Sequence And Series

Sequence : A sequence can be regarded as a function whose domain is the set of natural numbers or some subset of it of the type $\{1, 2, 3 \ldots k\}$. Sometimes, we use the functional notation $a(n)$ for a_n.

Series : Let $a_1, a_2, a_3 \ldots\ldots, a_n$ be a given sequence. Then, the expression $a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ is called the series associated with the given sequence.

$$\boxed{a_1 + a_2 + a_3 + \ldots + a_n = \sum_{k=1}^{n} a_k}$$

Arithmetic progression : $a,\ a+d,\ a+2d \ldots\ldots$

- **n^{th} term (General term)** $\boxed{a_n = a + (n-1)d}$ $\boxed{l = a + (n-1)d}$

- **The sum of n terms** $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$

- **Sum of A.P. when first and last term is given,** $\boxed{S_n = \frac{n}{2}[a + l]}$

a = first term
l = last term
d = common difference
n = the no. of terms
S_n = the sum of n terms

Arithmetic Mean (A.M.) $\boxed{A = \frac{a+b}{2}}$

a and b = two numbers
A = Arithmetic Mean

Geometric Progression (G.P.) $a,\ ar,\ ar^2,\ ar^3, \ldots\ldots$

- **General term of G.P.** $\boxed{a_n = ar^{n-1}}$ r = common ratio

- **Sum of n terms of G.P.** $\boxed{S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}}$

Case I If $r = 1$ $\boxed{S_n = na}$

Case II If $r \neq 1$ $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$ $\to r<1$ OR $\boxed{S_n = \frac{a(r^n - 1)}{r-1}}$ $\to r>1$

Geometric Mean (G.M.) $\boxed{G = \sqrt{ab}}$; $a, b > 0$

Relationship between A.M and G.M. $\boxed{A - G = \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0}$; $a, b > 0$

Sum of first n natural numbers

$S_n = 1 + 2 + 3 + \ldots + n$; $\boxed{S_n = \frac{n(n+1)}{2}}$

Sum of squares of the first n natural numbers

$S_n = 1^2 + 2^2 + 3^2 + \ldots + n^2$; $\boxed{S_n = \frac{n(n+1)(2n+1)}{6}}$

Sum of cubes of the first n natural numbers

$S_n = 1^3 + 2^3 + 3^3 + \ldots + n^3$; $\boxed{S_n = \frac{[n(n+1)]^2}{4}}$