bilateral elements.
- It consist of energy storage element inductor and capacitor but initial condition zero.
- The n/w b/w the source :

Concept of Symmetry two port n/w:

A two port n/w is said to symmetrical. if the ratio of excitation to response is remains constant at the both the port independent w.r.t defined circuit condⁿ such as open ckt or short ckt.



$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

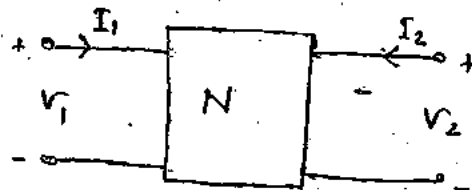
..

Z-parameters, or Impedance parameter or O.C. parameters.

$$(V_1, V_2) = f(I_1, I_2)$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

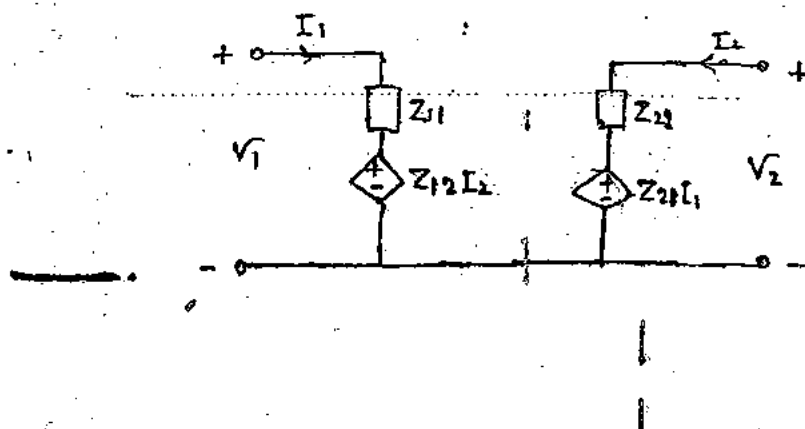
$$[V] = [Z][I]$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} \quad [\text{driving point i/p impedance}]$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} \quad \begin{matrix} \text{O.C. Reverse} \\ \text{transfer point} \end{matrix} \quad [\text{transfer impedance}]$$

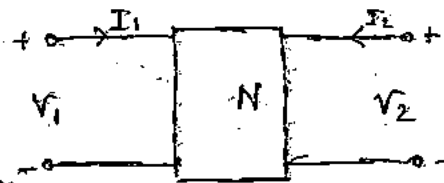
$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} \quad [\text{O.C. forward-transfer impedance}]$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} \quad [\text{O.C. driving point o/p impedance}]$$



Y Parameters, Admittance parameters or S.C Parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = f \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



$V_1, V_2 \rightarrow$ Independent variables &c

$I_1, I_2 \rightarrow$ dependent variables

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

$$[V] = [Y]^{-1}[I]$$

$$[V] = [Z][I]$$

$$[Z] = [Y]^{-1}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{Y_{11}Y_{22} - Y_{21}Y_{12}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$Z_{11} = \frac{Y_{22}}{|Y|}$$

$$Z_{12} = \frac{-Y_{12}}{|Y|}$$

$$Z_{21} = \frac{-Y_{21}}{|Y|}$$

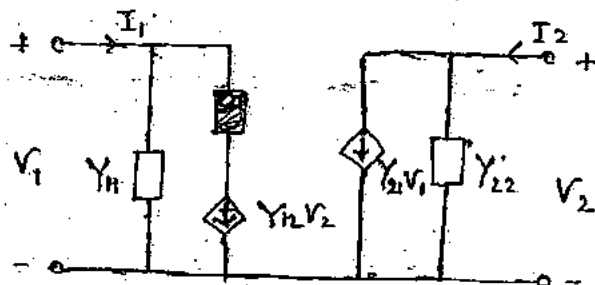
$$Z_{22} = \frac{Y_{11}}{|Y|}$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} \quad \text{S.c i/p admittance or driving point i/p admittance}$$

$$Y_{12} = \frac{I_2}{V_1} \Big|_{V_2=0} \quad \text{S.c Forward transfer admittance}$$

$$Y_{21} = \frac{I_1}{V_2} \Big|_{V_1=0} \quad \text{S.c Reverse " "}$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} \quad \text{S.c driving point o/p admittance}$$



H- parameter's -

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = f \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

V_1, I_1 = dependent variables

V_2, I_2 = Independent variables

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

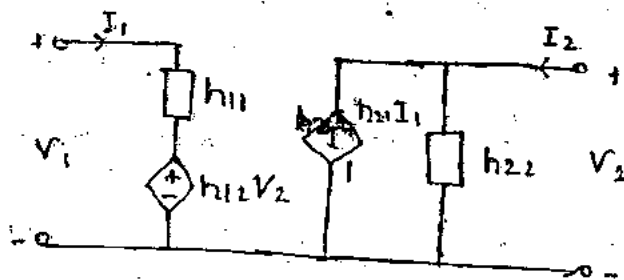
$$I_1 = h_{11} I_1 + h_{12} V_2$$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} = \frac{1}{\frac{I_1}{V_1} \Big|_{V_2=0}} = \frac{1}{Y_{11}} \quad (\text{s.c driving point i/p impedance})$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} \quad (\text{s.c Forward current gain})$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} \quad (\text{o.c Reverse voltage gain})$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{1}{\frac{V_2}{I_2} \Big|_{I_1=0}} = \frac{1}{Z_{22}} \quad (\text{o.c driving point admittance})$$



g - Parameters:

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = f \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

V_1, I_2 = Independent variables

V_2, I_1 = Dependent variables

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$[G] = [H]^{-1}$$

$$g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0}$$

O/c Driving point i/p admittance

$$g_{21} = \frac{V_2}{V_1} \Big|_{I_2=0}$$

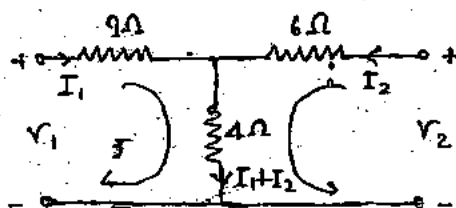
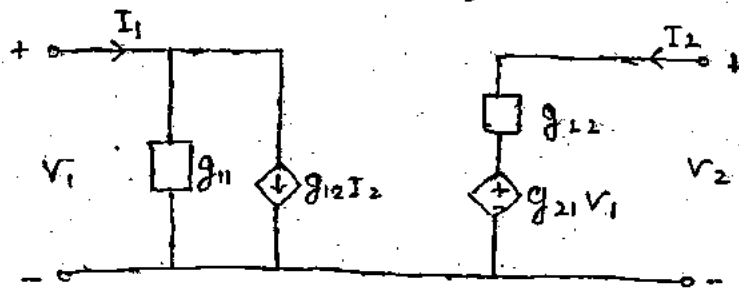
O/c Forward voltage gain

$$g_{12} = \frac{I_1}{I_2} \Big|_{V_1=0}$$

S.C Reverse current gain

$$g_{22} = \frac{V_2}{I_2} \Big|_{V_1=0}$$

S.C Driving point o/p impedance



Find the Z, Y, H parameters

Method 1: (using Mesh Analysis)

$$-V_1 + 9I_1 + 4(I_1 + I_2) = 0$$

$$V_1 = 6I_1 + 4I_2 \quad (1)$$

$$\boxed{V_1 = Z_{11}I_1 + Z_{12}I_2}$$

$$-V_2 + 6I_2 + 4(I_1 + I_2) = 0$$

$$V_2 = 4I_1 + 10I_2 \quad (2)$$

$$\boxed{V_2 = Z_{21}I_1 + Z_{22}I_2}$$

$$\hat{Z} = \begin{bmatrix} 6 & 4 \\ 4 & 10 \end{bmatrix}$$

$$[Y] = -[Z]^{-1}$$

$$= \frac{1}{60-16} \begin{bmatrix} 10 & -4 \\ -4 & 6 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 5/22 & -1/11 \\ -1/11 & 3/22 \end{bmatrix}$$

From Eqn (2)

$$10I_2 = V_2 - 4I_1$$

$$I_2 = 0.1V_2 - 0.4I_1 \quad (3)$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$(3) \rightarrow (1)$$

$$V_1 = 6I_1 + 4[0.1V_2 - 0.4I_1]$$

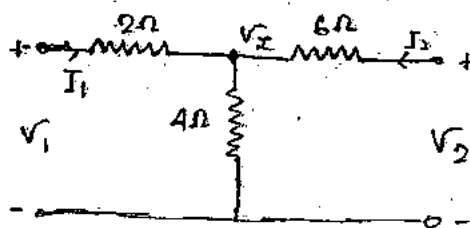
$$= 6I_1 - 1.6I_1 + 0.4V_2$$

$$V_1 = 4.4I_1 + 0.4V_2 \quad (4)$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$H = \begin{bmatrix} 4.4 & 0.4 \\ -0.4 & 0.4 \end{bmatrix}$$

Using Nodal Analysis



$$\frac{V_x - V_1}{2} + \frac{V_x}{4} + \frac{V_x - V_2}{6} = 0$$

$$V_x \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6} \right] - \frac{V_1}{2} - \frac{V_2}{6} = 0$$

$$V_x \left[\frac{11}{12} \right] = \frac{V_1}{2} + \frac{V_2}{6}$$

$$V_x = \left(\frac{12}{11} \right) \left[\frac{V_1}{2} + \frac{V_2}{6} \right] \quad \text{--- (1)}$$

Now

$$\begin{aligned} I_1 &= \frac{V_1 - V_x}{2} = \frac{V_1}{2} - \frac{1}{2} \cdot \frac{12}{11} \left[\frac{V_1}{2} + \frac{V_2}{6} \right] \\ &= \frac{V_1}{2} - \frac{6}{22} V_1 - \frac{V_2}{11} \end{aligned}$$

$$I_1 = \frac{5}{22} V_1 - \frac{1}{11} V_2 \quad \text{--- (2)}$$

$$I_2 = \frac{V_2 - V_x}{6} = \frac{V_2}{6} - \frac{1}{6} \cdot \frac{12}{11} \left(\frac{V_1}{2} + \frac{V_2}{6} \right)$$

$$I_2 = -\frac{1}{11} V_1 + \frac{3}{22} V_2 \quad \text{--- (3)}$$

$$Y = \begin{bmatrix} 5/22 & -1/11 \\ -1/11 & 3/22 \end{bmatrix}$$

$$Z = [Y]^{-1}$$

$$= \begin{bmatrix} 6 & 4 \\ 4 & 10 \end{bmatrix}$$

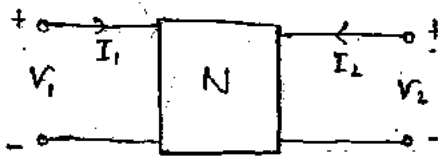
$$V_1 = 4.4 I_1 + 0.4 V_2$$

$$I_2 = -0.4 I_1 + 0.1 V_2$$

$$H = \begin{bmatrix} 4.4 & 0.4 \\ -0.4 & 0.1 \end{bmatrix}$$

T- parameter's or Transmission parameter or ABCD parameter :

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = f \left(\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \right)$$



V_1, I_1 = Dependent variables

V_2, I_2 = Independent variables



$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad \text{O.C Reverse Voltage gain}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \quad \text{O.C Reverse transfer admittance}$$

$$B = \left. -\frac{V_1}{I_2} \right|_{V_2=0} \quad \text{S.C Reverse } \therefore \text{ impedance}$$

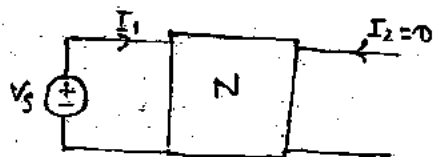
$$D = \left. -\frac{I_1}{I_2} \right|_{V_2=0} \quad \text{S.C Reverse trans current gain}$$

dition for symmetry:

$$1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{--- (1)}$$

$$2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--- (2)}$$

LI

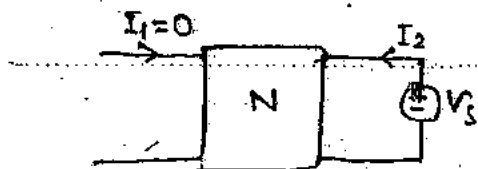


$$I_2 = 0 \quad \text{--- (1)}$$

$$V_S = Z_{11}I_1 + (0)Z_{12}$$

$$Z_{11} = \frac{V_S}{I_1} \quad \text{--- (3)}$$

II



$$I_1 = 0 \rightarrow \text{--- (2)}$$

$$V_S = Z_{22}I_2$$

$$Z_{22} = \frac{V_S}{I_2} \quad \text{--- (4)}$$

For n/w to be symmetry

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\boxed{Z_{11} = Z_{22}}$$

$$Z_{11} = \frac{Y_{22}}{|Y|}$$

$$Z_{22} = \frac{Y_{11}}{|Y|}$$

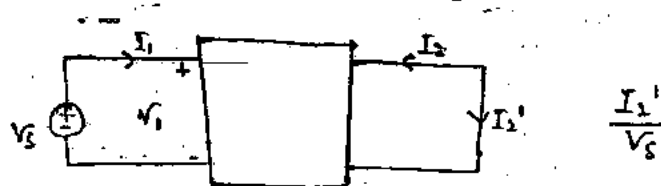
$$\frac{Y_{11}}{|Y|} = \frac{Y_{22}}{|Y|}$$

$$\boxed{Y_{11} = Y_{22}}$$

Condition of Reciprocity :

A n/w is said to be reciprocal, if the ratio of response to excitation remains the same even after posⁿ of response and excitation are interchange.

case I :



$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--- (2)}$$

$$V_1 = Z_{11}I_1 - Z_{12}I_2' \quad \text{--- (3) (From Eqn (1))}$$

$$0 = Z_{21}I_1 - Z_{22}I_2' \quad \text{--- (4) (From Eqn (2))}$$

$$Z_{21}I_1 = Z_{22}I_2'$$

$$I_1 = \frac{Z_{22}I_2'}{Z_{21}} \quad \text{--- (5)}$$

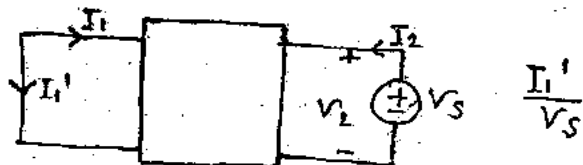
⑤ → ③

$$V_S = Z_{11} \left[\frac{Z_{22}}{Z_{21}} I_2' \right] - Z_{12} I_2'$$

$$V_S = \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \right] I_2'$$

$$\boxed{\frac{I_2'}{V_S} = \frac{Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}} \quad - \alpha$$

Q80



$$V_2 = V_S \rightarrow \textcircled{2}$$

$$V_S = Z_{11} I_1' + Z_{12} I_2$$

$$V_1 = 0 \rightarrow \textcircled{1}$$

$$0 = -Z_{11} I_1' + Z_{12} I_2$$

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1'$$

$$V_S = -Z_{21} I_1' + \frac{Z_{11}}{Z_{12}} Z_{22} I_1'$$

$$\boxed{\frac{I_1'}{V_S} = \frac{Z_{12}}{Z_{11} Z_{22} - Z_{21} Z_{12}}} \quad - \beta$$

— condition for Reciprocity

$$\frac{I_2'}{V_S} = \frac{I_1'}{V_S}$$

$$\frac{Z_{21}}{|Z|} = \frac{Z_{12}}{|Z|}$$

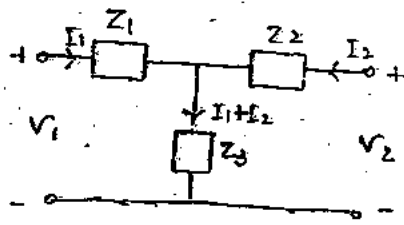
$Z_{12} = Z_{21}$ in term of Z-parameter

$$Z_{12} = -\frac{Y_{12}}{|Y|} \quad Z_{21} = -\frac{Y_{21}}{|Y|}$$

$$-\frac{Y_{12}}{|Y|} = -\frac{Y_{21}}{|Y|}$$

$Y_{12} = Y_{21}$ in term of Y-parameter

For T- π /w



$$V_1 = (Z_1 + Z_3)I_1 + Z_3 I_2$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = (Z_2 + Z_3)I_2 + Z_3 I_1$$

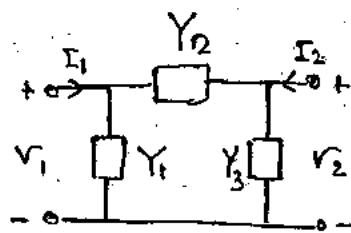
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$$

$$Z_{11} = Z_1 + Z_3 \quad ; \quad Z_{12} = Z_{21} = Z_3$$

$$Z_{22} = Z_2 + Z_3$$

11-11/10



$$I_1 = V_1 Y_1 + (V_1 - V_2) Y_2$$

$$I_1 = (Y_1 + Y_2) V_1 - V_2 Y_2$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = V_2 Y_3 + (V_2 - V_1) Y_2$$

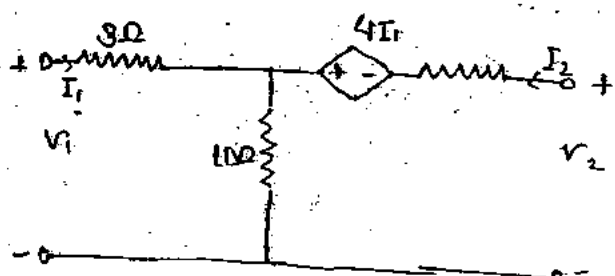
$$I_2 = -Y_2 V_1 + (Y_2 + Y_3) V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_2 + Y_3 \end{bmatrix}$$

$$\boxed{Y_{11} = Y_1 + Y_2 \quad ; \quad Y_{22} = Y_2 + Y_3}$$

$$Y_{21} = Y_{12} = -Y_2$$



Find Z, Y, H, T parameters

$$-V_1 + 3I_1 + 10(I_1 + I_2) = 0$$

$$V_1 = 13I_1 + 10I_2$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$-V_2 + 2I_2 - 4I_1 + 10(I_1 + I_2) = 0$$

$$V_2 = 6I_1 + 12I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z = \begin{bmatrix} 13 & 10 \\ 6 & 12 \end{bmatrix}$$

$$Y = [Z]^{-1}$$

$$= \frac{1}{156 - 60} \begin{bmatrix} 12 & -10 \\ -6 & 13 \end{bmatrix}$$

H-parameter

$$12I_2 = V_2 - 6I_1$$

$$I_2 = \frac{1}{12}V_2 - \frac{1}{2}I_1$$

$$V_1 = 13I_1 + 10 \left[\frac{1}{12}V_2 - \frac{1}{2}I_1 \right]$$

$$= 8I_1 + \frac{5}{6}V_2$$

$$V_1 = 8I_1 + \frac{5}{6}V_2$$

$$H = \begin{bmatrix} 8 & 5/6 \\ -1/2 & 1/12 \end{bmatrix}$$

T parameter

$$6I_1 = V_2 - 12I_2$$

$$I_1 = \frac{1}{6}V_2 - 2I_2$$

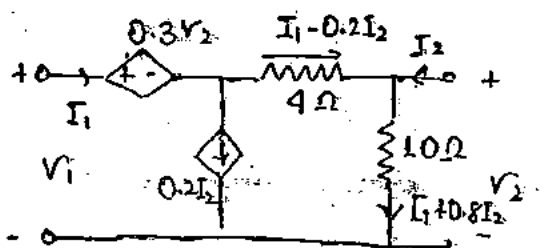
$$= \frac{1}{6}V_2 + 2(-I_2)$$

$$V_1 = 13 \left[\frac{1}{6} V_2 - 2I_2 \right] + 10I_2$$

$$= \frac{13}{6} V_2 - 26I_2 + 10I_2$$

$$V_1 = \frac{13}{6} V_2 + 16(-I_2)$$

$$T = \begin{bmatrix} 13/6 & 16 \\ 1/6 & 2 \end{bmatrix}$$



$$V_2 = 10(I_1 + 0.8I_2)$$

$$V_2 = 10I_1 + 8I_2$$

$$-V_1 + 0.3V_2 + 4(I_1 - 0.2I_2) + V_2 = 0$$

$$V_1 = 1.3V_2 + 4I_1 - 0.8I_2$$

$$V_1 = 1.3(10I_1 + 8I_2) + 4I_1 - 0.8I_2$$

$$V_1 = 13I_1 + 10.4I_2 + 4I_1 - 0.8I_2$$

$$V_1 = 17I_1 + 9.6I_2$$

$$Z = \begin{bmatrix} 17 & 9.6 \\ 10 & 0 \end{bmatrix}$$

$$Y = [Z]^{-1}$$

$$\Rightarrow 8I_2 = V_2 - 10I_1$$

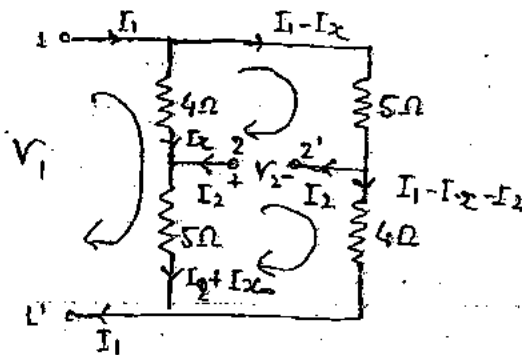
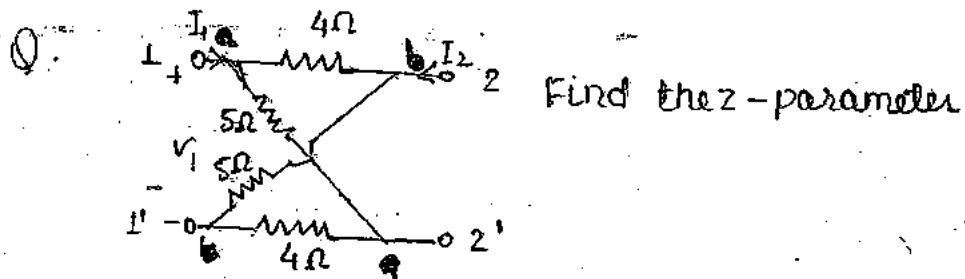
$$I_2 = \frac{1}{8}V_2 - \frac{5}{4}I_1$$

$$V_1 = 17I_1 + \frac{48}{5} \left[\frac{1}{8}V_2 - \frac{5}{4}I_1 \right]$$

$$= 17I_1 + \frac{6}{5}V_2 - 12I_1$$

$$V_1 = 5I_1 + \frac{6}{5}V_2$$

$$H = \begin{bmatrix} 5 & 6/5 \\ -5/4 & -3/8 \end{bmatrix}$$



$$V_1 = 4I_x + 5(I_x + I_2)$$

$$V_1 = 9I_x + 5I_2 \quad \text{--- (1)}$$

$$V_2 = -4I_x + 5(I_1 - I_x)$$

$$V_2 = 5I_1 - 9I_x \quad \text{--- (2)}$$

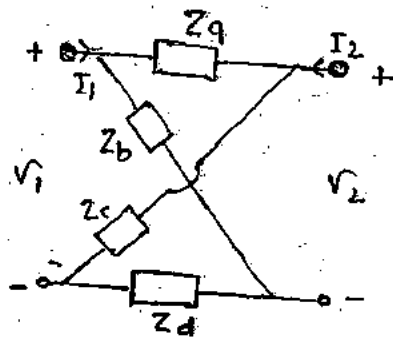
$$V_2 + 4(I_1 - I_x - I_2) - 5(I_2 + I_x) = 0$$

$$V_2 + 4I_1 - 9I_2 = 9I_x$$

$$V_1 = \frac{g}{2} I_1 + \frac{1}{2} I_2$$

$$V_2 = \frac{1}{2} I_1 + \frac{g}{2} I_2$$

N.B.



$$\left. \begin{array}{l} Z_a = Z_d \\ Z_b = Z_c \end{array} \right\} \text{Symmetrical}$$

$$Z_{11} = Z_{22} = \frac{-Z_a + Z_b}{2}$$

$$Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}$$

Similarly:

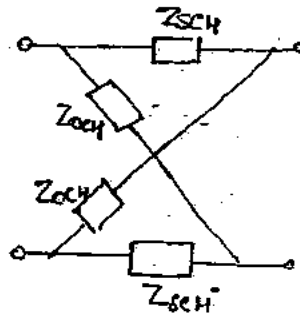
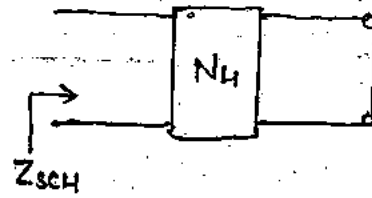
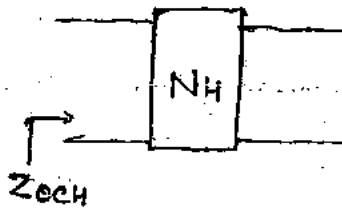
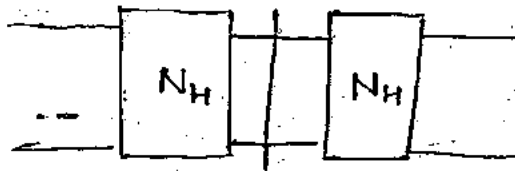
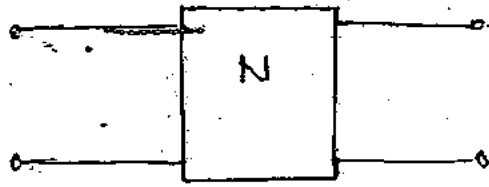
$$Y_{11} = Y_{22} = \frac{Y_a + Y_b}{2}$$

$$Y_{12} = Y_{21} = \frac{Y_b - Y_a}{2}$$

$$\left. \begin{array}{l} Y_a = Y_d \\ Y_b = Y_c \end{array} \right\} \text{Symmetrical}$$

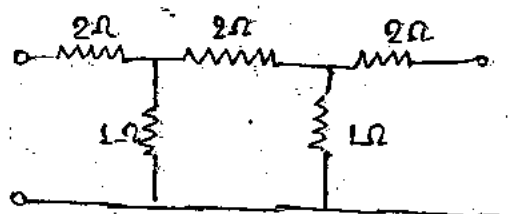
Bartlett's Bisection theorem:-

- 1. It is used for designing lattice n/w.
- 2. It is applicable for symmetrical n/w.
- 3. In this theorem, we will separate the n/w in two equal parts and then we will bisect the n/w from middle.

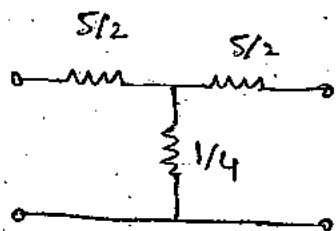
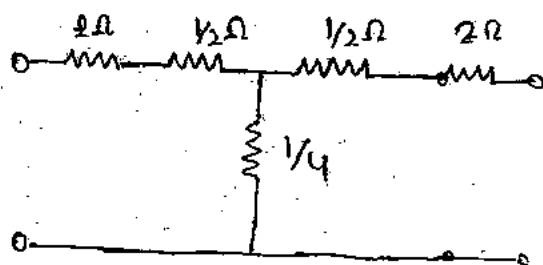
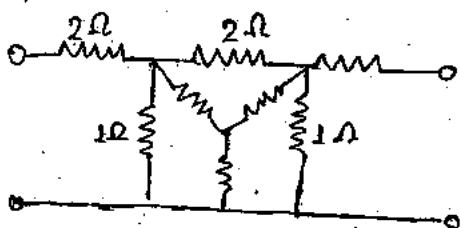


$$Z_{11} = \frac{Z_{sch} + Z_{och}}{2} = Z_{22}$$

$$Z_{21} = Z_{12} = \frac{Z_{och} - Z_{sch}}{2}$$



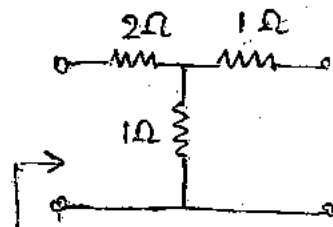
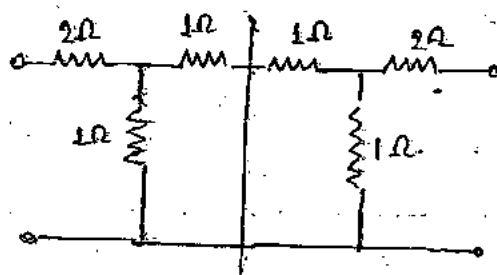
Method 1



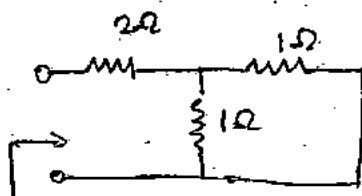
$$Z_{11} = 5/2 + 1/4 = 11/4 = 2.75$$

$$Z_{12} = 1/4 = 0.25$$

2nd method

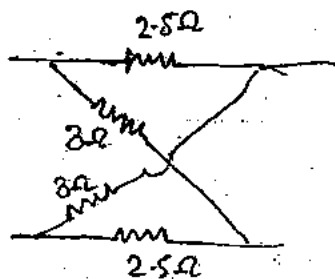


$$Z_{OCH} = 2 + 1 = 3\Omega$$



$$Z_{SCH} = 2 + (1 \parallel 1)$$

$$= 2.5\Omega$$

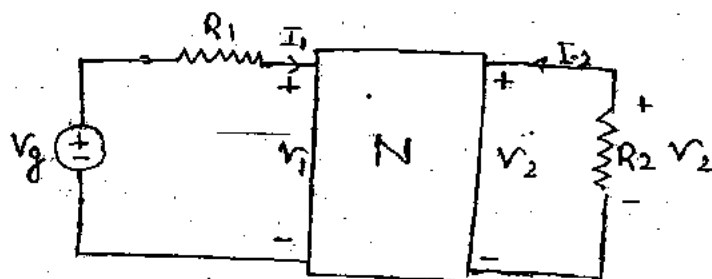


$$Z_{11} = Z_{22} = \frac{2.5 + 3}{2} = 2.75\Omega$$

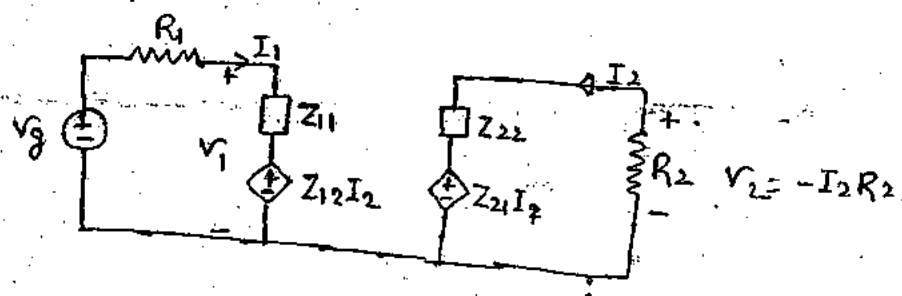
$$Z_{12} = Z_{21} = \frac{3 \cdot 0.5}{2} = 0.25$$

} $\cdot \frac{1}{2} \cdot \frac{1}{2}$

Q. In the below given n/w find $\frac{V_2}{V_g}$ in term of z-parameters



Soln: Method 1



$$-V_g + I_1 R_1 + I_1 z_{11} + I_2 z_{12} = 0$$

$$V_g = (R_1 + z_{11}) I_1 + I_2 z_{12} \quad \text{--- (1)}$$

$$I_2 z_{22} + z_{21} I_1 + I_2 R_2 = 0$$

$$I_1 = -\frac{(R_2 + z_{22})}{z_{21}} I_2 \quad \text{--- (2)}$$

$$\therefore V_g = -(R_1 + z_{11}) \times \frac{(R_2 + z_{22})}{z_{21}} I_2 + z_{12} I_2$$

$$V_g = I_2 \left[\frac{-(R_1 R_2 + R_1 z_{22} + z_{11} R_2 + z_{11} z_{22}) + z_{12} z_{21}}{z_{21}} \right]$$

$$V_g = \frac{-V_2}{-R_2} \left[\frac{Z_{21} + R_1 Z_{22} + R_2 Z_{11} + R_1 R_2}{Z_{21}} \right]$$

$$\frac{V_2}{V_g} = \frac{R_2 Z_{21}}{Z_{21} + R_1 Z_{22} + R_2 Z_{11} + R_1 R_2}$$

2nd method :-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

$$\begin{aligned} V_g &= V_1 + I_1 R_1 \\ &= (Z_{11} I_1 + Z_{12} I_2) + I_1 R_1 \\ &= (Z_{11} + R_1) I_1 + Z_{12} I_2 \quad \text{--- (3)} \end{aligned}$$

$$V_2 = -I_2 R_2$$

$$I_2 = -\frac{V_2}{R_2}$$

$$V_g = (Z_{11} + R_1) I_1 + Z_{12} \left(-\frac{V_2}{R_2} \right) \quad \text{--- (4)}$$

$$Z_{21} I_1 = \left(\frac{R_2 + Z_{22}}{R_2} \right) V_2$$

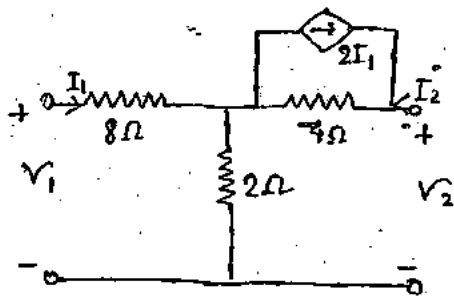
$$I_1 = \left(\frac{R_2 + Z_{22}}{R_2 Z_{21}} \right) V_2 \quad \text{--- (5)}$$

$$V_g = \frac{(R_1 + Z_{11}) \cdot (R_2 + Z_{22})}{R_1 Z_{21}} V_2 - \frac{Z_{12}}{R_2} \cdot V_2$$

$$V_g = V_2 \left[\frac{R_1 R_2 + R_1 Z_{22} + R_2 Z_{11} + Z_{11} Z_{22} - Z_{12} Z_{21}}{R_2 Z_{21}} \right]$$

$$\frac{V_2}{V_g} = \frac{R_2 Z_{21}}{Z_{21} + R_1 R_2 + R_1 Z_{22} + R_2 Z_{11}}$$

Ans



Calculate y and z
Parameter.

soln.

$$-V_1 + 8I_1 + 2(I_1 + I_2) = 0$$

$$V_1 = 10I_1 + 2I_2 \quad \text{--- (1)}$$

$$-V_2 + 4(2I_1 + I_2) + 2(I_1 + I_2) = 0$$

$$V_2 = 10I_1 + 6I_2$$

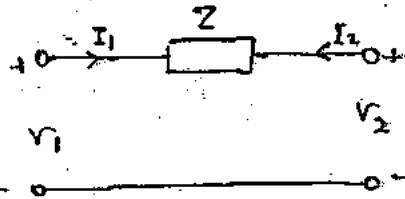
$$Z = \begin{bmatrix} 10 & 2 \\ 10 & 6 \end{bmatrix}$$

$$Y = [Z]^{-1}$$

$$Y = \frac{1}{40} \begin{bmatrix} 6 & -2 \\ -10 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{20} & -\frac{1}{20} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$I_2 \neq V_2 - 10I_1$$



Calculate Y and Z parameters.

Soln.

$$I_1 = \frac{V_1 - V_2}{Z} = \frac{V_1}{Z} - \frac{V_2}{Z}$$

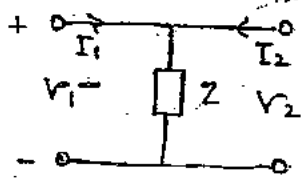
$$I_2 = \frac{V_2 - V_1}{Z} = -\frac{V_1}{Z} + \frac{V_2}{Z}$$

$$Y = \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}$$

$$[Z] = [Y]^{-1}$$

$$|Y| = 0$$

Inverse not possible



Calculate Y and Z parameters.

Soln

$$-V_1 + (I_1 + I_2)Z = 0$$

$$V_1 = ZI_1 + I_2Z$$

Similarly $V_2 = ZI_1 + \frac{1}{Z}I_2$

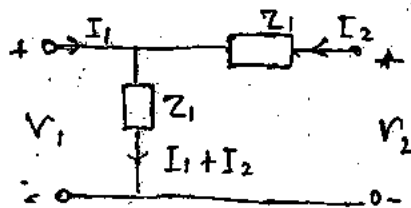
$$Z = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

$$Y = [Z]^{-1}$$

$$|Z| = 0$$

inverse not possible

∴ Y-parameter doesn't exist



calculate Z-parameter?

$$V_1 = I_1 Z_1 + I_2 Z_1$$

$$V_2 = Z_1 I_1 + (Z_1 + Z_2) I_2$$

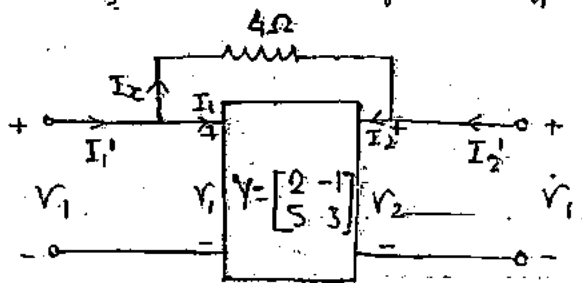
$$Z = \begin{bmatrix} Z_1 & Z_1 \\ Z_1 & Z_1 + Z_2 \end{bmatrix}$$

$$|Z| = Z_1^2 + Z_1 Z_2 - Z_1^2 = Z_1 Z_2$$

$$Y = [Z]^{-1}$$

$$= \frac{1}{Z_1 Z_2} \begin{bmatrix} Z_1 + Z_2 & -Z_1 \\ -Z_1 & Z_1 \end{bmatrix}$$

Q. Find the y parameters of the n/w given.



Soln.

$$I_1 = 2V_1 - V_2 \quad (1)$$

$$I_2 = 5V_1 + 3V_2 \quad (2)$$

$$I_x = \frac{V_1 - V_2}{4} = \frac{V_1}{4} - \frac{V_2}{4} \quad (3)$$

$$I_1' = I_1 + I_x$$

$$= 2V_1 - V_2 + \frac{V_1}{4} - \frac{V_2}{4}$$

$$I_1' = \frac{9V_1}{4} - \frac{5V_2}{4}$$

$$I_2' + I_x = I_2$$

$$I_2' = I_2 - I_x$$

$$= 5V_1 + 3V_2 - \frac{V_1}{4} + \frac{V_2}{4}$$

$$I_2' = \frac{19}{4}V_1 + \frac{13}{4}V_2$$

$$Y = \begin{bmatrix} 9/4 & -5/4 \\ 19/4 & 13/4 \end{bmatrix} \quad \underline{\underline{\text{Ans}}}$$

conversion of parameter

Γ to Z-parameter:

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

$$\rightarrow F_2 CV_2 = I_1 + DI_2$$

$$\boxed{V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2}$$

$$V_1 = AV_2 - BI_2$$

$$= A \left(\frac{1}{C} I_1 + \frac{D}{C} I_2 \right) - BI_2$$

$$= \left(\frac{A}{C} \right) I_1 + \left(\frac{AD}{C} - B \right) I_2$$

$$\boxed{V_1 = \left(\frac{A}{C} \right) I_1 + \left(\frac{AD - BC}{C} \right) I_2}$$

$$Z_{11} = \frac{A}{C} \quad ; \quad Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C} \quad Z_{22} = \frac{D}{C}$$

symmetry

$$Z_{11} = Z_{22}$$

$$\frac{A}{C} = \frac{D}{C}$$

$$\boxed{A = D}$$

Reciprocity

$$Z_{12} = Z_{21}$$

$$\frac{AD - BC}{C} = \frac{1}{C}$$

$$\boxed{AD - BC = 1}$$

2. H to Y-parameter:

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

① →

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2$$

Substitute this value in Eqⁿ (2)

$$I_2 = h_{21} \left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \right] + h_{22} V_2$$

$$= \frac{h_{21}}{h_{11}} V_1 - \frac{h_{12} h_{21}}{h_{11}} V_2 + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}} V_2$$

$$Y_{11} = \frac{1}{h_{11}}$$

$$Y_{12} = -\frac{h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}}$$

Symmetry

$$Y_{11} = Y_{22}$$

$$\frac{1}{h_{11}} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}}$$

$$h_{11} h_{22} - h_{12} h_{21} = 1$$

$$\boxed{|H| = 1}$$

Reciprocity

$$Y_{12} = Y_{21}$$

$$-\frac{h_{12}}{h_{11}} = -\frac{h_{12}}{h_{11}}$$

$$\boxed{h_{12} = -h_{21}}$$

off our power:
Symmetry

RECIPROCAL

1) $Z_{11} = Z_{22}$

(U) $Z_{12} = Z_{21}$

$Y_{11} = Y_{22}$

$Y_{21} = Y_{12}$

3) $|H| = 1$

(U) $h_{12} = -h_{21}$

$|G| = 1$

$g_{12} = -g_{21}$

3) $A = D$

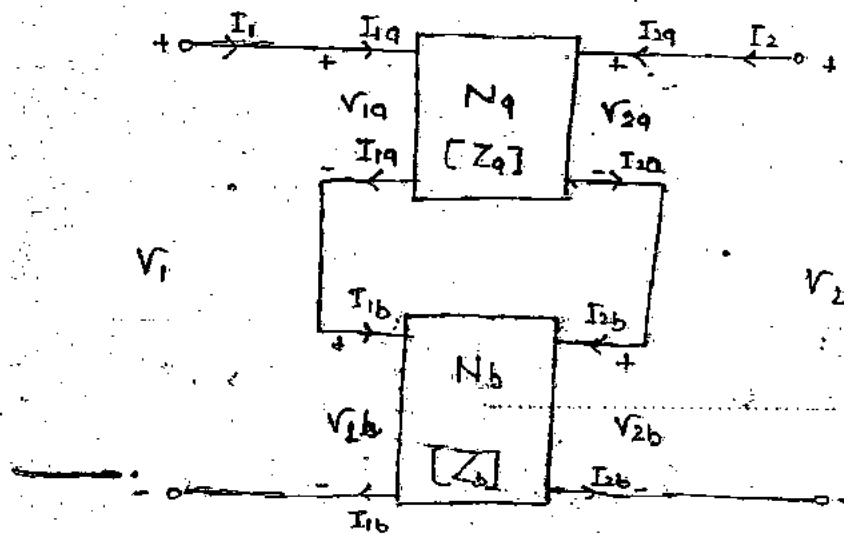
(U) $|T| = 1$

$a = d$

$|t| = 1$

Interconnection of η/ω :-

Series - Series Interconnection:



$$I_{1a} = I_{1b} = I_1$$

$$I_{2a} = I_{2b} = I_2$$

$$V_1 = V_{1a} + V_{1b}$$

$$V_2 = V_{2a} + V_{2b}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

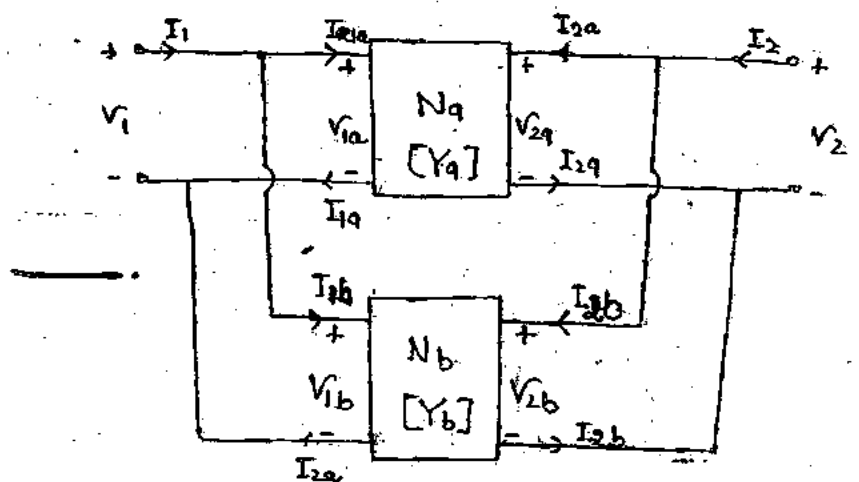
$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z_a] \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + [Z_b] \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \{ [Z_a] + [Z_b] \} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boxed{[Z] = [Z_a] + [Z_b]}$$

11-11 Interconnection



$$V_1 = V_{1a} = V_{1b} \quad ; \quad V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \quad -I_2 = I_{2a} + I_{2b}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

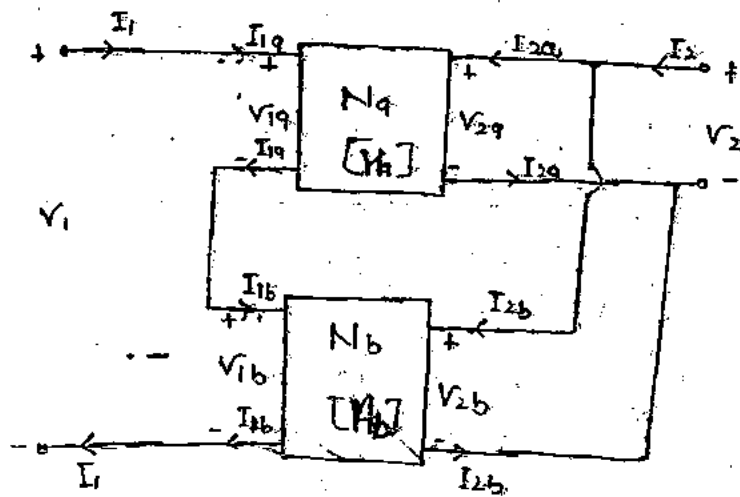
$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y_a] \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + [Y_b] \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \{ [Y_a] + [Y_b] \} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[Y] = [Y_a] + [Y_b]$$

3 Series-|| Interconnection



$$V_1 = V_{1a} + V_{1b}$$

$$V_{2a} = V_{2b} = V_2$$

$$I_{1b} = I_{1a} = I_1$$

$$I_2 = I_{2a} + I_{2b}$$

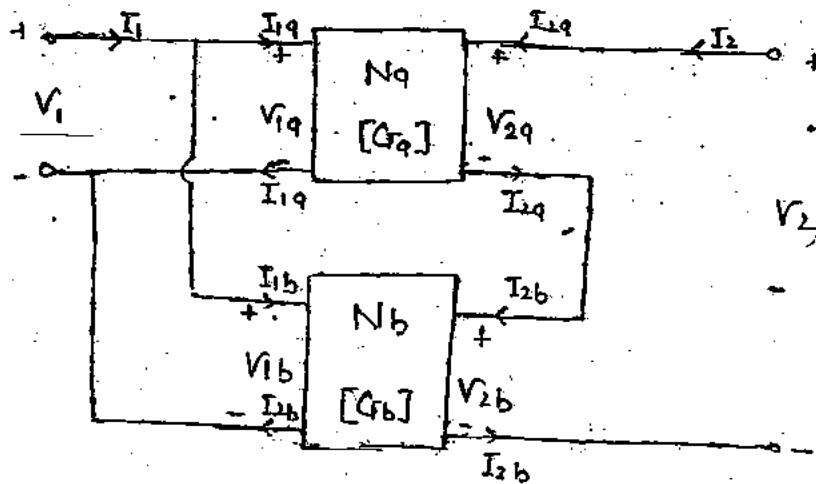
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$= [H_a] \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} + [H_b] \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \{ [H_a] + [H_b] \} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$[H] = [H_a] + [H_b]$$

11 - Series Interconnection



$$V_1 = V_{1a} + V_{1b} \quad ; \quad V_2 = V_{2a} + V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \quad I_2 = I_{2a} = I_{2b}$$

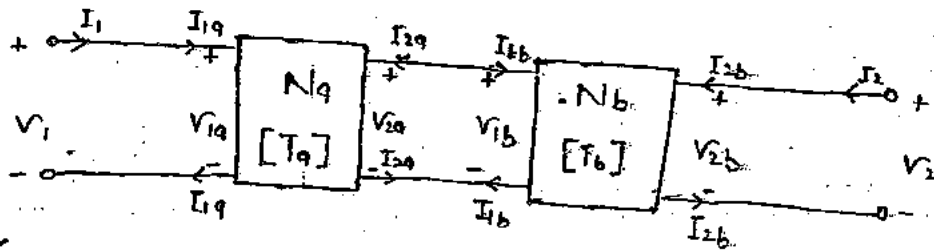
$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$= [G_{1a}] \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + [G_{1b}] \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \{ [G_{1a}] + [G_{1b}] \} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$[G] = [G_{1a}] + [G_{1b}]$$

5. cascade Interconnection :-



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix}$$

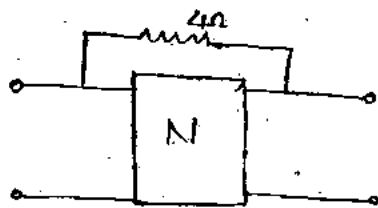
$$= [T_a] \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$= [T_a] \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= [T_a] \left\{ [T_b] \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} \right\}$$

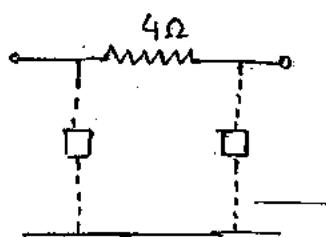
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a] [T_b] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$[T] = [T_a] [T_b]$$



$$[Y] = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$$

117.

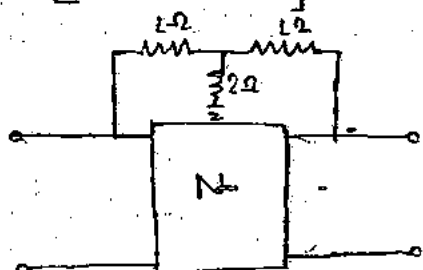


$$Y' = \begin{bmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{bmatrix}$$

$$Y = [Y'] + [Y_N]$$

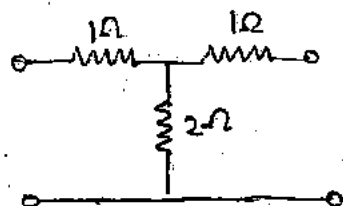
$$= \begin{bmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9/4 & -5/4 \\ 19/4 & 13/4 \end{bmatrix}$$



$$Y_N = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$$

119.



$$Z = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$Y = [Z]^{-1}$$

$$= \frac{1}{5} \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$$

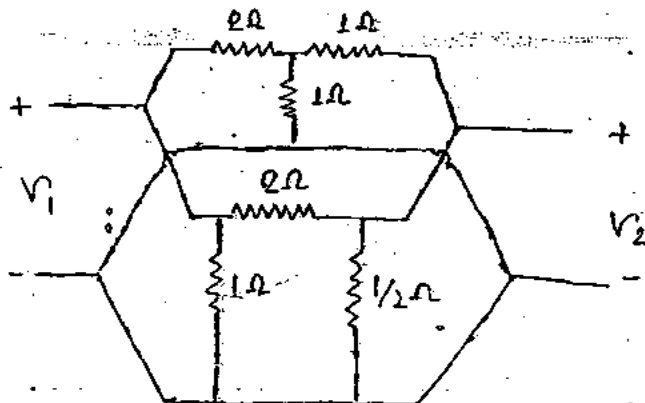
$$Y' = \begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$$[Y] = [Y_N] + [Y']$$

$$= \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 13/5 & -7/5 \\ 23/5 & 18/5 \end{bmatrix}$$

Q.



Find the overall Z-parameter of the n/w

Soln:

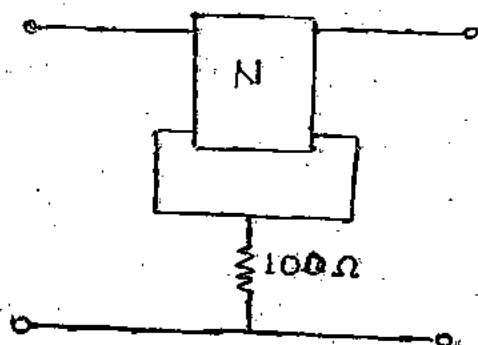
$$Y_a = Z = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$Y_a = \frac{1}{5} \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$Y_b = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 3/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix} + \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix} = \begin{bmatrix} 19/10 & -7/10 \\ -7/10 & 31/10 \end{bmatrix}$$

$$= \begin{bmatrix} 19/10 & -7/10 \end{bmatrix}$$

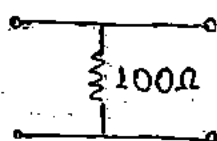


$$Y_N = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix} \text{ mS}$$

Find overall y-parameters.

Ans:

Y_b



$$Z_b = \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix}$$

$$Y_N = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix} \text{ mS}$$

$$Z_n = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix}$$

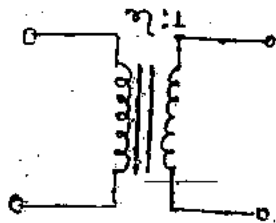
$$Z = [Z_n] + [Z_b]$$

$$= \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

$$Y = [Z]^{-1}$$

Q. #

Q. Find the Z, Y and T-parameter of Ideal X^r:



From above n/w it is not possible to find impedance & Admittance value. Since for Ideal X^r self & mutual inductance are ∞ . Therefore Y and Z-parameter not defined.

$$\frac{V_1}{V_2} = \frac{n}{1} \quad ; \quad \frac{I_1}{I_2} = \frac{1}{n}$$

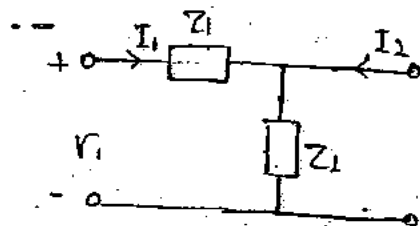
$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

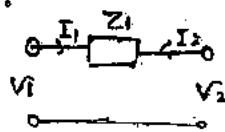
$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad \therefore \boxed{A = n}$$

$$D = \left. \frac{-I_1}{I_2} \right|_{V_2=0} \quad \boxed{D = 1/n}$$

Q. Find the T-parameter of the given n/w



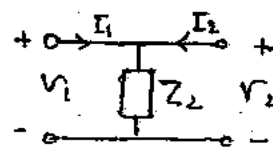
Soln



$$V_1 = -I_2 Z_1 + V_2$$

$$I_1 = -I_2$$

$$T_a = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}$$



$$V_1 = V_2$$

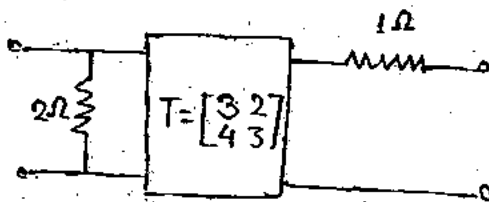
$$I_1 = \frac{V_2}{Z_2} - I_2$$

$$T_b = \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix}$$

$$[T] = [T_a][T_b]$$

$$= \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ 1/Z_2 & 1 \end{bmatrix} \text{ Ans}$$



Find overall T-parameter of the n/w.

soln.

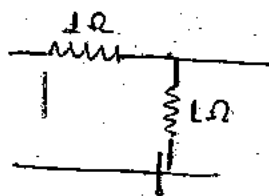
$$[T] = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 1/2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

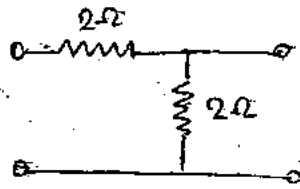
$$= \begin{bmatrix} 3 & 5 \\ 1/2 & 19/2 \end{bmatrix}$$

The T-parameter of n/w consisting of only impedances is ABCD. if all impedances of n/w is double then the new T-parameter are _____?

soln. let



$$T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$



$$T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

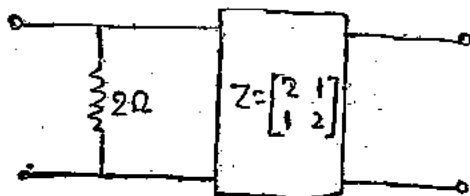
$$A = A'$$

$$B = 2B'$$

$$C = C'/2$$

$$D = D'$$

Q.

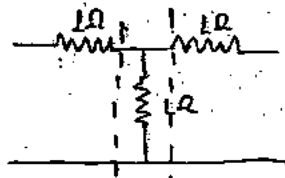


Find overall T_{eq}/w of the n/w.

Soln:

$$T_a = \begin{bmatrix} 1 & \\ & 1/2 \end{bmatrix}$$

$$Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$



$$T_b = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

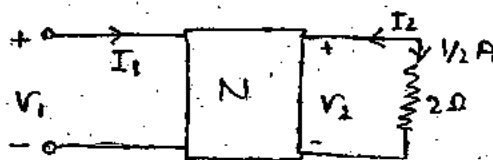
$$= \begin{bmatrix} 2 & 3 \\ 2 & 7/2 \end{bmatrix}$$

1. A 2-parameter of η/w is given as

$$[Z] = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

If the port 2 of this η/w is terminated by 2Ω . It draws a current of $1/2 A$ from the η/w . then determine the i/p supply voltage at port 1.

Soln:



$$V_1 = 4I_1 + 2I_2$$

$$V_2 = 3I_1 + I_2$$

given

$$\boxed{V_2 = \frac{1}{2} \times 2 = 1V}$$

$$\boxed{I_2 = -\frac{1}{2} A}$$

$$1 = 3I_1 - \frac{1}{2}$$

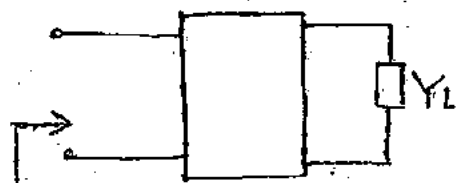
$$3I_1 = \frac{3}{2}$$

$$I_1 = \frac{1}{2} A$$

$$V_1 = 4 \times \frac{1}{2} + 2 \times -\frac{1}{2}$$

$$\boxed{V_1 = 1V} \quad \text{Ans}$$

GYRATOR:-

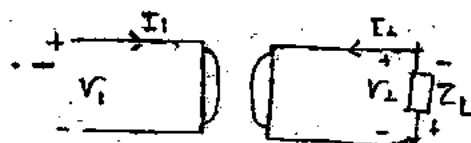


$$Z_{in} = a^2 Y_L$$

$$a \rightarrow \Omega$$

↳ gyration Resistance

It is 4-terminal device or two port device. Dirⁿ of gyration is important as it is not reciprocal device. If load is capacitive then I/P impedance inductive. It is difficult to use in IC chips because it is bulky. So we can simulate this ^{with} gyrator. It is made up of OP-amp & RC elements.



$$\frac{V_1}{I_1} = Z_{in} = a^2 Y_L = \frac{a^2}{Z_L}$$

$$\text{As } V_2 = -I_2 Z_L$$

$$\frac{V_1}{I_1} = \frac{a^2}{-V_2/I_2}$$

$$\frac{V_1}{I_1} = -\frac{I_2 a^2}{V_2} = -\frac{a I_2}{V_2/a}$$

$$V_1 = -QI_2$$

$$VI_1 = V_2/a$$

$$V_1 = -QI_2$$

$$V_2 = QI_1$$

$$T = \begin{bmatrix} 0 & Q \\ 1/a & 0 \end{bmatrix}$$

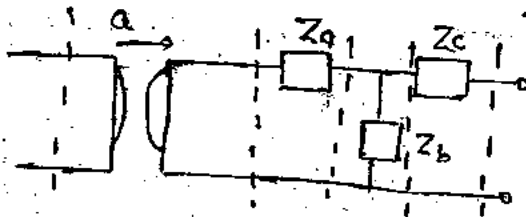
is passive and lossless.

$$V_1 I_1 + V_2 I_2 = -QI_2 \times \frac{V_2}{a} + V_2 I_2$$

$$= 0 \text{ Passive \& lossless.}$$



Find overall T-parameter of n/w shown in figure.



$$T = \begin{bmatrix} 1 & Z_g \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_b & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_c \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + Z_g/Z_b & Z_g \\ 1/Z_b & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_c \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{Z_g + Z_b}{Z_b} & \frac{Z_g Z_b + Z_b Z_c + Z_c Z_g}{Z_b} \\ 1/Z_b & \frac{Z_c + Z_b}{Z_b} \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & a \\ 1/a & 0 \end{bmatrix}$$

$$\therefore T = \begin{bmatrix} 0 & a \\ 1/a & 0 \end{bmatrix} \begin{bmatrix} \frac{Z_g + Z_b}{Z_b} & \frac{Z_g Z_b + Z_b Z_c + Z_c Z_g}{Z_b} \\ 1/Z_b & \frac{Z_c + Z_b}{Z_b} \end{bmatrix}$$

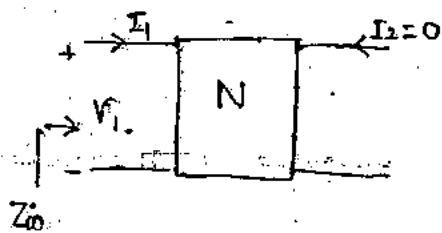
$$T = \begin{bmatrix} \frac{Q}{Z_b} & \frac{Q(Z_b + Z_c)}{Z_b} \\ \frac{Z_a + Z_b}{QZ_b} & \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{QZ_b} \end{bmatrix}$$

Short circuit & open circuit in terms of ABCD parameter

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

1. Dc i/p impedance

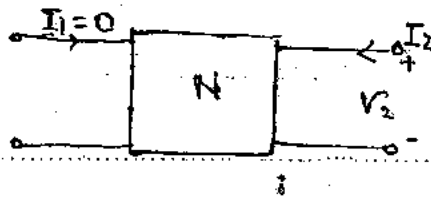


$$Z_{io} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$= \frac{AV_2 - BI_2}{CV_2 - DI_2} \Big|_{I_2=0}$$

$$\boxed{Z_{io} = \frac{A}{C}}$$

2. ac O/P impedance



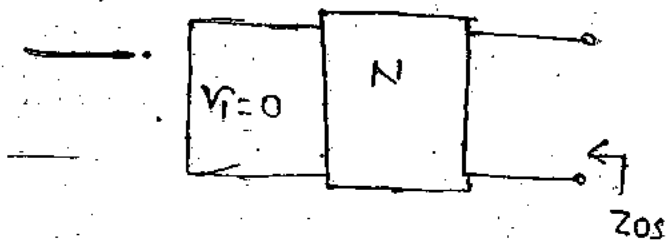
$$Z_{oo} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

$$0 = CV_2 - DI_2$$

$$\frac{V_2}{I_2} = \frac{D}{C}$$

$$\boxed{Z_{oo} = \frac{D}{C}}$$

b) S.C O/P impedance:



$$0 = AV_2 - BI_2$$

$$AV_2 = BI_2$$

$$\frac{V_2}{I_2} = \frac{B}{A}$$

$$Z_{os} = \frac{B}{A}$$

c) S.C I/P impedance:

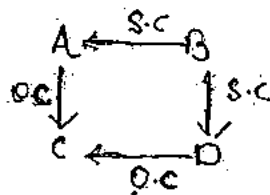


$$Z_{is} = \frac{V_1}{I_1} \Big|_{V_2=0}$$

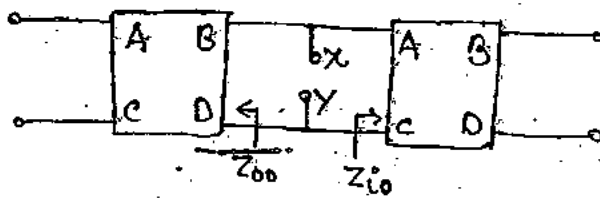
$$Z_{is} = \frac{AV_2 - BI_2}{CV_2 - DI_2} \Big|_{V_2=0}$$

$$Z_{is} = \frac{B}{D}$$

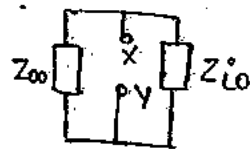
Short circuit



Q. Find the equivalent impedance b/w x & y



Soln.

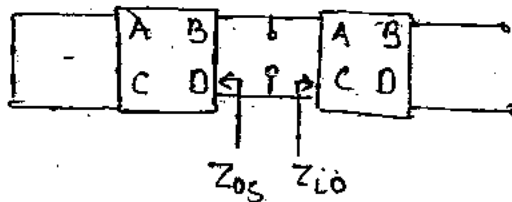


$$Z_{00} = \frac{D}{C} \quad Z_{0i} = \frac{A}{C}$$

$$Z_{xy} = Z_{00} \parallel Z_{0i}$$

$$= \frac{AD}{C(A+D)}$$

Q. Find the equivalent impedance b/w x & y

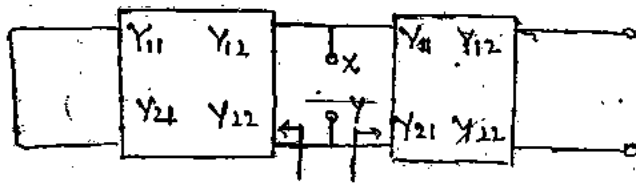


Soln.

$$Z_{0s} = \frac{B}{A} \quad Z_{0i} = \frac{A}{C}$$

$$Z_{xy} = \frac{\frac{B}{A} \cdot \frac{A}{C}}{\frac{B}{A} + \frac{A}{C}} = \frac{BA}{BC + A^2}$$

Find the equivalent impedance with x & y .



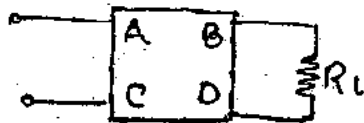
of y

$$Z_{22} = \frac{1}{Y_{22}} \quad \text{and} \quad Z_{11} = \frac{Y_{22}}{|Y|}$$

$$Z_{xy} = \frac{\frac{1}{Y_{22}} \cdot \frac{Y_{22}}{|Y|}}{\frac{1}{Y_{22}} + \frac{Y_{22}}{|Y|}}$$

$$Z_{xy} = \frac{Y_{22}}{|Y| + Y_{22}^2}$$

Find the i/p impedance



if

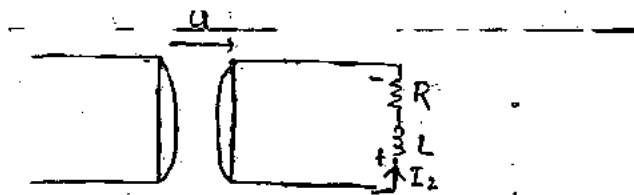
$$\frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$V_2 = -I_2 R_L \quad \text{given}$$

$$\frac{V_1}{I_1} = \frac{-AR_L - BI_2}{-CI_2 R_L - DI_2}$$

$$Z = \frac{AR_L + B}{CR_L + D}$$

Q



Find the ~~imp~~ i/p impedance.

Soln: $Z_{in} = Q^2 Y_L$
 $= \frac{Q^2}{R + j\omega L}$

2nd method

$$V_1 = -Q I_2$$

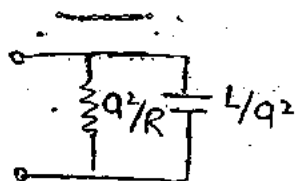
$$I_1 = V_2 / Q = -I_2 (R + j\omega L) / Q$$

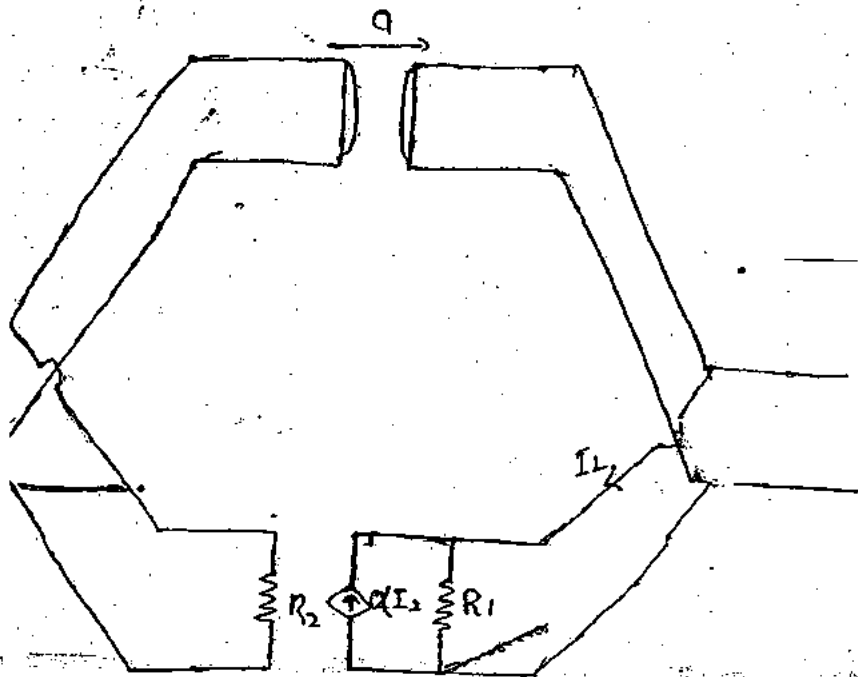
$$Z_{in} = \frac{V_1}{I_1} = \frac{-Q I_2}{-I_2 (R + j\omega L) / Q} = \frac{Q^2}{R + j\omega L}$$

$$Z_{in} = \frac{1}{\frac{R}{Q^2} + \frac{j\omega L}{Q^2}} = \frac{1}{Y_{in}}$$

$$\therefore Y_{in} = \frac{R}{Q^2} + \frac{j\omega L}{Q^2}$$

$$= G + j\omega C$$





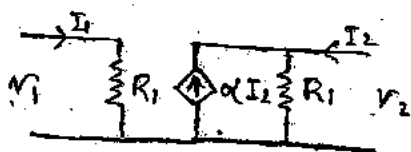
Find overall y -parameter?

Ans:-

$$V_1 = -\alpha I_2 \Rightarrow I_2 = -\frac{V_1}{\alpha}$$

$$V_2 = -\alpha I_1 \Rightarrow I_1 = -\frac{V_2}{\alpha}$$

$$Y_q = \begin{bmatrix} 0 & 1/\alpha \\ -1/\alpha & 0 \end{bmatrix}$$



$$I_1 = \frac{V_1}{R_2}$$

$$\text{and } V_2 = (I_2 + \alpha I_1) R_1$$

$$\therefore I_2 = \frac{V_2}{R_1(1+\alpha)}$$

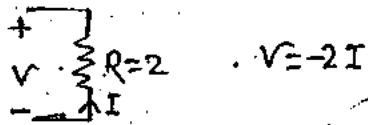
$$Y_b = \begin{bmatrix} 1/R_2 & 0 \\ 0 & 1/R_1(1+\alpha) \end{bmatrix}$$

$$Y = Y_a + Y_b$$

$$= \begin{bmatrix} \frac{1}{R_2} & \frac{1}{q} \\ -\frac{1}{q} & \frac{1}{R_1(1+\alpha)} \end{bmatrix}$$

Chapter 1

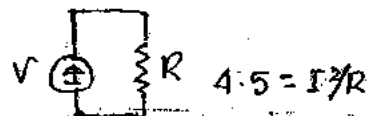
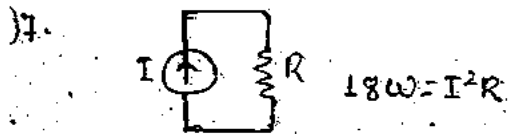
4. $V = 4I - 9$



$$-2I_1 = 4I - 9$$

$$6I_1 = 9$$

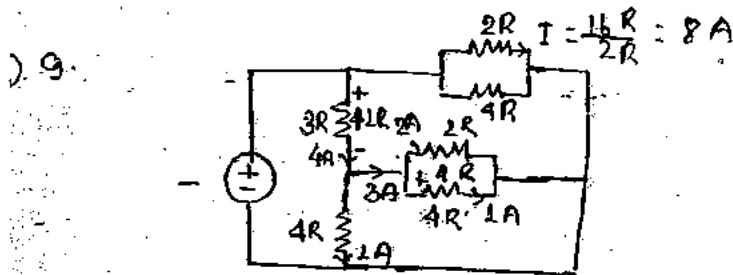
$$I_1 = 1.5 \quad (C)$$



$$I^4 = 18 \times 4.5 = 81$$

$$I = 3A$$

$$R = 2\Omega$$



Ans 8A

14.

$$V(t) = L \frac{di}{dt}$$

$$= L \frac{d}{dt} [e^{at} + e^{bt}]$$

$$= a e^{at} + b e^{bt}$$

Q15. Ans (9)

Q16 Ans: $\frac{V_x - 4}{2} + \frac{V_x}{2} + \frac{V_x + 2}{2} = 0$

$$\frac{3V_x}{2} - 1 = 0$$

$$V_x = \frac{2}{3}$$

$$V_o = -V_x$$

$$V_o = -\frac{2}{3}V \text{ Ans.}$$

Q17 Ans 80W (Nodal Analysis)

Q20 Ans: P, V, f

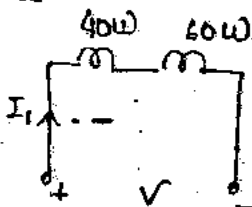
$$\uparrow P = \frac{V^2}{R \downarrow}$$

40W 60W :

$$R_{40W} < R_{60W}$$

$$R_{40} > R_{60}$$

(a) Series



$$P_1 = I^2 R_{40}$$

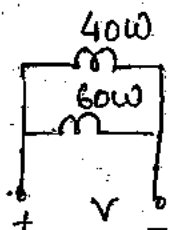
$$P_2 = I^2 R_{60}$$

$$\text{As } R_{40} > R_{60}$$

$$I^2 R_{40} > I^2 R_{60}$$

40W brighter.

parallel

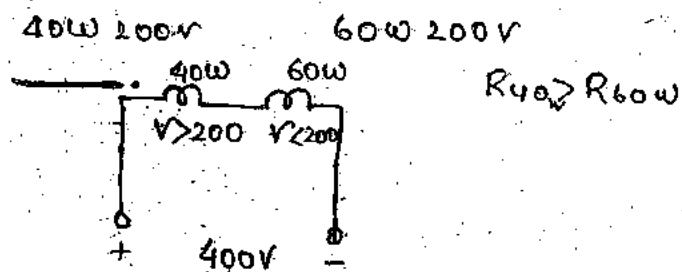


$$P_1 = \frac{V^2}{R_{40}}$$

$$P_2 = \frac{V^2}{R_{60}}$$

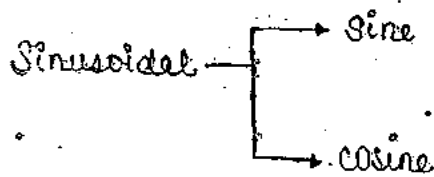
$$P_{60} > P_{40}$$

60W brighter



40W bulb destroy first.

SINUSOIDAL STEADY STATE ANALYSIS:

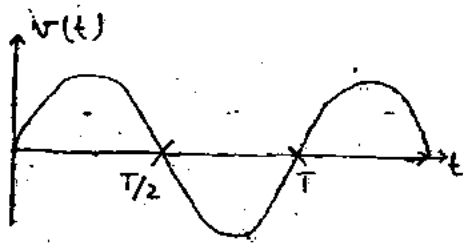
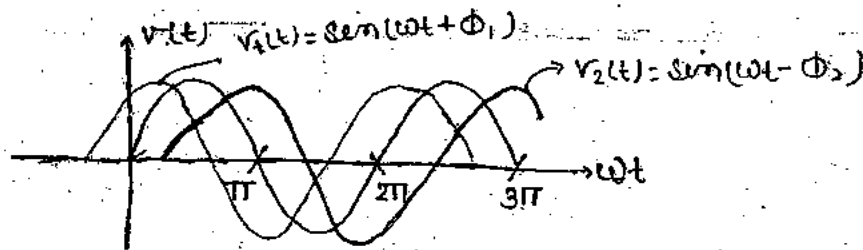


$$v(t) = V_m \sin \omega t$$

$V_m \rightarrow$ Max^m amplitude

$\omega \rightarrow$ Angular frequency (Rad/s)

$\omega t \rightarrow$ Argument



we compare phases of two sinusoidal

- ① both $v_1(t)$ and $v_2(t)$ have same frequency
- ② both $v_1(t)$ and $v_2(t)$ are expressed as either sine or cosine function.
- ③ Both $v_1(t)$ and $v_2(t)$ are written with +ve amplitude

$$v(t) = V_m \cos(\omega t + \theta)$$

$$= \operatorname{Re}\{V_m e^{j(\omega t + \theta)}\}$$

$$= \operatorname{Re}\{V_m e^{j\theta} \cdot e^{j\omega t}\}$$

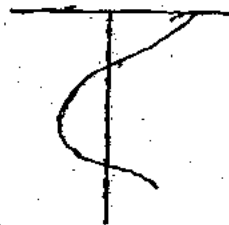
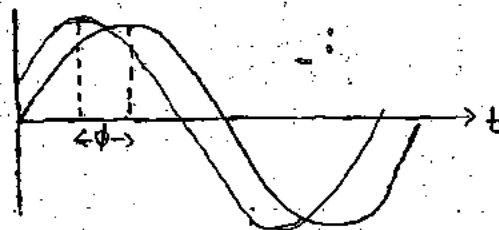
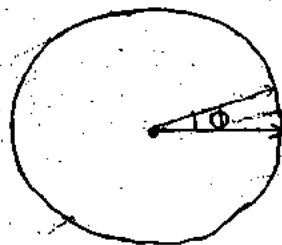
$$\boxed{V_m e^{j\theta} = V_m \angle \theta}$$

$$\text{Hints: } e^{j\theta} = \cos\theta + j\sin\theta$$

$$= \sqrt{\cos^2\theta + \sin^2\theta} \angle \tan^{-1} \frac{\sin\theta}{\cos\theta}$$

$$e^{j\theta} = 1 \angle \theta$$

By suppressing the time factor, we transform sinusoids from the time domain to the phasor domain. Thus the phasor transform sinusoidal sig from the time domain to the complex domain which is also called as frequency domain.



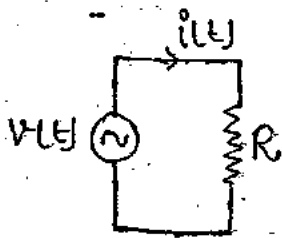
Difference b/w $v(t)$ & $\bar{v}$

$v(t)$ is the instantaneous or time domain representation while $\bar{v}$ is frequency or phasor domain.

$v(t)$ is time dependent while $\bar{v}$ is not.

$v(t)$ is generally real while $\bar{v}$ is complex.

1. Resistor:



$$v(t) = V_m \sin \omega t$$

$$i(t) = \frac{V_m}{R} \sin \omega t$$

$$P_{av} = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt$$

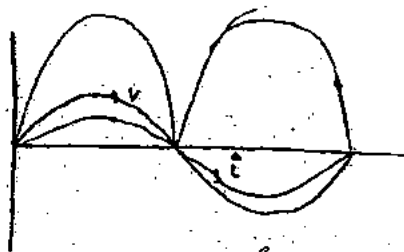
$$= \frac{1}{T} \int_0^T V_m \sin \omega t \cdot I_m \sin \omega t dt$$

$$= \frac{V_m I_m}{T} \int_0^T \sin^2 \omega t dt$$

$$= \frac{V_m I_m}{2T} \int_0^T (1 - \cos 2\omega t) dt$$

$$P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$$

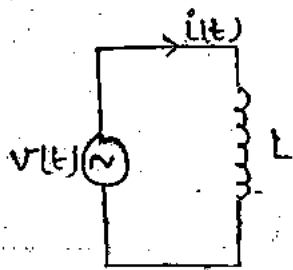
$$P_{av} = V_{rms} \cdot I_{rms}$$



$$f_{vi} = \omega$$

$$f_p = 2\omega$$

INDUCTOR:



$$i(t) = I_m \sin \omega t$$

$$v(t) = L \frac{di}{dt}$$

$$= L \frac{d(I_m \sin \omega t)}{dt}$$

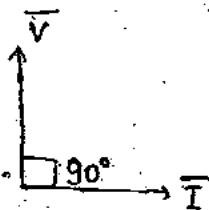
$$= L I_m \omega \cos \omega t$$

$$= L \omega I_m \sin(\omega t + \pi/2)$$

$$= L \omega I_m \sin \omega t \cdot j$$

$$\therefore v(t) = j \omega L I_m \sin \omega t$$

$$v(t) = j X_L I_m \sin \omega t$$

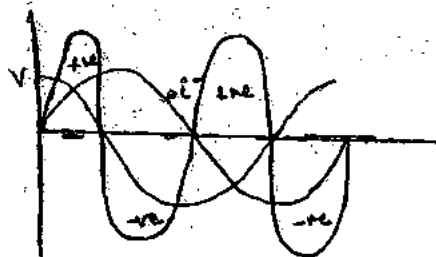


X_L = Inductive reactance

$$X_L = \omega L$$

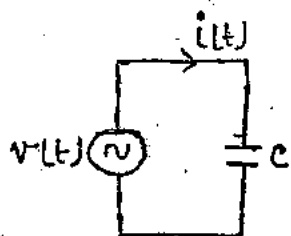
$$\begin{aligned} P_{av} &= \frac{1}{T} \int_0^T v(t) i(t) dt = \frac{1}{T} \int_0^T V_m \cos \omega t \cdot I_m \sin \omega t dt \\ &= \frac{V_m I_m}{T} \int_0^T \sin 2\omega t dt \end{aligned}$$

$$P_{av} = 0$$



$$P_p = 2 f_{v/i}$$

3. CAPACITOR:



$$v(t) = V_m \sin \omega t$$

$$i = C \frac{dv}{dt}$$

$$= C \frac{d}{dt} (V_m \sin \omega t)$$

$$= C V_m \omega \cos \omega t$$

$$= C V_m \omega \sin (\omega t + 90^\circ)$$

$$= j \omega C V_m \sin \omega t$$

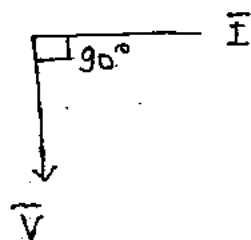
$$i = j \frac{1}{\omega C} V_m \sin \omega t$$

$$i = j \frac{V_m}{X_C} \sin \omega t$$

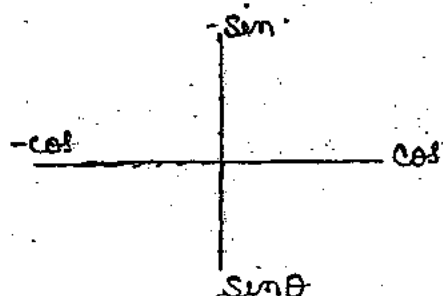
where $\frac{1}{\omega C} = X_C$ → Capacitive reactance

$$\text{Par} = 0$$

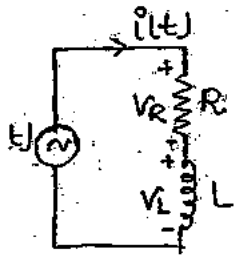
$$f_p = 2 f_{ri}$$



Hint: trigonometric conversion



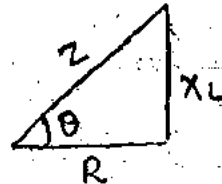
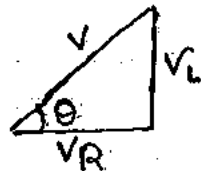
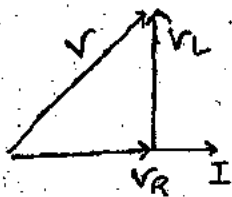
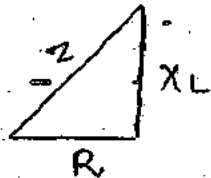
series R-L circuit:



$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$$\bar{I}Z = \bar{I}R + \bar{I}(jX_L)$$

$$Z = R + jX_L$$



$$V = \sqrt{V_R^2 + V_L^2}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1} \frac{V_L}{V_R}$$

$$\theta = \tan^{-1} \frac{X_L}{R}$$



$$S = \sqrt{P_{av}^2 + Q_L^2}$$

$$\theta = \tan^{-1} \frac{Q_L}{P_{av}}$$

$$P_{av} = \frac{1}{T} \int_0^T v(t) i(t) dt$$

$$= \frac{1}{T} \int_0^T V_m \sin(\omega t + \theta) \cdot I_m \sin \omega t dt$$

$$= \frac{V_m I_m \cos \theta}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta = V_{rms} \cdot I_{rms} \cos \theta$$

$$P_{av} = V_{rms} \cdot I_{rms} \cdot \cos \theta$$

POWER FACTOR:

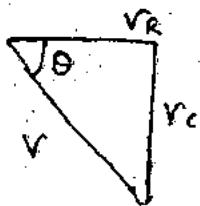
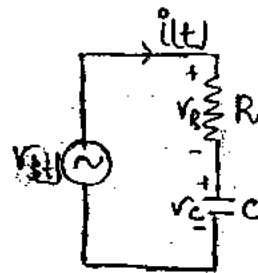
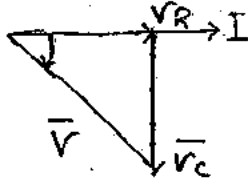
- Power factor indicates position of current phasor w.r.t. voltage phasor.
- By defining P.F, voltage is always taken as reference phasor, because real time system load connected in it.
- It tells us how much apparent power is utilized.
- If P.F is high then active power delivered to the load is high.

$$\cos \theta = \frac{V_R}{V} \text{ (lagging)} ; \cos \theta = \frac{R}{Z} \text{ (lagging)} ; \cos \theta = \frac{P}{S} \text{ (lagging)}$$

Series RC circuit:-

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

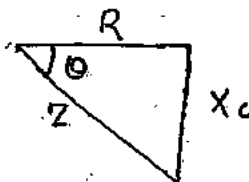
$$IZ = IR - jX_C I$$



$$V = \sqrt{V_R^2 + V_C^2}$$

$$\tan \theta = \frac{V_C}{V_R}$$

$$\cos \theta = \frac{V_R}{V} \text{ (leading)}$$

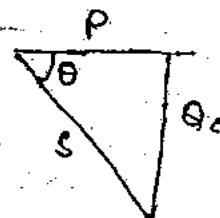


$$Z = R - jX_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$\tan \theta = -\frac{X_C}{R}$$

$$\cos \theta = \frac{R}{Z} \text{ (leading)}$$

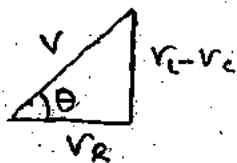
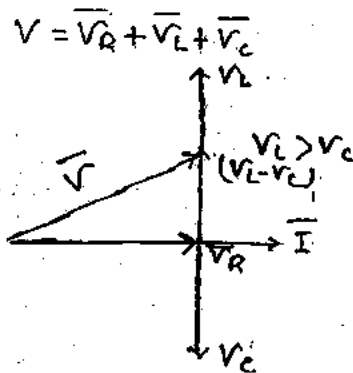
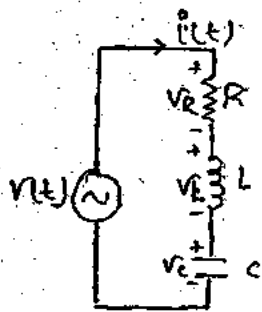


$$S = \sqrt{P^2 + Q_C^2}$$

$$\tan \theta = \frac{Q_C}{P}$$

$$\cos \theta = \frac{P}{S} \text{ (leading)}$$

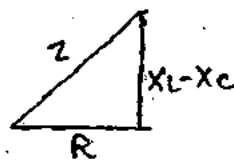
Series RLC circuit:-



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan \theta = \frac{V_L - V_C}{V_R}$$

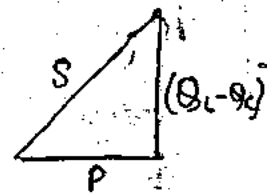
$$\cos \theta = \frac{V_R}{V} \text{ (lagging)}$$



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan \theta = \frac{X_L - X_C}{R}$$

$$\cos \theta = \frac{R}{Z} \text{ (lagging)}$$

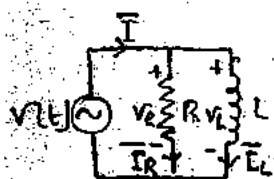


$$S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\tan \theta = \frac{Q_L - Q_C}{P}$$

$$\cos \theta = \frac{P}{S} \text{ (lagging)}$$

II RL circuit:

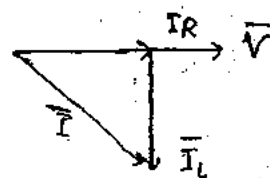


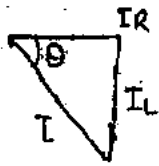
$$\bar{I} = \bar{I}_R + \bar{I}_L$$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{jX_L}$$

$$\frac{1}{Z} = \frac{1}{R} - \frac{j}{X_L}$$

$$Y = G - jB_L$$





$$I = \sqrt{I_R^2 + I_L^2}$$

$$\tan \theta = \frac{I_L}{I_R}$$

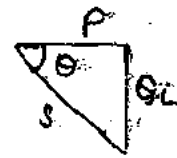
$$\cos \theta = \frac{I_R}{I} \text{ (lagging)}$$



$$Y = \sqrt{G^2 + B_L^2}$$

$$\tan \theta = \frac{B_L}{G}$$

$$\cos \theta = \frac{G}{Y} \text{ (lagging)}$$

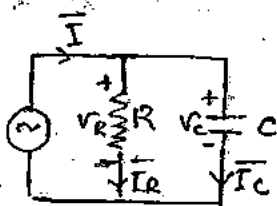


$$S = \sqrt{P^2 + Q_L^2}$$

$$\tan \theta = \frac{Q_L}{P}$$

$$\cos \theta = \frac{P}{S} \text{ (lagging)}$$

II RC Circuit:

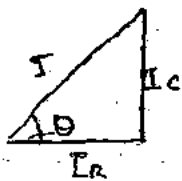
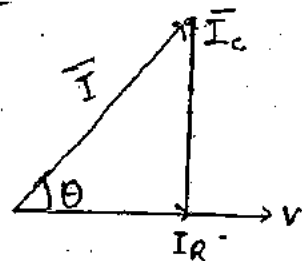


$$\bar{I} = \bar{I}_R + \bar{I}_C$$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{-jX_C}$$

$$\frac{1}{Z} = \frac{1}{R} + j\frac{1}{X_C}$$

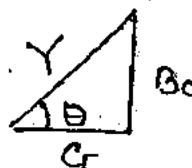
$$Y = G + jB_C$$



$$I = \sqrt{I_R^2 + I_L^2}$$

$$\tan \theta = \frac{I_L}{I_R}$$

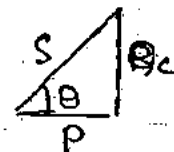
$$\cos \theta = \frac{I_R}{I} \text{ (leading)}$$



$$Y = \sqrt{G^2 + B_C^2}$$

$$\tan \theta = \frac{B_C}{G}$$

$$\cos \theta = \frac{G}{Y} \text{ (leading)}$$

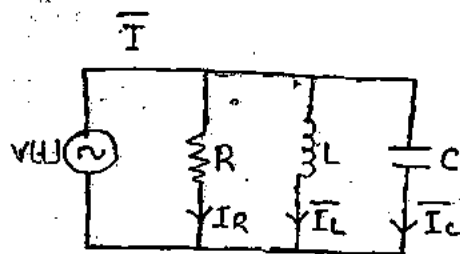


$$S = \sqrt{P^2 + Q_C^2}$$

$$\tan \theta = \frac{Q_C}{P}$$

$$\cos \theta = \frac{P}{S} \text{ (leading)}$$

Parallel R-L-C circuit:

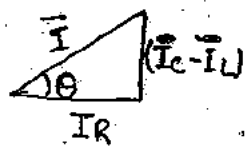
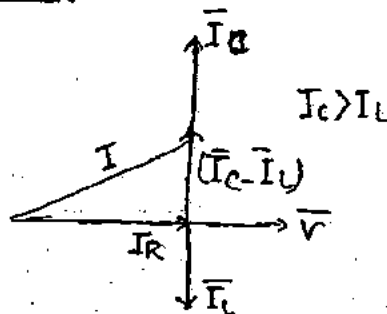


$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C}$$

$$\frac{1}{Z} = \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

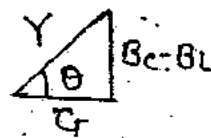
$$Y = G + j(B_C - B_L)$$



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\tan \theta = \frac{I_C - I_L}{I_R}$$

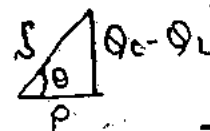
$$\cos \theta = \frac{I_R}{I}$$



$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$

$$\tan \theta = \frac{B_C - B_L}{G}$$

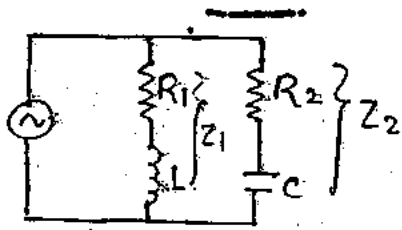
$$\cos \theta = \frac{G}{|Y|}$$



$$S = \sqrt{P^2 + (Q_C - Q_L)^2}$$

$$\tan \theta = \frac{Q_C - Q_L}{P}$$

$$\cos \theta = \frac{P}{S}$$



$$Z_1 = R_1 + jX_L$$

$$Y_1 = \frac{1}{R_1 + jX_L} \times \frac{R_1 - jX_L}{R_1 - jX_L}$$

$$= \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j \frac{X_L}{R_1^2 + X_L^2}$$

$$Y_1 = G_1 - jB_L$$

Similarly

$$Z_2 = R_2 - jX_C$$

$$Y_2 = \frac{1}{R_2 - jX_C} \times \frac{R_2 + jX_C}{R_2 + jX_C}$$

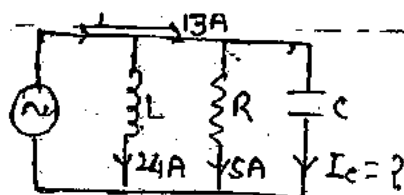
$$= \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j \frac{X_C}{R_2^2 + X_C^2}$$

$$Y_2 = G_2 + jB_C$$

Total admittance of η/w

$$Y = Y_1 + Y_2$$

$$= (G_1 + G_2) + j(B_C - B_L)$$



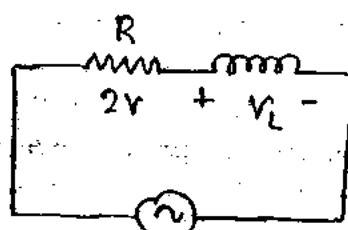
Find I and I_C ?

17.

$$13 = \sqrt{5^2 + I_C^2}$$

$$\therefore I_C = \sqrt{13^2 - 5^2} = 12 \text{ A}$$

$$I = \sqrt{5^2 + (12-24)^2} = 13 \text{ A}$$



Find the voltage across inductor?

20V (p.p)

Soln:

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_L = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{20^2 - 10^2} = \sqrt{300} = 17.32 \text{ V}$$

$$2 V_m = 20 \text{ V}$$

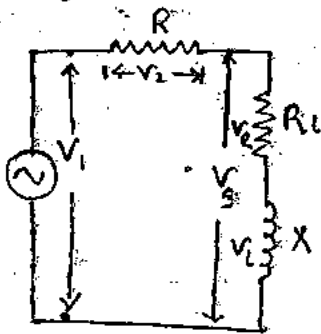
$$V_m = 10 \text{ V}$$

$$V_{rms} = \frac{10}{\sqrt{2}}$$

$$V_L = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 - 2^2} = \sqrt{50 - 4} = \sqrt{46} \text{ Volts}$$

Workbook Page 10:

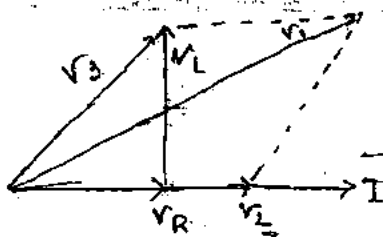
470.



① $V_1 = 220V$ $V_2 = 112V$ $V_3 = 136V$

$$Z = R_L + jX_L$$

$$\vec{V}_1 = \vec{V}_2 + \vec{V}_3$$



$$V_1 = \sqrt{V_2^2 + V_3^2 + 2V_2V_3\cos\phi}$$

$$(220)^2 = (112)^2 + (136)^2 + 2(112)(136)\cos\phi$$

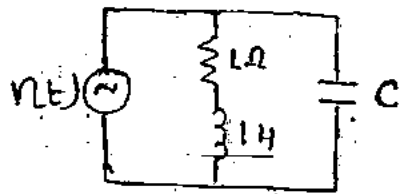
$$\cos\phi = 0.45$$

② $\cos\phi = \frac{V_R}{V_3}$

$$V_R = 0.45 \times 136V = 61.2V$$

$$P = \frac{V_R^2}{R_L} = \frac{(61.2)^2}{5} = 750W$$

7. Find the value of 'C' when P.F. of the circuit is 0.8 lagging.



$$v(t) = \sin t$$

Soln:

$$Y_1 = G_1 - jB_1$$

$$G_1 = \frac{R_1}{R_1^2 + X_L^2}$$

$$X_L = \omega L = 1 \times 1 = 1$$

$$G_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$B_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$Y_1 = \frac{1}{2} - j\frac{1}{2}$$

$$Y_2 = j\omega C = jC$$

$$Y = Y_1 + Y_2$$

$$= \frac{1}{2} + j\left(C - \frac{1}{2}\right)$$

$$\cos \phi = \frac{G}{|Y|}$$

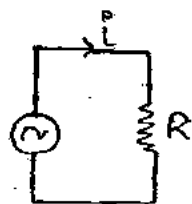
$$0.8 = \frac{1/2}{\sqrt{\frac{1}{4} + \left(C - \frac{1}{2}\right)^2}}$$

$$C = \frac{1}{8}, \frac{7}{8}$$

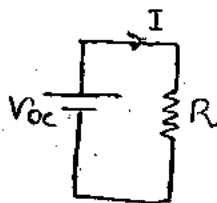
$$C = \frac{1}{8} \text{ F lagging.}$$

— RMS value: RMS value is defined based on heating effect
— of the waveform.

- The voltage at which heat dissipation in a.c. ckt is equal to heat dissipated in d.c. ckt. is called as RMS value. provided both a.c. & d.c. have equal value of R and operated for same time



$$W_{ac} = I^2 R t$$



$$W_{dc} = I^2 R t$$

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2 dt} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2 d\omega t}$$

AVERAGE VALUE:-

- It is based on the charge transfer in ckt.
- The voltage at which charge transfer in a.c. ckt is equal to charge transfer in d.c. ckt is called average value. provided both a.c. & d.c. have same value of R and operated for same time.

Symmetric

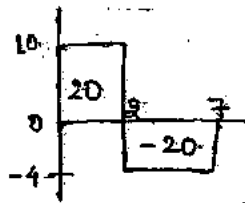
$$V_{av} = \frac{1}{T/2} \int_0^{T/2} v dt \quad \text{Half cycle}$$

$$= 0 \rightarrow \text{Full cycle}$$

nsymmetric

$$Var = \frac{1}{T} \int_0^T v dt$$

* $|A_1| = |A_2|$ then the waveform is symmetric *

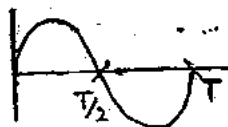


m Factor:

is the ratio of r.m.s value of waveform to the average value of waveform.

ak factor:

is the ratio of max^m value of waveform to the r.m.s value of waveform.

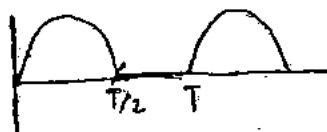


RMS

$$\frac{V_m}{\sqrt{2}}$$

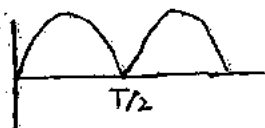
Average value

$$\frac{2V_m}{\pi}$$



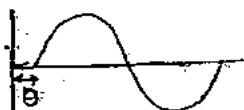
$$\frac{V_m}{2}$$

$$\frac{V_m}{\pi}$$



$$\frac{V_m}{\sqrt{2}}$$

$$\frac{2V_m}{\pi}$$



$$\frac{V_m}{\sqrt{2}}$$

$$\frac{2V_m}{\pi}$$

$$V_{rms} = \sqrt{\frac{1}{10} \left[\int_0^5 10^2 dt + \int_5^{10} \left(-\frac{2}{5}t + 2\right)^2 dt \right]} = 7.118$$

$$Var = \frac{1}{T} \int_0^T v dt = \frac{1}{10} \left[\int_0^5 10 dt + \int_5^{10} \left(-\frac{2}{5}t + 2\right) dt \right] = 4.5 \text{ Avg}$$

$v(t) = v_0 + v_1 \sin \omega t + v_2 \sin 3\omega t$ (for diffⁿ frequency)

$$Var = v_0$$

$$V_{rms} = \sqrt{v_0^2 + v_{rms2}^2 + \dots + v_{rmsn}^2}$$

$$= \sqrt{v_0^2 + \left(\frac{v_1}{\sqrt{2}}\right)^2 + \left(\frac{v_3}{\sqrt{2}}\right)^2}$$

$v(t) = v_0 + v_1 \sin(\omega t + \phi_1) + v_2 \sin(\omega t + \phi_2)$ (For same frequency)

$\downarrow$ $\downarrow$
 $v_1 \angle \phi_1$ $v_2 \angle \phi_2$

$$v(t) = v_0 + (v_1 \angle \phi_1 + v_2 \angle \phi_2)$$

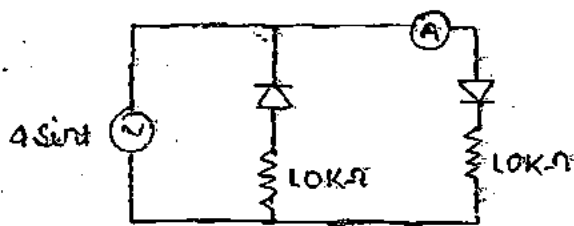
$$= v_0 + v \angle \phi = v_0 + v \sin(\omega t + \phi)$$

$$V_{rms} = \sqrt{v_0^2 + \left(\frac{v}{\sqrt{2}}\right)^2}$$

* $P_{av} = \frac{V_{rms}^2}{R}$

* $P_{dc} = I_{av}^2 R$

* $P_{ac} = I_{rms}^2 R$



The given ckt having ideal diodes and ammeter which reads the average value. Find reading of the ammeter.

Soln: $i = \frac{v(t)}{10k} = \frac{4 \sin t}{10k} = 0.4 \sin t \text{ mA}$

$$I_{av} = \frac{0.4}{\pi} \text{ mA}$$

Q. Find the power of an element, if the voltage across it

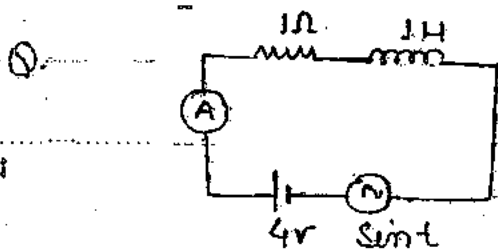
$$v(t) = 100 \sin \omega t + 40 \cos(3\omega t - 30^\circ) + 5 \sin(5\omega t + 45^\circ)$$

and current through it, $i(t) = 8 \sin(\omega t + 10^\circ) + 6 \sin(3\omega t + 40^\circ) + 2 \cos(5\omega t - 120^\circ)$

Soln: $v(t) = 100 \sin \omega t + 40 \sin(3\omega t + 60^\circ) + 5 \sin(5\omega t + 45^\circ)$
 $i(t) = 8 \sin(\omega t + 10^\circ) + 6 \sin(3\omega t + 40^\circ) + 2 \sin(5\omega t - 30^\circ)$

$$P_{av} = \frac{100}{\sqrt{2}} \times \frac{8}{\sqrt{2}} \cos 10^\circ + \frac{40}{\sqrt{2}} \times \frac{6}{\sqrt{2}} \cos 20^\circ + \frac{5}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \cos 75^\circ$$

$$= 507.99 \text{ W}$$



Find the reading of Ammeter and power factor of ckt.

Soln: Case I : 4V

As DC $L \rightarrow S.C$

$$I_{DC} = \frac{4}{1} = 4 \text{ A}$$

Case 2: Sin t

$$Z = R + j\omega L$$

$$= 1 + j1$$

$$I_{ac} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2} A$$

$$I_{rms} = \sqrt{I_{rms1}^2 + I_{rms2}^2} \quad (\text{Current through Ammeter})$$

$$= \sqrt{4^2 + (1/2)^2} = \sqrt{16.25} A$$

$$\cos \phi = \frac{P}{S} = \frac{I^2 R}{VI} = \frac{I_{rms} \cdot R}{V_{rms}}$$

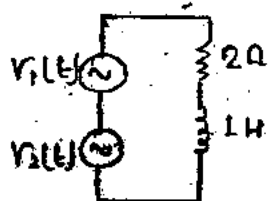
$$V_{rms} = \sqrt{4^2 + (1/2)^2}$$

$$= \sqrt{16.5} V$$

$$\cos \phi = \frac{\sqrt{16.25} \cdot 1}{\sqrt{16.5}}$$

$$\cos \phi = 0.9923$$

$$\phi = 7.07^\circ$$



$$V_1(t) = 10 \sin 4t$$

$$V_2(t) = 20 \sin 2t$$

Find average power in the circuit

$$\text{Case I } V(t) = 10 \sin 4t$$

$$Z = R + j\omega L$$

$$= 2 + j4$$

$$I_1 = \frac{10 \sin 4t}{4.47 \angle 63.43^\circ} = \frac{10 \angle 0}{4.47 \angle 63.43^\circ} = 2.23 \sin(\omega t - 63.43^\circ)$$

case 2: $V_2(t) = 20 \sin 2t$

$$Z = 2 + j2$$

$$= 2\sqrt{2} \angle 45^\circ$$

$$I_2(t) = \frac{20 \angle 0^\circ}{2\sqrt{2} \angle 45^\circ}$$

$$= 7.07 \angle -45^\circ$$

$$i_2(t) = 7.07 \sin(\omega t - 45^\circ)$$

$$P = \frac{10}{\sqrt{2}} \times \frac{2.23}{\sqrt{2}} \cos 63.43^\circ + \frac{20}{\sqrt{2}} \times \frac{7.07}{\sqrt{2}} \cos 45^\circ$$

$$= 55 \text{ W} \quad \underline{\text{Ans}}$$

Complex power:

$$S = VI^*$$

$$S = |S| \angle \theta$$



Apperant power

Q. ~~$V(t) = 60$~~ The voltage across load is $V(t) = 60 \cos(\omega t - 10^\circ)$ and the current in the dirⁿ of voltage drop $i(t) = 1.5 \cos(\omega t + 50^\circ)$. Find complex and apperant power and real & reactive power?

Soln: $V(t) = 60 \angle -10^\circ$, $I(t) = 1.5 \angle 50^\circ$

$$S = \frac{60}{\sqrt{2}} \angle -10^\circ \times \frac{1.5}{\sqrt{2}} \angle -50^\circ$$

$$= 45 \angle -80^\circ$$

$$|S| = 45 \text{ VA}$$

$$S = 45 \angle -60^\circ$$

$$= 22.5 - j38.97$$

$$S = P + jQ$$

$$P = 22.5 \text{ W}$$

$$Q = -38.97 \text{ VAR}$$

THEOREMS:

Superposition theorems:

$$\begin{array}{c} R \\ \text{---} \text{---} \text{---} \\ \text{I} \quad \text{V} \end{array} \quad V = I R$$

$$\begin{array}{c} R \\ \text{---} \text{---} \text{---} \\ \text{I}_2 \quad \text{V}_2 \end{array} \quad V_2 = I_2 R$$

$$\begin{array}{c} R \\ \text{---} \text{---} \text{---} \\ \alpha I \end{array} \quad V'' = (\alpha I) R$$

$\underbrace{\quad}_{\alpha V}$

$$\begin{array}{c} R \\ \text{---} \text{---} \text{---} \\ \text{I}_1 \quad \text{V}_1 \end{array} \quad V_1 = I_1 R$$

$$\begin{array}{c} R \\ \text{---} \text{---} \text{---} \\ \text{I}_1 + \text{I}_2 \end{array} \quad V' = (I_1 + I_2) R$$

$$= I_1 R + I_2 R$$

$$= V_1 + V_2$$

(Principle of Additivity)

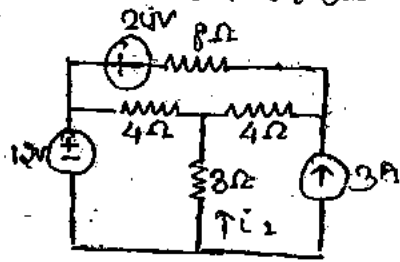
In linear bilateral n/w containing more than one independent voltage & current source, The response in any element is the sum of response obtained with one source acting at a time and other source being deactivated.

Deactivation means all independent sources is replace by their internal resistance. voltage source by s.c. and current source replace by o.c.

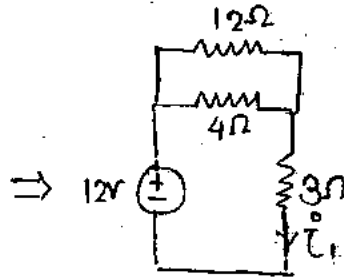
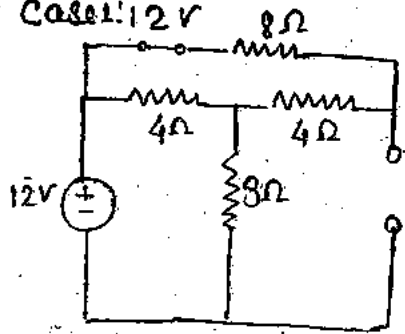
N.B Dependent source is as remain as it is.

Superposition theorem not applicable for power calculation.

Q. Find the current in 3Ω resistor?

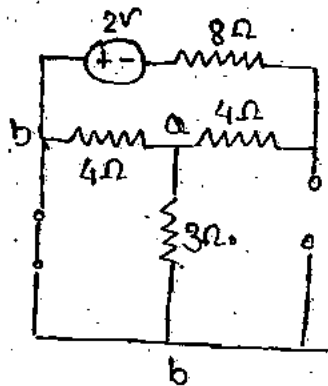


Soln: - Case 1: 12V

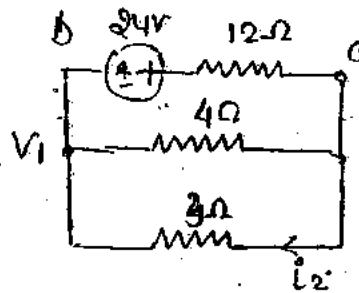


$$i_1 = \frac{12}{6} = 2A$$

Case 2: 24V



$\Rightarrow$



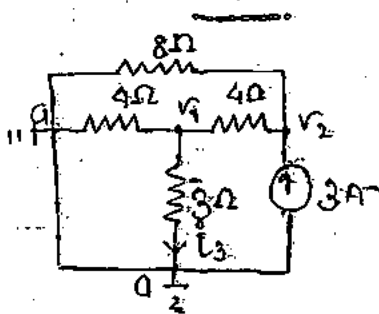
$$\frac{V_1 - 24}{12} + \frac{V_1}{4} + \frac{V_1}{3} = 0$$

$$V_1 = 3V$$

$$i_2 = \frac{0 - V_1}{3} = \frac{0 - 3}{3} = -1A$$

∴ when ever find current in the ckt always prefer Superposition theorem.

Case 3 3A



NOTE: Point a is equipotential.
Take the both point as reference.

$$\frac{V_1}{4} + \frac{V_1}{3} + \frac{V_1 - V_2}{4} = 0$$

$$V_1 \times \left[\frac{10}{12} \right] = \frac{V_2}{4}$$

$$V_2 = \frac{10}{3} V_1 \quad \text{--- (1)}$$

$$\frac{V_2}{8} + \frac{V_2 - V_1}{4} = 3$$

$$V_2 + 2V_2 - 2V_1 = 24$$

$$3 \left[\frac{10}{3} V_1 \right] - 2V_1 = 24$$

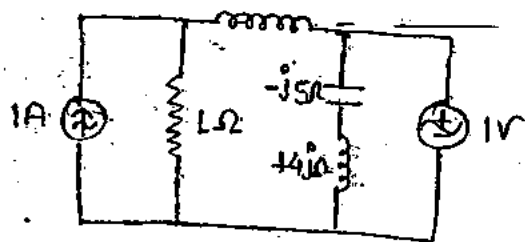
$$V_1 = 3V$$

$$I_3 = \frac{3}{3} = 1A$$

By using Superposition theorem

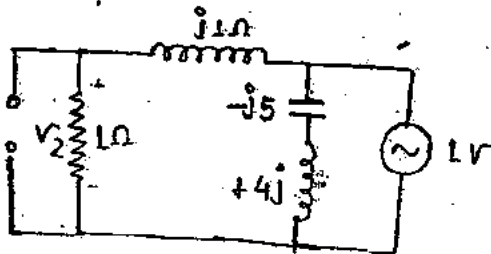
$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 2 - 1 + 1 = 2A \quad \text{Ans.} \end{aligned}$$

N.B For incoming current take -ve sign.
For outgoing current take +ve sign.



calculate voltage drop across 1Ω resistor...

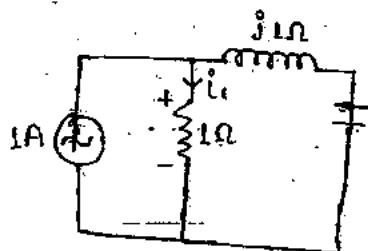
47: Case: $1V$



$$V_2 = \frac{j}{1+j} \times 1$$

$$V_2 = \frac{j}{1+j} \text{ Volts}$$

Case: $1A$



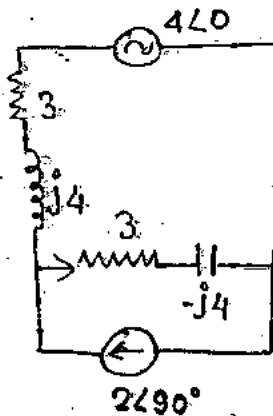
$$i_1 = \frac{1 \times j}{1+j}$$

$$V_1 = \frac{j}{1+j} \times 1 = \frac{j}{1+j} \text{ Volts}$$

$$V = V_1 + V_2$$

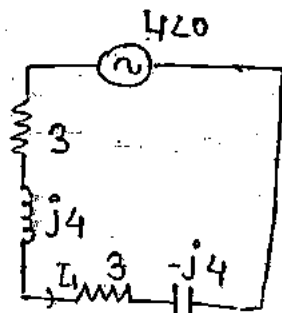
$$= \frac{j}{1+j} + \frac{j}{1+j} = 1 \text{ Volts}$$

Q.



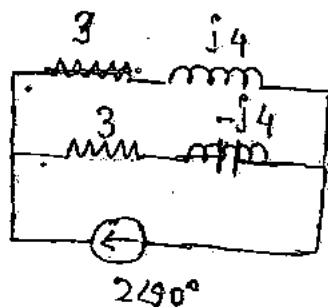
Determine using superposition theorem.

Solⁿ: Case I $4\angle 0$



$$I_1 = \frac{4\angle 0}{6 + j(4-4)} = \frac{4}{6} \angle 0 = \frac{2}{3} \text{ A}$$

Case $2\angle 90^\circ$



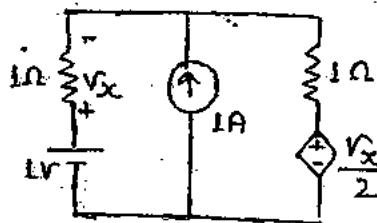
$$I_2 = 2\angle 90^\circ \cdot \frac{3 + j4}{3 + j4 + 3 - j4}$$

$$= \frac{-4 + j3}{3}$$

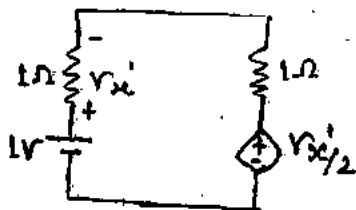
$$I = I_1 + I_2$$

$$= \frac{2}{3} - \frac{4}{3} + j$$

$$= -\frac{2}{3} + j \text{ A}$$



Case I: 1V



$$-1 + I_x \times 1 + I_x \times 1 + \frac{V_{x'}}{2} = 0$$

$$-1 + V_{x'} + I_x \times 1 + \frac{V_{x'}}{2} = 0$$

$$2I_x = 1 - \frac{V_{x'}}{2}$$

$$2I_x = 1 - \frac{V_{x'}}{2}$$

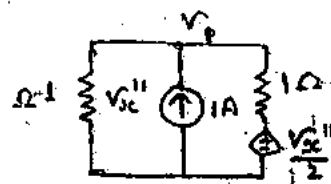
$$V_{x'} = 1$$

$$2V_{x'} = 1 - \frac{V_{x'}}{2}$$

$$4V_{x'} = 2 - V_{x'}$$

$$V_{x'} = \frac{2}{5} \text{ V}$$

Case II: 1A



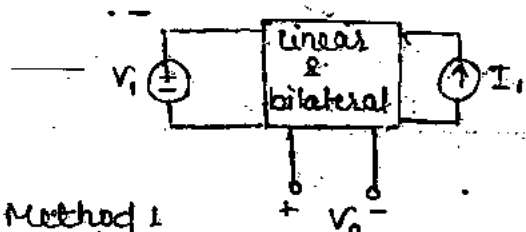
$$\frac{V_1}{1} + \frac{V_1 - V_{x''}}{1} = 0$$

$$2V_1 - \frac{V_{x''}}{2} = 0$$

Ex:

$$V_1 = 5V \quad I_1 = 0A \quad V_0 = 3V$$

$$V_1 = 0 \quad I_1 = 2A \quad V_0 = 6V$$



Find the O/P voltage when $V_1 = 10V$ and $I_1 = 8A$

Method 1

Soln:

$$\begin{aligned} \textcircled{1} \quad V_1 &= 5V & V_0 &= 3V \\ &\downarrow (2) & &\downarrow (2) \\ V_1 &= 10 & V_{01} &= 6V \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad I_1 &= 2A & V_0 &= 6V \\ &\downarrow (4) & &\downarrow (4) \\ I_1 &= 8A & V_{02} &= 24V \end{aligned}$$

$$\begin{aligned} V_0 &= V_{01} + V_{02} \\ &= 6 + 24 = 30V \end{aligned}$$

Method 2:

$$V_0 = \alpha V_1 + \beta I_1$$

$$3 = 5\alpha + \beta(0)$$

$$\alpha = \frac{3}{5}$$

$$V_0 = \alpha V_1 + \beta I_1$$

$$6 = 0 + 2\beta$$

$$\beta = 3$$

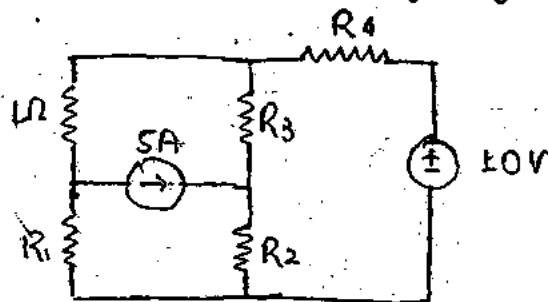
$$V_0 = \alpha V_1 + \beta I_1$$

$$= \frac{3}{5} \times 10 + 3 \times 8$$

$$= 30V \text{ Ans}$$

in n/w having linear and bilateral n/w then based the homogeneity principle if excitation (source) is multiply constant (K), then response of each element is also multiply with constant K

Power dissipation in 10 resistor is 576W, when the voltage source acting alone and the power dissipation is 1W, when current source acting alone. Find the total power dissipation in 10 resistor when both source acting together?



Formula:

$$P_0 = (\pm \sqrt{P_1} \pm \sqrt{P_2} \pm \sqrt{P_3} \pm \dots \pm \sqrt{P_n})^2$$

$$P_1 = I^2 R$$

$$I_1 = \sqrt{\frac{P_1}{R}}$$

$$P_2 = i^2 R$$

$$I_2 = \sqrt{\frac{P_2}{R}}$$

$$I = \pm I_1 \pm I_2$$

$$P = I^2 R$$

$$P = (\pm I_1 \pm I_2)^2 R$$

$$P = \left(\pm \sqrt{\frac{P_1}{R}} \pm \sqrt{\frac{P_2}{R}} \right)^2 R$$

$$\begin{aligned}
 P &= (\pm \sqrt{P_1} \pm \sqrt{P_2})^2 \\
 &= (\pm \sqrt{576} \pm \sqrt{1})^2 \\
 &= 625 \text{ W} \quad \text{Ans}
 \end{aligned}$$

Chapter 7:

Q8. $P = (\pm \sqrt{P_1} \pm \sqrt{P_2} \pm \sqrt{P_3})^2$

For max^m power

$$\begin{aligned}
 P &= (\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3})^2 \\
 &= (\sqrt{18} + \sqrt{50} + \sqrt{98})^2 = 475 \text{ W}
 \end{aligned}$$

For min^m power

$$\begin{aligned}
 P &= (\sqrt{P_1} - \sqrt{P_2} - \sqrt{P_3})^2 \\
 &= (\sqrt{98} - \sqrt{50} - \sqrt{18})^2 = 2 \text{ W}
 \end{aligned}$$

Chapter 4

Q 4:

$$-V_1 - V_0 + V_R = 0$$

$$V_0 = V_R - V_1$$

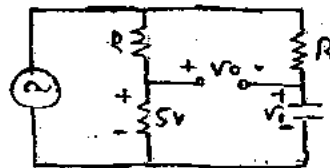
$$V_1 = \frac{10 \times (-jX_C)}{R - jX_C}$$

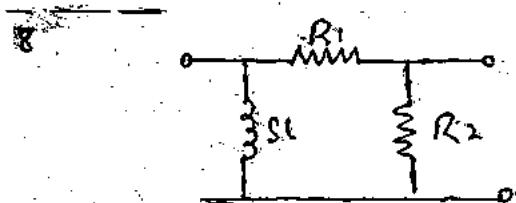
$$V_0 = 5 + \frac{j10X_C}{R - jX_C}$$

$$= \frac{5R + j5X_C}{R - jX_C}$$

$$V_0(j\omega) = 5 \left[\frac{R + jX_C}{R - jX_C} \right]$$

$$|V_0(j\omega)| = 5 \frac{\sqrt{R^2 + X_C^2}}{\sqrt{R^2 + X_C^2}} = 5 \text{ V}$$





$$Z_{11} = \frac{SL(R_1 + R_2)}{SL + R_1 + R_2}$$

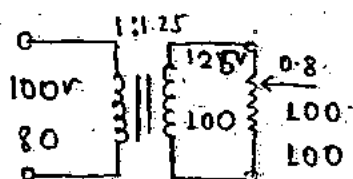
$$= \frac{S(R_1 + R_2)}{S + \frac{R_1 + R_2}{L}}$$

$$Z_{22} = \frac{R_2(R_1 + SL)}{R_1 + R_2 + SL}$$

$$= \frac{R_2(S + R_1/L)}{S + \frac{R_1 + R_2}{L}}$$

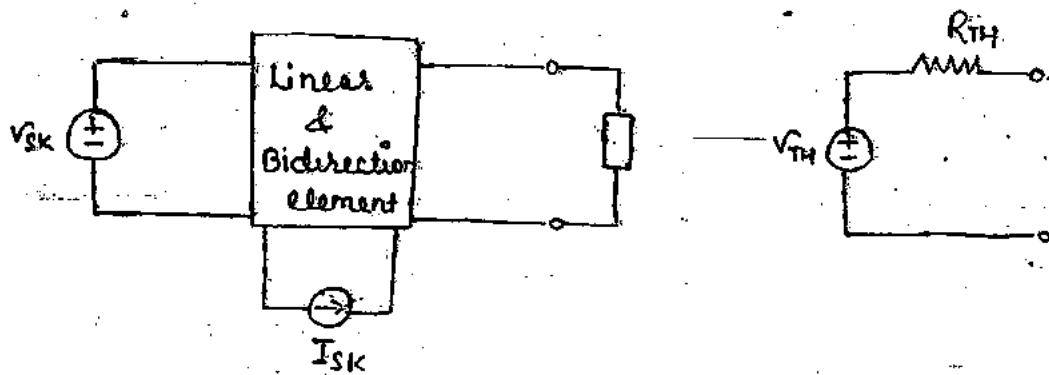
→ (c) Ans

2.



$$\frac{100}{100} \text{ and } \frac{80}{100} \quad \text{Ans}$$

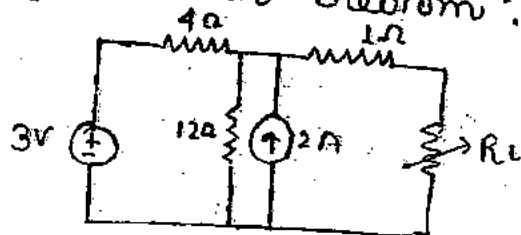
THEVENIN THEOREM: (HELMHOLTZ)



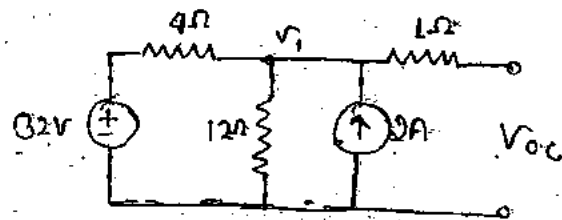
A two terminal n/w N_A containing linear & bidirectional element and independent source is equivalent to a single n/w containing an equivalent voltage source V_{th} in series with the resistance R_{th} .

The source voltage is the O.C voltage across the terminal of n/w N_A and Resistance R_{th} is the O/P terminal of N_A measured at terminal after deactivating all independent source in N/w (N_A).

Q Find the current through R_L when R_L is 6Ω and 36Ω by using thevenin theorem.



an: case V_{TH}



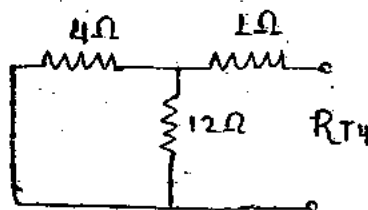
$$\frac{V_1 - 32}{4} + \frac{V_1}{12} - 3 = 0$$

$$3V_1 - 96 + V_1 = 24$$

$$4V_1 = 120$$

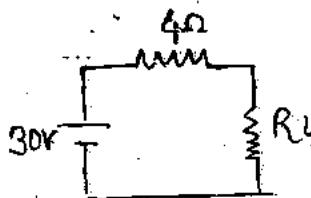
$$V_1 = V_{OC} = 30V$$

case R_{TH}



$$R_{TH} = (12 \parallel 4) + 1 \Omega$$

$$= 3 + 1 = 4 \Omega$$



$$I_b = \frac{30}{4 + R_L}$$

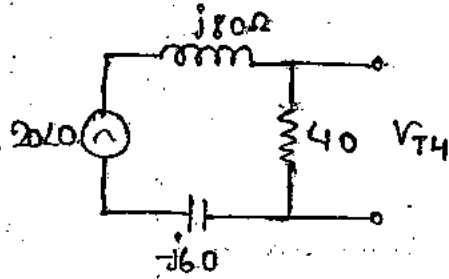
(a) $R_L = 6 \Omega$

$$I = \frac{30}{10} = 3A$$

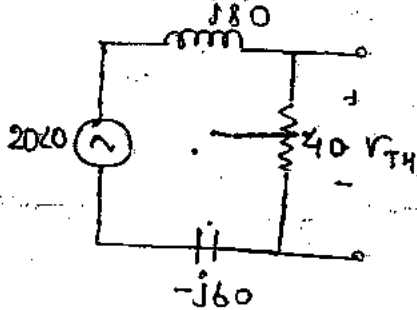
(b) $R_L = 36 \Omega$

$$I = \frac{30}{40} = 0.75A$$

Q. Find V_{TH} and R_{TH}



Soln: Case: V_{TH}



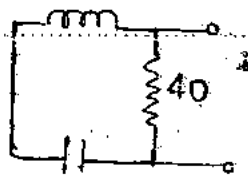
$$I = \frac{20\angle 0}{40 + j20}$$

$$= \frac{1\angle 0}{2 + j} = 0.447\angle -26.5^\circ$$

$$V_{TH} = 40 \times 0.447\angle -26.5^\circ$$

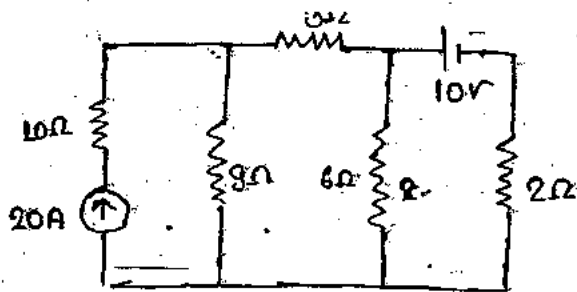
$$= 17.8\angle -26.5^\circ$$

Case R_{TH}



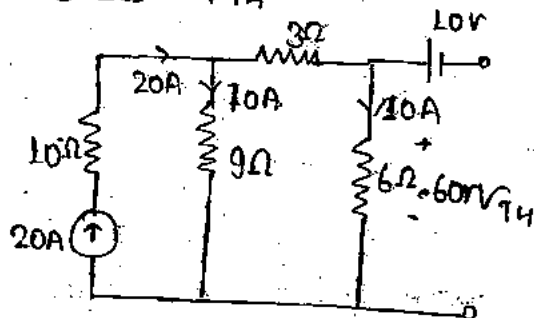
$$R_{TH} = 40 \parallel (j80 - j60)$$

$$= \frac{j20 \times 40}{40 + j20} = \frac{j40}{2 + j}$$



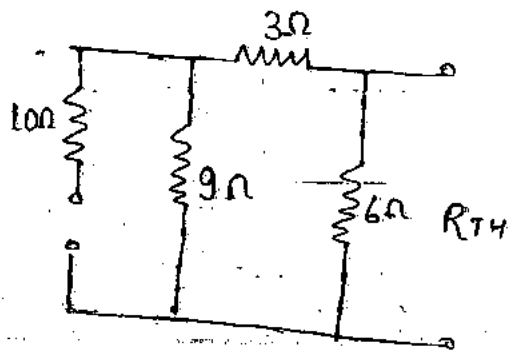
Find current flowing through 2Ω resistor using thevenin theorem

Soln: Case 1: V_{TH}



$$V_{TH} = 60 - 10 = 50 \text{ Volts}$$

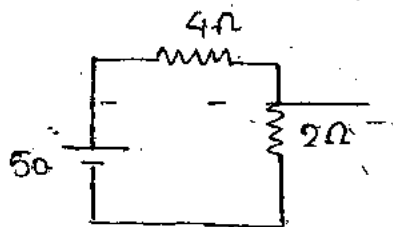
Case 2: R_{TH}



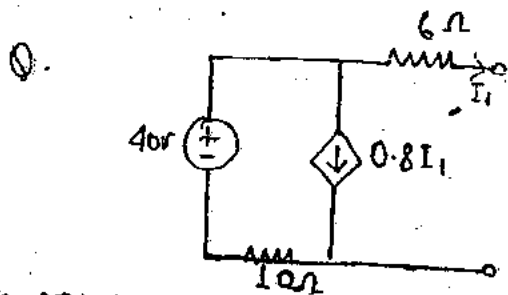
$$R_{TH} = (9 \parallel 3) \parallel 6$$

$$= 12 \parallel 6$$

$$= \frac{12 \times 6}{18} = 4 \Omega$$

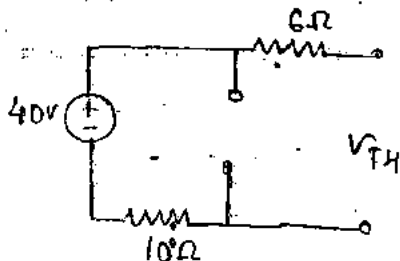


$$I = \frac{50}{6} = 8.33 \text{ A}$$



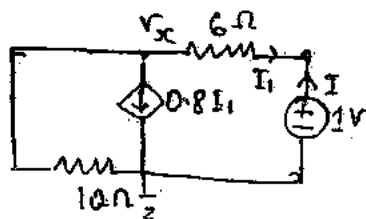
open

case I - V_{TH}



$$V_{TH} = 40 \text{ V}$$

case II - R_{TH}



$$\frac{V_x}{10} + 0.8I_1 + \frac{V_x - 1}{6} = 0$$

$$\frac{V_x}{10} + 0.8 \left(\frac{V_x - 1}{6} \right) + \frac{V_x - 1}{6} = 0$$

$$24V_x = 18$$

$$V_x = 3/4 \text{ volts}$$

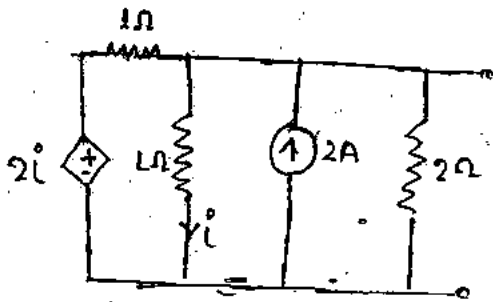
$$I = \frac{1 - V_{oc}}{6}$$

$$= \frac{1 - 3/4}{6}$$

$$= \frac{1}{24} \text{ A}$$

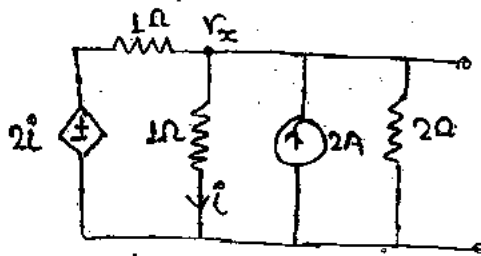
$$R_{TH} = \frac{V}{I} = \frac{1}{1/24} = 24 \Omega$$

Q.



Find thevenin resistance

Soln. Case 1 V_{TH}



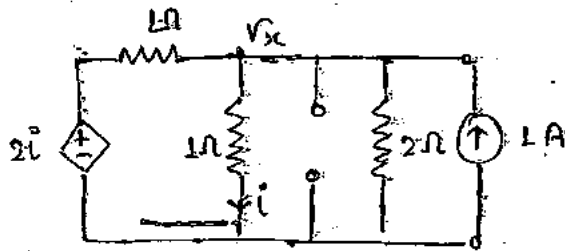
$$\frac{V_x - 2i}{1} + \frac{V_x}{1} + \frac{V_x}{2} = 2$$

$$i = \frac{V_x}{1}$$

$$2V_x + \frac{V_x}{2} - 2V_x = 2$$

$$\therefore V_{oc} = 4V$$

Case R_{TH}



$$\frac{V_x - 2i}{1} + \frac{V_x}{1} + \frac{V_x}{2} = 0$$

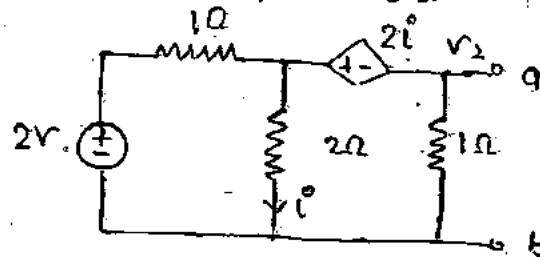
$$i = \frac{V_x}{1}$$

$$V_x = 2V$$

$$V = 2V$$

$$R_{TH} = \frac{V}{I} = \frac{2}{1} = 2\Omega$$

Q. Develop thevenin equivalent



Soln: Case V_{TH}

$$\frac{V_2}{1} + \frac{V_1 - 2}{2V} + \frac{V_1}{2} = 0$$

$$\frac{3V_1}{2} + \frac{V_2}{2} - 2 = 0$$

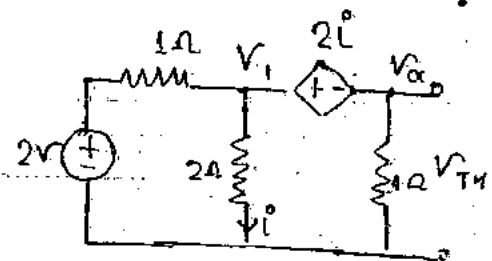
$$V_2 = 4 - 3V_1$$

$$\frac{2V_1}{1} = 0$$

$$V_1 - V_2 = 2i$$

$$V_1 - V_2 = 2 \frac{V_1}{2}$$

$$V_2 = 2V_1$$



$$\frac{3V_1}{2} + 2V_1 = 2$$

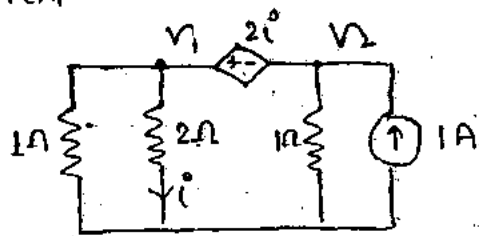
$$7V_1 = 4$$

$$V_1 = \frac{4}{7}$$

$$V_2 = \frac{4}{7} \times 2 = \frac{8}{7}$$

$$V_{TH} = \frac{8}{7}$$

Case 2: R_{TH}



$$\frac{V_1}{1} + \frac{V_2}{2} + \frac{V_2}{1} = 1$$

$$V_2 + \frac{3V_2}{2} = 1$$

$$i^\circ = \frac{V_1}{2}$$

$$V_2 - V_1 = 2i^\circ$$

$$V_2 - V_1 = 2 \frac{V_1}{2}$$

$$V_2 = 2V_1$$

$$2V_1 + \frac{3V_2}{2} = 1$$

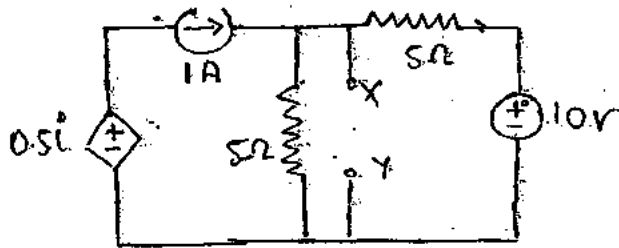
$$7 \frac{V_1}{2} = 1$$

$$V_1 = \frac{2}{7}$$

$$V_2 = \frac{4}{7}$$

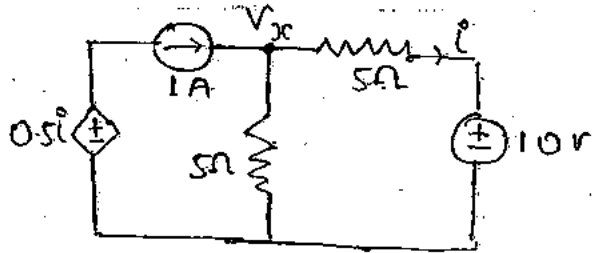
$$R_{TH} = \frac{V}{I} = \frac{4}{7}$$

①



Find thevenin Equivalent

Soln: Case V_{TH}



$$\frac{V_X - 0.5 \times 5}{5} + \frac{V_X - 10}{5} = 1$$

$$\frac{V_X - 0.5 \times 5}{5} + \frac{V_X - 10}{5} = 1$$

$$V_{X_{oc}} \quad \frac{V_X}{5} + \frac{V_X - 10}{5} = 1$$

$$\frac{2V_X}{5} - 2 = 1$$

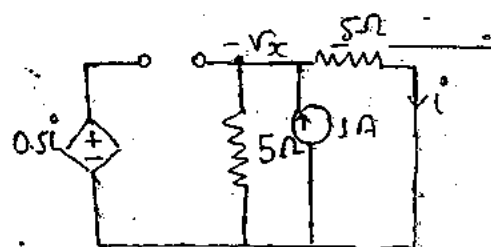
$$2V_X = 15$$

$$V_X = \frac{15}{2} = 7.5$$

$$V_{TH} = V_X - V_Y$$

$$= 7.5 - 0 = 7.5V$$

Case R_{TH}



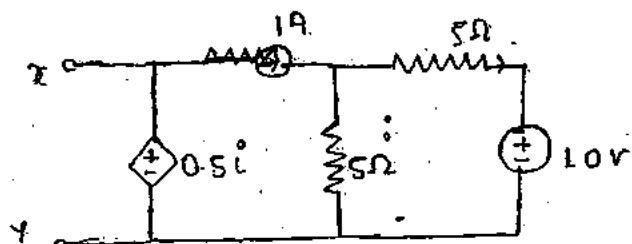
$$\frac{V_x}{5} + \frac{V_x}{5} = 1$$

$$\frac{2V_x}{5} = 1$$

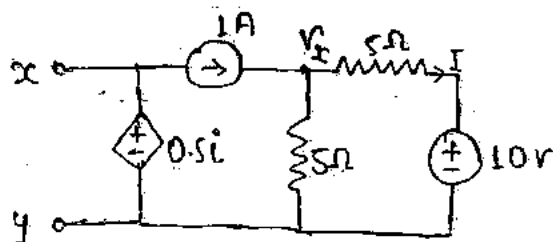
$$V_x = 5/2$$

$$R_{TH} = \frac{V}{I} = \frac{5}{2} \Omega$$

①



Soln: Case V_{TH}



$$\frac{V_x}{5} + \frac{V_x - 10}{5} = 1$$

$$\frac{2V_x}{5} = 3$$

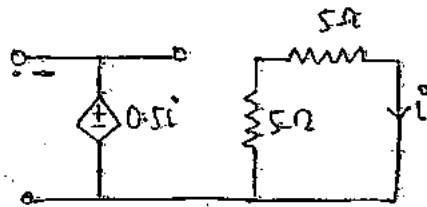
$$V_x = 7.5V$$

$$I_1 = \frac{V_x}{5} = \frac{7.5}{5} = 1.5$$

$$V = V_x - I_1 \cdot 5 = 7.5 - 1.5 \cdot 5 = 0V$$

$$\begin{aligned}
 V_{TH} &= 0.5i \\
 &= 0.5 \left(\frac{7.5 - 10}{5} \right) \\
 &= 0.1 \times -2.5 = -0.25 \text{ Volts}
 \end{aligned}$$

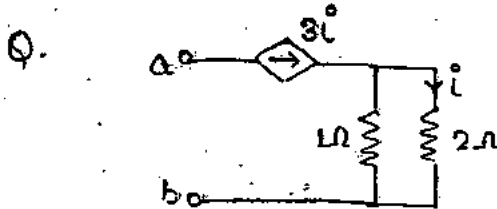
Case R_{TH}



$$\begin{aligned}
 \text{As } i &= 0 \\
 0.5i &= 0
 \end{aligned}$$

$$R_{TH} = 0 \Omega \text{ Ans}$$

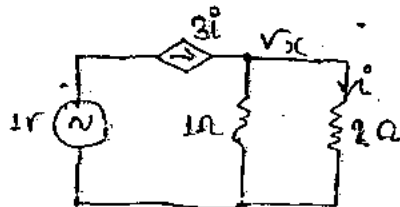
N.B For calculating R_{TH} never connected voltage source in || with dependent sources.



N.B: When there is no independent source present in the n/w. V_{TH} is always zero.

$$V_{TH} = 0$$

• Never connected current source in series with dependent current source while calculating R_{TH}



$$\frac{V_X}{1} + \frac{V_X}{2} = 3i$$

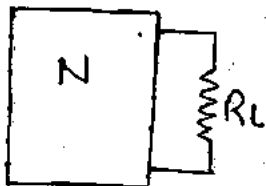
$$\frac{3V_X}{2} = 3i$$

$$\frac{3V_X}{2} = \frac{3V_X}{2}$$

$$V_X = 0$$

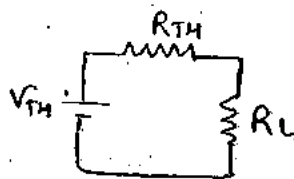
∴ R_{TH} can be not be determined.

When a complex n/w N is connected to load resistor R_L , the power dissipation in load is P . When two identical n/w N is connected to load resistor R_L , find power dissipation in the load.



- (a) P
- (b) $2P$
- (c) $4P$
- (d) $P+4P$

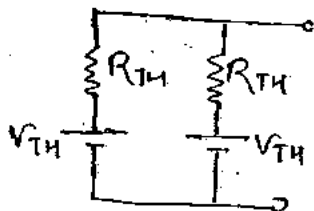
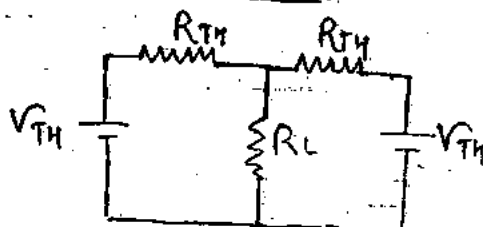
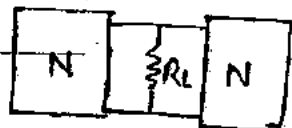
17%



$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

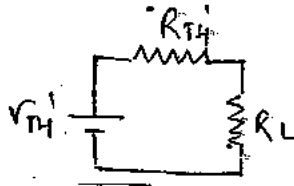
$$P = I^2 R_L$$

$$= \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$



$$V_{TH}' = V_{TH}$$

$$R_{TH}' = \frac{R_{TH}}{2}$$



$$P = I^2 R_L$$

$$= \left(\frac{V_{TH}'}{R_{TH}' + R_L} \right)^2 \cdot R_L$$

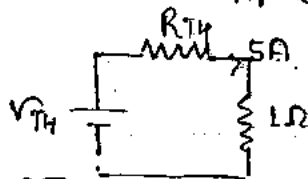
$$= \left(\frac{V_{TH}}{\frac{R_{TH}}{2} + R_L} \right)^2 R_L$$

$$P = \left(\frac{2V_{TH}}{R_{TH} + 2R_L} \right)^2 R_L$$

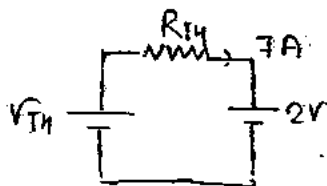
Ans $P + 0.4P$:

- Q. A Battery charger drives a current of 5A. the load resistance $R_L = 1\Omega$. When same battery charger is used for charging of ideal 2V source at current of 7A. Find V_{TH} and R_{TH} .

Soln.



$$5 = \frac{V_{TH}}{1 + R_{TH}} \therefore V_{TH} = 5 + 5R_{TH} \quad (1)$$



$$V_{TH} = 7R_{TH} + 2 \quad (2)$$

$$5R_{TH} + 5 = 7R_{TH} + 2$$

$$3 = 2R_{TH}$$

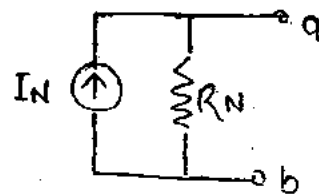
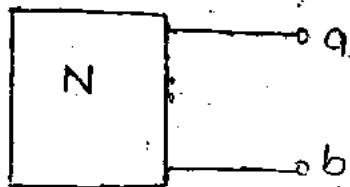
$$\therefore R_{TH} = 3/2$$

$$V_{Th} = 7 \times \frac{3}{2} + 2 = 12.5 \text{ volts}$$

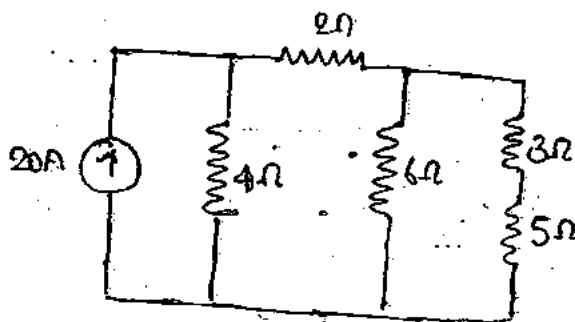
NORTON'S THEOREM:-

A two-terminal N/W NA containing linear, bidirectional elements, and independent source is equivalent to similar N/W containing an independent current source (I_N) in || with a resistance R_N .

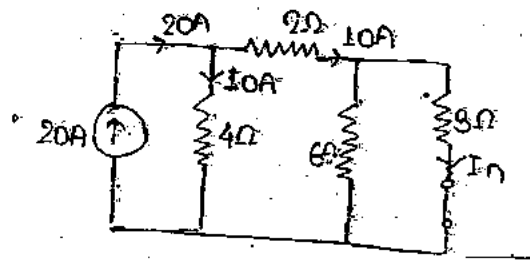
Source current is the s.c current across NA and R_N is resistance looking into the terminal after deactivating all independent sources in a N/W NA.



Q. Find the current flowing 5Ω resistance by using Norton's theorem.

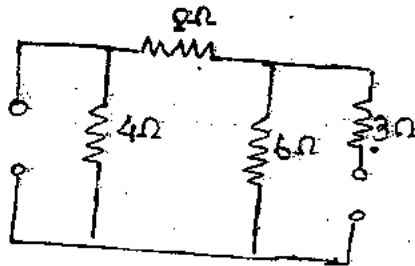


Soln: Case I_N

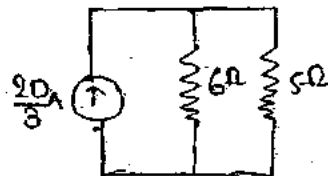


$$I_N = 10 \times \frac{6}{2+6} = \frac{20}{3} \text{ A}$$

Case R_N

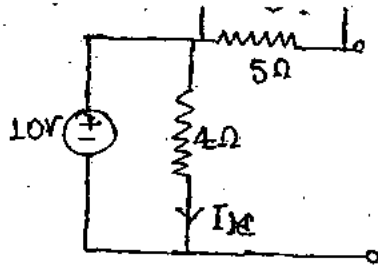


$$R_N = 6\Omega$$



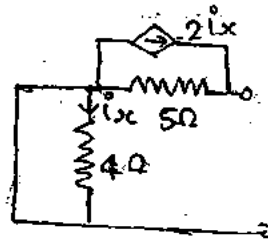
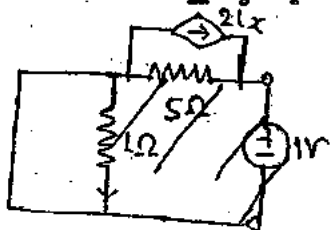
$$I = \frac{20}{3} \cdot \frac{6}{11} = \frac{40}{11} \text{ A}$$

Q.



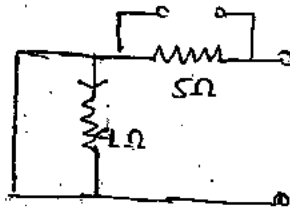
Develop the Norton equivalent⁴⁴ across the terminal?

Soln: Case of R_N



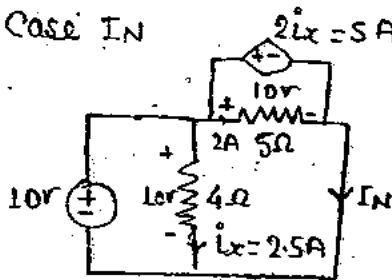
1.6.1

As $i_x = 0$



$$R_N = 5\Omega$$

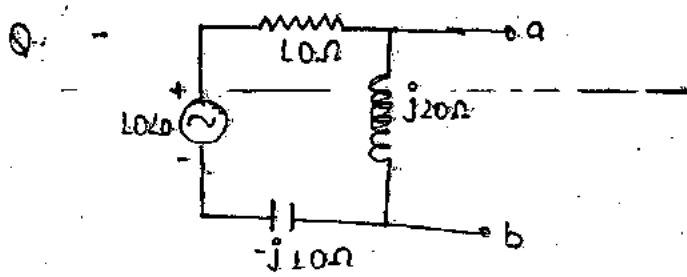
Case I_N



$$2i_x + 2 = I_N$$

$$I_N = 2 \times 2.5 + 2 = 7A$$

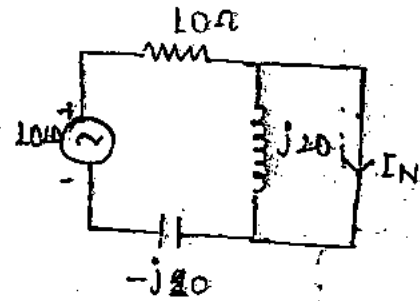
1.6 When short circuit voltage source connected in || to the resistance. then internal resistance of voltage source is 0 zero. Since voltage source and Resistance connected in ||. then equivalent resistance Zero.



Soln: Case I_N

$$I = \frac{10\angle 0^\circ}{10 + j10} = \frac{10\angle 0^\circ}{10 - 10j}$$

$$\therefore \frac{1\angle 0^\circ}{1-j} = \frac{1}{\sqrt{2}} \angle 45^\circ$$



Case R_N

$$\therefore \frac{1}{Z_N} = \frac{1}{10 - j10} + \frac{1}{j20}$$

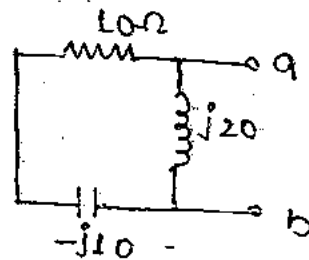
$$= \frac{1}{20} \left[\frac{2}{1-j} + \frac{1}{j} \right]$$

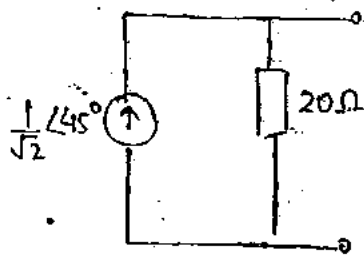
$$= \frac{1}{20} \left[\frac{2j + 1 - j}{1 - j \cdot j} \right]$$

$$= \frac{1}{20} \left[\frac{1 + j}{1 - j} \times \frac{j}{j^2} \right]$$

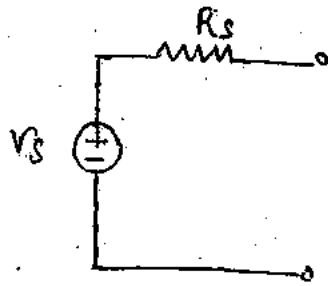
$$= \frac{1}{20} \left[\frac{1 + j}{1 + j} \right]$$

$$Z_N = 20\Omega$$

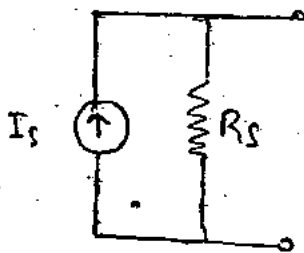
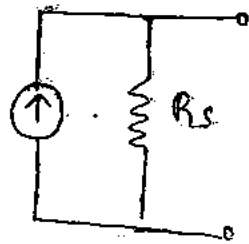




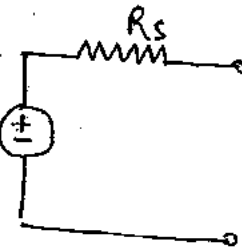
Source transformation:-



$$= I_N = \frac{V_S}{R}$$



$$= V_S = I_S R_S$$



$$R_{TH} = \frac{V_{TH}}{I_N} = \frac{V_{OC}}{I_{SC}}$$

Q. Find the current from 4Ω resistor by using following data:

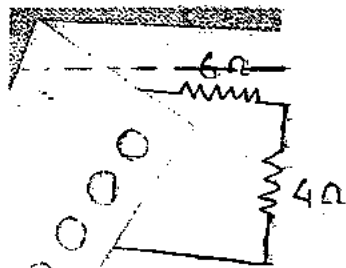
V	60V	0
I	0A	10A

Soln:

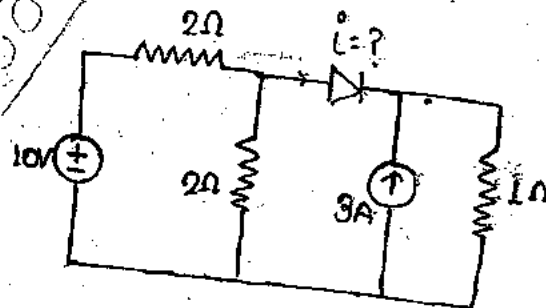
$$V_{OC} = 60V = V_{TH}$$

$$I_{SC} = 10A = I_N$$

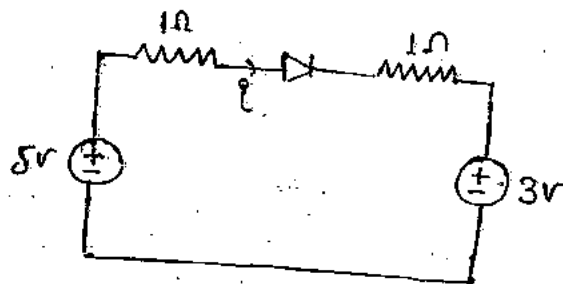
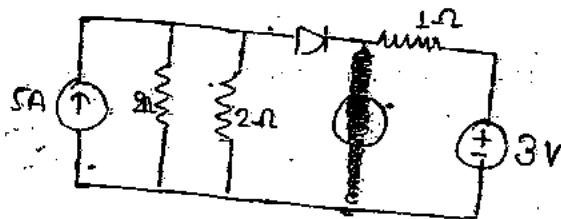
$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{60}{10} = 6\Omega$$



$$I = \frac{60}{10} = 6A$$



Soln:

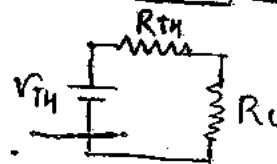


$$I = \frac{5-3}{1+1} = \frac{2}{2} = 1A$$

Q. The N/w N contains Resistor and independent source.
FOR $R=0.2 \Omega$ & $I=3A$ & $5A$. Find the value of I : $R=1\Omega$



Soln:-



$$I = \frac{V_{TH}}{R_{TH}}$$

$$\therefore V_{TH} = 3R_{TH}$$

$$1.5 = \frac{V_{TH}}{R_{TH} + 2}$$

$$1.5 R_{TH} + 3 = V_{TH}$$

$$1.5 R_{TH} + 3 = 3 R_{TH}$$

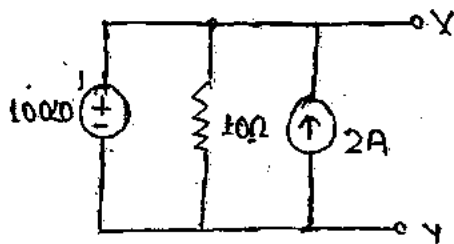
$$3 = 1.5 R_{TH}$$

$$R_{TH} = 2 \Omega$$

$$V_{TH} = 6V$$

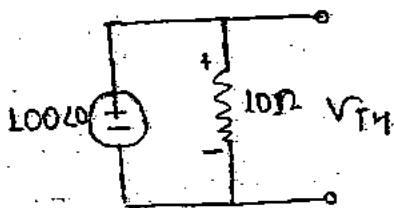
$$I = \frac{6}{2+1} = 2A \quad \underline{\text{Ans}}$$

Q.



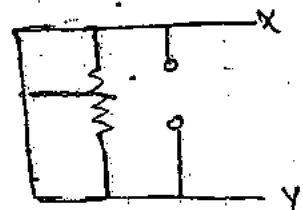
Develop the thevenin and Norton equivalent cir.

Soln: case V_{TH}



$$V_{TH} = 100 \text{ volts}$$

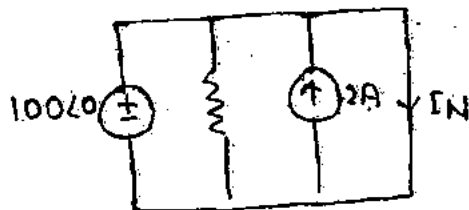
case R_{TH}



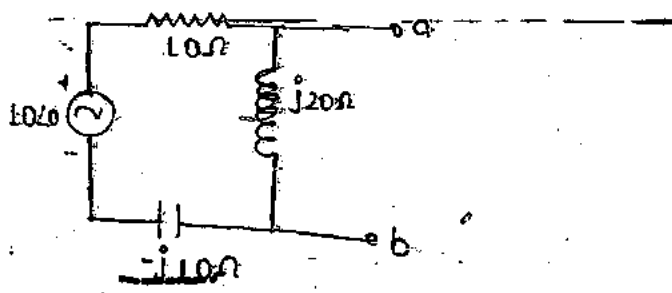
$$i = 0$$

$$R_{TH} = 10 \Omega$$

case I_N



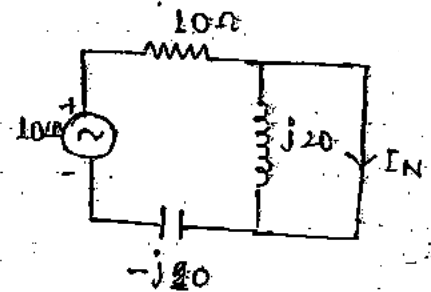
Norton can be develop



Soln: Case I_N

$$I = \frac{10\angle 0}{10 + j10} = \frac{10\angle 0}{10 - 10j}$$

$$= \frac{1\angle 0}{1 - j} = \frac{1}{\sqrt{2}} \angle 45^\circ$$



Case R_N

$$\frac{1}{Z_N} = \frac{1}{10 - j10} + \frac{1}{j20}$$

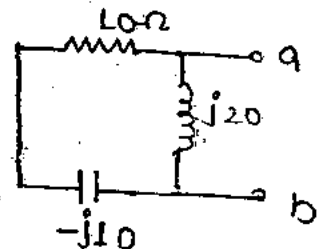
$$= \frac{1}{20} \left[\frac{2}{1 - j} + \frac{1}{j} \right]$$

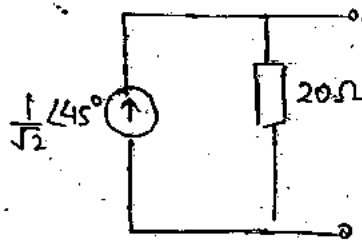
$$= \frac{1}{20} \left[\frac{2j + 1 - j}{(1 - j)j} \right]$$

$$= \frac{1}{20} \left[\frac{1 + j}{1 - j} \times \frac{j}{j^2} \right]$$

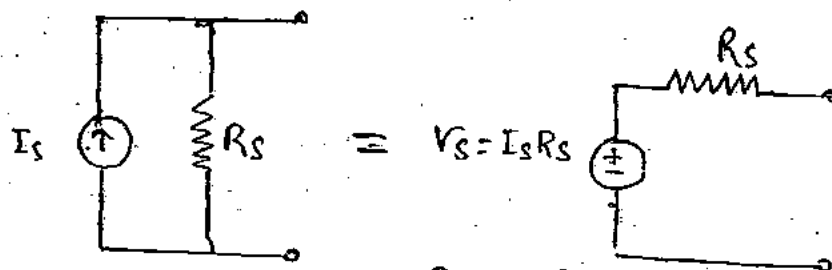
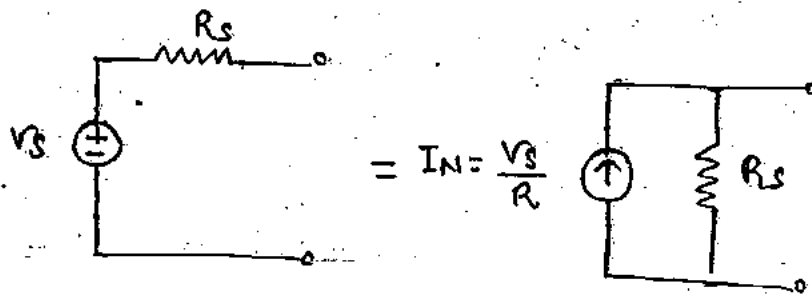
$$= \frac{1}{20} \left[\frac{1 + j}{1 + j} \right]$$

$$Z_N = 20\Omega$$





Source transformation:-



$$R_{TH} = \frac{V_{TH}}{I_N} = \frac{V_{OC}}{I_{SC}}$$

Q. Find the current from 4Ω resistor by using following data:

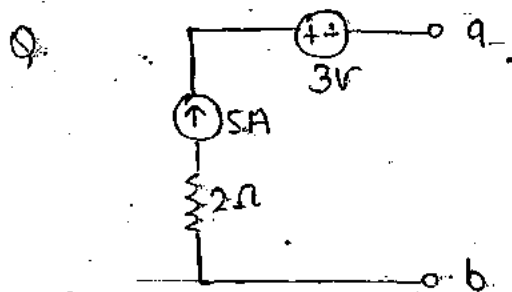
V	60V	0
I	0A	10A

Soln

$$V_{OC} = 60V = V_{TH}$$

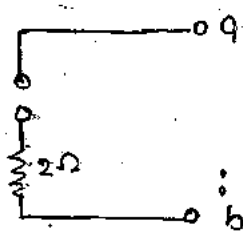
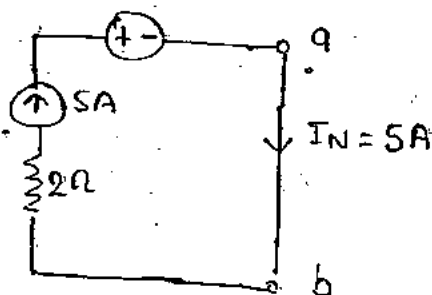
$$I_{SC} = 10A = I_N$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{60}{10} = 6\Omega$$



- Develop the Norton and
Theremin Equivalent Ckt.

Soln.

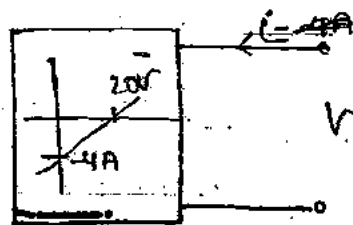


$$R_{TH} = \frac{V_{TH}}{I_N} \neq 0\Omega$$

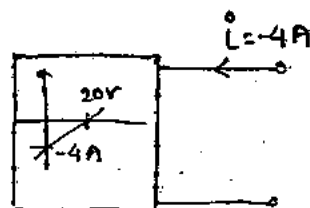
$$R_{TH} =$$

Theremin equivalent ~~to~~ ckt can not valid because
it does not obey the KCL law.

Q.



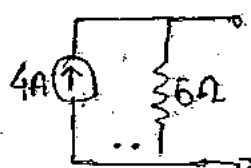
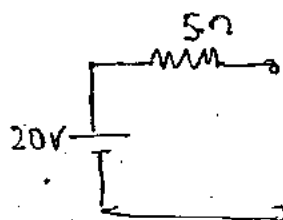
Soln.



$$V_{TH} = 20V$$

$$I_N = 4A$$

$$R_N = R_{TH} = \frac{V_{TH}}{I_N} = \frac{20}{4} = 5\Omega$$



RECIPROCITY THEOREM:

In linear bilateral single source n/w. ratio of the response to excitation is constant even when the position the response & excitation is interchanged.

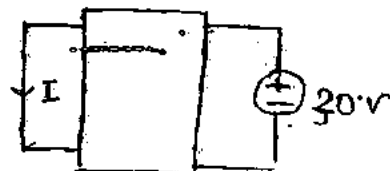
In any linear bilateral n/w, if single voltage source V_x in branch x produces current response I_y in branch y . then the removal of voltage source from branch x and insertion in branch y will produce the current I_x in branch x .

It is not applicable

- (1) When more than one independent voltage & current source present.
- (2) When dependent source is present.
- (3) When ratio of response & excitation is not $\propto$ mho.



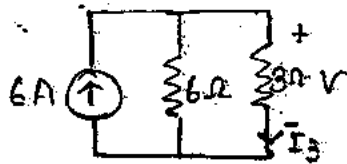
Q. Find I



$$\frac{2}{10} = \frac{I}{30}$$

$$I = 6A$$

Q.

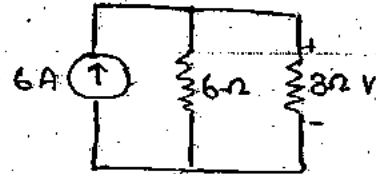


Verify the reciprocity theorem

Soln: $I_3 = 6 \cdot \frac{6}{9} = 4 \text{ A}$

$V = I_3 R = 4 \times 3 = 12 \text{ volts}$

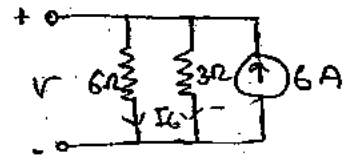
$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2$



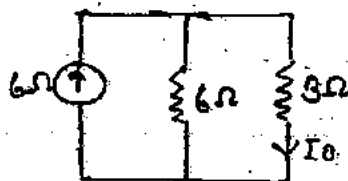
$I_6 = 6 \times \frac{3}{9} = 2 \text{ A}$

$V = I_6 \times 6$
 $= 2 \times 6 = 12 \text{ volts}$

$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2 \text{ A}$

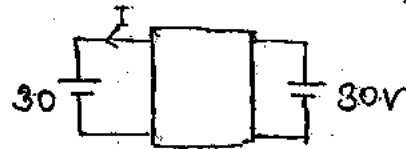
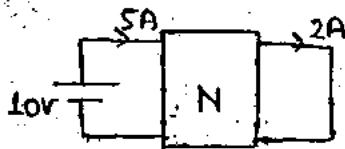


Q.



Soln: Reciprocity theorem not valid because ratio of excitation & response is not Ω or V

Q. The given N/w N satisfy reciprocity theorem then find value of I.



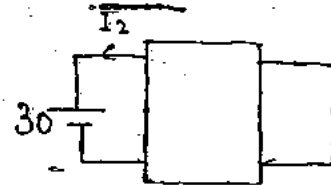
Soln: Case I 30V



$$\frac{2}{10} = \frac{I_1}{30}$$

$$I_1 = 6A$$

Case 2 30V



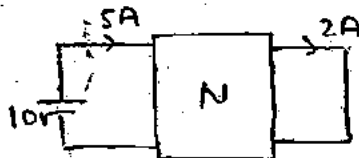
$$\frac{I_2}{30} = -\frac{5}{10}$$

$$I_2 = -15A$$

$$I = I_1 + I_2$$

$$= -15 + 6 = -9A$$

Q. The given n/w N satisfy reciprocity theorem then find value of I.



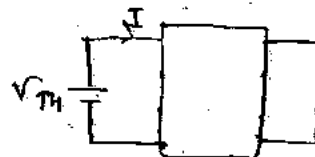
Soln: Case I 30V



$$\frac{I}{30} = \frac{2}{10}$$

$$I = 6A$$

Case 4Ω

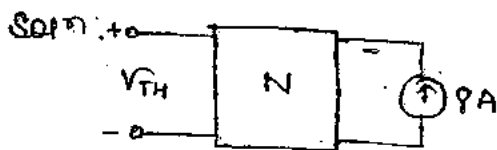
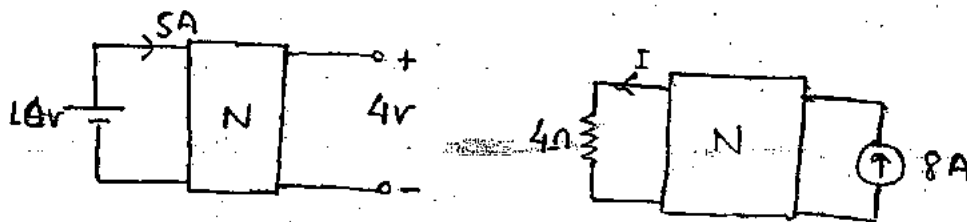


$$R_{th} = \frac{10}{5} = 2\Omega$$



$$I_{4\Omega} = \frac{6 \times 2}{6} = 2A$$

Q. The below two satisfy the Reciprocity theorem then find value of I

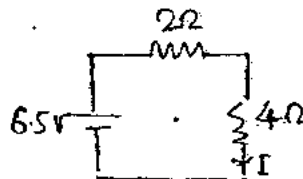


$$\frac{4}{5} = \frac{V_{TH}}{8}$$

$$V_{TH} = \frac{32}{5} = 6.5 \text{ volts}$$



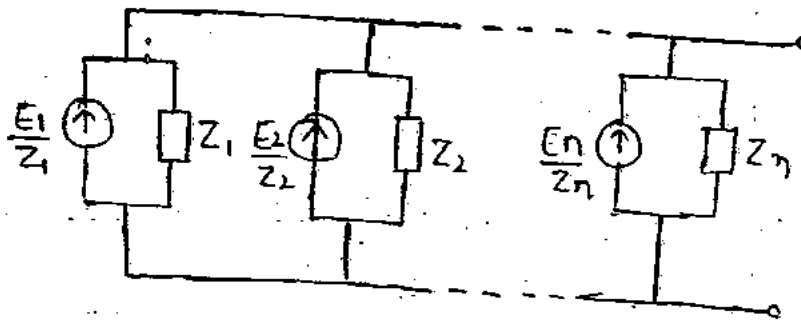
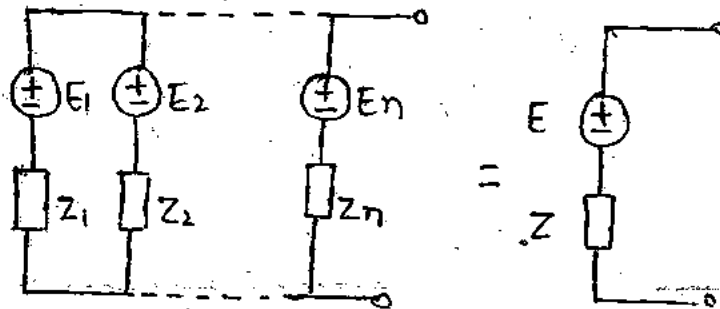
$$R_{TH} = \frac{10}{5} = 2\Omega$$



$$I_1 = \frac{6.5}{6} = 1.08A$$

● MILNANN'S THEOREM:-

If n voltage sources having voltages $E_1, E_2, \dots, E_n$ and internal impedances $Z_1, Z_2, \dots, Z_n$ are connected in // then this voltage sources can be replaced by single voltage source E with series impedances Z .



$$I_N = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \dots + \frac{E_n}{Z_n}$$

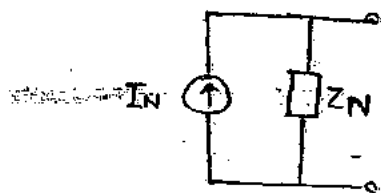
$$Z_n = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}}$$

$$E = I_N \cdot Z_N = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \dots + \frac{E_n}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}}$$

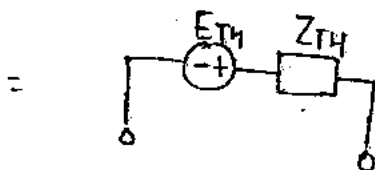
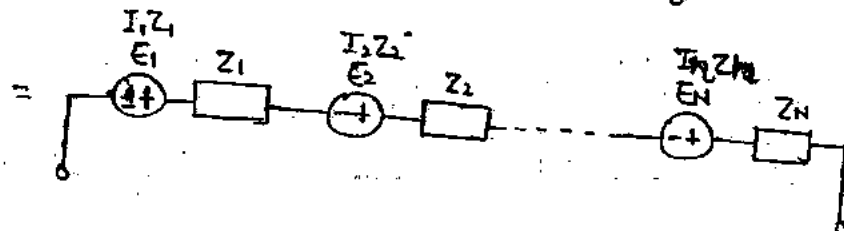
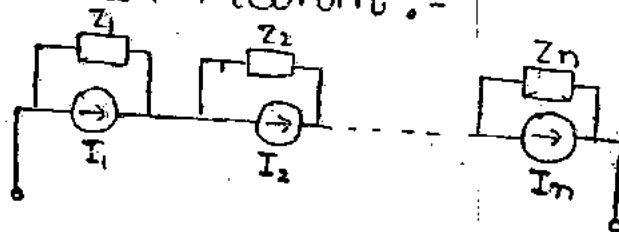
$$E = \frac{E_1 Y_1 + E_2 Y_2 + \dots + E_n Y_n}{Y_1 + Y_2 + \dots + Y_n}$$

$$E = \frac{\sum_{i=1}^n E_i Y_i}{\sum_{i=1}^n Y_i}$$

$$Z = Z_n = \frac{1}{Y_1 + Y_2 + \dots + Y_n} = \frac{1}{\sum_{i=1}^n Y_i}$$



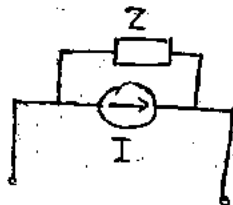
Dual Millman theorem :-



$$E_{TH} = E_1 + E_2 + \dots + E_n$$

$$= I_1 Z_1 + I_2 Z_2 + \dots + I_n Z_n$$

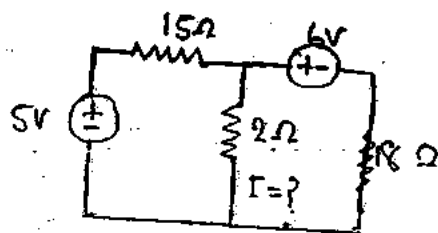
$$Z_{TH} = Z_1 + Z_2 + \dots + Z_n$$



$$I = \frac{E_{TH}}{Z_{TH}} = \frac{I_1 Z_1 + I_2 Z_2 + \dots + I_n Z_n}{Z_1 + Z_2 + \dots + Z_n}$$

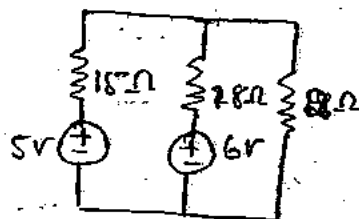
$$I = \sum_{i=1}^n \frac{I_i Z_i}{\sum_{i=1}^n Z_i}$$

Q.



Find the value of I using millman theorem

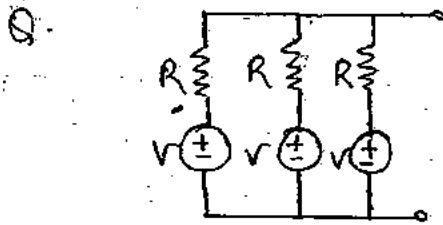
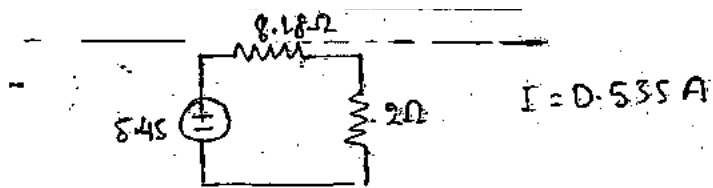
Soln.



$$E = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} = \frac{5}{15} + \frac{6}{18} = \frac{\frac{1}{3} + \frac{1}{3}}{\frac{1}{15} + \frac{1}{18}} =$$

$$E = 5.45V$$

$$R = \frac{1}{\frac{1}{15} + \frac{1}{18}} = \frac{15 \times 18}{18 + 15} = 8.18\Omega$$



Soln:

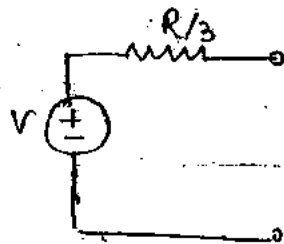
$$E = \frac{V}{R} + \frac{V}{R} + \frac{V}{R}$$

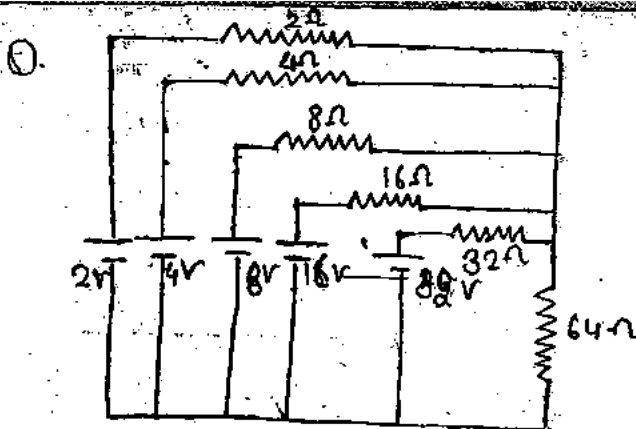
$$= \frac{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}}{3/R}$$

$$= \frac{3V/R}{3/R}$$

$E = V$

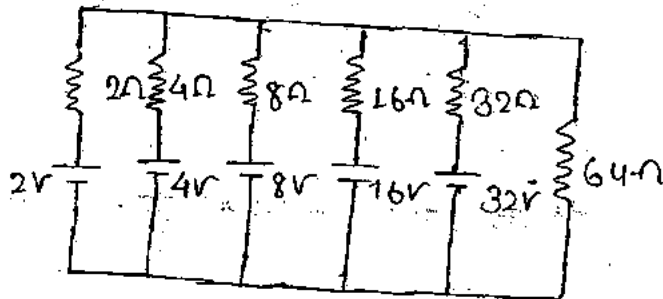
$R_{th} = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{R}{3}$





Find the current through 16Ω resistor

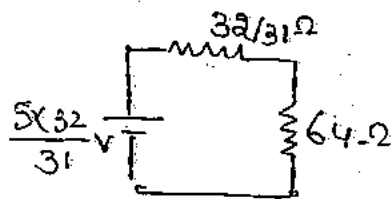
Soln:



$$E = \frac{\frac{2}{2} + \frac{4}{4} + \frac{8}{8} + \frac{16}{16} + \frac{32}{32}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}} :$$

$$E = \frac{5 \times 32}{31}$$

$$R = \frac{1}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}} = \frac{32}{31} \Omega$$

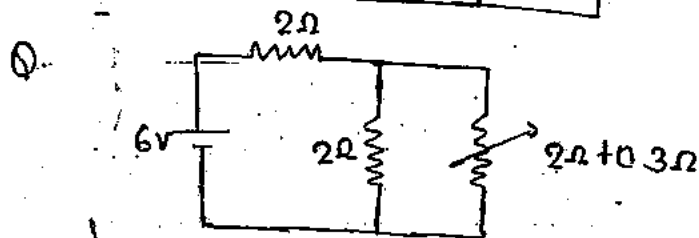
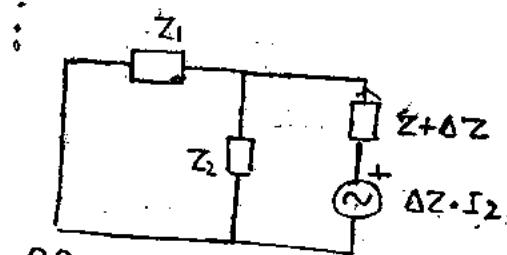
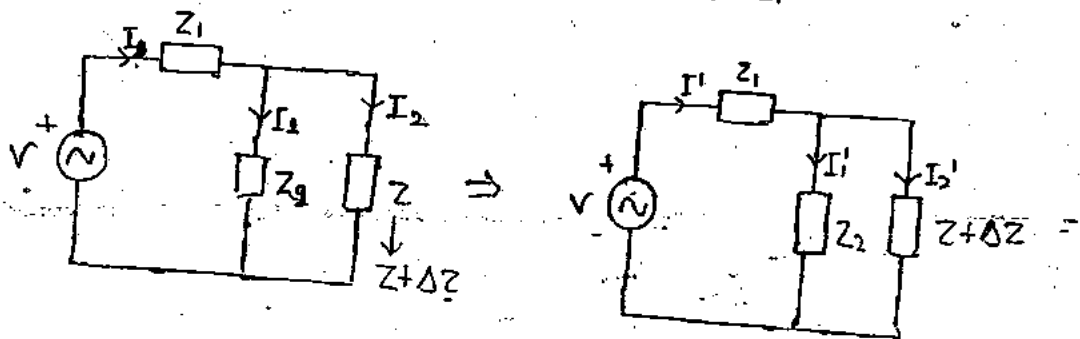


$$I = \frac{5 \times 32 / 31}{\frac{32}{31} + 64}$$

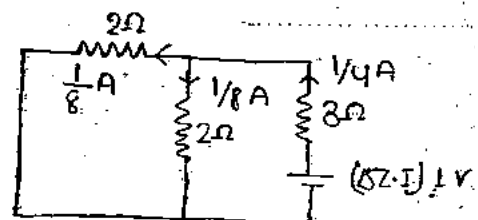
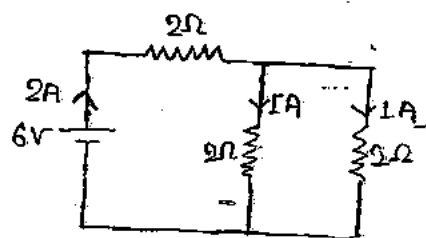
I

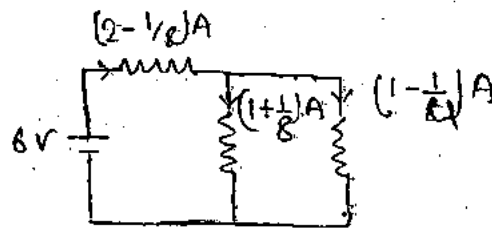
Compensation Theorem :-

In a linear n/w acted upon by independent sources. if an impedance is Z carrying current I . if change to $Z + \Delta Z$ the incremental changes will be identical to those produced by voltage source $\Delta Z \cdot I$ in series with the altered impedances $Z + \Delta Z$ with all the independent source deactivated.



Soln



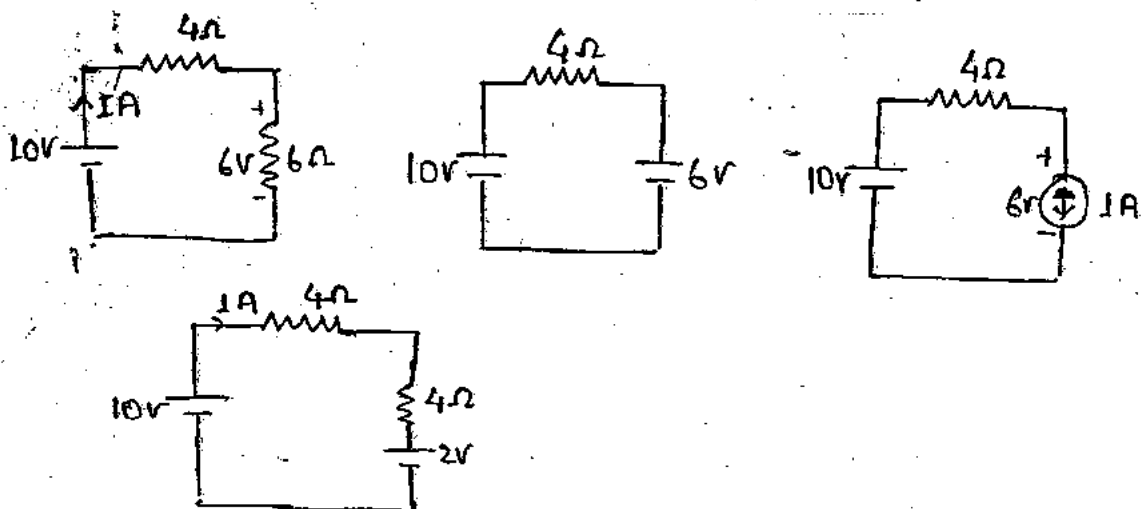


NOTE

1. This theorem is used to determine the steady state error introduced in the system by measuring instrument and obtained null deflection in the galvanometer of the bridge circuit.

SUBSTITUTION THEOREM:

Any branch in a n/w may be substituted by different branch without disturbing the voltages and current in the entire n/w provided that a new branch of set of same terminal voltage and current as the original branch.



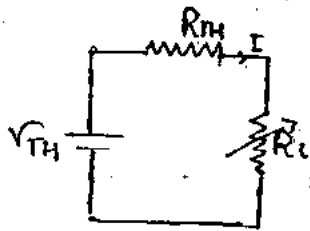
TELLEGEN'S THEOREM: -

In any n/w (linear, non linear, unidirectional, bidirectional, time invariant, time variant) the sum of total instantaneous power consumed by various element in various branches of ckt is always zero. This implies that total power delivered by active element is equal to total power absorbed by passive elements of the branches of n/w.

N.B

- For verification of dkt tellegen's theorem, KVL & KCL theorem are used.
- It is based on the law of conservation of energy.

Maximum Power Transfer:



$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P = I^2 R_L$$

$$P = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 \cdot R_L$$

$$\frac{dP}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - R_L \cdot 2(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right] = 0$$

$$R_{TH} + R_L - 2R_L = 0$$

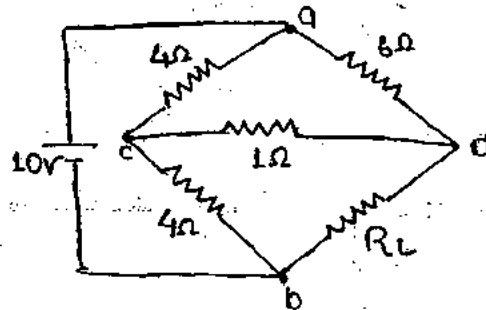
$$\boxed{R_{TH} = R_L}$$

$$P_{max} = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 \cdot R_L$$

$$P_{max} = \left(\frac{V_{TH}}{R_{TH} + R_{TH}} \right)^2 \cdot R_{TH}$$

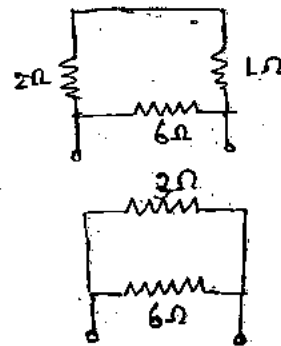
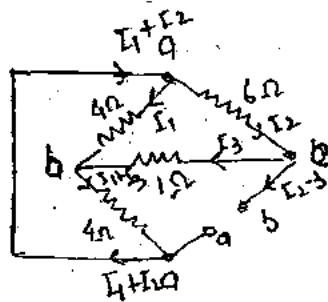
$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

Q.



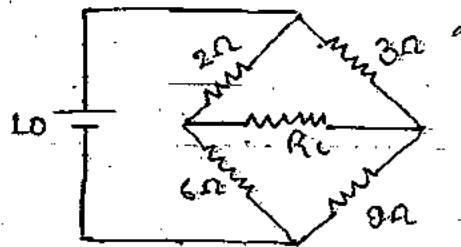
for what value of R_L
max power transfer

Soln:



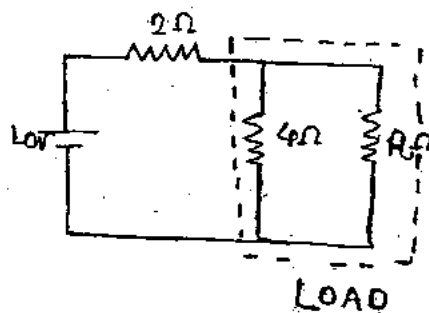
$$R_{TH} = R_L = \frac{3 \times 6}{9} = 2\Omega$$

Q. Find the value of R_L so that max^m power transfer to the load and also find max^m power.



Solⁿ: As the bridge is balanced, no current will flow from R_L and hence power dissipation across R_L is zero.

Q. Find the value R so that max^m power is transfer.



Solⁿ:

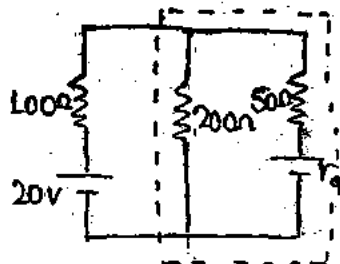
$$R_L = R_{TH}$$

$$2 = \frac{4 \times R}{4 + R}$$

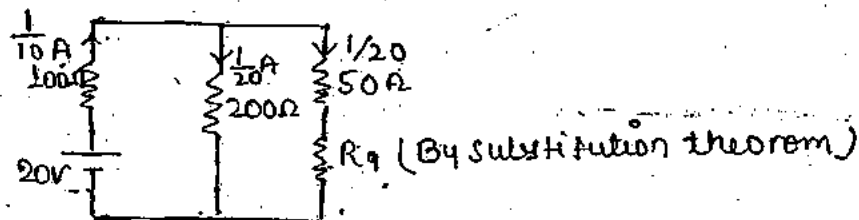
$$8 + 2R = 4R$$

$$R = 4\Omega$$

Q. What should be value of V_a that maxm power transfer



Soln:



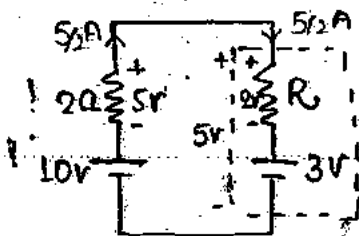
$$R_L = 200 \parallel (50 + R_a)$$

$$100 = 200 \parallel (50 + R_a)$$

$$R_a = 150 \Omega$$

$$V_a = IR_a = \frac{1}{20} \times 150 = 7.5V$$

Q.



Find the value of R so that maxm power transfer

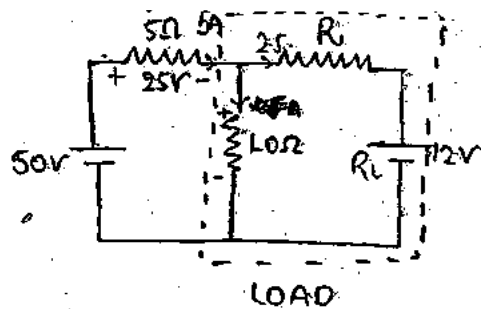
As we know $R_L = R_{Th}$

Voltage across R_L = Voltage across R_{Th}

Soln:

$$R = \frac{2}{5/2} = \frac{4}{5} = 0.8 \Omega$$

Q



For what value of R max power transfer.

Soln. Method 1:

$$R_L = \frac{12}{5/2} = \frac{24}{5} \Omega$$

$$\therefore R + R_L = 10$$

$$R = 10 - \frac{24}{5} = \frac{26}{5} = 5.2 \Omega$$

Method 2:

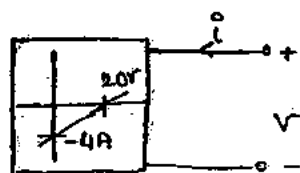
$$\frac{25-50}{50} + \frac{25}{10} + \frac{25-12}{R} = 0$$

$$-5 + 2.5 + \frac{13}{R} = 0$$

$$-2.5 + \frac{13}{R} = 0$$

$$R = \frac{13}{2.5} = 5.2 \Omega$$

Q

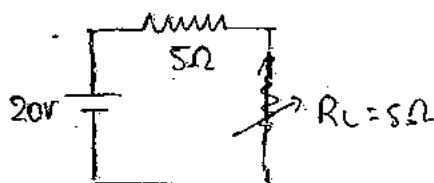


Find the max power that can be drawn from it.

$$V_{TH} = 20V$$

$$R_{TH} I = 4A$$

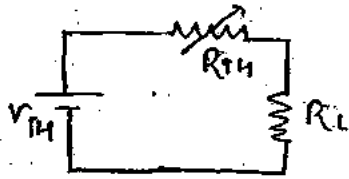
$$R_{TH} = \frac{20}{4} = 5 \Omega$$



$$P_{max} = \frac{20^2}{4 \times 5}$$

$$= \frac{20 \times 20}{20} = 20W$$

Q. Prove the max^m power transfer ^{not} valid if R_{TH} is variable.



Soln: $I = \frac{V_{TH}}{R_{TH} + R_L}$

$$P = I^2 R_L$$

$$= \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

$$\frac{dP}{dR_{TH}} = V_{TH}^2 R_L \left(\frac{-2}{(R_{TH} + R_L)^3} \right) = 0$$

$$R_{TH} \neq R_L$$

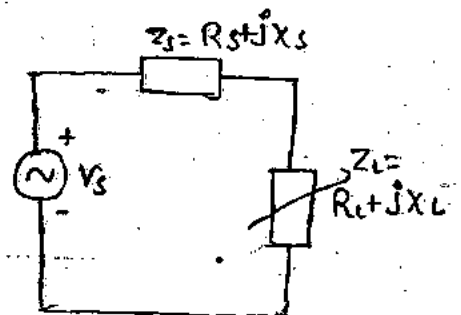
MPT is not valid.

~~Max^m~~ Power transfer (AC)

$$|I| = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

$$P = |I|^2 R_L$$

$$= \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \quad \text{--- (1)}$$



Case 1: When R_L X_L variables

$$\frac{\partial P}{\partial X_L} = V_S^2 R_L \left[\frac{-1}{(R_S + R_L)^2 + (X_S + X_L)^2} \right] \frac{\partial}{\partial X_L} [(X_S + X_L)^2] = 0$$

$$\Im(X_S + X_L) = 0$$

$$\boxed{X_L = -X_S} \quad \text{--- (2)}$$

$$\textcircled{2} \rightarrow \textcircled{1}$$

$$P = \frac{V_S^2 \cdot R_L}{(R_S + R_L)^2}$$

$$\frac{\partial P}{\partial R_L} = 0$$

$$R_L = R_S$$

$$Z_L = R_L + jX_L = R_S - jX_S = Z_S^*$$

$$\boxed{Z_L = Z_S^*}$$

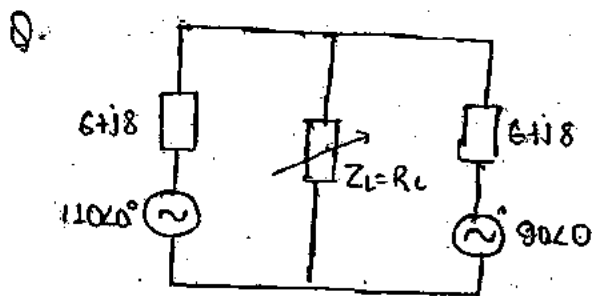
NOTE:

Case I: only R_L & X_L variables (X_L is constant)

$$* R_L = \sqrt{R_{TH}^2 + (X_{TH} + X_L)^2} \quad *$$

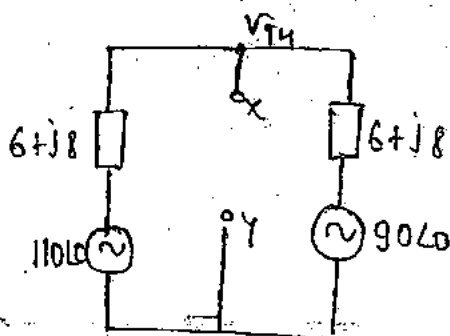
Case II: when load impedance is only resistive

$$* R_L = \sqrt{R_{TH}^2 + X_{TH}^2} \quad *$$



Find max power dissipation
in load impedance

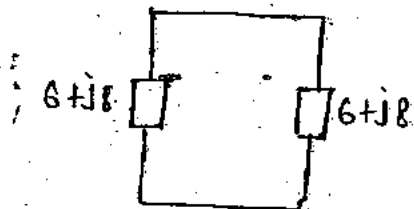
Soln:



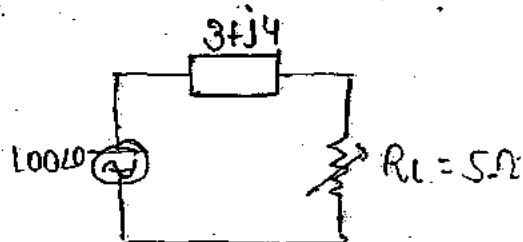
$$\frac{V_{TH} - 110\angle 0}{6 + j8} + \frac{V_{TH} - 90\angle 0}{6 + j8} = 0$$

$$2V_{TH} = 200\angle 0$$

$$V_{TH} = 100\angle 0$$



$$Z_{TH} = 3 + j4$$



$$R_L = \sqrt{3^2 + 4^2} = 5$$

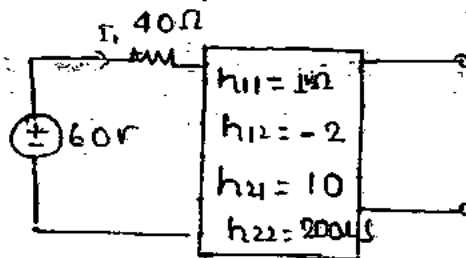
$$|I| = \frac{100 \angle 0}{8 + j4} = \frac{25 \angle 0}{2 + j1}$$

$$P = |I|^2 R_L$$

$$= \left(\frac{25}{\sqrt{2^2 + 1^2}} \right)^2 \cdot 5$$

$$= \frac{25 \times 25}{5} \times 5 = 625 \text{ W}$$

Determine the thevenin equivalent at o/p port of crt?



Soln:

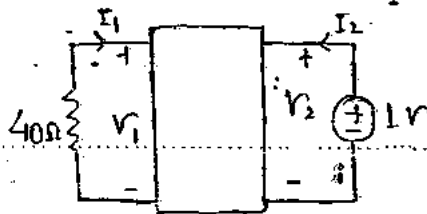
$$V_1 = 1 \text{ V} \quad I_1 = -2 \text{ V}$$

$$V_2 = 10 I_1 + 200 \mu \text{ S} V_2$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Case Z_{TH}



$$V_1 = -40 I_1 \quad V_2 = 1 \text{ V}$$

$$-40 I_1 = h_{11} I_1 + h_{12} V_2$$

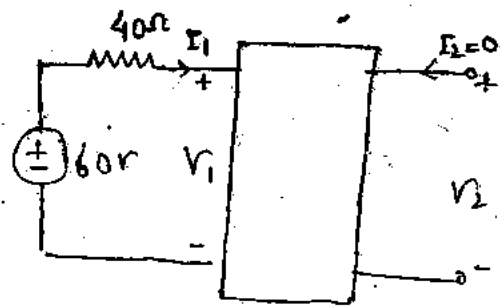
$$I_1 = \frac{-h_{12}}{-(40 + h_{11})}$$

$$I_2 = h_{21} \left(\frac{-h_{12}}{40 + h_{11}} \right) + h_{22} \times 1$$

$$Z_{TH} = \frac{V_2}{-I_2} = -\frac{V_2}{I_2}$$

$$Z_{TH} = 51.46 \Omega$$

Case 2: V_{TH}



$$I_2 = 0$$

$$60 = 40I_1 + V_1$$

$$60 = 40I_1 + h_{11}I_1 + h_{12}V_2$$

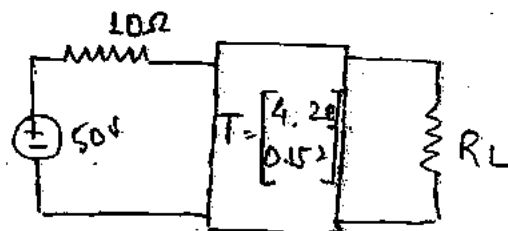
$$h_{12}V_2 = (60) + (-40 - h_{11})I_1$$

$$0 = h_{21}I_1 + h_{22}V_2$$

$$I_1 = -\frac{h_{22}}{h_{21}} V_2$$

$$V_2 = V_{TH} = -29.69V$$

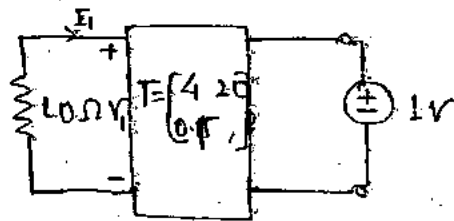
Q. Find the max^m power that can be transfer to the load.



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Solve Case RTH



$$V_1 = -10I_1 \quad V_2 = 1V$$

$$V_1 = 4V_2 - 20I_2 \quad (1)$$

$$I_1 = 0.1V_2 - 2I_2 \quad (2)$$

$$-10I_1 = 4V_2 - 20I_2$$

$$-10I_1 = 4 - 20I_2$$

$$I_1 =$$

$$V_1 = -10I_1$$

$$V_1 = -10(0.1V_2 - 2I_2)$$

$$V_1 = -V_2 + 20I_2 \quad (3)$$

$$(3) \rightarrow (1)$$

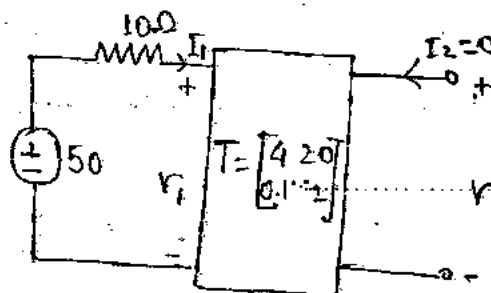
$$-V_2 + 20I_2 = 4V_2 - 20I_2$$

$$40I_2 = 5V_2$$

$$\frac{V_2}{I_2} = \frac{40}{5} = 8\Omega$$

$$Z_{TH} = 8\Omega$$

Case V_{TH}



$$I_2 = 0$$

$$50 = 10I_1 + V_1 \quad (1)$$

$$V_1 = 4V_2 - 20I_2$$

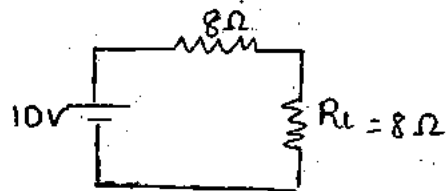
$$V_1 = 4V_2$$

$$50 = 10I_1 + 4V_2$$

$$50 = 10I_1 + 4V_2$$

$$50 = 10(0.1)V_2 + 4V_2$$

$$V_{Th} = 10V$$

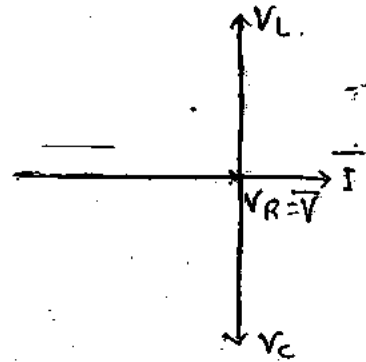
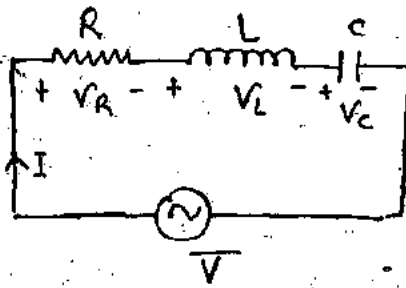


Maxm power transfer $R_L = R_{Th}$

$$P_{max} = \frac{10^2}{4 \times 8} = \frac{100}{4 \times 8} = \frac{100}{32} = 3.125W$$

Zero state response :- ———

RESONANCE:



At Resonance

$$V_C = V_L$$

$$IX_L = IX_C$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

At Resonance

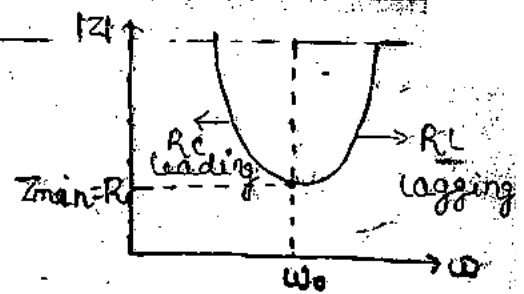
$$Z = R$$

- In case of Resonance voltage across L and C is zero. there phase difference is opposite so cancel each other.

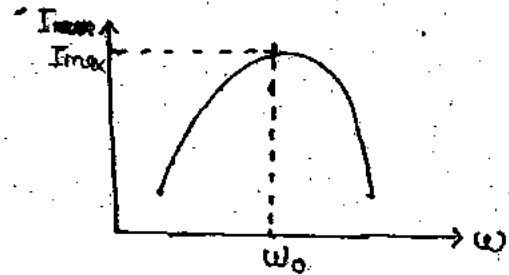
- For occurrence of Resonance in any system, two energy are required. In an electrical system these two energy provided by L & C in the form magnetic & electrical field.
- The circuit is said to be resonance, when $\bar{V}$ and $\bar{I}$ are in phase.
- It indicates, the rate at which energy transformation takes place b/w L & C.

$$(1) \quad Z = R + j(X_L - X_C) \\ = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

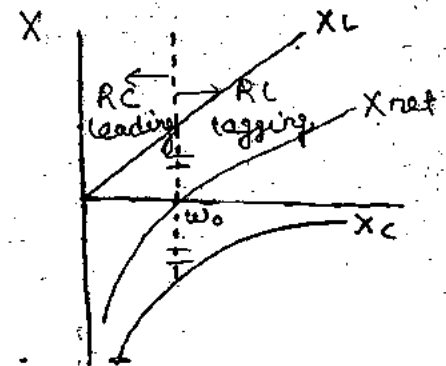


$$(2) \quad I = \frac{V}{|Z|}$$



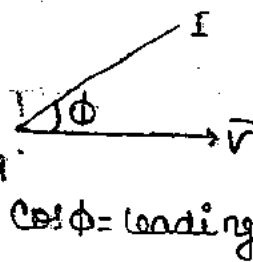
$$(3) \quad X_L = \omega L \quad X_C = \frac{1}{\omega C}$$

$$X_{net} = X_L - X_C = \omega L - \frac{1}{\omega C}$$



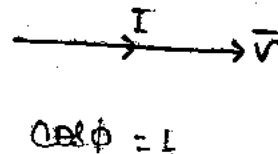
For $\omega < \omega_0$

$$Z = R - jX_C$$



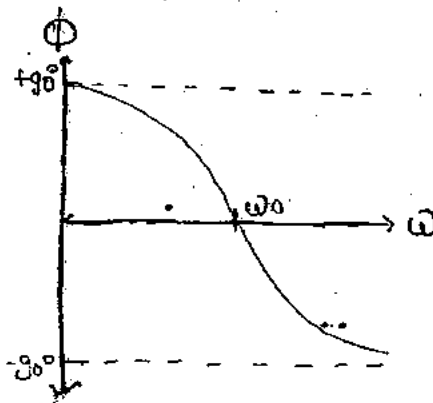
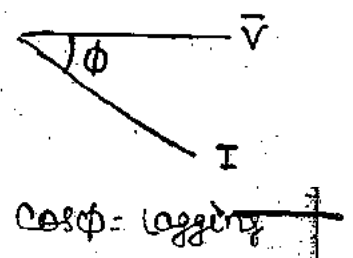
For $\omega = \omega_0$

$$Z = R$$



For $\omega > \omega_0$

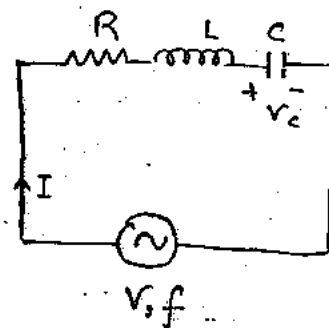
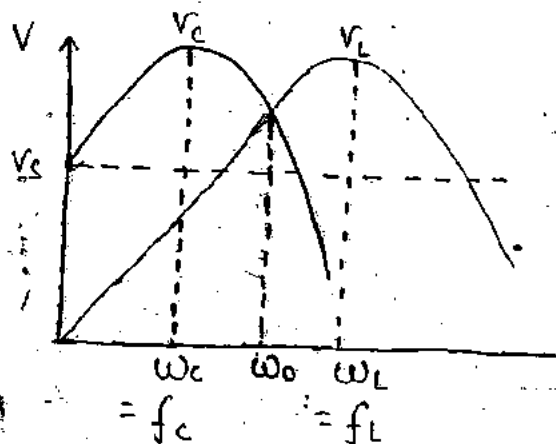
$$Z = R + jX_L$$



From the graph we conclude following points:

1. For $\omega < \omega_0$, the Capacitive reactance is higher than X_L i.e. $X_C > X_L$. Therefore net reactance is capacitance in nature. Hence I leads applied voltage V and gives leading P.F.
 2. At $\omega = \omega_0$, $X_L = X_C$ and hence $Z_{min} = R$, the I and V are in phase hence $\cos \phi = 1$.
 3. For $\omega > \omega_0$, $X_L > X_C$. Therefore net reactance is inductive in nature and current I lags the applied voltage V and giving lagging P.F.
- At Resonance Reactive power is zero.

$$Z = R$$



$$I = \frac{V}{Z} \quad ; \quad Z = R + j(X_L - X_C)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$|I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_L = I \cdot X_L$$

$$V_L = \frac{V \cdot X_L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

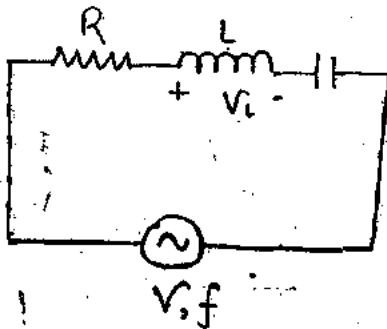
$$V_L = \frac{V}{\omega C \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

since V_L is changes with ω hence

$$\frac{dV_L}{d\omega} = 0$$

$$f_c = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{2L}}$$

N.B. At resonance, i.e. $\omega = \omega_0$, V_L and V_C is equal and may be greater than V_S . It may be happenen
- $V_L = V_C = V_S$ which depends upon quality factor.



$$I = \frac{V}{Z}$$

$$Z = R + j(X_L - X_C)$$

$$|I| = \frac{V}{R + j(X_L - X_C)} = \frac{V}{R + j(\omega L - \frac{1}{\omega C})}$$

$$V_L = I \cdot X_L$$

$$= \frac{V \cdot X_L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V \cdot \omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\frac{dV_L}{d\omega} = 0$$

$$f_c = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{RC}{2L}}$$

$$f_c < f_0 < f_L$$

f_c = frequency at which v_c is max^m

f_L = frequency at which v_L is max^m

f_0 = Resonant frequency.

Quality factor:

Quality factor is the ratio of the max^m energy stored in the ckt to the energy dissipated in ckt in one time period.

$$Q = 2\pi \frac{\text{Max^m energy stored in ckt}}{\text{Energy dissipated per cycle of sec.}}$$

Let $i(t) = i_m \sin \omega t$

$$W_g = \frac{1}{2} Li^2 + \frac{1}{2} Cv_c^2$$

$$v_c = \frac{1}{C} \int i dt$$

$$i = \frac{1}{C} \int i_m \sin \omega t dt$$

$$v_c = -\frac{i_m}{\omega C} \cos \omega t$$

$$W_g = \frac{1}{2} L i_m^2 \sin^2 \omega t + \frac{1}{2} C \cdot \frac{i_m^2}{\omega^2 C^2} \cos^2 \omega t$$

$$W_g = \frac{1}{2} L i_m^2 \sin^2 \omega t + \frac{1}{2} \frac{i_m^2}{\omega^2 C} \cos^2 \omega t$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$W_g = \frac{1}{2} L i_m^2 \sin^2 \omega t + \frac{1}{2} L i_m^2 \cos^2 \omega t$$

$$W_g = \frac{1}{2} L i_m^2$$

$$Q = 2\pi \cdot \frac{\frac{1}{2} L i_m^2}{\left(\frac{i_m}{\sqrt{2}}\right)^2 R \cdot \frac{1}{f}}$$

$$Q = \frac{\omega \cdot L}{R} = \frac{1}{R} \frac{L}{\sqrt{LC}}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega L}{R} = \frac{X_L}{R} = \frac{I X_L}{I R} = \frac{V_L}{V_R}$$

At Resonance $V_L = V_C$

$$Q = \frac{V_C}{V_R} = \frac{I X_C}{I R} = \frac{1}{\omega R C}$$

$$Q = \frac{\omega L}{R} = \frac{1}{\omega R C}$$

$$Q = \frac{\text{Reactive component of voltage}}{\text{Active " " "}}$$

At $\omega = \omega_0$.

$$(1) V_R = I_R \cdot R = I \cdot R = \frac{V}{R} \cdot R = V \quad \boxed{V_R = V}$$

$$(2) V_C = -jX_C \cdot I_C = -jX_C \cdot I = -j \left(\frac{1}{\omega R C} \right) V = -jQV$$

$$(3) V_L = +jX_L \cdot I_L = j \left(\frac{\omega L}{R} \right) V = jQV$$

Amplification factor

$$V = V_R + V_C + V_L$$

$$V = V_R + jQV - jQV$$

$$\boxed{V = V_R} \text{ at } \omega = \omega_0$$

BANDWIDTH:

Bandwidth represent the range of freq. for which power level is at least half of the max^m power.

$$\boxed{Q = \frac{\omega_0}{B.W}}$$

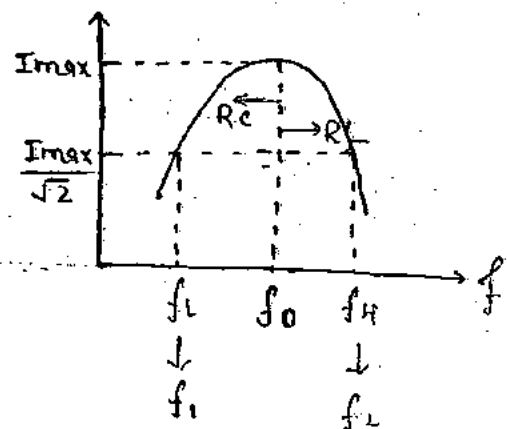
$$f_0 \rightarrow I = I_{\max} \quad Z = R$$

$$f_1, f_2 \rightarrow I = \frac{I_{\max}}{\sqrt{2}} \quad Z = \sqrt{2}R$$

$$Z = R + jX$$

$$\sqrt{R^2 + X^2} = \sqrt{2}R$$

$$R^2 + X^2 = 2R^2$$



$$R^2 = X^2$$

$$X = \pm R$$

$$\text{At } f_1 \quad Z_1 = R - jR$$

$$\text{At } f_2 \quad Z_2 = R + jR$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad (1)$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \quad (2)$$

$$(1) + (2)$$

$$L(\omega_1 + \omega_2) - \frac{1}{C} \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) = 0$$

$$L(\omega_1 + \omega_2) - \frac{1}{C} \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \right) = 0$$

$$L - \frac{1}{C \omega_1 \omega_2} = 0$$

$$L = \frac{1}{C \omega_1 \omega_2}$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$\boxed{\begin{aligned} \omega_1 \omega_2 &= \omega_0^2 \\ \omega_0 &= \sqrt{\omega_1 \omega_2} \end{aligned}}$$

$$(1) - (2)$$

$$L(\omega_1 - \omega_2) + \frac{1}{C} \left(\frac{1}{\omega_2} - \frac{1}{\omega_1} \right) = -2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C} \left(\frac{1}{\omega_1} - \frac{1}{\omega_2} \right) = 2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C} \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right) = 2R$$

$$(\omega_2 - \omega_1) \left(L + \frac{1}{C \omega_1 \omega_2} \right) = 2R$$

$$(\omega_2 - \omega_1) \left[L + \frac{1}{Q \cdot \frac{1}{LQ}} \right] = 2R$$

$$(\omega_2 - \omega_1)(2L) = 2R$$

$$\boxed{\omega_2 - \omega_1 = \frac{R}{L}} \text{ rad/sec}$$

$$\text{where } \omega_2 - \omega_1 = B \cdot W$$

$$B \cdot W = \frac{R}{L}$$

$$\text{As we know } Q = \omega \frac{L}{R}$$

$$\therefore \frac{R}{L} = \frac{\omega}{Q}$$

$$\boxed{Q = \frac{\omega_0}{B \cdot W}}$$

$$\begin{aligned} \omega_1 &= \omega_0 - \frac{B \cdot W}{2} \\ \omega_2 &= \omega_0 + \frac{B \cdot W}{2} \end{aligned}$$

Q - high

$Q \gg 1$

* "For higher value of Q it considers as Resonant frequency is equal to arithmetic mean of the corner frequency".

$$\omega_1 + \omega_2 = 2\omega_0$$

$$\boxed{\omega_0 = \frac{\omega_1 + \omega_2}{2}}$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$\boxed{\omega_2 - \omega_1 = \frac{R}{L} = \alpha} \quad \text{--- (A)}$$

$$\therefore (\omega_2 + \omega_1)^2 - (\omega_2 - \omega_1)^2 = 4\omega_1\omega_2$$

$$\boxed{\omega_2 + \omega_1 = \sqrt{(\omega_2 - \omega_1)^2 + 4\omega_1\omega_2}} \quad \text{--- (B)}$$

Solving α and β we get ω_1 and ω_2 .

Objecting

(i) $\omega = \omega_1$ $Z_1 = R - jR$ (capacitive in nature)

$$\theta_1 = \tan^{-1} \frac{-R}{R}$$

$$\theta_1 = -45^\circ$$

$$\cos \theta_1 = \frac{1}{\sqrt{2}} = 0.707 \text{ (leading) } ;$$

(2) $\omega = \omega_2$ $Z_2 = R + jR$ (inductive in nature)

$$\theta_2 = \tan^{-1} \frac{R}{R}$$

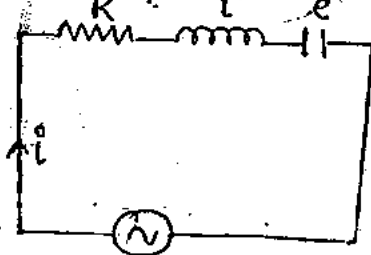
$$\theta_2 = 45^\circ$$

$$\cos \theta_2 = \frac{1}{\sqrt{2}} = 0.707 \text{ (lagging)}$$

N.B. (1) If P.F. angle is opposite to impedance angle then P.F. is leading.

(2) If P.F. angle is same as impedance angle then P.F. is lagging.

SERIES RESONANCE:



v, f_0

$$v = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$\frac{dv}{dt} = R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C}$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = \frac{1}{L} \frac{dv}{dt}$$

characteristic eqn

$$s^2 + \left(\frac{R}{L}\right)s + \left(\frac{1}{LC}\right) = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$2\zeta\omega_n = \frac{R}{L}$$

$$\zeta = \frac{R}{2L} \cdot \frac{1}{\omega_n}$$

$$\zeta = \frac{R}{2L} \cdot \sqrt{LC}$$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\text{or } \zeta = \frac{1}{2Q}$$

Objective 1

$R \uparrow$

$$B.W. = \frac{R}{L}$$

$Q \downarrow$

$f_1 \downarrow$

$f_2 \uparrow$

(At resonance)

$$\text{Losses} \propto \frac{1}{Q}$$

$$Q = \frac{V_L \text{ or } V_C}{V_R} = \frac{V_L \text{ or } V_C}{V_R} > 1$$

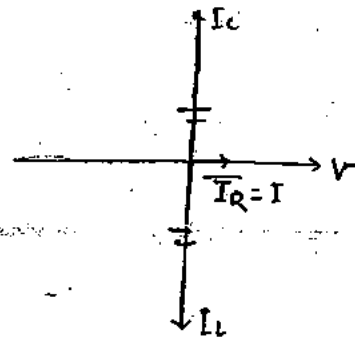
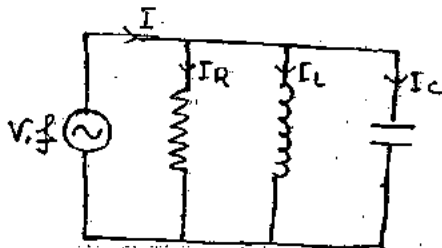
$$Q = \frac{X_L \text{ or } X_C}{R} > 1$$

interview question

$X_L > R$
 $X_C > R$

condition for
Series resonance

II Resonance crt:-



$$I = I_R + I_L + I_C$$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C}$$

$$Y = G + j(B_C - B_L)$$

$$\text{At } \omega = \omega_0$$

$$B_C = B_L$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

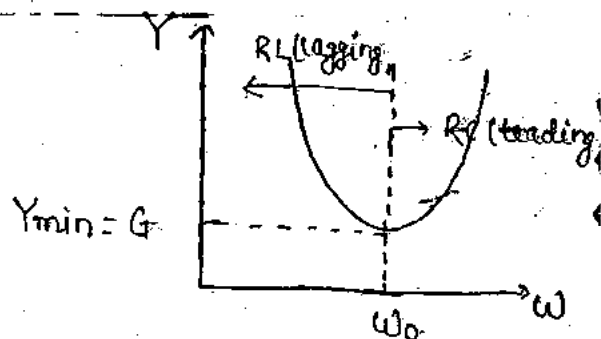
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ rad/sec}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

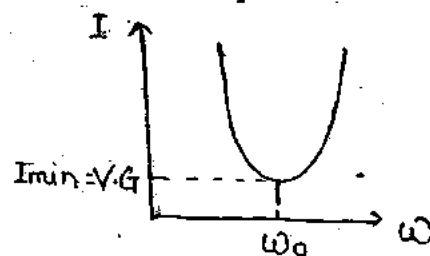
(1) $Y = G + j(\omega C - \frac{1}{\omega L})$

$Y \neq G + j(\omega C - \frac{1}{\omega L})$

$|Y| = \sqrt{G^2 + (\omega C - \frac{1}{\omega L})^2}$



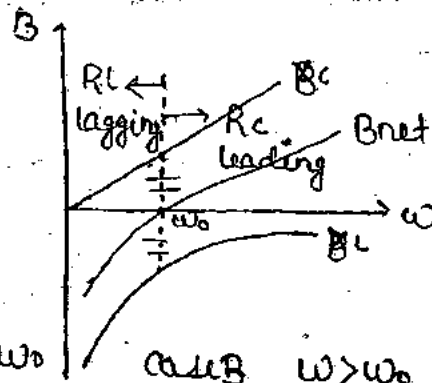
(2) $I = \frac{V}{|Z|} = V|Y|$



(3) $B_{net} = B_C - B_L = \omega C - \frac{1}{\omega L}$

$B_C = \omega C$

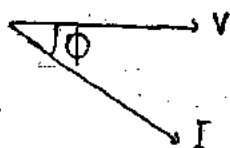
$B_L = \frac{1}{\omega L}$



case 1

$\omega < \omega_0$

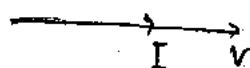
$Y = G - jB$



$\cos \phi = \text{lagging}$

case 2: $\omega = \omega_0$

$Y = G$



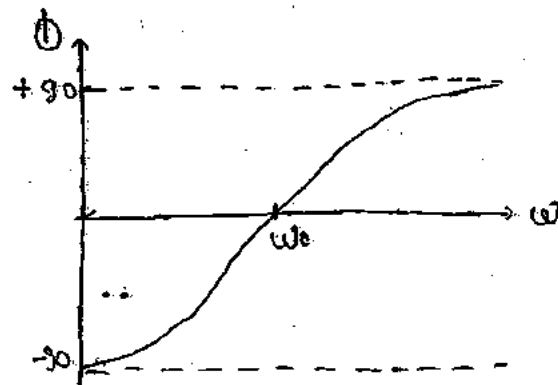
$\cos \phi = 1$

case 3: $\omega > \omega_0$

$Y = G + jB$



$\cos \phi = \text{leading}$



$$Q = \frac{\text{Reactive component of current}}{\text{Active component of current}}$$

$$Q = \frac{I_L \text{ or } I_C}{I_R} = \frac{\frac{V}{X_L} \text{ or } \frac{V}{X_C}}{V/R}$$

$$Q = \frac{R}{X_L} \text{ or } \frac{R}{X_C}$$

$$\boxed{Q = \frac{R}{\omega L}}$$

$$\boxed{Q = \omega R C}$$

$$Q = \frac{R}{L} \cdot \sqrt{L C}$$

$$\boxed{Q = R \cdot \sqrt{\frac{C}{L}}}$$

At $\omega = \omega_0$

$$I_R = V_R \cdot G = G \cdot V = \left(\frac{1}{R} \right) G \cdot V = I$$

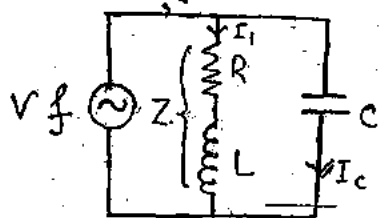
$$I_L = -j\omega C V_L = -j\omega C \cdot V = -j \frac{1}{\omega C} \cdot I_R = -j \left(\frac{R}{\omega C} \right) I = -j Q I$$

$$I_C = j\omega C V_C = j\omega C \cdot V = j(\omega C R) I = j Q I \quad \xrightarrow{\text{Amplification factor}}$$

$$I = I_R + I_L + I_C = I_R - j Q I + j Q I = I_R$$

$$\boxed{I = I_R} \quad \text{at } \omega = \omega_0$$

→ Practical tank circ:-



$$Y = Y_1 + Y_2$$

$$= R + j\omega L + j\omega C$$

$$= R + j(\omega C - \omega L)$$

At $\omega = \omega_0$

$$B_L = B_C$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$R^2 + X_L^2 = X_L X_C$$

$$\Rightarrow R^2 + \omega^2 L^2 = \omega L \cdot \frac{1}{\omega C}$$

$$\Rightarrow R^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

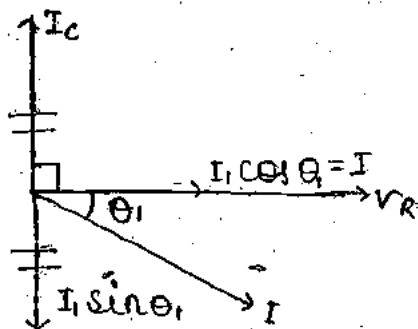
$$\Rightarrow \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

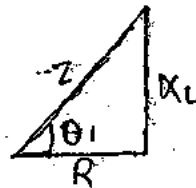
$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{L}}$$

Calculation of Z_{04} By tank circ:



At re



$$\tan \theta_1 = \frac{X_L}{R}$$

$$\cos \theta_1 = \frac{R}{Z_1}$$

$$\sin \theta_1 = \frac{X_L}{Z_1}$$

At Resonance frequency $\omega = \omega_0$

$$I_c = I_1 \sin \theta$$

$$\frac{V}{X_c} = \frac{V}{Z_1} \cdot \frac{X_L}{Z_1}$$

$$Z_1^2 = X_L \cdot X_c = \omega L \cdot \frac{1}{\omega C}$$

$$Z_1 = \sqrt{\frac{L}{C}}$$

$$I = I_1 \cos \theta$$

$$I = \frac{V}{Z_1} \cdot \frac{R}{Z_1}$$

$$\frac{V}{I} = \frac{Z_1^2}{R} = \frac{L}{CR}$$

$$Z_{04} = \frac{L}{RC}$$

* Z_{04} is the impedance seen by tank circ at Resonance frequency ($\omega = \omega_0$).

Calculation of Quality factor or Q-factor

$$Q = \frac{I_c}{I} \text{ or } \frac{I_1 \sin \phi_1}{I}$$

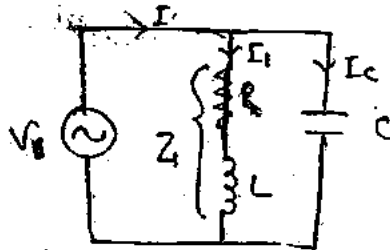
$$= \frac{V/X_c}{V/Z_{DY}} \text{ or } \frac{\frac{V}{Z_1} \cdot \frac{X_L}{Z_1}}{V/Z_{DY}}$$

$$= \frac{Z_{DY}}{X_c} \text{ or } \frac{Z_0 X_L}{Z_1^2}$$

$$Q = \frac{\omega L}{R}$$

objecting

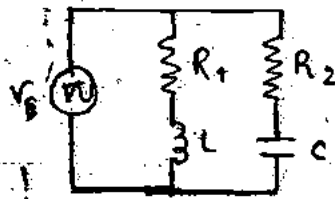
Ideal tank crt :- ($R=0$)



At $\omega = \omega_0$

$$Z_{DY} = \infty$$

* tank crt acts as open crt *
ie there is no current flow through crt



$$Y = Y_1 + Y_2$$

$$= (G_1 + G_2) + j(B_c - B_L)$$

At Resonance $\omega = \omega_0$

$$B_c = B_L$$

$$\frac{X_c}{R^2 + X_c^2} = \frac{X_L}{R^2 + X_L^2}$$

$$f = \frac{1}{2\pi \sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$$

This crt is in resonance at any frequency

When $R_1^2 = R_2^2 = L/C$

$$\frac{1/\omega C}{\frac{L}{C} + \frac{1}{\omega^2 C^2}} = \frac{\omega L}{\frac{L}{C} + \omega^2 L^2}$$

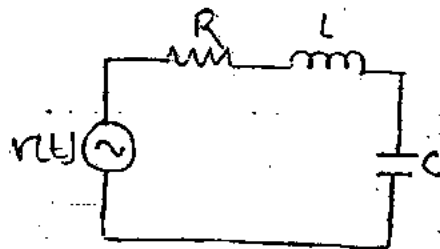
$$\frac{1}{\omega L + \frac{1}{\omega C}} = \frac{\omega L}{\omega L + \frac{1}{\omega C}}$$

Q. In the given crt.

(a) ω_0 and Half power frequency,

(b) B.W and Q-factor,

(c) Amplitude of current at $\omega = \omega_0$ and Half power frequency.



$$v(t) = 20 \sin \omega t$$

$$R = 2\Omega \quad L = 1\text{mH}$$

$$C = 0.4\mu\text{F}$$

Solⁿ (a) $\omega_0 = \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 10^{-6} \times 1 \times 10^{-3}}} = \frac{1}{\sqrt{4 \times 10^{-10}}}$

$$= \frac{1}{2 \times 10^{-5}} = \frac{10^5}{2} = 50\text{KRad/sec}$$

(b) $\text{B.W} = \frac{R}{L} = \frac{2}{1 \times 10^{-3}} = 2000 = 2\text{Krad/sec}$

$$Q = \frac{\omega L}{R} = \frac{\omega_0}{\text{B.W}} = \frac{50}{2} = 25$$

⑧ for Half power frequency

$$At \quad Z_1 = \sqrt{2} R \quad \omega_1 \omega_2$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} = \sqrt{2} R$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L} \right)^2 + \frac{1}{LC}}$$

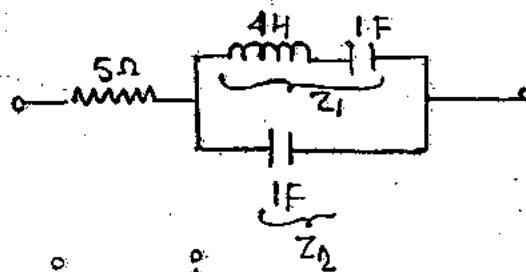
$$\omega_2 = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L} \right)^2 + \frac{1}{LC}}$$

⑨ $\omega_1 = \omega_0 - \frac{B\omega}{2} = 50 - 1 = 49 \text{ K rad/sec}$ } this is valid when $Q > 7$
 $\omega_2 = \omega_0 + \frac{B\omega}{2} = 50 + 1 = 51 \text{ K rad/sec}$

⑩ $I_{\omega_0} = \frac{V}{R} = \frac{20}{2} = 10 \text{ A}$

$$I_{\omega_1 \omega_2} = \frac{V}{\sqrt{2} R} = \frac{20}{\sqrt{2} \times 2} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

Q. Find the Resonant frequency of the given crt :



Soln

$$Z_1 = j4\omega - \frac{j}{\omega}$$

$$Z_2 = -\frac{j}{\omega}$$

$$Z_{eq} = Z_1 || Z_2$$

$$= j(4\omega - \frac{1}{\omega}) || -j/\omega$$

$$Z_{eq} = \frac{(j4\omega - \frac{j}{\omega}) \cdot -j/\omega}{j4\omega - \frac{j}{\omega} - \frac{j}{\omega}}$$

$$Z_{eq} = \frac{j(1 - 4\omega^2)}{\omega^2(4\omega - 2)}$$

$$\bar{Z} = 5 + Z_{eq}$$

$$\bar{Z} = 5 + j \frac{(1 - 4\omega^2)}{\omega^2(4\omega - 2)}$$

$$\omega_0 \quad 1 - 4\omega^2 = 0$$

$$\omega^2 = \frac{1}{4}$$

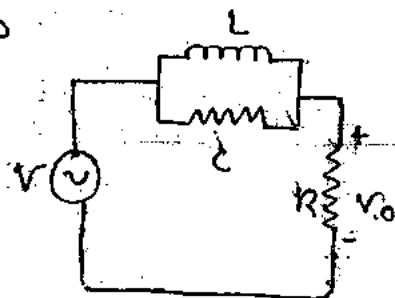
$$\omega = \frac{1}{2} = 0.5 \text{ rad/sec}$$

Q. Find the v_o from the crt at $\omega = \omega_0$ of tank crt

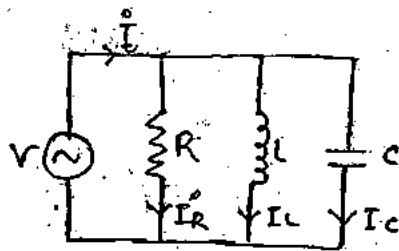
(a) v (b) $v/2$ (c) $2v$ (d) ∞

Ans $v_o = 0$ because $R = 0$

$$Z_{04} = \frac{L}{CR} = \infty$$



II RLC OFF:



$$i = I_R + I_L + I_C$$

$$= \frac{v}{R} + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$\frac{di}{dt} = \frac{1}{R} \frac{dv}{dt} + \frac{di}{dt} + \frac{i}{C}$$

$$i = \frac{v}{R} + \frac{1}{L} \int v dt + C \frac{dv}{dt}$$

$$\frac{di}{dt} = \frac{1}{R} \frac{dv}{dt} + \frac{v}{L} + C \frac{d^2v}{dt^2}$$

$$\frac{1}{C} \frac{di}{dt} = \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} + \frac{d^2v}{dt^2}$$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

compare with $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

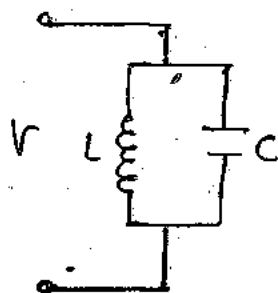
$$2\zeta\omega_n = \frac{1}{RC} \quad ; \quad \omega_n = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$\zeta = \frac{1}{2\omega_n RC}$$

$$\zeta = \frac{1}{2RC} \sqrt{\frac{L}{C}}$$

$$\text{or } \zeta = \frac{1}{2Q}$$

Q. In the given tank ckt the circulating current at Resonance will be equal to



(a) $\frac{V}{L}$

(b) $V \sqrt{\frac{C}{L}}$

(c) $V \sqrt{\frac{L}{C}}$

(d) 0

Soln.

$$I = \frac{V}{X_L}$$

$$= \frac{V}{\omega L}$$

$$= \frac{V}{\frac{L}{\sqrt{LC}}} = V \sqrt{\frac{C}{L}} \quad \underline{\text{Ans}}$$

Q. Page 7, 29, 32

w.B Page 9:

Q46: $P = V_{rms} I_{rms} \cos \phi$

$$= \frac{1}{\sqrt{2}} \cdot I$$

Page 10, 51, 50, 52

TRANSIENT PERIOD:

Defn:

Transient will be present in CRT, when CRT is subjected to any changes either by source magnitude or any CRT elements provided, the CRT consist of energy storage elements becoz.

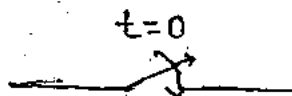
- (1) L does not allow sudden change in current and store energy in the form of ~~electrical~~ ~~force~~ magnetic field.
- (2) C does not allow sudden change in voltage and store energy in the form electrical field.
- (3) When CRT is purely resistive, then no transient are present because resistor allow sudden change in current & voltage.

$t(0^-)$ = just before the switch operation

$t(0)$ = exact instant of switch operation

$t(0^+)$ = just after the switch operation

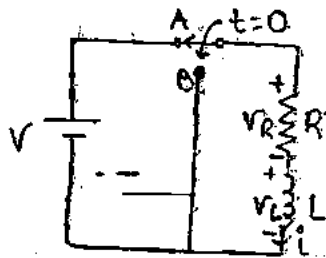
$t(\infty)$ = Steady state after the switch operation



$i_L(0^-) = i_L(0) = i_L(0^+)$ [because L does not allow sudden change in current]

$v_C(0^-) = v_C(0) = v_C(0^+)$ [because C does not allow sudden change in voltage]

Source free RL crt:



In case of DC, L is short circuited

$$\therefore I_0 = \frac{V}{R}$$

$$i(0^-) = i(0) = i(0^+)$$

For $t > 0$



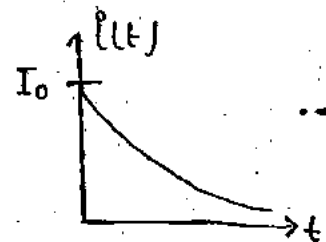
$$V_R + V_L = 0$$

$$iR + L \frac{di}{dt} = 0$$

$$iR = -L \frac{di}{dt}$$

$$\int_0^t -\frac{R}{L} dt = \int_{I_0}^{i(t)} \frac{1}{i} di$$

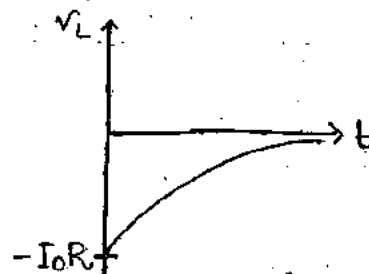
$$i(t) = I_0 e^{-R/L t}$$



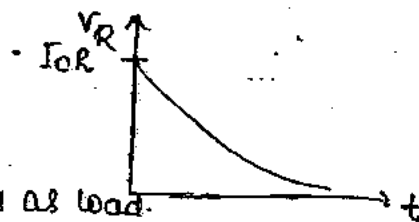
$$V_L = L \frac{di}{dt}$$

$$= L I_0 e^{-\frac{R}{L} t} \left(-\frac{R}{L} \right)$$

$$V_L = -I_0 R e^{-\frac{R}{L} t}$$



$$V_R = I_0 R e^{-\frac{R}{L} t}$$



- NB:
- During charging, L acts as load.
 - During discharging, L acts as source.

- In discharging L, the current dirⁿ does not change but polarity of voltage across the L changes

Proof The amount of energy lost by inductor is same as the amount of energy dissipated in inductor Resistor.

Soln
$$W_L(t) = \frac{1}{2} L i^2(t)$$

$$= \frac{1}{2} L (I_0 e^{-R/L t})^2$$

$$= \frac{1}{2} L I_0^2 e^{-\frac{2R}{L} t}$$

$$W_L(0) = \frac{1}{2} L I_0^2$$

$$W_L(t) = W_L(0) e^{-\frac{2R}{L} t} \quad \text{--- (1)}$$

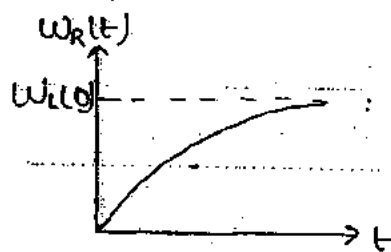
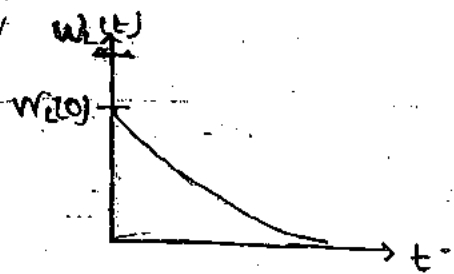
$$W_R(t) = \int P dt$$

$$= \int V_R(t) i(t) dt$$

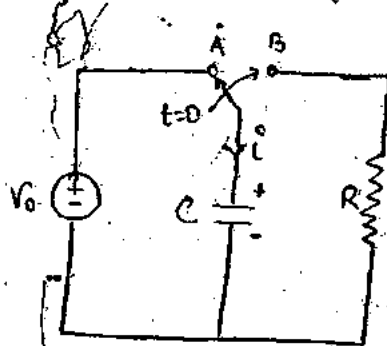
$$= \int_0^t (I_0 R e^{-\frac{R}{L} t}) (I_0 e^{-\frac{R}{L} t}) dt$$

$$= \frac{I_0^2 R \cdot e^{-\frac{2R}{L} t}}{-\frac{R}{L}} = \frac{1}{2} L I_0^2 (1 - e^{-\frac{2R}{L} t})$$

$$W_R(t) = W_L(0) (1 - e^{-\frac{2R}{L} t}) \quad \text{--- (2)}$$



SOURCE FREE RC CIRCUIT:



$$V(0^-) = V(0) = V(0^+) = V_0$$

for $t > 0$

$$i_C + i_R = 0$$

$$C \frac{dv_C}{dt} + \frac{V_C}{R} = 0$$

$$-\frac{V_C}{R} = C \frac{dv_C}{dt}$$

$$-\int_0^t \frac{1}{RC} dt = \int_{V_0}^{V(t)} \frac{1}{V_C} dV_C$$

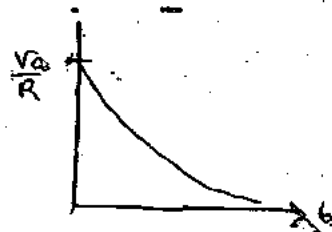
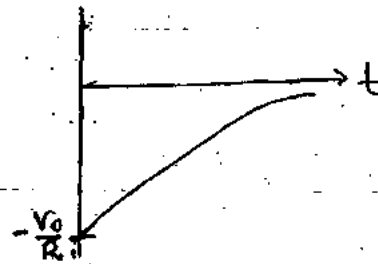
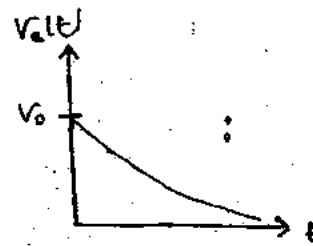
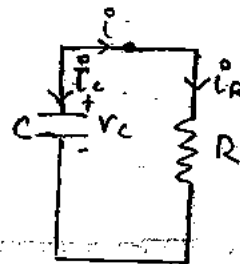
$$V(t) = V_0 e^{-t/RC}$$

$$i_C = C \frac{dv_C}{dt} = C \frac{d(V_0 e^{-t/RC})}{dt}$$

$$i_C = -\frac{V_0}{R} e^{-t/RC}$$

$$i_C = -i_R$$

$$i_R = \frac{V_0}{R} e^{-t/RC}$$



- during discharging, C acts as source.
- during charging, C acts as load.
- During discharge, the polarity of cap voltage across capacitor does not change but the ~~current~~ dirⁿ get reversed.

^{IES} The amount of energy of capacitor ~~is lost~~ is same as the amount of energy dissipated in

Proof: $W_C(t) = \frac{1}{2} C v_c^2$

$$= \frac{1}{2} C (V_0 e^{-t/RC})^2$$

$$W_C(t) = \frac{1}{2} C V_0^2 e^{-2t/RC}$$

$$W_C(0) = \frac{1}{2} C V_0^2$$

$$\therefore \boxed{W_C(t) = W_C(0) e^{-2t/RC}} \quad \text{--- (1)}$$

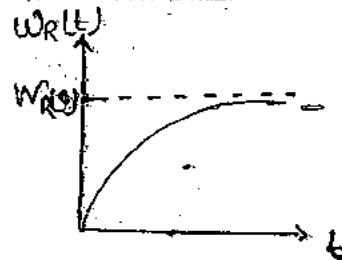
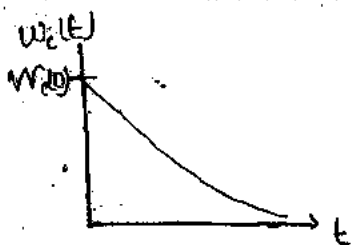
$$W_R(t) = \int_0^t P dt$$

$$= \int_0^t V_R(t) i_R(t) dt$$

$$= \int_0^t (V_0 e^{-t/RC}) \left(\frac{V_0}{R} e^{-t/RC} \right) dt$$

$$= \frac{1}{2} C V_0^2 [1 - e^{-2t/RC}]$$

$$\boxed{W_R(t) = W_R(0) [1 - e^{-2t/RC}]} \quad \text{--- (2)}$$



Time constant:

$$e^{-at} = e^{-t/\tau} \quad (1)$$

$$i(t) = I_0 e^{-Rt/L}$$

$$\boxed{\tau = L/R}$$

$$i(t) = I_0 e^{-t/RC}$$

$$\boxed{\tau = RC}$$

It is the time taken by the response to reach 36.7% of its initial value while discharging or decaying.

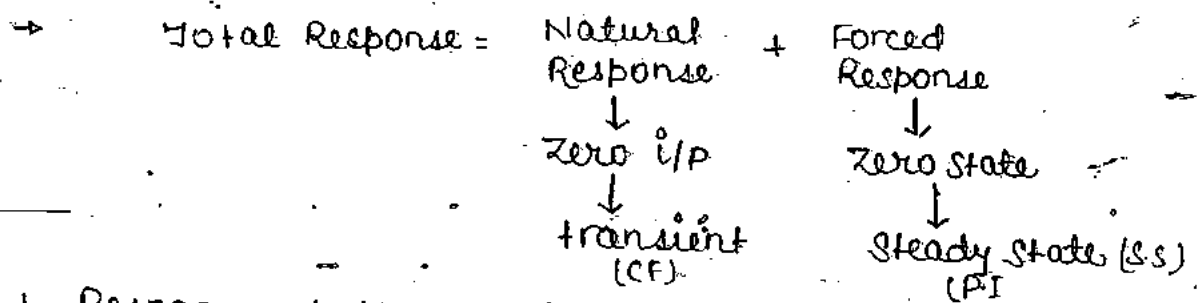
or

It is the time taken by the response to reach 63.3% of its steady state value while charging.

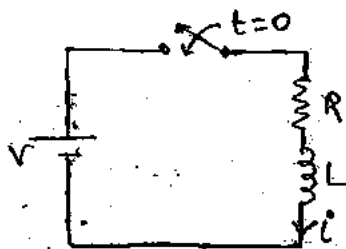
$$i(t) = I_0 e^{-t/\tau}$$

$$i(\tau) = I_0 e^{-\tau/\tau}$$

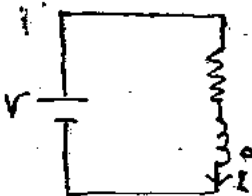
$$\boxed{i(\tau) = 0.367 I_0}$$



1. Response of the ckt & n/w with source present in it is called forced response which leads to ~~forced~~ ss response. This response independent of the nature of passive element and it only depends upon the type of i/p's. In the D.E this part found out by determining the P.I soln.
2. The response of ckt or n/w without source in it is called as Natural response which leads to transient response. This response purely depends on nature of passive element and independent from the i/p type. In the Differential eqn this part found out by determining the C.F soln.



$$i(0^-) = i(0^+) = 0 \text{ A}$$



$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{R}$$

$$T.R = C.F + P.I$$

① C.F -

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$i(t) = A e^{-\frac{R}{L}t}$$

or

(2) P.I -

$$\frac{di(t)}{dt} = \frac{V}{R}$$

$$i(t) = i(t)_{CF} + i(t)_{PI}$$

$$= A e^{-\frac{R}{L}t} + \frac{V}{R}$$

At $t=0$ $i=0$ A

$$A = (0 - \frac{V}{R})$$

$$A = [i(0) - i(\infty)]$$

$$i(t) = A e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$i(t) = \frac{V}{R} [1 - e^{-\frac{R}{L}t}]$$

*

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-\frac{R}{L}t}$$

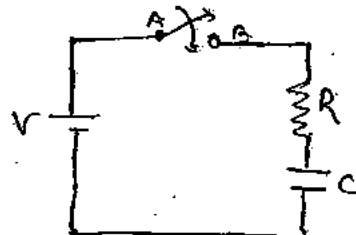
$$i(t) = (E \cdot V) + [E \cdot V - (E \cdot V)] e^{-\frac{R}{L}t}$$

*

This is applicable for dc & ult & also RC circuit.

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-\frac{t}{RC}}$$

for RC circuit



Q. If the initial voltage on the C = 150V

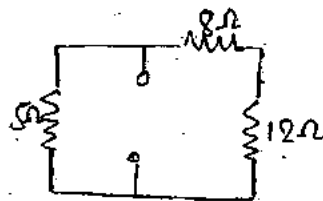
Find v_c , i_x $t > 0$

for $t > 0$ $V_0 = 15V$

$$V_c(t) = V_0 e^{-t/RC}$$

$$\tau = R_{eq} C$$

Calculate R_{eq}



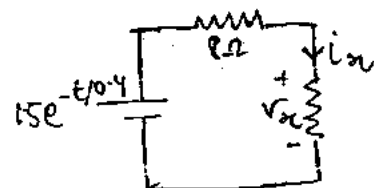
$$R_{eq} = 20 \parallel 5 = \frac{20 \times 5}{25} = 4 \Omega$$

$$\tau = 4 \times 0.1 = 0.4 \text{ sec}$$

$$\therefore V_c = 15 e^{-t/0.4}$$

$$i_x = 15 e^{-t/0.4} \cdot \frac{12}{20}$$

$$i_x = \frac{V_x}{20} = 0.75 e^{-t/0.4}$$



Q. Find the value of v_c , v_x , i_c w.r.t time when $V_0 = 3V$

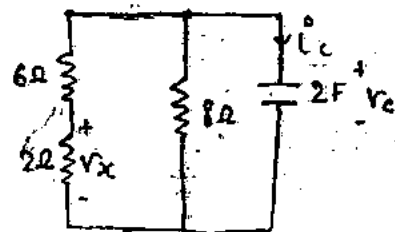
$$R_{eq} = 8 \parallel 8 = \frac{8 \times 8}{16} = 4 \Omega$$

$$\tau = 4 \times 2 = 8 \text{ sec}$$

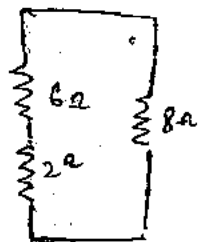
$$V_c(t) = V_0 e^{-t/RC} = 3 e^{-t/8}$$

$$i_c = \frac{-V_0}{R} e^{-t/RC} = -\frac{3}{4} e^{-t/8}$$

$$V_x = \frac{3}{4} e^{-t/8}$$

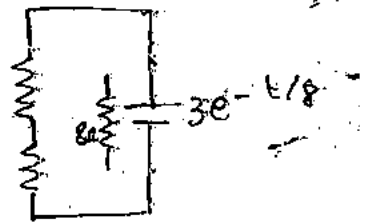


For R_{eq} (C is open)



$$V_x = - \frac{3}{4} e^{-t/8} \cdot \frac{2}{8}$$

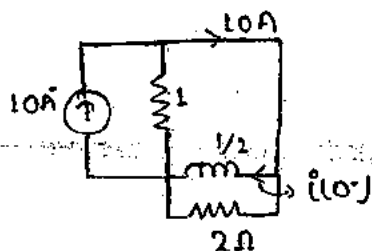
$$V_x = V_c(t) \cdot \frac{2}{8} \\ = 8e^{-t/8} \cdot \frac{2}{8} = \frac{3}{4} e^{-t/8}$$



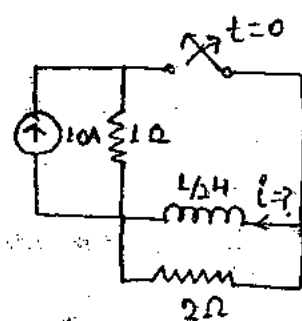
(because C and Resistor in ||)

Q. Find the value current when $t > 0$

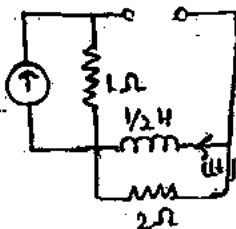
Soln. At $t = 0^-$



$$i(0^-) = i(0^+) = 10A$$



At $t > 0$



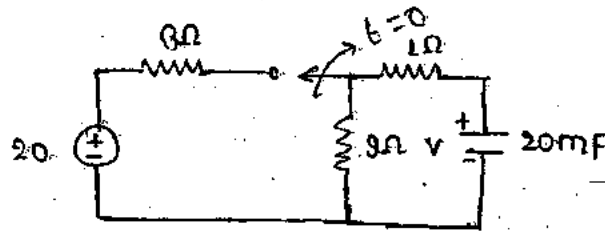
$$I = I_0 e^{-R/L t}$$

$$R = 2\Omega \quad L = 1/2$$

$$\tau = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$I = 10 e^{-4t} \text{ Ans}$$

Q. Find $v_c(t)$ when $t > 0$



Soln: $t < 0$

$$v_c(0^-) = v_c(0^+) = 25V$$

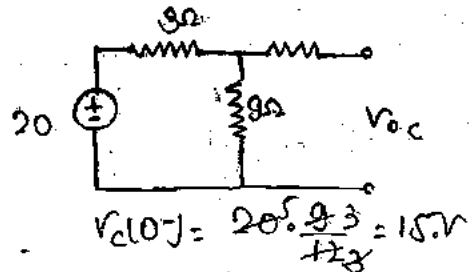
$t > 0$

$$v_c(t) = v_0 e^{-t/RC}$$

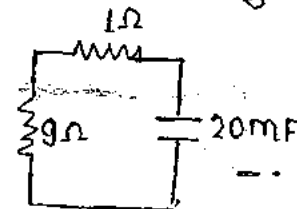
$$R_{eq} = 10\Omega \quad C = 20mF$$

$$RC = 10 \times 20 \times 10^{-3} = 0.2 \text{ sec}$$

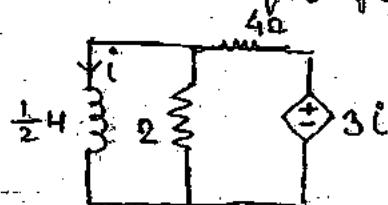
$$v_c(t) = 25 e^{-t/0.2} = 25 e^{-5t}$$



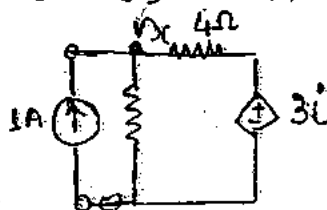
$$v_c(0^-) = 20 \times \frac{9 \times 3}{123} = 15V$$



Q. The initial value of current through inductor is 10 A. Find the value of i for $t > 0$



Soln: $i(0^-) = i(0^+) = 10A$



$$1 = \frac{v_x}{2} + 3i$$

$$1 = \frac{v_x}{2} + 3 \times 10$$

$$v_x = 8$$

$$1 = \frac{v_x}{2} + \frac{v_x - 3i}{4}$$

$$1 = \frac{v_x}{2} + \frac{v_x + 3}{4}$$

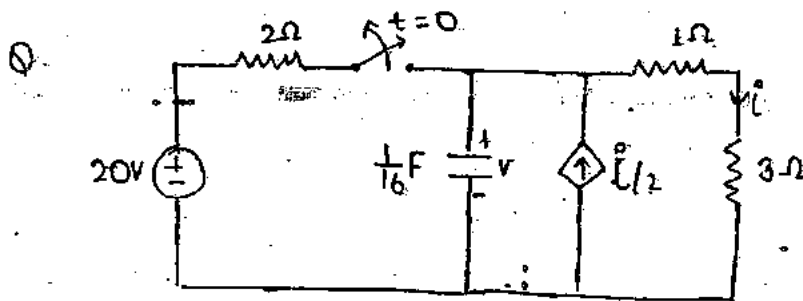
$$\frac{3V_{oc}}{4} = \frac{1}{4}$$

$$V_{oc} = 1/3 \text{ Volts}$$

$$R_{TH} = \frac{V_{oc}}{I} = \frac{1/3}{1} = \frac{1}{3} \Omega$$

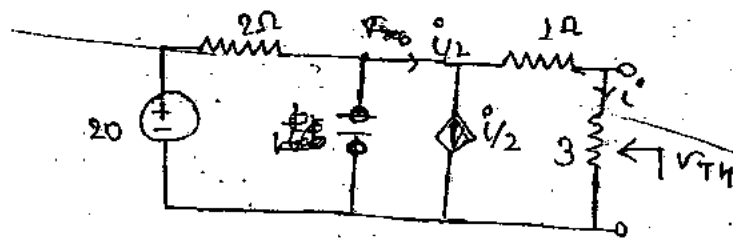
$$\tau = R_{TH} / L = \frac{1}{3} \times 2 = \frac{2}{3} \text{ sec}$$

$$I(t) = 10e^{-\frac{2}{3}t} \text{ A}$$



determine the current i in the given CT.

Soln: At $t=0^-$



$$V_c - 20 - 20 + 2i/2 + V_c = 0$$

$$5i = 20$$

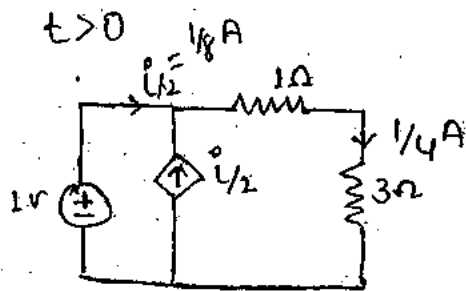
$$i = 4 \text{ A}$$

$$-20 + 2i/2 + V_c = 0$$

$$-20 + 2 \times 2 + V_c$$

$$V_c = 16 \text{ V}$$

$$V(0^-) = V(0^+) = 16 \text{ V}$$



$$R_{TH} = \frac{1}{1/8} = 8 \Omega$$

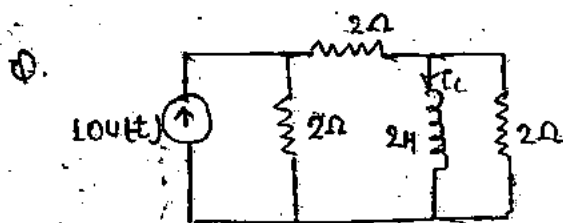
$$\tau = \frac{L}{R} = 8 \times \frac{1}{16} = \frac{1}{2} \text{ sec} \quad Z = RC = 8 \times \frac{1}{16} = \frac{1}{2} \text{ sec}$$

$$V(t) = V_0 e^{-t/\tau} = 16 e^{-2t/1} \text{ V}$$

$$i_L(t) = \frac{16 e^{-2t}}{3+1} = 4 e^{-2t} \text{ A}$$

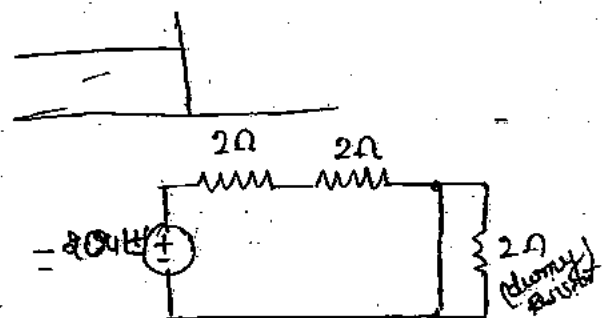
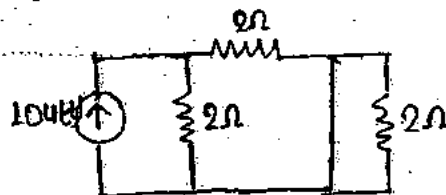
N.B (1) we never connected voltage source in || with dependent voltage source.

(2) we never connected current source in series with dependent current source.



Find the current response on the inductor

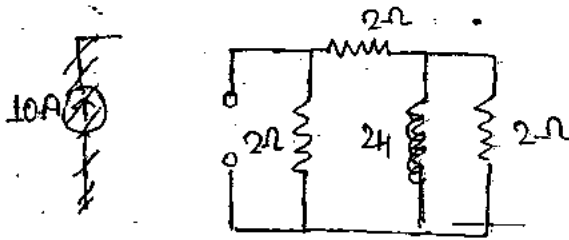
Soln: $t < 0$ or $t(0^-)$



$$I = \frac{20}{4} = 5 \text{ A}$$

$$I(0^-) = I(0^+) = 5 \text{ A}$$

$$t > 0$$

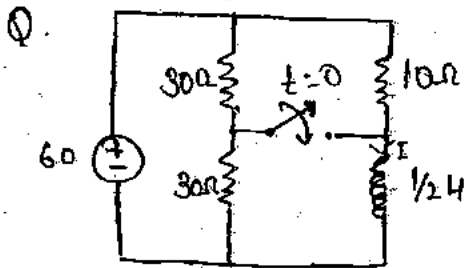


$$R_{eq} = 4 \parallel 2 = \frac{4 \times 2}{4 + 2} = \frac{4}{3}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{\frac{4}{3}}{2} = \frac{2}{3} \Rightarrow \tau = \frac{3}{2} \text{ sec}$$

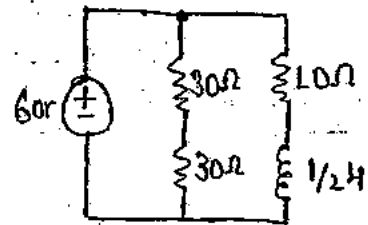
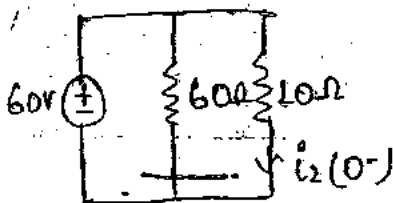
$$i(t) = I_0 e^{-t/\tau}$$

$$i(t) = 10 e^{-\frac{2}{3}t} \text{ A}$$



find the current through inductor?

at $t = 0^-$

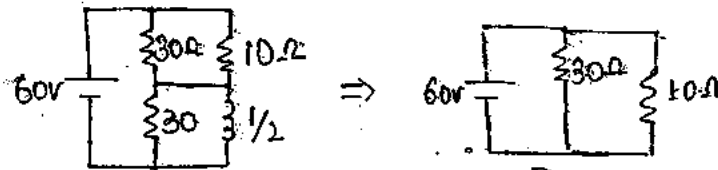


$$I = \frac{60 \times \frac{60 \times 10}{60 + 10}}{60 + 10} = \frac{60 \times 600}{70} =$$

$$i_L(0^-) = \frac{60}{10} = 6 \text{ A}$$

$$i_L(0^-) = i_L(0^+) = 6 \text{ A}$$

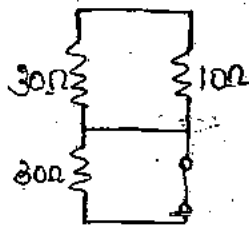
$$t > 0$$



$$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{R_t}{L} t}$$

$$i(\infty) = \frac{60}{30 \parallel 10}$$

$$i(\infty) = \frac{60}{30 \times 10} \times 40 = 8 \text{ A}$$



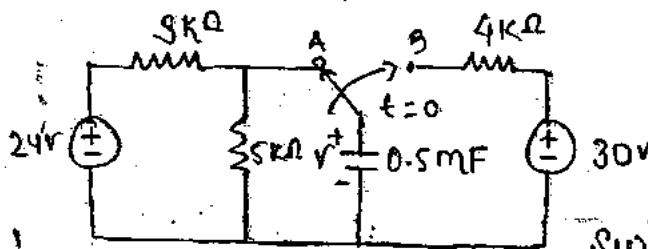
$$R_{eq} = 30 \parallel 30 \parallel 10$$

$$= \frac{30 \times 30}{60} \parallel 10$$

$$= 15 \parallel 10 = \frac{15 \times 10}{25} = 6 \Omega$$

$$i(t) = 8 + [6 - 8] e^{-\frac{L}{R} t}$$

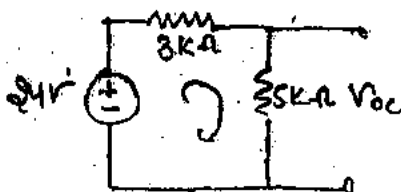
Q.



The switch in the as in posⁿ A for long time. At $t=0$ Switch is move to B posⁿ

Determine $v(t)$ for $t > 0$

Solⁿ At $t(0^-)$



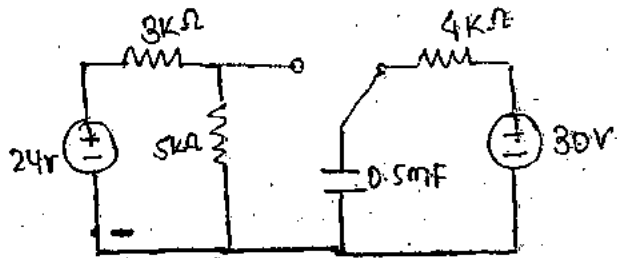
$$I = \frac{24}{8} = 3 \text{ A}$$

$$v_{oc} = 5 \times 3 = 15 \text{ V}$$

$$v(0^-) = 15 \text{ V}$$

$$v(\infty) = v(0^+) = 15V$$

At $t > 0$



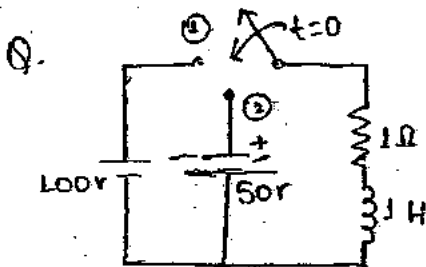
$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/RC}$$

$$v(t) = v(\infty) + [v(0^+) - v(\infty)]e^{-t/RC}$$

$$v(t) = \frac{30}{4} + [15 - \frac{30}{4}]e^{-\frac{t}{4 \times 0.5}}$$

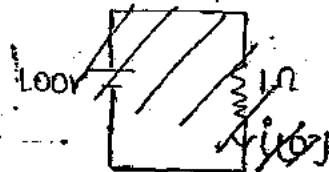
$$= 30 - 15e^{-t/2} \text{ volts}$$

$$= 15V$$



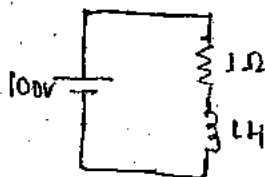
Q. A switch at $t = 0$ sec the switch connected to posⁿ 1 after one time constant the switch connected to ②. find the current response when switch connected to position ②.

Soln: $t > 0$ for $t > 0$



$$i(\infty) = 100A - i(0^+)$$

$t > 0$

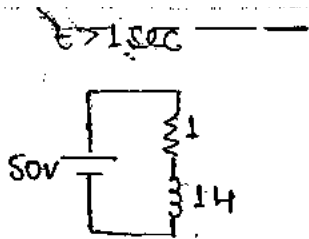


$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

$$i(\infty) = 100A$$

$$i(0^+) = i(0^-) = 0A$$

$$i(t) = 100(1 - e^{-t}) \quad \text{--- (1)}$$



$$i(t) = i(\infty) + [i(1^+) - i(\infty)]e^{-\frac{(t-1)}{\tau}}$$

$$i(\infty) = 50 \text{ A}$$

$$i(1^+) = 100[1 - e^{-1}]$$

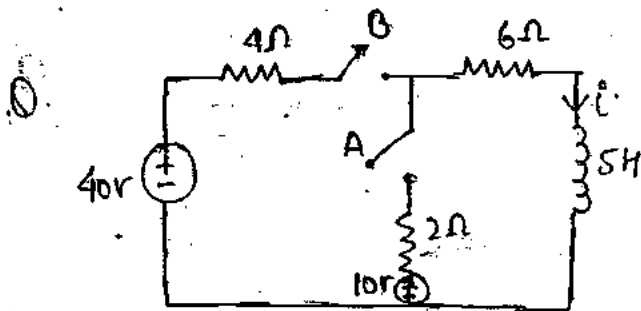
$$\approx 100[1 - 0.367]$$

$$= 63.3 \text{ A}$$

$$i(1) = 63.3 \text{ A}$$

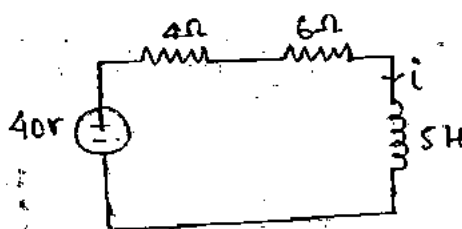
$$i(t) = 50 + [63.3 - 50]e^{-(t-1)}$$

$$i(t) = 50 + 13.3e^{-(t-1)} \text{ A} \quad \underline{\text{Ans}}$$



At $t=0$ switch B in the figure is closed and switch A 4 sec later find $i(t)$ for $t > 0$?

Soln: $t < 4 \text{ sec } t > 0$



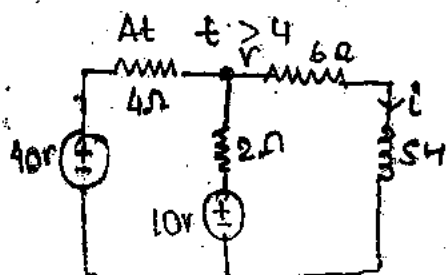
$$i(0^-) = i(0^+) = 0 \text{ A}$$

$$I = \frac{40}{10} = 4 \text{ A}$$

$$I(\infty) = 4 \text{ A}$$

$$i(t) = 4 - 4e^{-\frac{t}{\tau}}$$

$$\therefore i(4) = 4[1 - e^{-8}] \approx 4 \text{ A}$$



$$V_{6\Omega} = \frac{10}{2} = 5 \text{ V}$$

$$\frac{V-40}{4} + \frac{V-10}{2} + \frac{V}{6} = 0$$

$$V \left[\frac{1}{4} + \frac{1}{2} + \frac{1}{6} \right] = 10 + 5$$

$$V \left[\frac{3+6+2}{12} \right] = 15$$

$$V = 15 \times \frac{12}{11} = \frac{180}{11} \text{ volts}$$

$$i(\infty) = \frac{15 \times 12}{11 \times 6} = \frac{30}{11} \text{ A}$$

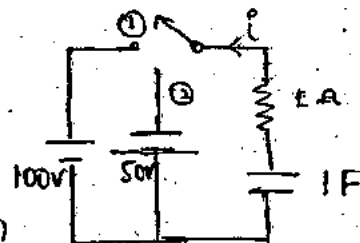
$$\tau = \frac{L}{R_{eq}}$$

$$R_{eq} = (4 \parallel 2) + 6 = \frac{22}{3}$$

$$\tau = \frac{5}{\frac{22}{3}} = \frac{15}{22} \text{ sec}$$

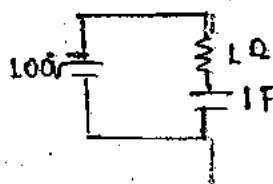
$$\begin{aligned} i(t) &= i(\infty) + [i(4) - i(\infty)] e^{-\frac{22}{15}(t-4)} \\ &= \frac{30}{11} + (4 - \frac{30}{11}) e^{-\frac{22}{15}(t-4)} \\ &= \frac{30}{11} - 2.727 + 1.272 e^{-\frac{22}{15}(t-4)} \text{ Ans.} \end{aligned}$$

Q. In the circ show $t=0$ sec the switch is connected at posⁿ 1 after 1 time constant the switch is move to (2). Find current response when switch at posⁿ (2).

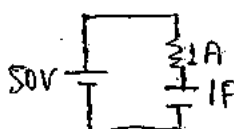


Soln: $V(0^-) = V(0^+) = 0V$ at $t < 0$

$t > 0$



$t > 1 \text{ sec}$



$$V(\infty) = 100V$$

$$\begin{aligned} V(t) &= 100 + (0 - 100) e^{-t} \\ &= 100(1 - e^{-t}) \end{aligned}$$

$$V(1^+) = 100(1 - e^{-1}) = 63.3 \text{ volts}$$

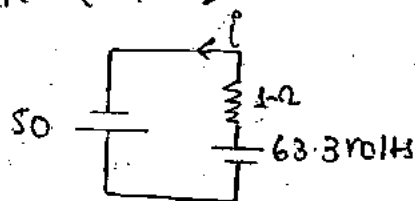
$$\begin{aligned} V(t) &= 50 + (63.3 - 50) e^{-(t-1)} \\ &= 50 + 13.3 e^{-(t-1)} \end{aligned}$$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-(t-1)/\tau}$$

$$i(\infty) = 0$$

$$i(t) =$$

At $t = 1^+$



$$i = 63.3 + 50 = 113.3 \text{ V}$$

$$i(t) = 0 + [113.3 - 0]e^{-(t-1)}$$

$$i(t) = 113.3e^{-(t-1)} \text{ Aug}$$

Q. Find the current response of $i(t)$ for $t > 0$ when capacitor of was initially unenergized.

Soln: Ex 0

$$v(0^-) = v(0^+) = 0 \text{ V}$$

$$v(t) = v(\infty) + [v(0^+) - v(\infty)]e^{-t/\tau}$$

$$= 10 + [0 - 10]e^{-t} = 10 - 10e^{-t}$$

$$i(t) = 10e^{-t}; 0 \leq t \leq 1$$

$t > 1 \text{ sec}$

$$v(1^+) = 10 - 10e^{-1} = 10(1 - 0.372) = 6.3 \text{ volts}$$

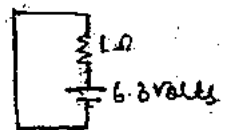
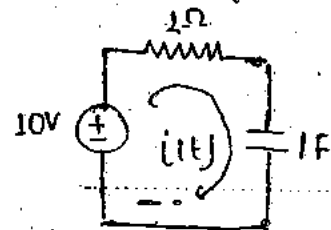
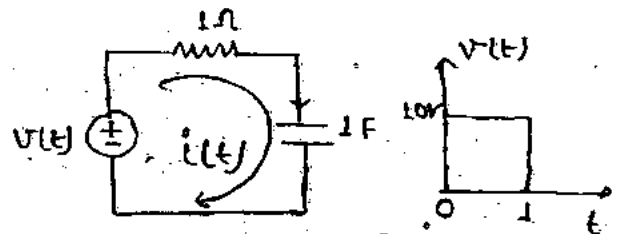
$$v(t) = v(\infty) + [v(1^+) - v(\infty)]e^{-(t-1)}$$

$$= 0 + [6.32 - 0]e^{-(t-1)}$$

$$= 6.32e^{-(t-1)}$$

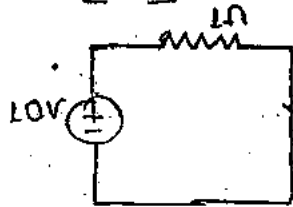
$$I(t) = C \frac{dv}{dt} = 1 \times -6.32e^{-(t-1)}$$

$$= -6.32e^{-(t-1)} \text{ Aug}$$

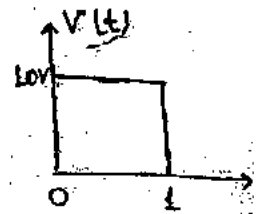
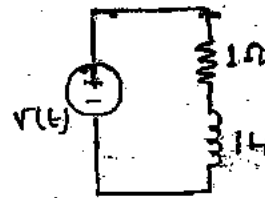


Q Find the current response $i(t)$ when initially inductor is unenergised.

Soln: $0 \leq t \leq 0$ $i(0^-) = i(0^+) = 0A$



$$i(\infty) = \frac{10}{1} = 10A$$



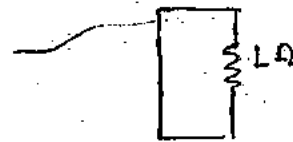
$$i(t) = 10 + (0 - 10)e^{-t} \\ = 10 - 10e^{-t}$$

$t > 1 \text{ sec}$

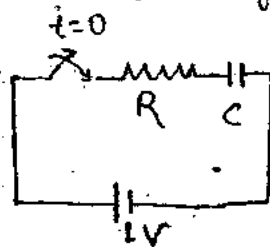
$$i(t) = 10 - 10e^{-1} = 6.32A$$

$$i(\infty) = 0$$

$$i(t) = 0 + (6.32 - 0)e^{-(t-1)} \\ = 6.32e^{-t} \text{ Ans}$$



Q Find the rate of rise of voltage across the capacitor at $t(0^+)$



Soln: $V(t) = 1 + (0 - 1)e^{-t/Rc}$

$$= 1 - e^{-t/Rc}$$

$$\frac{dV}{dt} = \frac{1}{Rc} e^{-t/Rc}$$

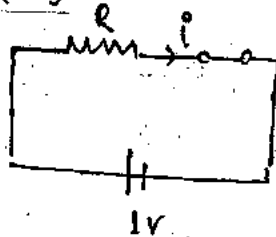
$$\frac{dV(0^+)}{dt} = \frac{1}{Rc} \text{ Ans}$$

Method : 2:

At $t(0^-)$

$$V(0^-) = V(0^+) = 0V$$

At $t(0^+)$



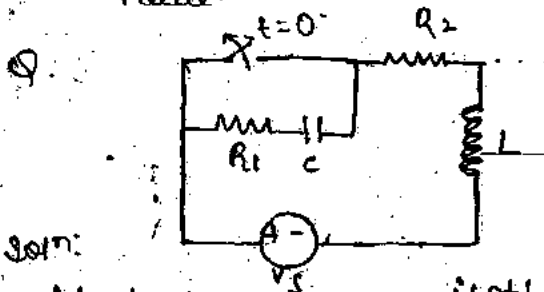
$$i = \frac{1}{R}$$

$$i = C \frac{dv}{dt}$$

$$\frac{1}{R} = C \frac{dv}{dt} \quad \therefore \frac{dv}{dt} = \frac{1}{RC}$$

N.B In this type Question at $t(0^+)$

- (1) Inductor is replaced by current source of its initial value.
- (2) capacitor is replaced by voltage source of its initial value.



Find the voltage across the switch for $t=0^+$

Soln:

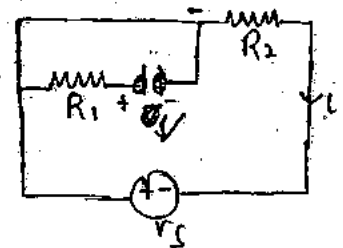
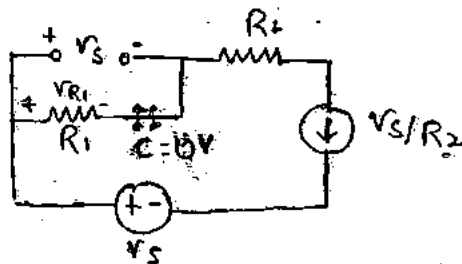
At $t=0^-$

$$i = \frac{V_s}{R_2}$$

$$i(0^-) = i(0^+) = \frac{V_s}{R_2}$$

$$V(0^-) = V(0^+) = 0V$$

At $t=0^+$

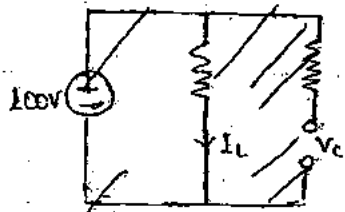
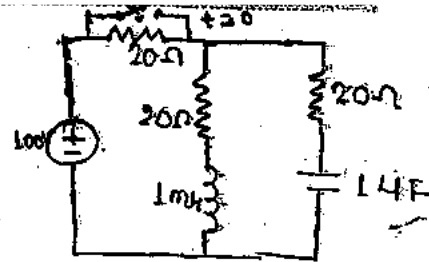


$$V_{R1} = \frac{V_s}{R_2} \times R_1$$

$$V_s = V_{R1} = \frac{V_s}{R_2} \times R_1$$

Q. Find the rate of rise of current through inductor at $t=0^+$

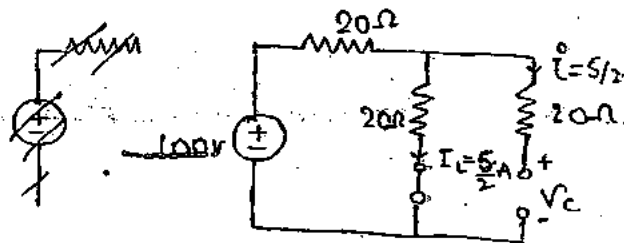
Soln: $\lambda = \phi$



$$I_L(0^-) = I_L(0^+) = \frac{100}{20} = 5A$$

$$V(0^-) = V(0^+) = 100V$$

$t=0^+$ $t=0^-$



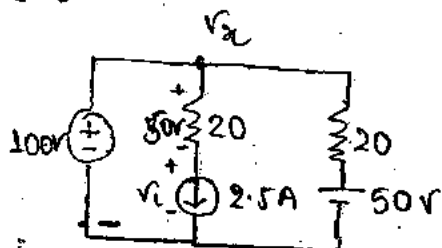
$$I_L = \frac{100}{40} = 2.5A$$

$$V_L(0^+) = I_L(0^+) = 2.5A$$

$$V(0^+) = V(0^+) = 50V$$

$t=0^+$

$$V_C = 20 \times \frac{5}{2} = 50V$$



$$\frac{V_L}{L} + \frac{V_L}{20} = \frac{100}{20} - \frac{50}{20}$$

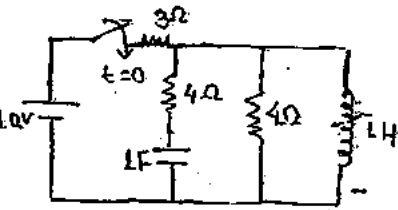
$$V_L = 50V$$

$$V_L = L \frac{di}{dt}$$

$$50 = L \times \frac{di}{dt}$$

$$\frac{di}{dt} = 50 \times 10^3 \text{ A/s}$$

- Q. Find the current flowing through capacitor and voltage across inductor at $t=0$ and also calculate the energy stored at $t(\infty)$?



Soln: $t = 0^-$

$$I(0^-) = I(0^+) = 0A$$

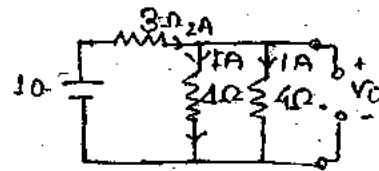
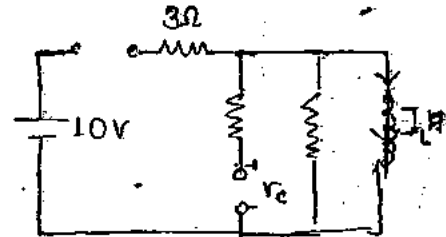
$$V(0^-) = V(0^+) = 0V$$

$$t = 0^+$$

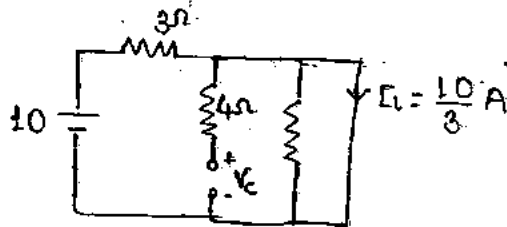
$$V_c = 10V$$

$$I_c = \frac{10}{5} = 2A$$

$$I_L = 1A$$



$$\text{At } t = \infty$$



$$I_L(\infty) = \frac{10}{3} A$$

$$V_c(\infty) = 0V$$

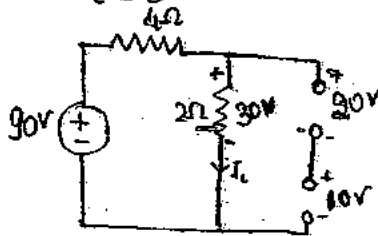
$$W_L(\infty) = \frac{1}{2} L I^2 = \frac{1}{2} \times \frac{10^2}{3} \times 1 = \frac{100}{18} = \frac{50}{9} J$$

$$W_C(\infty) = \frac{1}{2} C V^2 = 0J$$

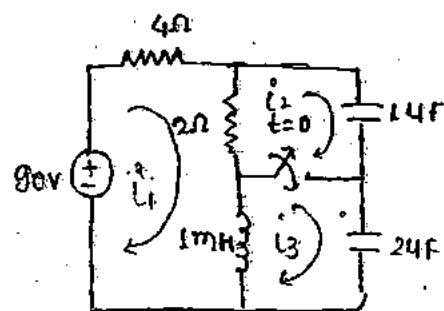
- Q. Find the current through i_1 , i_2 and i_3

Soln:

$$t = 0^-$$

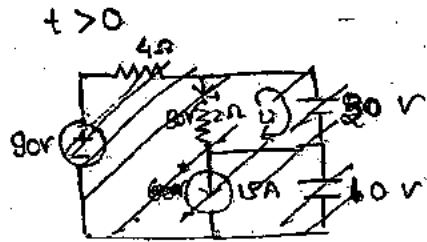


$$I_1 = \frac{30}{30} = 1A$$



$$I_L(0^-) = I_L(0^+) = 1A$$

$$I_C(0^-) = \dots$$

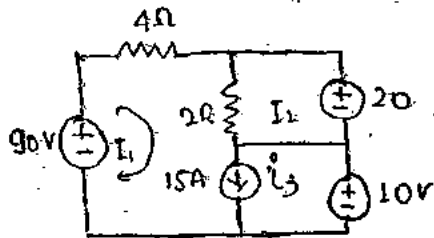


$$V_{C1} = \frac{30}{3} \times 2 = 20V$$

$$V_{C2} = \frac{30}{3} \times 1 = 10V$$

$$\begin{cases} V_{C1} = \frac{V_s}{C_1 + C_2} \cdot C_2 \\ V_{C2} = \frac{V_s}{C_1 + C_2} \cdot C_1 \end{cases}$$

At $t = 0^+$



Loop 1: $-90 + 4i_1 + 20 + 10 = 0$

$$4i_1 = 60$$

$$i_1 = \frac{60}{4} = 15A$$

Loop 2: $20 + (i_2 - i_1)20 = 0$

$$20 + 2i_2 - 30 = 0$$

$$2i_2 = 10A$$

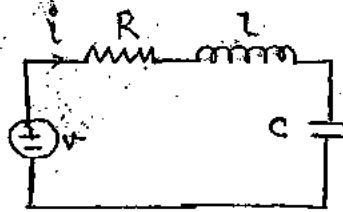
$$i_2 = \frac{10}{2} = 5A$$

Loop 3:

$$15 = i_1 - i_3$$

$$i_3 = 0A$$

Series RLC CRT:



$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$0 = \frac{dV}{dt} = \frac{d}{dt} \left(iR + L \frac{di}{dt} + \frac{1}{C} \int i dt \right) = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

taking laplace

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\alpha = -\frac{R}{2L} \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case 1: For overdamped $\left[\left(\frac{R}{2L}\right)^2 > \frac{1}{LC} \right]^*$

$$i(t) = C_1 e^{(\alpha + \beta)t} + C_2 e^{(\alpha - \beta)t}$$

Case 2: For critical damped

$$* \left(\frac{R}{2L}\right)^2 = \frac{1}{LC} *$$

$$s_{1,2} = -\frac{R}{2L}$$

$$i(t) = (C_1 + C_2 t) e^{\alpha t}$$

Case 3: underdamped

$$s_{1,2} = -\frac{R}{2L} \pm j \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

$$i(t) = (C_1 \cos \beta t + C_2 \sin \beta t) e^{\alpha t}$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{4L}}$$

$$\boxed{\omega_d = \omega_0 \sqrt{1 - \frac{R^2 C}{4L}}} \quad \text{or} \quad \boxed{\omega_d = \omega_0 \sqrt{1 - \xi^2}}$$

Damping coefficient $\alpha = \frac{R}{2L}$

$$\boxed{\zeta = \frac{2L}{R} = \frac{1}{\alpha}}$$

case 4: undamped ($R=0$)

$$s = \pm j \frac{1}{\sqrt{LC}}$$

$$\boxed{i(t) = C_1 \cos \beta t + C_2 \sin \beta t}$$

Q. II RLC crt:

$$I = I_R + I_L + I_C$$

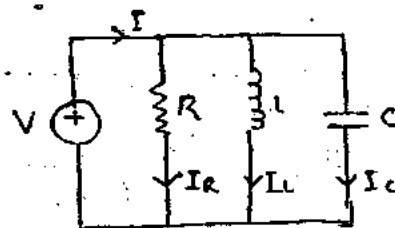
$$I = \frac{V}{R} + \frac{1}{L} \int v dt + C \frac{dv}{dt}$$

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{V}{L} = 0$$

$$\frac{d^2 v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{V}{LC} = 0$$

$$\boxed{s^2 + \frac{1}{RC} s + \frac{V}{LC} = 0}$$

$$s_{1,2} = \underbrace{-\frac{1}{2RC}}_{\alpha} \pm \sqrt{\underbrace{\left(\frac{1}{RC}\right)^2}_{\beta} - \frac{1}{LC}}$$



Case 1: overdamped $\left[\left(\frac{1}{2Rc}\right)^2 > \frac{1}{Lc}\right]$

$$v(t) = c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t}$$

Case 2: critically damped $\left[\left(\frac{1}{2Rc}\right)^2 = \frac{1}{Lc}\right]$

$$s_{1,2} = -\frac{1}{2Rc}$$

$$v(t) = (c_1 + c_2 t) e^{\alpha t}$$

Case 3: underdamped $\left[\left(\frac{1}{2Rc}\right)^2 < \frac{1}{Lc}\right]$

$$s_{1,2} = -\frac{1}{2Rc} \pm j \sqrt{\frac{1}{Lc} - \left(\frac{1}{2Rc}\right)^2}$$

$$v(t) = (c_1 \cos \beta t + c_2 \sin \beta t) e^{\alpha t}$$

$$\begin{aligned} \omega_d &= \sqrt{\frac{1}{Lc} - \left(\frac{1}{2Rc}\right)^2} \\ &= \frac{1}{\sqrt{Lc}} \sqrt{1 - \frac{L}{4R^2c}} \end{aligned}$$

$$\omega_d = \omega_0 \sqrt{1 - \frac{L}{4R^2c}}$$

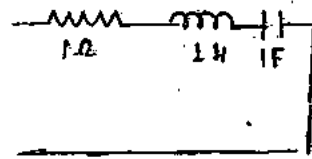
$$\boxed{\omega_d = \omega_0 \sqrt{1 - \xi^2}}$$

where $\boxed{\xi = \frac{1}{Rc} \sqrt{\frac{L}{c}}}$

$$\boxed{\tau = 2/Rc}$$

Q. The given n/w is

- (a) underdamped
- (b) overdamped
- (c) critically damped
- (d) undamped



soln: $s_{1,2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 1}$

$\left(\frac{1}{2}\right)^2 < 1$ underdamped

Or $\xi = \frac{R}{2L} \sqrt{\frac{C}{L}} = \frac{1}{2}$ When $0 < \xi < 1$ underdamped

Ac transient:

- Dc transient are most severe than Ac transient
- In the A-c ckt based on selection of crt parameters; operating frequency & switching operation, it is possible to obtain transient free response but in D-c crt it is not so.

Series R-L crt:

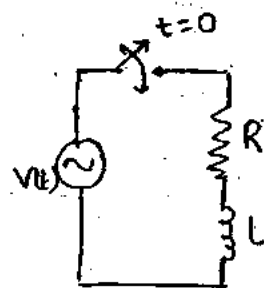
$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

$$i(t) = \underbrace{i(t)}_{\text{C-F}} + \underbrace{i(t)}_{\text{P-I}}$$

① $\frac{di}{dt} + \frac{R}{L} i = 0$

$$i_{\text{CF}}(t) = A \cdot e^{-\frac{Rt}{L}}$$

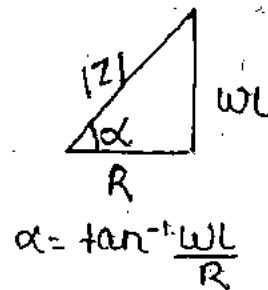


$$V(t) = V_m \sin(\omega t + \theta)$$

$$\hat{i}_{PR}(t) = \frac{V}{Z} = \frac{V}{|Z| \angle \alpha}$$

$$\hat{i}_{PR} = \frac{V_m \sin(\omega t + \theta)}{|Z| \angle \alpha}$$

$$\hat{i}_{PR}(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \alpha)$$



$$i(t) = A e^{-\frac{R}{L}t} + \frac{V_m}{|Z|} \sin(\omega t + \theta - \alpha) \quad \text{--- (1)}$$

At $t = 0$

$$0 = A \cdot 1 + \frac{V_m}{|Z|} \sin(\theta - \alpha)$$

$$A = -\frac{V_m}{|Z|} \sin(\theta - \alpha)$$

Substitutes this value in eqⁿ (1)

$$i(t) = \underbrace{-\frac{V_m}{|Z|} \sin(\theta - \alpha) e^{-\frac{R}{L}t}}_{\text{tr. response}} + \underbrace{\frac{V_m}{|Z|} \sin(\omega t + \theta - \alpha)}_{\text{S.S.}}$$

To make the transient response zero

$$\theta - \alpha = 0$$

$$\theta = \alpha$$

$$\boxed{\theta = \tan^{-1} \frac{\omega L}{R}} \quad \star$$

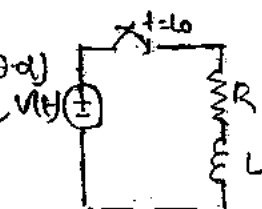
(ii)

if we take $v(t) = V_m \cos(\omega t + \theta)$

$$\boxed{\theta = \tan^{-1} \frac{\omega L}{R} + \pi/2}$$

(iii) if switching is happen at $t = t_0$ when $v(t) = V_m \sin(\omega t + \theta)$

$$i(t) = \underbrace{-\frac{V_m}{|Z|} \sin(\omega t_0 + \theta - \alpha) e^{-\frac{R}{L}(t-t_0)}}_{\text{tr. response}} + \underbrace{\frac{V_m}{|Z|} \sin(\omega t + \theta - \alpha)}_{\text{S.S.}}$$



To make transient zero

$$\omega t_0 + \theta - \alpha = 0$$

$$\omega t_0 = -\theta + \alpha = \tan^{-1} \frac{\omega L}{R} - \theta$$

Case 4: if $v(t) = V_m \cos(\omega t + \theta)$ and switching at $t = t_0$

$$\omega t_0 + \theta - \alpha = \frac{\pi}{2}$$

$$-\omega t_0 = \frac{\pi}{2} + \alpha - \theta$$

$$\omega t_0 = \frac{\pi}{2} + \tan^{-1} \left(\frac{\omega L}{R} \right) - \theta$$

Generalised formula for RC crt:

1. $t=0$, $v(t) = V_m \sin(\omega t + \theta)$, $\theta = \tan^{-1} \omega L$ or $\tan^{-1}(\omega R)$

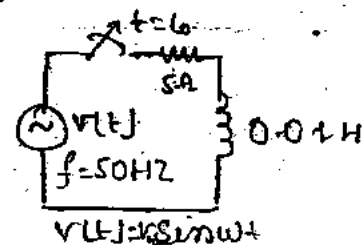
2. $t=0$, $v(t) = V_m \cos(\omega t + \theta)$, $\theta = \tan^{-1} \omega L + \frac{\pi}{2}$ or $\theta = \tan^{-1}(\omega R) + \frac{\pi}{2}$

3. $t=t_0$, $v(t) = V_m \sin(\omega t + \theta)$, $\theta = \tan^{-1} \omega L - \theta$ or $\theta = \tan^{-1}(\omega R) - \theta$

4. $t=t_0$, $v(t) = V_m \cos(\omega t + \theta)$, $\theta = \tan^{-1} \omega L - \theta + \frac{\pi}{2}$ or $\theta = \tan^{-1}(\omega R) - \theta + \frac{\pi}{2}$

N.B: For RL & RC crt the formula is same but here derived the expression for $i(t) = I_m \sin(\omega t + \theta)$ or $I_m \cos(\omega t + \theta)$ either at $t=0$, or $t=t_0$

Q. At what switching instant t the current in the crt free from transient



Soln: $\omega t_0 = \tan^{-1} \frac{\omega L}{R} - \theta$

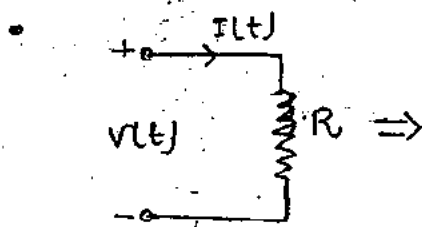
$$\omega t_0 = \tan^{-1} \frac{\omega L}{R}$$

$$t_0 = \frac{\tan^{-1} \omega L}{\omega}$$

$$= \frac{\tan^{-1} \left(\frac{3.14 \times 50 \times 0.01}{5} \right)}{2 \times 3.14 \times 50}$$

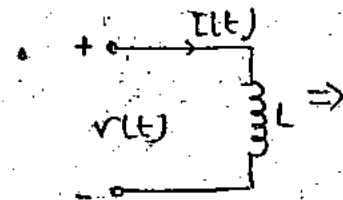
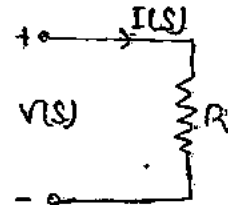
$$t_0 = 1.786 \text{ msec}$$

Laplace transform:-



$$v(t) = i(t) \cdot R$$

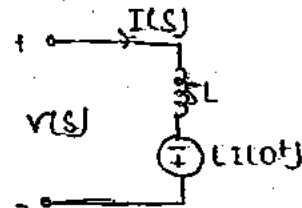
$$v(s) = i(s) \cdot R$$



$$v(t) = L \frac{di(t)}{dt}$$

$$v(s) = L[s i(s) - i(0^+)]$$

$$v(s) = L s i(s) - L i(0^+)$$

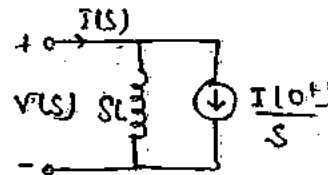


Transform impedance

$$Z(s) = \frac{v(s)}{i(s)} = L s - \frac{L i(0^+)}{i(s)}$$

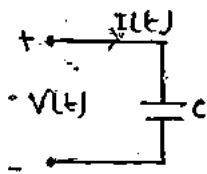
$$s L i(s) = v(s) + L i(0^+)$$

$$i(s) = \frac{v(s)}{s L} + \frac{i(0^+)}{s}$$



Transformed Admittance

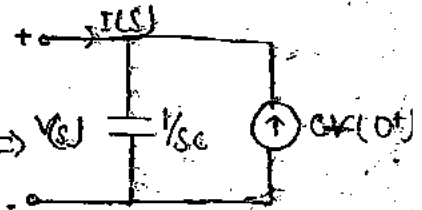
$$Y(s) = \frac{i(s)}{v(s)} = \frac{1}{s L} + \frac{i(0^+)}{s v(s)}$$



$$i(t) = C \frac{dv}{dt}$$

$$I(s) = C [sV(s) - V(0^+)]$$

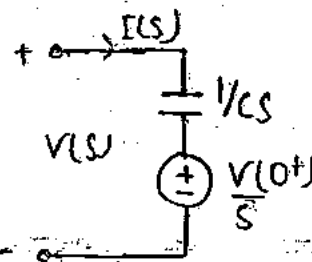
$$I(s) = \frac{V(s)}{1/sC} - \frac{CV(0^+)}{1/sC}$$



$$\frac{I(s)}{V(s)} = sC - \frac{CV(0^+)}{V(s)} \quad (\text{Transformed Admittance})$$

$$sC V(s) = I(s) + CV(0^+)$$

$$V(s) = \frac{I(s)}{sC} + \frac{V(0^+)}{s}$$



• Transform impedance

$$\frac{V(s)}{I(s)} = \frac{1}{sC} + \frac{V(0^+)}{sI(s)}$$

Q. Find current response $i(t)$ in the circuit

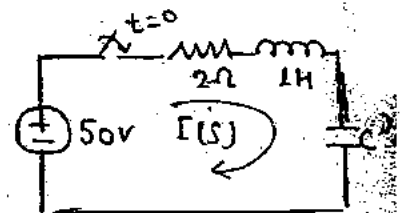
$$\text{Soln: } 50 = 2i + \frac{di}{dt} + \int i dt$$

$$\frac{50}{s} = I(s) \left[2 + s + \frac{2}{s} \right]$$

$$\frac{50}{s} = I(s) \frac{[2s + s^2 + 2]}{s}$$

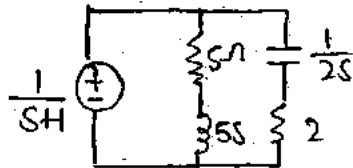
$$I(s) = \frac{50}{s^2 + 2s + 2}$$

$$\therefore i(t) = 50 \cdot e^{-t} \sin t$$



Q. For the crt shown in the below obtained on expression by the current delivered by source as function of time when switch is closed $t=0$. Assume the crt initially unenergised

Soln:



$$Z(s) = (5 + 5s) \parallel (2 + 1/2s)$$

$$= \frac{(5 + 5s) \times (4s + 1)}{10s + 10s^2 + 4s + 1} = \frac{20s^2 + 25s + 5}{10s^2 + 14s + 1}$$

$$Z(s) = \frac{V(s)}{I(s)}$$

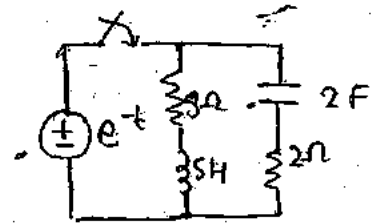
$$\text{or: } I(s) = \frac{V(s)}{Z(s)} = \frac{1}{s+1} \times \frac{10s^2 + 14s + 1}{20s^2 + 25s + 5}$$

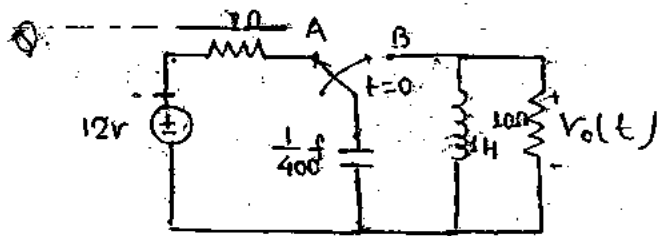
$$I(s) = \frac{10s^2 + 14s + 1}{5(s+1)^2 (4s+1)}$$

$$\frac{10s^2 + 14s + 1}{5(s+1)^2 (4s+1)} = \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{4s+1}$$

$$= \frac{1}{5(s+1)^2} + \frac{2}{3(s+1)} + \frac{2}{3(4s+1)}$$

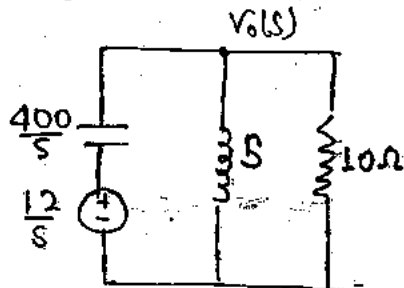
$$i(t) = \frac{1}{5} te^{-t} + \frac{2}{3} e^{-t} + \frac{2}{3} e^{-\frac{1}{4}t}$$



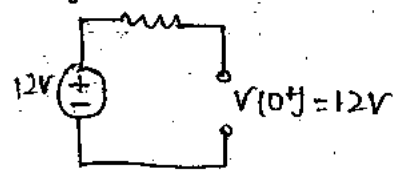


At $t=0^-$
 Given: $i(0^+) = i(0^-) = 0 \text{ A}$
 $V(0^+) = V(0^-) = 12 \text{ V}$

$t > 0$



Find $V_o(t)$ for $t > 0$, if the switch change at $t=0$. After having remain in the position A for the long time.



$$\frac{V_o(s) - 12/s}{400/s} + \frac{V_o(s)}{s} + \frac{V_o(s)}{10} = 0$$

$$\frac{sV_o(s)}{400} + \frac{V_o(s)}{s} + \frac{V_o(s)}{10} = 12/400$$

$$= V_o \left[\frac{s^2 + 400 + 40s}{400s} \right] = \frac{12}{400}$$

$$= V_o(s) = \frac{12s}{s^2 + 400 + 40s}$$

$$V_o(s) = \frac{12s}{(s+20)^2} = \frac{A}{(s+20)^2} + \frac{B}{s+20} = \frac{A + Bs + 20B}{(s+20)^2}$$

$$A = 12s \quad B = 12$$

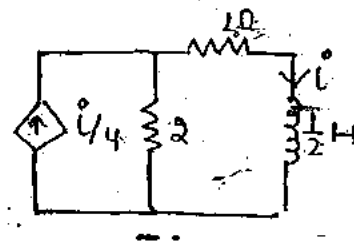
$$B = 12$$

$$A + 20B = 0$$

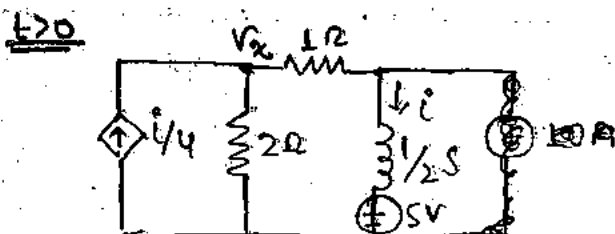
$$A = -240$$

$$V_o(s) = \frac{-240}{(s+20)^2} + \frac{12}{s+20} = 12e^{-20t} - 240te^{-20t} \quad \text{Ans}$$

Q. If the initial value of current through the inductance is 10A then find the current $t > 0$



Soln: $i(0^-) = i(0^+) = 10A$



Wrong Attempt

$$i = \frac{V_x + 5}{1 + \frac{1}{2}s}$$

$$\frac{V_x}{2} + \frac{V_x + 5}{1 + \frac{1}{2}s} = i/4$$

$$\frac{V_x(1 + \frac{1}{2}s) + 2V_x + 10}{2(1 + \frac{1}{2}s)} = \frac{V_x + 5}{(1 + \frac{1}{2}s)} = \frac{V_x + 5}{4(1 + \frac{1}{2}s)}$$

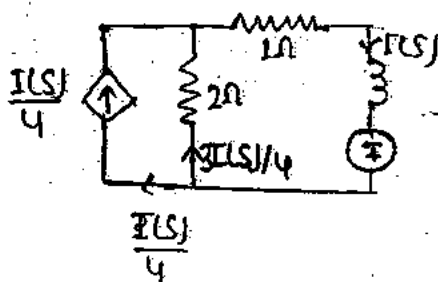
$$V_x + 2V_x + \frac{V_x s}{2} + 10 = 2V_x + 10$$

$$V_x + \frac{V_x s}{2} = 0$$

$$V_x = 0$$

$$i = \frac{5}{1 + \frac{1}{2}s} = 10e^{-2t}$$

Soln:



$$2 \times \frac{3I(s)}{4} + I(s) + \frac{5I(s)}{2} = 5$$

$$I(s) \left[\frac{3}{2} + 1 + \frac{s}{2} \right] = 5$$

$$I(s) [5 + s] = 10$$

$$I(s) = \frac{10}{s+5} = 10e^{-5t} \text{ Ang } \frac{1}{s}$$

• Response of Impulse excitation but $v(t) = \delta(t)$ —

$$(1) i = \frac{1}{L} \int_{-\infty}^t v dt$$

Substitutes this value in eq.

$$= \frac{1}{L} \left[\int_{-\infty}^{0^-} v dt + \int_{0^-}^{\infty} v dt \right] \quad (1)$$

$$i(0^+) = i(0^-) + \frac{1}{L} \int_{0^-}^{0^+} v dt$$

$$i(0^-) = 0$$

$$i(0^+) = \frac{1}{L}$$

$$i(t) = i(0^-) + \int_{-0^-}^{\infty} v dt$$

$$W_L(0^+) = \frac{1}{2} L i(0^+)^2 = \frac{1}{2L}$$

$$i(0^-) = i(0^+)$$

$$W_L = \frac{1}{2L} \text{ Joules}$$

Hence the L allow instantaneous change in current for voltage impulse function.

(2) Capacitor:

$$v = \frac{1}{C} \frac{di}{dt} = \frac{1}{C} \int_{-\infty}^t i dt$$

$$v = \frac{1}{C} \int_{-\infty}^{0^+} i dt + \int_{0^+}^{\infty} i dt$$

$$v(0^+) = v(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i dt \quad (1)$$

$$v(0^-) = v(0^+)$$

$$\text{Let } i = \delta(t)$$

$$v(0^+) = v(0^-) + \frac{1}{C} \int_{0^-}^{0^+} \delta(t) dt$$

$$v(0^-) = 0$$

$$v(0^+) = \frac{1}{C}$$

$$W_C(0^+) = \frac{1}{2} C v(0^+)^2$$

$$W_C(0^+) = \frac{1}{2C} \text{ Joules}$$

Hence the capacitor allows instantaneous changes in voltage for the current impulse function.

NETWORK SYNTHESIS:

Positive Real function

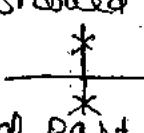
ex: $\frac{1}{s+2j}$, $\frac{1}{s-2j}$

but $-1+2j$ is not PRF because Real function in -ve.

Properties of Hurwitz polynomials: (always says about simple pole)

1. All the coefficient of the polynomial should be real & positive

$$P(s) = s^3 + 3s^2 + 5s + 1$$

2. Polynomial $P(s)$ should be real for real value of s .
3. odd & even part of $P(s)$ should give roots on $j\omega$ axis. ex: $M(s) = s^2 + 1$, $N(s) = s^3 + s$

4. The continued fraction of even & odd part should be +ve residue.
5. There should not be any missing power b/w highest and lower power of s except all the power either even or odd.
6. If two polynomials are Hurwitz, then their multiplication is also Hurwitz.

$$P(s) = P_1(s) \cdot P_2(s)$$

Thus a polynomial is Hurwitz if

- (a) $P(s)$ is real when s is real
- (b) The roots of $P(s)$ have real parts which are to be zero or -ve.

Properties of PRF:

$$F(s) = \frac{P(s)}{Q(s)}$$

1. Both $P(s)$ & $Q(s)$ are Hurwitz polynomials
2. Only simple poles with +ve real residue can exist on jw axis
3. The highest and lowest of power of $P(s)$ & $Q(s)$ differ by unity. This condⁿ prohibited multiple poles and zero at $s=0, \infty$

$$\text{ex: } F(s) = \frac{s+4}{s^2+3s+2}$$

4. if $F(s)$ is +ve real function then reciprocal of $F(s)$ is also +ve real function.
[This implies that if driving point impedance is a +ve real function then driving point admittance +ve real function]

5. if $F_1(s) = 2+j1$
 $F_2(s) = 3+j1$

The sum of +ve real function is also +ve real function but their difference may or may not be +ve real function

6. Procedures for testing of PRF:

- 1) If $F(s) = \frac{P(s)}{Q(s)}$ then $P(s)$ & $Q(s)$ must be Hurwitz
- 2) $F(s)$ must have simple pole on jw axis with +ve and real residue.
- 3) If $F(s) = \frac{P(s)}{Q(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$

Where $M(s)$ = even part

$N(s)$ = odd part

• the test of Hurwitz of FES

$$|A(s)| = M_1 M_2 - N_1 N_2$$

$$\operatorname{Re}[A(j\omega)] \geq 0 \quad \forall \omega$$

Find whether the below given polynomial is Hurwitz or not

(1) $P(s) = s^4 + s^3 + 2s^2 + 3s + 2$

Soln: $M(s) = s^4 + 2s^2 + 2$

$N(s) = s^3 + 3s$

Case 1: $\frac{M(s)}{N(s)}$

$$\begin{array}{r} s^3 + 3s \overline{) s^4 + 2s^2 + 2} \quad (s) \quad \text{* sign not be change*} \\ \underline{s^4 + 3s^2} \\ -s^2 + 2 \end{array}$$

it is not Hurwitz polynomials due * condition

(2) $P(s) = s^4 + s^3 + 5s^2 + 3s + 4$

$M(s) = s^4 + 5s^2 + 4$

$N(s) = s^3 + 3s$

$\frac{M(s)}{N(s)}$

$$\begin{array}{r} s^3 + 3s \overline{) s^4 + 5s^2 + 4} \quad (s) \\ \underline{s^4 + 3s^2} \\ 2s^2 + 4 \end{array}$$

$$\begin{array}{r} 2s^2 + 4 \overline{) s^3 + 3s} \quad \left(\frac{s}{2}\right) \\ \underline{s^3 + 2s} \\ s + 4 \end{array}$$

$$\begin{array}{r} s \overline{) 2s^2 + 4} \quad (2s) \\ \underline{2s^2 + 4s} \\ -4s + 4 \end{array}$$

$$\begin{array}{r} 4 \overline{) -4s + 4} \quad \left(\frac{-s}{1}\right) \\ \underline{-4s + 4} \\ 0 \end{array}$$

Hence it is Hurwitz polynomials.

(iii) - $P(s) = s^4 + s^3 + 6s + 4$ case 2:

Soln: As a power of $P(s)$ is missing in the above polynomial. Hence above polynomial is not Hurwitz.

(iv) $P(s) = s^4 + 3s^2 + 4$

Soln: case 3:

$$\frac{dP(s)}{ds} = 4s^3 + 6s$$

Let $4s^3 + 6s \overline{) s^4 + 3s^2 + 4} \left(\frac{s}{4} \right)$ Sign changes.

$$\underline{s^4 + 3s^2}$$

$$\frac{3s^2 + 4}{2} \overline{) 4s^3 + 6s} \left(\frac{8s}{3} \right)$$

$$\underline{4s^3 + 3s}$$

$$\frac{3s}{2} \overline{) 3s^2 + 4} \left(-\frac{9}{28}s \right)$$

Hence Above polynomial is not Hurwitz.

(v) $P(s) = s^4 + 3s^2 + 2$

$$\frac{dP(s)}{ds} = 4s^3 + 6s$$

$4s^3 + 6s \overline{) s^4 + 3s^2 + 2} \left(\frac{4s}{4} \right)$

$$\underline{s^4 + 3s^2}$$

$$\frac{3}{2}s^2 + 2 \overline{) 4s^3 + 6s} \left(\frac{8s}{3} \right)$$

$$\underline{4s^3 + 16s}$$

$$\frac{2}{3}s \overline{) \frac{3}{2}s^2 + 2} \left(\frac{9}{4}s \right)$$

$$\underline{\frac{3}{2}s^2 + \frac{9}{2}s}$$

$$2 \overline{) \frac{2}{3}s} \left(\frac{8}{3} \right)$$

$$\underline{\frac{2}{3}s}$$

Above polynomial is Hurwitz.

$$(VI) P(s) = 5s^3 + 6s^2 + 9s + 4$$

$$M(s) = 6s^2 + 4$$

$$N(s) = 5s^3 + 9s$$

$$\begin{array}{r} 5s^3 + 9s \overline{) 6s^2 + 4} \quad \left(\frac{5}{3}s \right) \quad 6s^2 + 4 \\ \underline{5s^3 + 10s} \\ 17s + 4 \end{array}$$

$$\frac{17s}{3} \overline{) 17s + 4} \quad \left(\frac{18}{17}s \right)$$

$$\begin{array}{r} \frac{17s}{3} \overline{) \frac{17s}{3} + \frac{18}{17}s} \\ \underline{\frac{17s}{3}} \\ \frac{18}{17}s \end{array}$$

Hurwitz polynomial

Short cuts: (i) $as^3 + bs^2 + cs + d$ (only valid for this eqn)

$$* \boxed{bc > ad} * \text{stable}$$

$$\boxed{54 > 20} \text{ above question}$$

Stable

$$\boxed{bc < ad} \text{ unstable}$$

$$\boxed{bc = ad} \text{ marginally stable}$$

$$(ii) as^2 + bs + c = 0$$

a, b, c should be real, then system is stable

(vi) $P(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 4s + 8$

Soln: Case 3: $2s^4 + 6s^2 + 8 \overline{) s^5 + 3s^3 + 4s} \quad (s/2)$
 $\underline{s^5 + 3s^3 + 4s}$

$38 \overline{) 2s^4 + 6s^2 + 2} \quad (2s/3)$

Since instant termination then another method to solve this question.

$$\boxed{\begin{aligned} P_1(s) &= \frac{P(s)}{W(s)} \\ P(s) &= P_1(s) \cdot W(s) \end{aligned}}$$

$2s^4 + 6s^2 + 8 \overline{) s^5 + 2s^4 + 6s^2 + 4s + 8} \quad (s/2) + 1$
 $\underline{s^5 + 3s^3 + 4s}$

$\underline{2s^4 + 6s^2 + 8}$
 $\underline{2s^4 + 6s^2 + 8}$

X

$P_1(s) = \frac{s}{2} + 1$ Hurwitz (Pole in the -jw axis or)
 Pole is -ve value

$W(s) = 2s^4 + 6s^2 + 8$

$\frac{dW(s)}{ds} = 8s^3 + 12s$

$8s^3 + 12s \overline{) 2s^4 + 6s^2 + 8} \quad (s/4)$
 $\underline{2s^3 + 3s}$

$\underline{-3s^2 + 8}$
 $\underline{-3s^2 + 8} \quad (8s/3)$
 $\underline{8s^3 + 64s}$

$\underline{-28s}$
 $\underline{-28s} \quad (3s^2 + 8) \quad (-\frac{9}{28}s)$

$W(s)$ is not Hurwitz

$P(s) = P_1(s) \cdot W(s)$ is not Hurwitz.

$$P(S) = S^3 + 4S^2 + 2S + 8$$

$$M(S) = 4S^2 + 8$$

$$N(S) = S^3 + 2S$$

$$\begin{array}{r} 4s^2 + 8s + 8 \overline{) 4s^3 + 28s^2 + 45s + 4} \\ \underline{4s^3 + 8s^2} \\ 20s^2 + 45s + 4 \\ \underline{20s^2 + 40s} \\ 5s + 4 \end{array}$$

$$P_1(s) = \frac{P(s)}{W(s)}$$

$$4s^2 + 8 \int \frac{1}{s^2 + 4s^2 + 2s + 8} \left(\frac{s}{4} + 1 \right) ds$$

$$\begin{array}{r} 4s^2 + 8 \\ 4s^2 + 8 \\ \hline X \end{array}$$

PILS is Hurwitz

WCSJ = $4s^2 + 8$ Hurwitz (no missing terms) $s = \pm j\sqrt{\frac{8}{4}} = \pm j\sqrt{2}$
Simple pole, one pair of conjugate poles

$$P(s) = P_1(s) \cdot W(s) \quad \text{Hurwitz}$$

Q. Find a for which $P(s)$ is Hurwitz:

$$PLSJ = 2s^4 + s^3 + 9s^2 + s + 2$$

$$M(S) = 2S^4 + 0S^2 + 2$$

NUSJ = 9345

$$s^3 + s \mid 2s^4 + as^2 + 2 \quad (2s)$$

$$\frac{(a-2)s^2 + 2}{s^3 + \frac{2s}{a-2}}$$

$$\frac{S(1 - \frac{1}{a-2})}{(a-2)^2} \left[(a-2)^{k+2} - \frac{(a-2)^2}{(a-2)^2} \right]$$

Q. $F(s) = \frac{s^2 + 10s + 4}{s + 2}$ Find the whether the below given function is a PRF or not?

Soln: $P(s) = s^2 + 10s + 4$

$Q(s) = s + 2$

$M_1(s) = s^2 + 4$ $N_1(s) = 10s$

$M_2(s) = 2$ $N_2(s) = s$

$A(s) = M_1 M_2 - N_1 N_2$

$= (s^2 + 4)2 - 10s \cdot s$

$= 2s^2 + 8 - 10s^2$

$A(s) = -8s^2 + 8$

$A(j\omega) = -8(-j\omega)^2 + 8 = 8\omega^2 + 8$

$A(j\omega) \geq 0 \quad \forall \omega \quad \text{P.R.F.}$

Q. $F(s) = \frac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$

Soln: $P(s) = s^3 + 5s^2 + 9s + 3$

$M_1(s) = 5s^2 + 3$ $N_1(s) = s^3 + 9s$

$Q(s) = s^3 + 4s^2 + 7s + 9$

$M_2(s) = 4s^2 + 9$ $N_2(s) = s^3 + 7s$

$A(s) = M_1 N_2 - N_1 M_2$

$= (5s^2 + 3)(4s^2 + 9) - (s^3 + 9s)(s^3 + 7s)$

$= 20s^4 + 12s^2 + 45s^2 + 27 - s$

In case of $P(s)$

$bc > ad \Rightarrow 46 > 3$ stable Hurwitz

In case of $Q(s)$

$bc < ad \Rightarrow 28 > 9$ stable Hurwitz

$F(s) = \frac{P(s)}{Q(s)}$ Hurwitz

$$A(s) = (5s^2 + 3)(4s^2 + 9) - (s^2 + 2s)(s^3 + 7s)$$

$$= 20s^4 + 57s^2 + 27 - (s^6 + 8s^4 + 63s^2)$$

$$= -s^6 + 4s^4 - 6s^2 + 27$$

$$A(j\omega) = \omega^6 + 4\omega^4 + 6\omega^2 + 27$$

$$A(j\omega) \geq 0 \quad \forall \omega \quad \text{PRF} \quad \underline{A(j\omega)}$$

(1)

- 1) Foster I $\rightarrow Z(s) \rightarrow \text{Series}$
 11) Foster II $\rightarrow Y(s) \rightarrow \text{II}$ } Partial fraction

(2)

Cauer-I
 Cauer-II } $\rightarrow$ ladder N/w $\rightarrow$ continued fraction

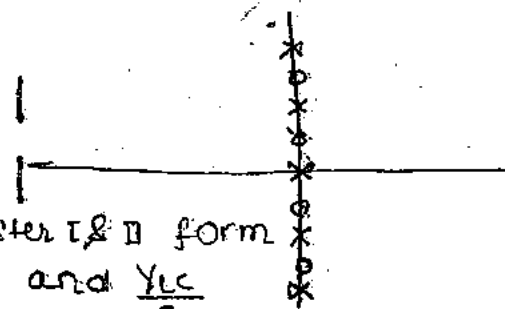
FE

Properties of LC n/w:-

- (1) Either pole or zero should be present in origin.
- (2) " " " " " " " " at ∞ infinity.
- (3) Poles and zero are arranged alternatively on $j\omega$ axis.
- (4) The transfer function consist of
 $\frac{\text{even}}{\text{odd}}$ or $\frac{\text{odd}}{\text{even}}$ power of s

(5) In the partial fraction of expansion Residue should be positive Real.

(6) $\frac{dx}{d\omega} > 0$ ie slope is positive



N.B In case Foster I & II form
 derive $\frac{Z_L}{s}$ and $\frac{Y_C}{s}$

Q. $Z(s) = \frac{s(s^2 + 4)}{2(s^2 + 1)(s^2 + 9)}$

Synthesize the n/w in Foster I, Foster II and canonical form.

Soln: Foster I

$s_z = 0, \pm j2$, $s_p = \pm j$, $\pm j3$



$$\frac{Z(s)}{s} = \frac{1}{2} \left[\frac{s^2 + 4}{(s^2 + 1)(s^2 + 9)} \right]$$

$$\frac{s^2 + 4}{(s^2 + 1)(s^2 + 9)} = \frac{a}{s^2 + 1} + \frac{b}{s^2 + 9}$$

Put $s^2 = x$

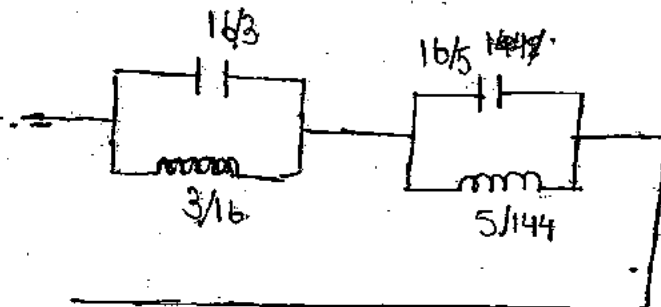
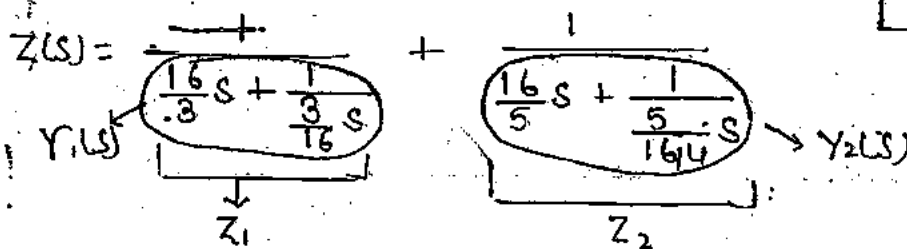
$$\frac{x + 4}{(x + 1)(x + 9)} = \frac{a}{x + 1} + \frac{b}{x + 9}$$

$\therefore a = \frac{3}{8}$, $b = \frac{5}{8}$

$$\frac{Z(s)}{s} = \frac{1}{2} \left[\frac{3/8}{s^2 + 1} + \frac{5/8}{s^2 + 9} \right]$$

$$Z(s) = \frac{3/16 s}{s^2 + 1} + \frac{5/16 s}{s^2 + 9}$$

Z	Y
$1/s$	$1/s$
$1/cs$	cs
R	R



Foster-II

$$Y(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$\frac{Y(s)}{s^2} = 2 \left[\frac{(s^2+1)(s^2+9)}{s^2(s^2+4)} \right]$$

Put $s^2 = x$

$$\frac{(x+1)(x+9)}{x(x^2+4)} = \frac{A}{x} + \frac{B}{x+4}$$

$$\frac{(x+1)(x+9)}{x(x^2+4)} = \frac{x^2+10x+9}{x^3+4x} = 1 + \frac{6x+9}{x^2+4x}$$

$$\frac{6x+9}{x^2+4x} = \frac{A}{x} + \frac{B}{x+4}$$

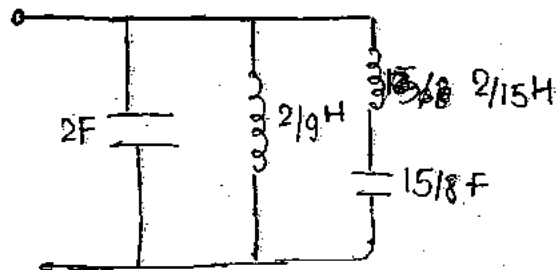
$$A = 9/4 \quad B = 15/4$$

$$\frac{Y(s)}{s^2} = 2 \left[1 + \frac{9/4}{s^2} + \frac{15/4}{s^2+4} \right]$$

$$Y(s) = 2s + \frac{9}{2s} + \frac{15/2s}{s^2+4}$$

$$= \frac{Y_1}{Z_1} + \frac{Y_2}{Z_2} + \frac{Y_3}{Z_3}$$

$$= 2s + \frac{1}{\frac{2}{9}s} + \frac{\frac{2}{15}s + \frac{1}{8}s}{\frac{2}{15}s + \frac{1}{8}s}$$



Convert

$$Z(s) = \frac{s^3 + 4s}{2s^4 + 20s^2 + 18}$$

$$Z(s) = \frac{1}{Y \left[\frac{2s^4 + 20s^2 + 18}{s^3 + 4s} \right]} = \frac{1}{\dots}$$

$$\begin{array}{r} s^3 + 4s \overline{) 2s^4 + 20s^2 + 18} \quad (2s \rightarrow Y \rightarrow C \\ \underline{2s^4 + 8s^2} \\ 12s^2 + 18 \end{array}$$

$$\begin{array}{r} s^2 + 3s \overline{) 12s^2 + 18} \quad (12s/12 \rightarrow Z \rightarrow L \\ \underline{12s^2 + 36s} \\ -24s + 18 \end{array}$$

$$\begin{array}{r} \frac{5}{2}s \overline{) -24s + 18} \quad (\frac{24}{5}s \rightarrow Y \rightarrow C \\ \underline{-24s + 18} \\ 0 \end{array}$$

$$Z(s) = \frac{1}{\frac{2s^4 + 20s^2 + 18}{s^3 + 4s}}$$

$$= \frac{1}{2s + \frac{12s^2 + 18}{s^3 + 4s}}$$

$$= \frac{1}{2s + \frac{1}{\frac{s^3 + 4s}{12s^2 + 18}}}$$

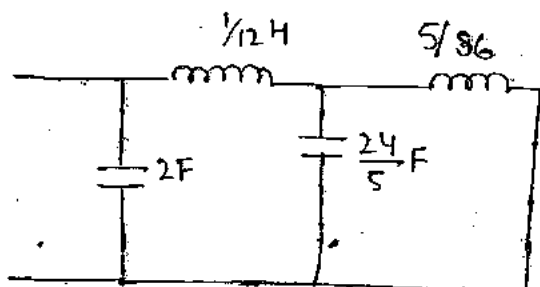
$$= \frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{5/2s}{12s^2 + 18}}}$$

$$= \frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{12s^2 + 18}{5/2s}}}}$$

$$\frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{24s}{5} + \frac{1}{18}}}} = \frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{24s}{5} + \frac{1}{18}}}}$$

$$\frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{24s}{5} + \frac{1}{18}}}} = \frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{24s}{5} + \frac{1}{18}}}}$$

$$\frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{24s}{5} + \frac{1}{18}}}} = \frac{1}{2s + \frac{1}{\frac{s}{12} + \frac{1}{\frac{24s}{5} + \frac{1}{18}}}}$$



Case II

$$Z(s) = \frac{s^3 + 4s}{2s^4 + 20s^2 + 18} = \frac{1}{\frac{2s^4 + 20s^2 + 18}{s^3 + 4s}} = \frac{1}{\frac{18 + 20s^2 + 2s^4}{4s^3 + 4s}}$$

$$4s + s^3 \overline{) 18 + 20s^2 + 2s^4} \left(\frac{9}{2s} \rightarrow Y \rightarrow L \right)$$

$$\underline{18 + 9s^2}$$

$$- - -$$

$$\frac{31}{2}s^2 + 2s^4 \overline{) 4s + s^3} \left(\frac{8}{31}s \rightarrow Z \rightarrow C \right)$$

$$\underline{4s + \frac{16}{31}s^3}$$

$$- - -$$

$$\frac{15}{31}s^3 \overline{) \frac{31}{2}s^2 + 2s^4} \left(\frac{961}{30s} \rightarrow Y \rightarrow L \right)$$

$$\frac{31}{2}s^2 \overline{) \frac{15}{31}s^3}$$

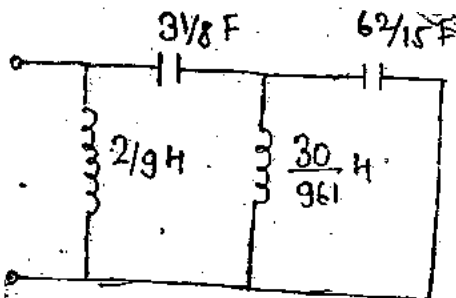
$$\underline{\frac{15}{2}s^2}$$

$$- - -$$

$$\frac{15}{31}s^3 \overline{) \frac{15}{62}s^3} \rightarrow Z \rightarrow C$$

$$\underline{\frac{15}{31}s^3}$$

$$- - -$$

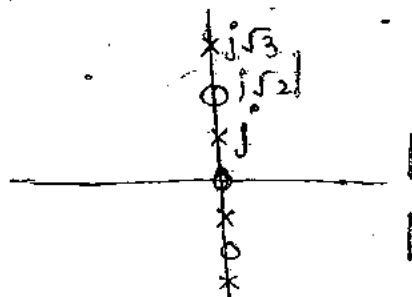


Q. Synthesize the n/w in Foster I, Foster II, Case I and Case II form:

$$Z(s) = \frac{s(s^2 + 2)}{(s^2 + 1)(s^2 + 3)}$$

Soln: $s_z = 0, \pm j\sqrt{2}$

$s_p = \pm j, \pm j\sqrt{3}$



Foster I

$$\frac{Z(s)}{s} = \frac{s^2 + 2}{s(s^2 + 1)(s^2 + 3)}$$

Put $s^2 = x$

$$\frac{x+2}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

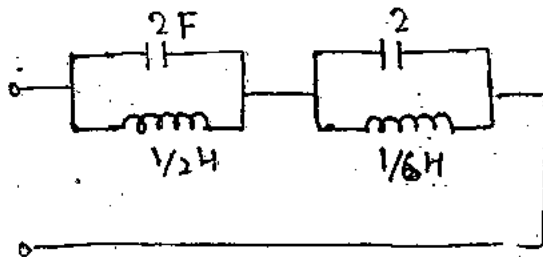
$$A = \frac{1}{2} \quad B = \frac{1}{2}$$

$x \rightarrow \infty$

$$\frac{Z(s)}{s} = \frac{1/2}{s^2+1} + \frac{1/2}{s^2+3}$$

$$Z(s) = \left[\frac{1/2 s}{s^2+1} + \frac{s/2}{s^2+3} \right]$$

$$Z(s) = \frac{1}{Y_1 \left(2s + \frac{1}{s/2} \right)} + \frac{1}{Y_2 \left(2s + \frac{1}{\frac{2}{3}s} \right)}$$



Foster II For

$$Y(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

$$\frac{Y(s)}{s} = \frac{(s^2+1)(s^2+3)}{s^2(s^2+2)}$$

Put $s^2 = x$

$$\frac{(x+1)(x+3)}{x(x+2)} = \frac{x^2+4x+3}{x^2+2x} = 1 + \frac{2x+3}{x^2+2x}$$

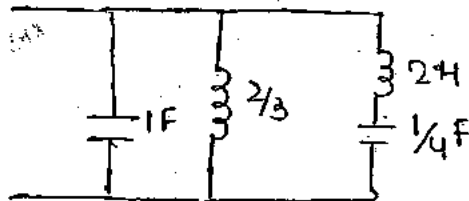
$$\frac{2x+3}{x^2+2x} = \frac{A}{x} + \frac{B}{x+2}$$

$$A = 3/2 \quad B = 1/2$$

$$Y(s) = \left[1 + \frac{3/2}{s^2} + \frac{1/2}{s^2+2} \right]$$

$$Y(s) = \left[s + \frac{3/2 s}{s^2} + \frac{1/2 s}{s^2+2} \right]$$

$$Y(s) = \left[s + \frac{1}{\frac{2}{3}s} + \frac{1}{2s + \frac{1}{4s}} \right]$$



∴ Case I

$$Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)} = \frac{s^3+2s}{s^4+4s^2+3}$$

$$Z(s) = \frac{1}{Y - \left[\frac{s^4+4s^2+3}{s^3+2s} \right]}$$

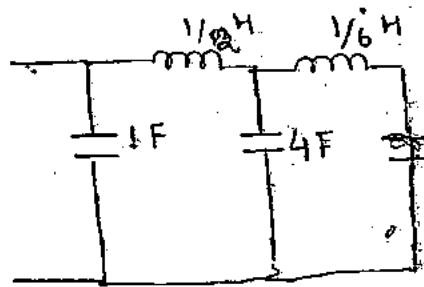
$$s^3+2s \overline{) s^4+4s^2+3} \quad (s \rightarrow Y \rightarrow c$$

$$\underline{2s^2+3} \quad 2s^2+3 \overline{) s^3+2s} \quad (\frac{1}{2}s \rightarrow Z \rightarrow L$$

$$\underline{\frac{1}{2}s} \quad \frac{1}{2}s \overline{) 2s^2+3} \quad (4s \rightarrow Y \rightarrow c$$

$$3) \frac{1}{2}s \overline{) \frac{1}{2}s} \quad (\frac{1}{6}s \rightarrow Z \rightarrow L$$

$$Z(s) = \frac{1}{s + \frac{1}{\frac{1}{2}s + \frac{1}{4s + \frac{1}{6s}}}}$$



Case II

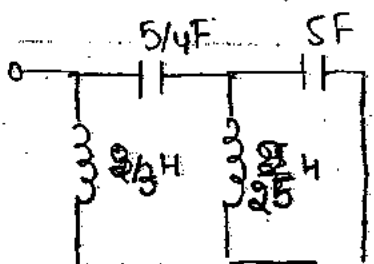
$$Z(s) = \frac{1}{\frac{3 + 4s^2 + s^4}{2s + s^3}}$$

$$\begin{array}{r} 2s + s^3 \overline{) 3 + 4s^2 + s^4} \left(\frac{3}{2}s - Y \rightarrow E \right. \\ \underline{3 + \frac{3}{2}s^2} \\ \frac{5}{2}s^2 + s^4 \end{array}$$

$$\begin{array}{r} \frac{5}{2}s^2 + s^4 \overline{) 2s + s^3} \left(\frac{4}{5}s \rightarrow Z \rightarrow C \right. \\ \underline{2s + \frac{4}{5}s^3} \\ \frac{1}{5}s^3 \end{array}$$

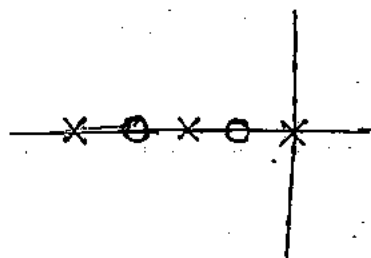
$$\begin{array}{r} \frac{1}{5}s^3 \overline{) \frac{5}{2}s^2 + s^4} \left(\frac{25}{2}s \rightarrow Y \rightarrow C \right. \\ \underline{\frac{5}{2}s^2} \\ \phantom{\frac{5}{2}s^2 + } \frac{1}{5}s^4 \end{array}$$

$$\begin{array}{r} s^4 \overline{) \frac{1}{5}s^3} \left(\frac{1}{5}s \rightarrow Z \rightarrow C \right. \\ \underline{\frac{1}{5}s^3} \\ \phantom{\frac{1}{5}s^3 + } X \end{array}$$



Properties of Z_{RC} Y_{RL}

- (1) The lowest frequency is due to pole and it may be present at origin or nearer to origin.
- (2) The highest critical frequency is due to zero and it may be present at infinity or near to infinity ^{alternatively}.
- (3) Poles and zero are arranged in -ve real axis.
- (4) Slope $\left(\frac{dZ_{RC}}{ds} < 0, \frac{dY_{RL}}{ds} < 0 \right)$ -ve.



Properties of Z_{RL} & Y_{RC}

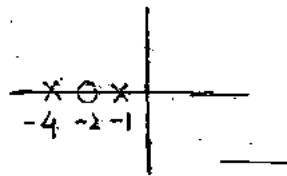
1. Highest critical frequency is due to pole and it may be present at either at ∞ or nearer to ∞ .
2. The lowest critical f is due to zero and it may be present at origin or nearer to origin.
3. Poles and zero are arranged alternatively in -ve real axis.
4. Slope is +ve.

$$\frac{dZ_{RL}}{ds} > 0, \frac{dY_{RC}}{ds} > 0$$

Note: For Foster I & II form we will do partial fraction expansion for Z_{RC} Y_{RL} $\frac{Z_{RL}}{s}$ $\frac{Y_{RC}}{s}$

Q. $Z(s) = \frac{3(s+2)}{(s+1)(s+4)}$

Soln: ^{Foster I}
 $S_Z = -2$ $S_p = -1, -4$



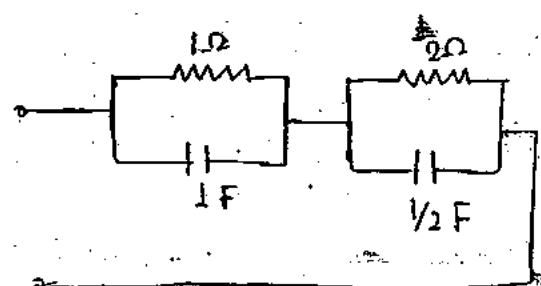
$Z(s) = \frac{3(s+2)}{(s+1)(s+4)}$

$Z_{RC} = \frac{a}{s+1} + \frac{b}{s+4}$

$a = 1$ $b = 2$

$Z_{RC} = \frac{1}{s+1} + \frac{2}{s+4}$

$= \frac{1}{s+1} + \frac{1}{2+s/2}$



Foster II -

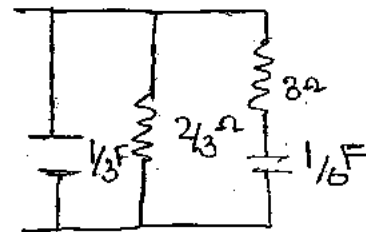
$Y(s) = \frac{s^2+5s+4}{3(s+2)}$

$\frac{Y(s)}{s} = \frac{s^2+5s+4}{s^2+2s} = \frac{1}{3} \left[1 + \frac{3s+4}{s^2+2s} \right]$

$\frac{3s+4}{s^2+2s} = \frac{A}{s} + \frac{B}{s+2}$; $A = 2$ $B = 2$

$Y(s) = \left[\frac{s}{3s} + \frac{2}{3} + \frac{s}{3s+6} \right]$

$Y_{RC} = \frac{2}{3} + \frac{1}{3}s + \frac{1}{3 + \frac{1}{6}s}$



Cauer I:

$$Z = \frac{1}{Y} = \frac{s^2 + 5s + 4}{3s + 6}$$

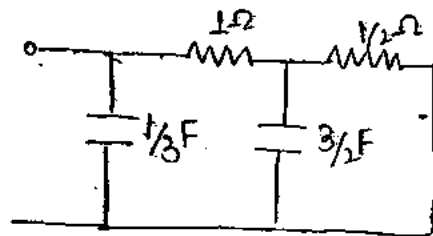
$$3s + 6 \overline{) s^2 + 5s + 4} \quad (\frac{1}{3}s \rightarrow Y \rightarrow C)$$

$$\underline{s^2 + 2s} \quad \underline{3s + 4} \quad (1 \rightarrow Z \rightarrow R)$$

$$\underline{3s} \quad 2) \underline{3s + 4} \quad (\frac{2}{3}s \rightarrow Y \rightarrow C)$$

$$\underline{2s} \quad 4) \underline{2s} \quad (\frac{1}{2} \rightarrow Z \rightarrow R)$$

$$Z_{RC} = \frac{1}{\frac{1}{3}s + \frac{1}{1 + \frac{1}{\frac{3}{2}s + \frac{1}{1/2}}}}$$



Cauer II:

$$Z = \frac{1}{Y} = \frac{4 + 5s + s^2}{6 + 3s}$$

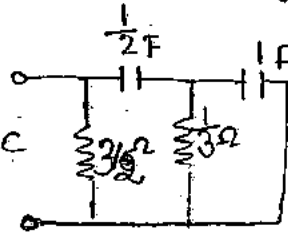
$$6 + 3s \overline{) 4 + 5s + s^2} \quad (\frac{2}{3} \rightarrow Y \rightarrow R)$$

$$\underline{4 + 2s} \quad 3s + s^2 \overline{) 4 + 3s} \quad (\frac{2}{3}s \rightarrow Y \rightarrow C)$$

$$\underline{3s} \quad 5) \underline{3s + s^2} \quad (3 \rightarrow Y \rightarrow R)$$

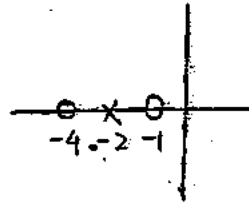
$$\underline{s^2} \quad s^2) \underline{s^2} \quad (\frac{1}{s} \rightarrow Y \rightarrow C)$$

$$Z_{RC} = \frac{1}{\frac{2}{3} + \frac{1}{\frac{2}{3} + \frac{1}{\frac{1}{s} + \frac{1}{3 + \frac{1}{s}}}}}$$



$$Z(s) = \frac{(s+1)(s+4)}{(s+2)}$$

Soln: $s_z = -1, -4$
 $s_p = -2$



$$Z(s) = \frac{s^2 + 4s + 4}{s + 2}$$

Foster I

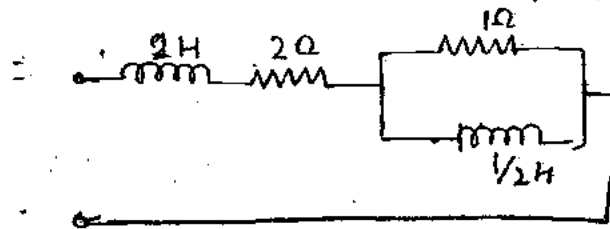
$$\frac{Z(s)}{s} = \frac{s^2 + 4s + 4}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$\frac{Z(s)}{s} = \frac{s^2 + 4s + 4}{s^2 + 2s} = 1 + \frac{2s + 4}{s^2 + 2s}$$

$$\frac{2s + 4}{s^2 + 2s} = \frac{A}{s} + \frac{B}{s+2} = \frac{2}{s} + \frac{1}{s+2}$$

$$Z(s) = \left[s + 2 + \frac{s}{s+2} \right] = \left[s + 2 + \frac{1}{1 + 2/s} \right]$$

$\begin{matrix} Y_1 & Y_2 \end{matrix}$



Foster II

$$\frac{Y(s)}{s} = \frac{s + 2}{s(s+1)(s+4)}$$

$$\frac{Y(s)}{s} = \frac{s + 2}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

A =

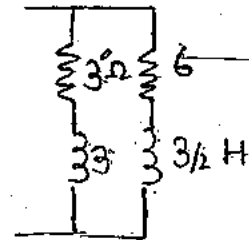
Foster II

$$Y_{RC}(s) = \frac{s+2}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$$

$$A = 1/3$$

$$B = 2/3$$

$$Y_{RC}(s) = \frac{1}{3s+3} + \frac{2}{3s+6}$$



Exer I

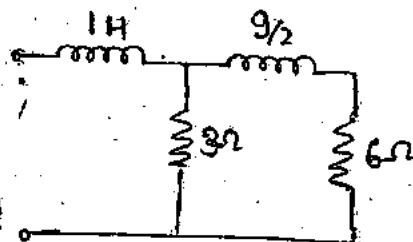
$$Z = \frac{s+2}{(s+1)(s+4)}$$

$$Z(s) = \frac{s^2+5s+4}{s^2+5s+4} \quad (s \rightarrow Z)$$

$$\frac{s^2+5s+4}{s^2+5s+4} \rightarrow Y$$

$$\frac{2}{3} \bigg| \frac{s^2+5s+4}{s^2+5s+4} \rightarrow Z$$

$$4) \frac{2}{3} \bigg| \frac{1}{6} \rightarrow Y$$



Exer II

$$Z(s) = \frac{4+5s+s^2}{2+s}$$

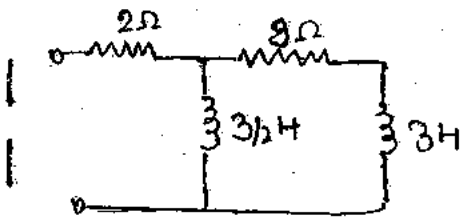
$$2+s \bigg| 4+5s+s^2 \rightarrow Z$$

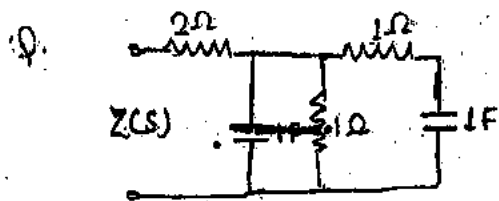
$$\frac{4+5s}{2+s}$$

$$3s+s^2 \bigg| 4+5s \rightarrow Y$$

$$\frac{2}{3} \bigg| \frac{3s+s^2}{3s+s^2} \rightarrow Z$$

$$s \bigg| \frac{s}{3} \rightarrow Y$$





Realization of driving point impedance $Z = \frac{\alpha s^2 + 7s + 3}{s^2 + 3s + 1}$ is given above. find the value of α and β .

Soln:

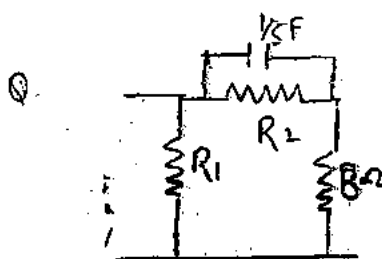
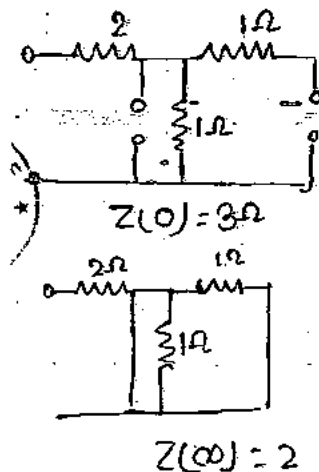
Note: In this type question always find $Z(s)$ value at $s=0, \infty$ where capacitor acts as O.C and S.C respectively.

$$Z(0) = \frac{3}{\beta} = 3$$

$$\boxed{\beta = 1}$$

$$Z(\infty) = \frac{\alpha}{\beta} = 2$$

$$\boxed{\alpha = 2}$$

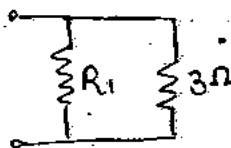


$$Z(0) = 3$$

$$Z(\infty) = 2$$

Find the value of R_1, R_2

Soln: $Z(\infty)$

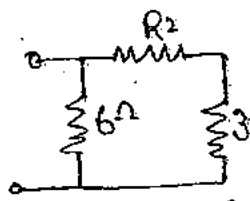


$$Z(\infty) = \frac{R_1 \cdot 3}{R_1 + 3} = 2$$

$$2R_1 + 6 = 3R_1$$

$$R_1 = 6$$

$Z(0)$



$$Z(0) = \frac{6(R_2 + 3)}{9 + R_2} = 3$$

$$6R_2 + 18 = 27 + 3R_2$$

$$3R_2 = 9$$

$$R_2 = 3$$

Dot polarity:

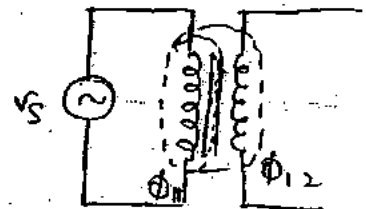
Principle of Induction states that if we have current carrying coil near each other then they affect one and another.

Faraday's law of Induction states that whenever a coil in M.F and field is time varying then voltage induced in the coil.

$$\Phi = \Phi_{11} + \Phi_{12}$$

leakage flux

Magnetising flux



$$e_2 = N_2 \frac{d\Phi_{12}}{dt}$$

$$e_2 = N_2 \frac{d\Phi_{12}}{di_1} \times \frac{di_1}{dt}$$

where

$$M_{12} = \frac{N_2 d\Phi_{12}}{di_1}$$

$$e_2 = M_{12} \frac{d\Phi_{12}}{dt}$$

$$e_2 = M_{12} \frac{di_1}{dt}$$

Similarly

$$e_1 = M_{21} \frac{d\Phi_{21}}{dt}$$

$$e_1 = M_{21} \frac{di_2}{dt}$$

$$M_{21} = \frac{N_1 d\Phi_{21}}{di_2}$$

$$M_{12} \cdot M_{21} = N_2 \frac{d\Phi_{12}}{dt_1} \cdot N_1 \frac{d\Phi_{21}}{dt_2}$$

$$M_1 \cdot M_2 = N_2 K_1 \frac{d\Phi_1}{dt_1} \cdot N_1 K_2 \frac{d\Phi_2}{dt_2}$$

$$= K_1 K_2 \left(N_1 \frac{d\Phi_1}{dt_1} \right) \left(N_2 \frac{d\Phi_2}{dt_2} \right)$$

$$= K_1 K_2 \cdot L_1 L_2$$

$$\text{Let } M_{12} = M_{21} = M$$

$$K = K_1 = K_2$$

$$M^2 = K_1 K_2 L_1 L_2$$

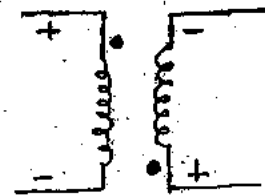
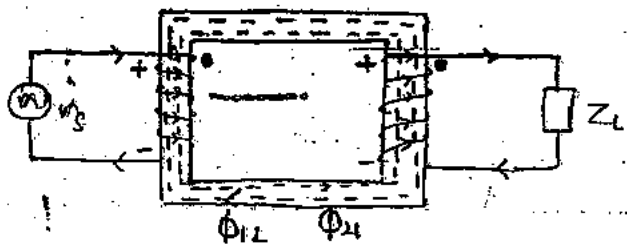
$$M^2 = K^2 L_1 L_2$$

$$M = K \sqrt{L_1 L_2}$$

where $K = \frac{\text{useful flux}}{\text{Total flux}}$ (coupling coefficient)

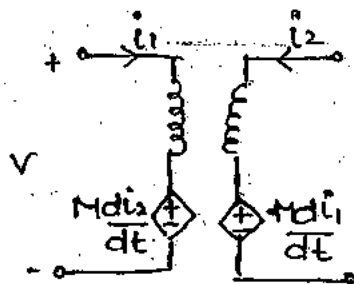
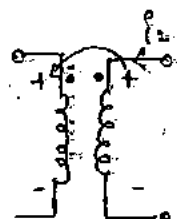
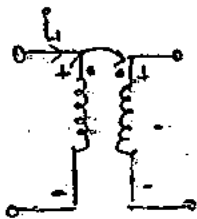
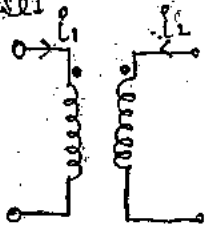
$$0 \leq K \leq 1$$

Polarity concept

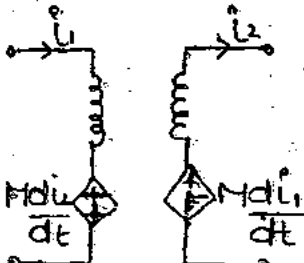
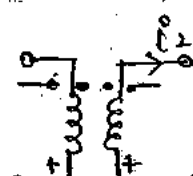
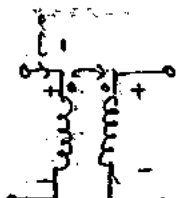
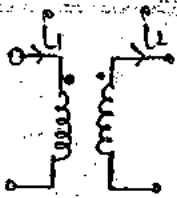


Dot Polarity should have same polarity.

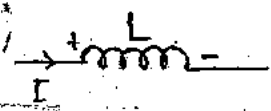
Case 1



Case 2



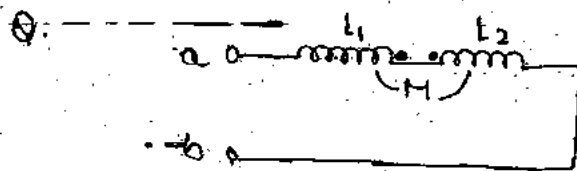
*



$$V = L \frac{di}{dt}$$

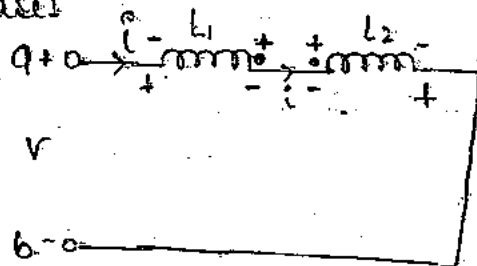
$$= j\omega L i$$

$$= s L i$$

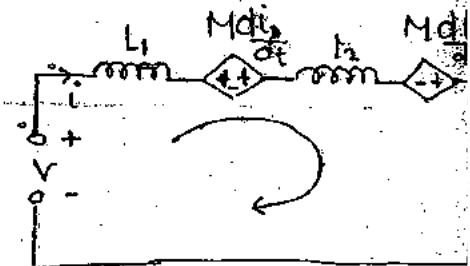


Find equivalent inductance

Soln: case 1



$\Rightarrow$



$\therefore$ Apply KCL

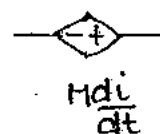
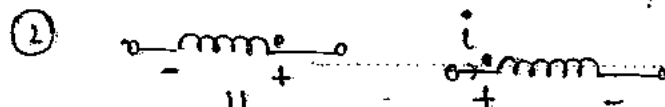
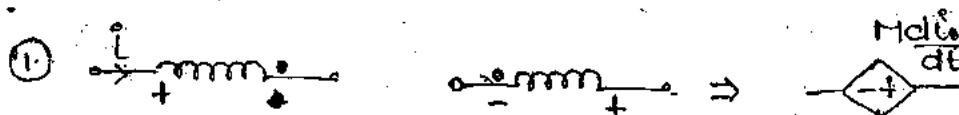
$$-V = L_1 \frac{di}{dt} - M \frac{di}{dt} + L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$V = (L_1 + L_2 - 2M) \frac{di}{dt}$$

$$V = L_{eq} \frac{di}{dt}$$

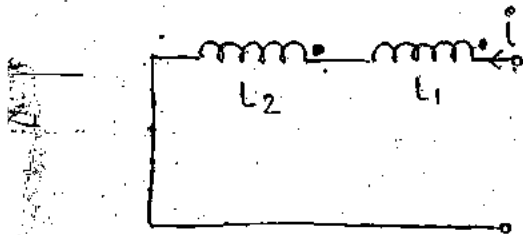
where

$$L_{eq} = L_1 + L_2 - 2M$$

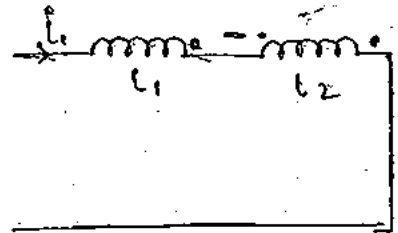


Note:

If the current enters or exits both dots, then mutual inductance M is additive.

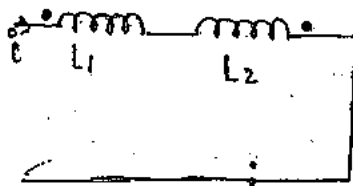


$$L_{eq} = L_1 + L_2 + 2M$$

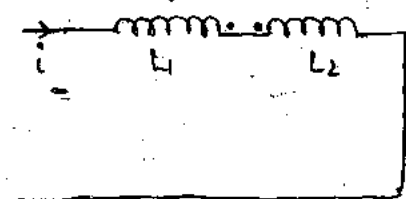


$$L_{eq} = L_1 + L_2 + 2M$$

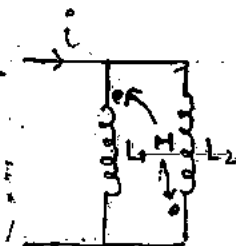
If the current enters from one dot and leaves at the other, then mutual inductance is subtractive in nature.



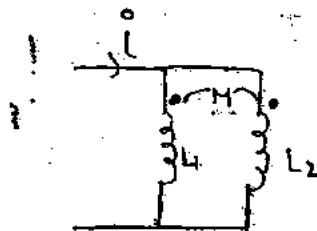
$$L_{eq} = L_1 + L_2 - 2M$$



$$L_{eq} = L_1 + L_2 - 2M$$

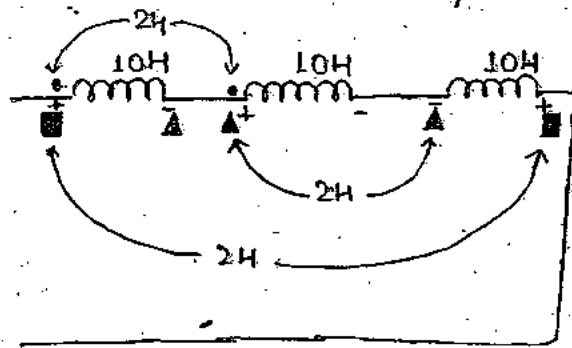


$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

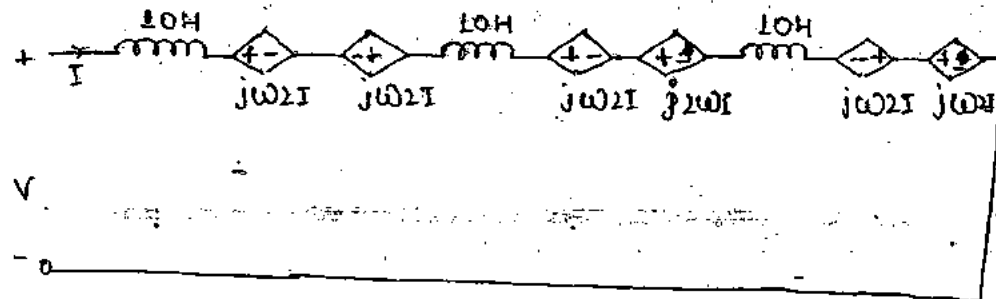


$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Q. Find the inductance at i/p terminals.



Soln:



$$V = (j\omega 10I + j\omega 2I - j\omega 2I) + (j\omega 10I + j\omega 2I + j\omega 2I) + (j\omega 10I - j\omega 2I + j\omega 2I)$$

$$V = j\omega 34I$$

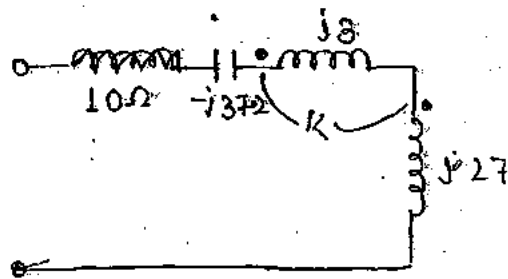
$$\therefore L_{eq} = 34H$$

$$L_{eq} = L_1 + L_2 + L_3 + 2H - 2H + 2H$$

$$= 10 + 10 + 10 + 2 \times 2$$

$$= 34H$$

Determine the k for series resonance?



Soln:

$$L_{eq} = L_1 + L_2 + 2M$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 + 2\omega M$$

$$X_L = (3 + 27) + 2X_m \quad \text{--- (1)}$$

As we know

$$M = k \sqrt{L_1 L_2}$$

$$\omega M = k \sqrt{\omega L_1 \cdot \omega L_2}$$

$$X_m = k \sqrt{3 \times 27} = 9k$$

$$X_L = (3 + 27) + 2 \times 9k$$

$$X_L = 30 + 18k$$

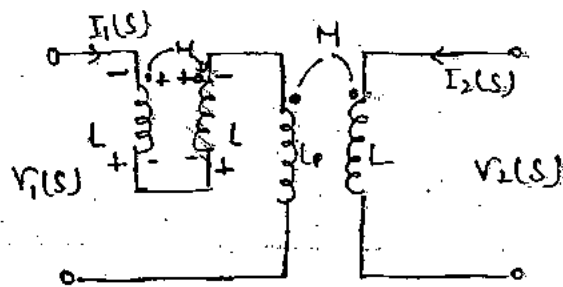
At series resonance

$$X_L = X_C$$

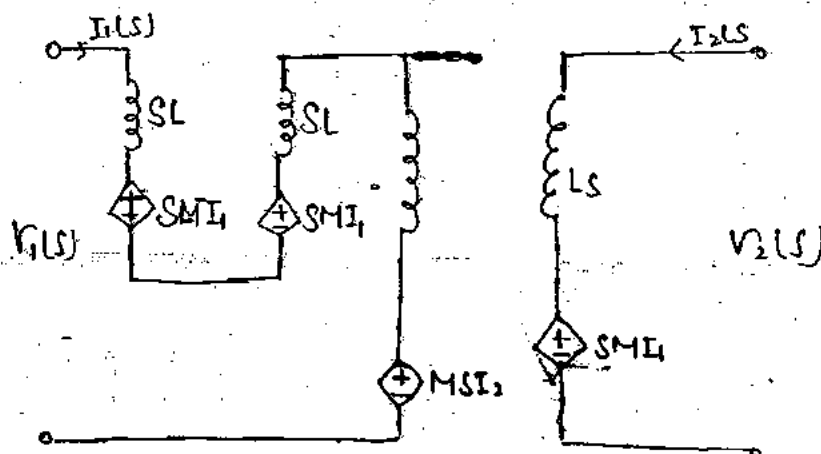
$$30 + 18k = 37.2$$

$$\boxed{k = 0.4} \quad \text{Ans}$$

Find the Z parameter of two port n/w given



Soln:



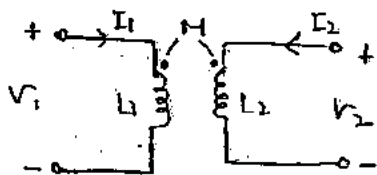
$$V_2(s) = I_2(s)SL + SM I_1$$

$$V_1(s) = SL I_1 - SM I_1 - SM I_1 + SL I_1 + SL I_1 + SM I_2$$

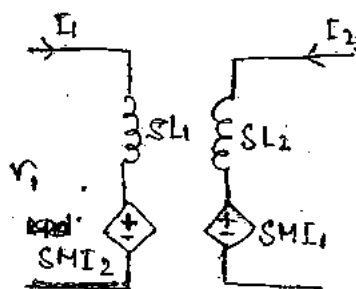
$$V_1(s) = I_1(3SL - 2SM) + SM I_2$$

$$Z = S \begin{bmatrix} 3L - 2M & M \\ M & L \end{bmatrix}$$

Q. Determine the equivalent circuit of λ^r



Soln:



$$V_1 = SL_1 I_1 + SM I_2$$

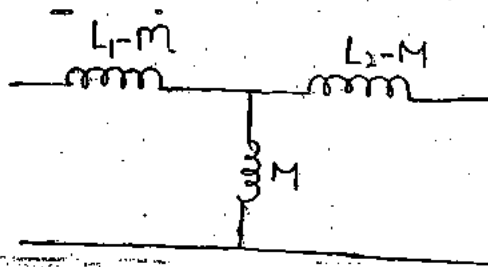
$$V_2 = SM I_1 + SL_2 I_2$$

$$Z = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}$$

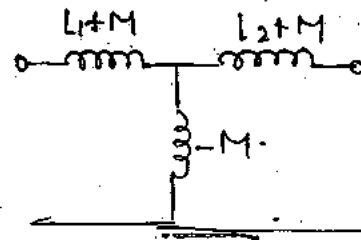
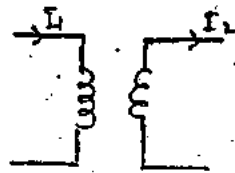
$$Z_{11} = L_1$$

$$Z_{12} = Z_{21} = M$$

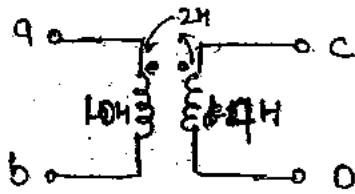
$$Z_{22} = L_2$$



In case



Q. Find the equivalent inductance b/w a & b when terminal c & d short circuited

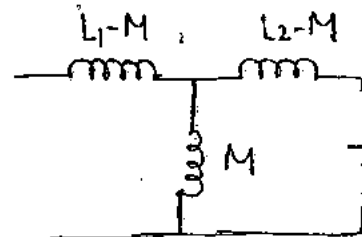


Soln: case I (supporting)

$$L_{eq} = L_1 - M + \frac{M(L_2 - M)}{M + L_2 - M}$$

$$= L_1 - M + \frac{ML_2 - M^2}{L_2}$$

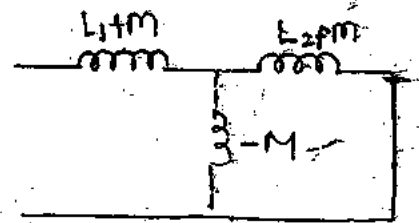
$$= \frac{L_1 L_2 - L_2 M - M^2 + M L_2}{L_2}$$



$$= L_1 - \frac{M^2}{L_2}$$

Case (opposing current) ———

$$L_{eq} = L_1 - \frac{M^2}{L_2}$$



$$10 - \frac{4^2}{40} = 9.4$$

• Notes: ———

1. If both dots are positive or negative same sign w.r.t to each other.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

If different sign then

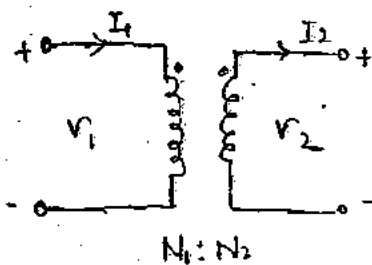
$$\frac{V_1}{V_2} = -\frac{N_1}{N_2}$$

2. If I_1 and I_2 both leaves or enters the dotted terminal then

$$\frac{I_2}{I_1} = -\frac{N_1}{N_2}$$

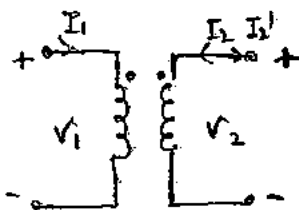
If one current enters at dotted terminal & other current leaves dotted terminals

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$



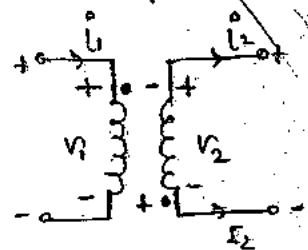
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$



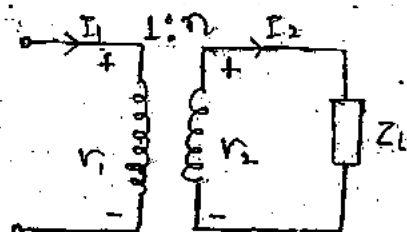
$$\frac{V_1}{V_2} = -\frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = -\frac{N_2}{N_1}$$



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$



$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{1}{n}$$

$$v_1 = v_2/n$$

$$Z_{in} = \frac{v_1}{I_1} = \frac{v_2}{n} \cdot \frac{1}{nI_2}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

FILTER'S

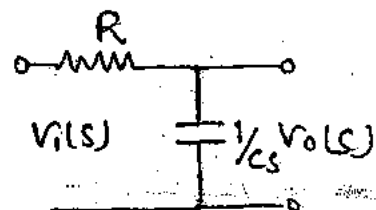
1. Based on the components present in the filter they classified as active or passive elements.
2. In the passive filter it is not possible to ↑ the gain.

1. Low pass filter:

$$\frac{V_o(s)}{V_i(s)} = \frac{1/cs}{R + 1/cs}$$

$$= \frac{1}{1 + RCS}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + ZS}}$$



Shortcuts:

$$f = s = j\omega = j2\pi f$$

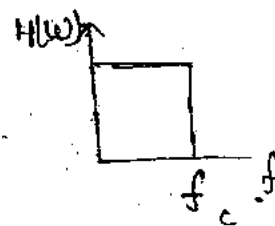
So f

At $s=0$ $f=0$

$s=\infty$ $f=\infty$

Capacitor acts as open circuit $V_i(s) = V_o(s)$

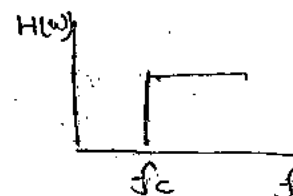
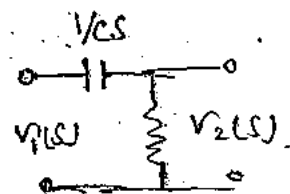
" " " " S.O.T $V_o(s) = 0$



100% low pass filter:

2. High pass filter:

$$\begin{aligned} \frac{V_o(s)}{V_i(s)} &= \frac{R}{R + \frac{1}{cs}} \\ &= \frac{RCS}{1 + RCS} = \frac{ZS}{1 + ZS} \end{aligned}$$



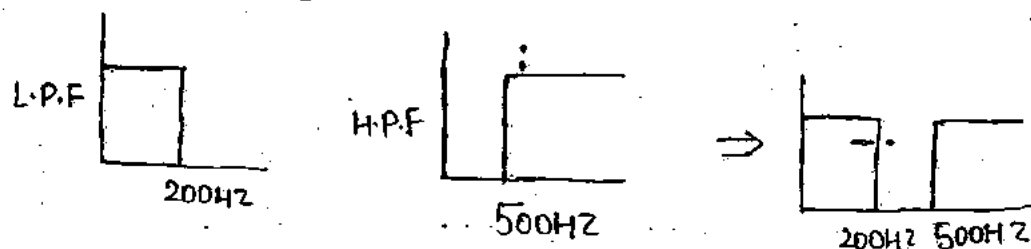
3. Band pass filter

Band pass filter can be obtained by the combination of LPF and HPF. and the f_c of low pass filter is greater than f_c of HPF.



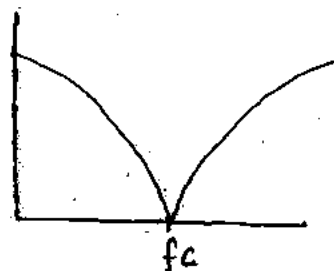
Band elimination

Band elimination filter can be obtained by LPF and HPF. and f_c of low pass filter should be less than H.P.F.



|| combination

Note: Notch filter



1st order filter:

1. $\frac{1}{1+zs}$ LPF

2. $\frac{zs}{1+zs}$ HPF

3. $\frac{1-zs}{1+zs}$ All pass filter

Note:

In the APF poles and zero are symmetric to imaginary axis. and poles are present left hand of s-plane and zero's are present at right hand of s-plane.

2nd order filter:

1. $\frac{p}{s^2+as+b}$ LPF

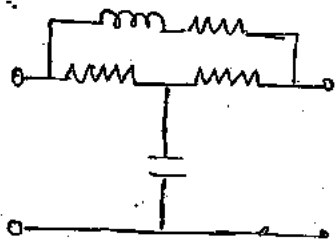
2. $\frac{ps^2}{s^2+as+b} = \text{H.P.F.}$

3. $\frac{ps}{s^2+as+b} = \text{BPF}$

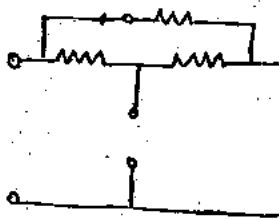
4. $\frac{ps^2+q}{s^2+as+b} = \text{BEF}$

5. $\frac{s^2-ps+q}{s^2+as+b} = \text{All pass filter}$

Q.

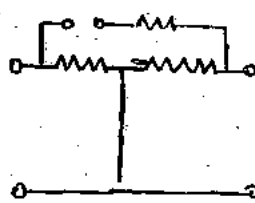


Soln: $S = 0$



Pass

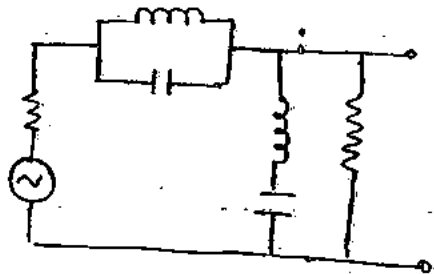
$S = \infty$



Stop

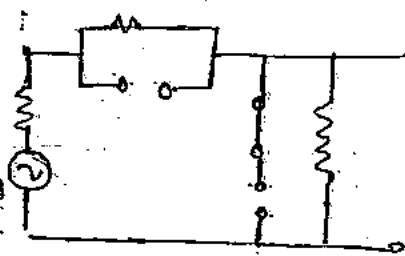
→ LPF

Q.



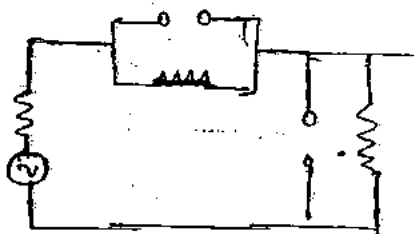
Soln:

$S = 0$



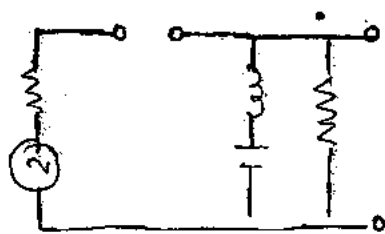
Pass

$S = \infty$

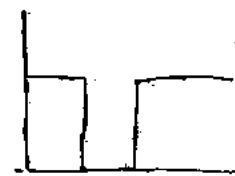


Pass

$f = f_r$

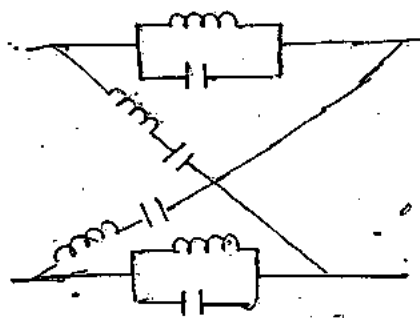


Stop



BE

Q.



All pass filter

Q1.

$$\left(\frac{1}{1+2s} \right) \cdot \left(\frac{1}{1+2s} \right)$$

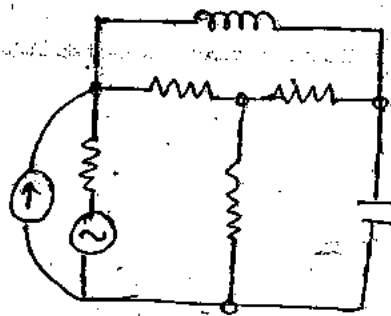
$$\frac{1}{(2s+1)^2} \quad s = -\frac{1}{2}, \frac{1}{2} \quad \text{critically damped}$$

∴ Graph theory and n/w Topology

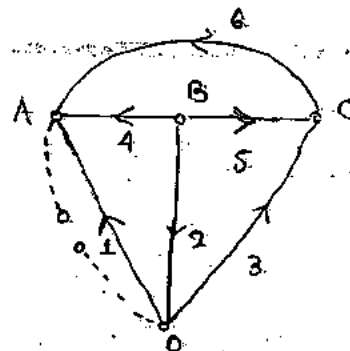
- N/w topology is the study of n/w properties by investigating the interconnection b/w branches and node.

In the n/w topology any n/w is replaced by the graph, to develop the graph, each element is replaced by st and arc is replaced by semi-circle and independent source are replaced by internal resistance. The graph retain all the node of original n/w.

Ex:



=



* No of branch of n/w $\geq$ no of branch in the graph.

Augmented incidence matrix

Notes: Incoming $\rightarrow$ +ve
outgoing $\rightarrow$ -ve

$[A_a] =$

	1	2	3	4	5	6
A	+1			+1		+1
B		-1		-1	-1	
C			+1		+1	-1
D	-1	+1	-1			

For given graph, A_0 matrix is always unique.

Properties of $[A_0]$:

1. The unique entry in a column identify nodes of branch which is connected.
2. The unique entry in a row identify the branches incident at node.
3. If the degree of the node two, then it implies that two branches are incident at node & these are in series.
4. Column of $[A_0]$ with unique entry into identical row correspond to two branches with same end nodes and hence they are in parallel.
5. Given the $[A_0]$ the corresponding graph can be easily constructed, since $[A_0]$ is complete mathematical description of n/w.

Reduce incidence matrix

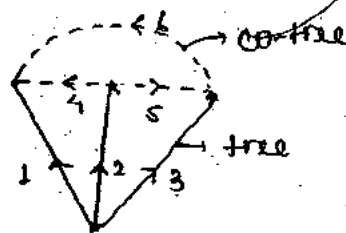
$[A_0] =$

	1	2	3	4	5	6
A	+1			+1		+1
B		-1		-1	-1	
C			+1		+1	-1

0 $\rightarrow$ reference node

Tree and co-tree:

- Tree is connected subgraph, it connects all nodes of n nodes but it does not consist of any closed path.
- The set of branches which are disconnected to form a tree is called co-tree.



Tree branch and co-tree branches:

The branch which forms a tree is called twig or tree branch. It is generally represented by solid or thick line.

$$\boxed{\text{Total no tree branches} = n - 1}$$

where n = no of nodes

ex: Tree $\rightarrow \{1, 2, 3\}$

The branch which is disconnected to form a tree is called link or chord. It is generally represented by dotted line.

$$\boxed{TNL = b - (n - 1)}$$

ex: Co-tree branches = $\{4, 5, 6\}$

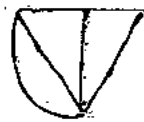
b = total no of branches

n = total no of nodes

Total no of possible trees = n^{n-2}

The above formula can be applied when,

- ① All the nodes should be connected whether at least one.
- ② No repeated branch should be present b/w nodes.



{ The above formula

Total no of possible tree of any graph

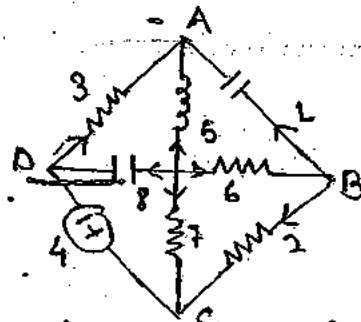
$$\text{Det}(A A^T) \quad A_r = \text{reduce incident matrix}$$

$$\text{Total no of node pair voltages} = N C_2 = \frac{N(N-1)}{2}$$

$$\text{Total no of edges} = N C_2 = \frac{N(N-1)}{2}$$

branch

Q1.

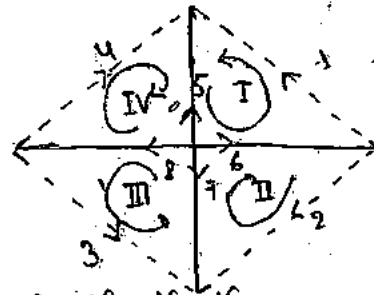
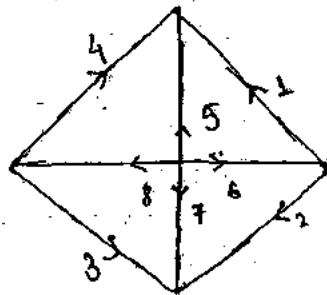


Develop tie-set matrix for the N/w

Procedure:

1. Develop the graph from n/w directed
2. Select the tree & co-tree from graph.
3. Identify the total no of links
4. The no of tie set current to be assign is equal to total no of links
5. Identify the fundamental loop or f-loop or basic loop. (basic loop is a loop which consist of only one link.)
6. The dirn of tie set current (basic loop current) is the same as the dirn of link.

7. Assign the +1 in the matrix if the dirn of tie-set current and branch is same



[B] =

	V_1 1	V_2 2	V_3 3	V_4 4	V_5 5	V_6 6	V_7 7	V_8 8
I	+1				-1	-1		
II		+1				-1	+1	
III			+1				+1	+1
IV				+1	-1			+1

KVL

$$V_1 - V_5 - V_6 = 0$$

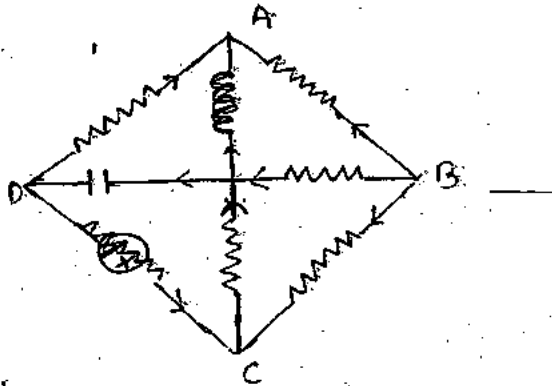
$$V_2 - V_6 + V_7 = 0$$

$$V_3 + V_7 + V_8 = 0$$

$$V_4 - V_5 + V_8 = 0$$

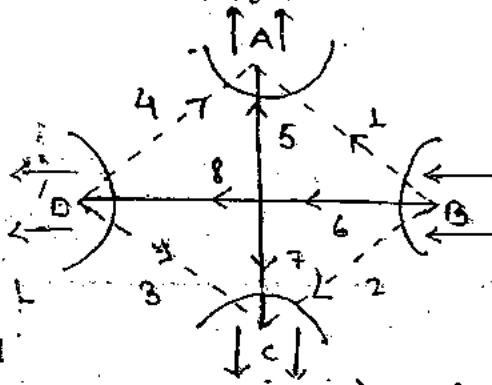
$$[B] = \{ [U] : [G] \}$$

Q. Develop the cut set matrix for given η_{tw} :-



Procedure:

1. Develop the graph for η_{tw} .
2. Develop the tree from graph.
3. Identify total no of basic cut-set, FC'S, & F-cut set.
4. Basic cut set consist of only one free-branch.
Total no basic cut set is equal to total no free branch = $N-1$
5. Basic cut-set dirⁿ is same as the free-branch current dirⁿ.



$[C] =$

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
A	+1			+1	+1			
B	+1	+1				+1		
C		-1	-1				+1	
D			-1	-1				+1

K-C1

$$i_1 + i_4 + i_5 = 0$$

$$i_1 + i_2 + i_6 = 0$$

$$-i_2 - i_3 + i_7 = 0$$

$$-i_3 - i_4 + i_8 = 0$$

$$[C] = \{ [C_b] : U \}$$

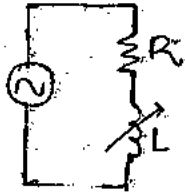
Conclusion:

1. Total no of possible tree = N^{n-2}
2. Tie-set matrix is not unique. it is equal to ~~N^{n-2}~~
 N^{n-2}
3. Cut-set matrix is not unique. it is equal to N^{n-2}
4. $[B] = [U : B_L]$
 $[C] = [C_b : U]$

$$[C_b] = -[B_L]^T$$
$$[B_L] = -[C_b]^T$$
5. Rank of cut-set matrix is equal to tree branches
ie $N-1$
6. Rank of tie-set matrix is equal to total no of
links. ie $b - (N-1)$
7. Rank of incident matrix = $N-1$

LOCUS DIAGRAM:-

The path traced by the terminals of trace vector by varying any one of the CRT elements or by varying frequency is called current locus.



Initially

$$L = 0 \quad \omega = 0$$

$$X_L = 0$$

$$Z = R$$

$$I = \frac{V}{R}$$

$$\theta = \tan^{-1} \frac{\omega L}{R} = 0^\circ$$

$$L \uparrow \quad \omega \uparrow$$

$$X_L \uparrow$$

$$Z \uparrow$$

$$I \downarrow$$

$$\theta \uparrow$$

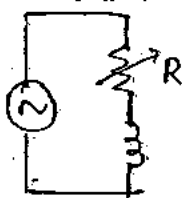
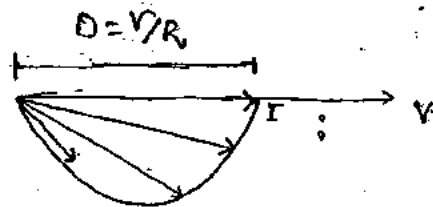
$$L = \infty \quad \omega = \infty$$

$$X_L = \infty$$

$$Z = \infty$$

$$I = 0$$

$$\theta = 90^\circ$$



$$R = 0$$

$$Z = X_L$$

$$I = \frac{V}{X_L}$$

$$\theta = \tan^{-1} \frac{\omega L}{R} = 90^\circ$$

$$R \uparrow$$

$$Z \downarrow$$

$$I \downarrow$$

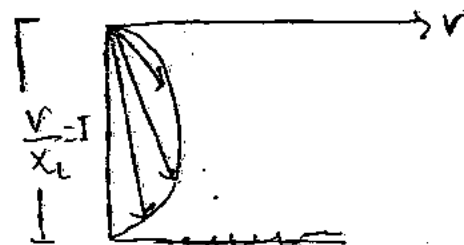
$$\theta \downarrow$$

$$R = \infty$$

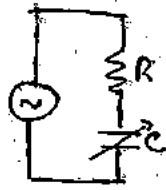
$$Z = \infty$$

$$I = 0$$

$$\theta = 0$$



Q.



$$X_C = \frac{1}{\omega C}$$

$$\text{At } C = 0$$

$$X_C = \infty$$

$$Z = \infty$$

$$I = 0$$

$$\theta = \tan^{-1} \left(\frac{X_C}{R} \right) = 90^\circ$$

$$C \uparrow$$

$$X_C \downarrow$$

$$Z \downarrow$$

$$I \uparrow$$

$$\theta \downarrow$$

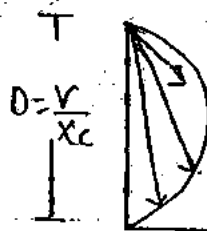
$$C = \infty$$

$$X_C = 0$$

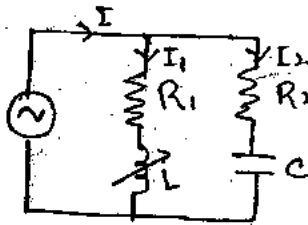
$$Z = R$$

$$I \uparrow$$

$$\theta = 0^\circ$$



Q.



$$L = 0$$

$$X_L = 0$$

$$Z = R$$

$$I_1 = \frac{V}{R}$$

$$\theta_1 = 0^\circ$$

$$I = \bar{I}_1 + \bar{I}_2$$

$$L \uparrow$$

$$X_L \uparrow$$

$$Z \uparrow$$

$$I_1 \downarrow$$

$$\theta = \uparrow$$

$$\bar{I} \downarrow$$

$$L = \infty$$

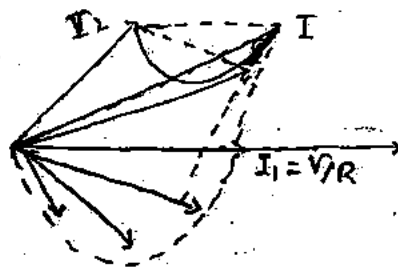
$$X_L = \infty$$

$$Z = \infty$$

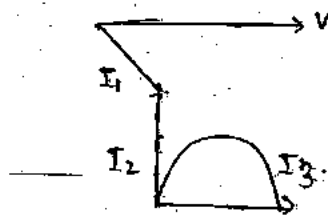
$$I_1 = 0$$

$$\theta = 90^\circ$$

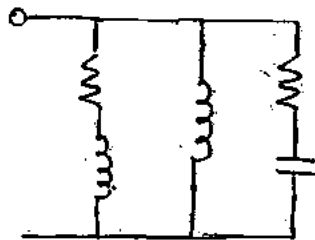
$$I = I_2$$



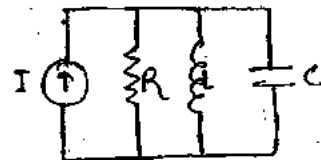
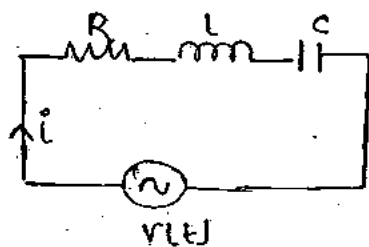
ex: design the approximate net for the given $e(t)$ -



Soln:



Dual N/w:



$$V = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$\tilde{i} = \frac{V}{R} + C \frac{dv}{dt} + \frac{1}{L} \int v dt$$

Duality

Series $\rightarrow$ ||

Voltage $\rightarrow$ current

KVL $\rightarrow$ KCL

Loop $\rightarrow$ node

R, L, C $\rightarrow$ G, C, L

O.C $\rightarrow$ S.C

Thermin $\rightarrow$ Norton

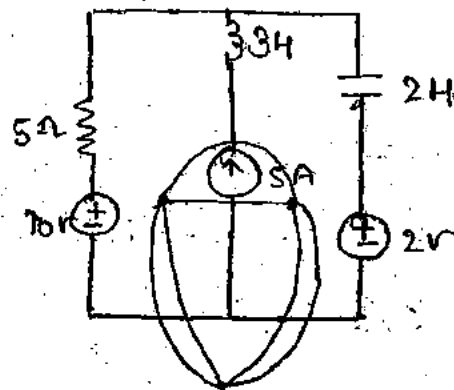
Foster I $\rightarrow$ Foster II

$\frac{dV}{dt} \rightarrow \frac{di}{dt}$

$V \int dt \rightarrow I \int dt$

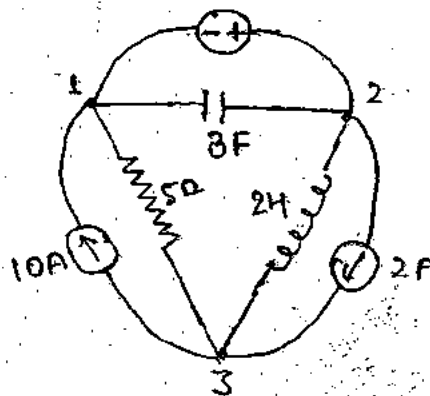
Out-Set $\rightarrow$ In-Set

Duality does not mean equivalence but it means mathematical representation of both n/w is same.



Notes:

1. When voltage source drives the I in $\odot$ dirn Arrow mark of current source is indicated towards respective node.
2. When current source drives the current in $\odot$ dirn the sign is given to respective node.



14
15
16
17