Short Answer Type Questions – II

[3 MARKS]

Que 1. What must be subtracted from $\boldsymbol{x^4 + 3x^3 + 4x^2 - 3x - 6}$ to get $\boldsymbol{3x^3 + 4x^2 - x + 3}$?

Sol. Let $p(x)$ be the required polynomial.

$$Then, x^4 + 3x^3 + 4x^2 - 3x - 6 - p(x) = 3x^3 + 4x^2 - x + 3$$

$$\therefore\ p(x) = x^4 + 3x^3 + 4x^2 - 3x - 6 - 3x^3 - 4x^2 + x - 3$$

$$= x^4 - 2x - 9$$

Que 2. What must be added to $\boldsymbol{2x^2 - 5x + 6\ to\ get\ x^3 - 3x^2 + 3x - 5}$?

Sol. Let $p(x)$ be added.

$$Than, 2x^2 - 5x + 6 + p(x) = x^3 - 3x^2 + 3x - 5$$

$$\therefore\ p(x) = x^3 - 3x^2 + 3x - 5 - 2x^2 + 5x - 6$$

$$= x^3 - 5x^2 + 8x - 11$$

Que 3. If x $\boldsymbol{x + 2k\ is\ a\ factor\ of\ f(x) = x^4 - 4k^2x^2 + 2x + 3k + 3, find\ k}$.

Sol. Here, $f(x) = x^4 - 4k^2x^2 + 2x + 3k + 3$

Since (x – 2k) is a factor of f(x), so by factor theorem,

$$f(-2k) = 0$$

$$(-2k)^4 - 4k^2(-2k)^2 + 2(-2k) + 3k + 3 = 0$$

$$16k^4 - 16k^4 - 4k + 3k + 3 = 0$$

$$\Rightarrow -k + 3 = 0 \quad \Rightarrow -k = -3 \quad \Rightarrow k = 3$$

Que 4. Find the remainder when $\boldsymbol{f(x) = 9x^3 - 3x^2 + 14x - 3\ is\ divided\ by\ g(x) = (3x - 1)}$.

Sol. Taking g(x) = 0 we have,

$$3x - 1 = 0 \quad \Rightarrow x = \frac{1}{3}$$

By remainder theorem when $f(x)$ is divided by $g(x)$, the remainder is equal to $f\left(\frac{1}{3}\right)$

Now,

$$f(x) = 9x^3 - 3x^2 + 14x - 3$$

$$f\left(\frac{1}{3}\right) = 9\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right)^2 + 14\left(\frac{1}{3}\right) - 3$$

$$= 9 \times \frac{1}{27} - 3 \times \frac{1}{9} + \frac{14}{3} - 3 = \frac{1}{3} - \frac{1}{3} + \frac{14}{3} - 3 \Rightarrow f\left(\frac{1}{3}\right) = \frac{5}{3}$$

Hence, required remainder = $= \frac{5}{3}$

Que 5. Check whether polynomial $p(x) = 2x^3 - 9x^2 + x + 12$ is a multiple of 2x – 3 or not.

Sol. The polynomial p(x) will be a multiple of $2x - 3\ if\ (2x - 3)$ divides p(x) completely.

Now, 2x – 3 = 0 $\Rightarrow x = \frac{3}{2}$

Also,

$$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12$$

$$= 2 \times \frac{27}{8} - 9 \times \frac{9}{4} + \frac{3}{2} + 12$$

$$= \frac{54}{8} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{54-162+12+96}{8} = \frac{162-162}{8} = \frac{0}{8}$$

$$p\left(\frac{3}{2}\right) = 0$$

As (2x – 3) divides p(x) completely, therefore p(x) is a multiple of (2x – 3).

Que 6. Show that $2x + 1$ is a factor of polynomial $2x^3 - 11x^2 - 4x + 1$.

Sol.

$$Let, p(x) = 2x^3 - 11x^2 - 4x + 1 \quad and\ g(x) = 2x + 1$$

By factor theorem (2x – 1) will be a factor of p(x) if $p\left(\frac{-1}{2}\right) = 0$

Now,

$$p(x) = 2x^3 - 11x^2 - 4x + 1$$

$$\Rightarrow p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) + 1$$

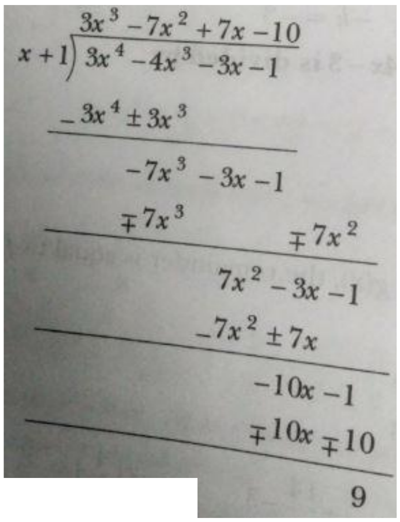
$$= 2\left(\frac{-1}{8}\right) - 11 \times \frac{1}{4} + 4 \times \frac{1}{2} + 1 = -\frac{1}{4} - \frac{11}{4} + 2 + 1$$

$$= \frac{-1 - 11 + 8 + 4}{4} = \frac{-12 + 12}{4} \Rightarrow p\left(\frac{-1}{2}\right) = 0$$

As $p\left(\frac{-1}{2}\right) = 0$, therefore (2x – 1) is a factor of $2x^3 - 11x^2 - 4x + 1$.

Que 7. By actual division, find the quotient and remainder when $3x^4 - 4x^3 - 3x - 1$ is divided by x + 1.

Sol. By long division, we have



$$\begin{array}{r|l} & 3x^3 - 7x^2 + 7x - 10 \\ x+1 & 3x^4 - 4x^3 - 3x - 1 \\ & \underline{_-3x^4 \pm 3x^3} \\ & -7x^3 - 3x - 1 \\ & \underline{\mp 7x^3 \quad \mp 7x^2} \\ & 7x^2 - 3x - 1 \\ & \underline{_-7x^2 \pm 7x} \\ & -10x - 1 \\ & \underline{\mp 10x \mp 10} \\ & 9 \end{array}$$

Quotient = $= 3x^3 - 7x^2 + 7x - 10, Remainder = 9$

Que 8. If $\sqrt{m} + \sqrt{n} - \sqrt{p} = 0$, then find the value of $(m + n - p)^2$.

Sol. We have $\sqrt{m} + \sqrt{n} - \sqrt{p} = 0$

$\Rightarrow$ $\sqrt{m} + \sqrt{n} = \sqrt{p}$

Squaring both the sides, we get

$$\left(\sqrt{m} + \sqrt{n}\right)^2 = \left(\sqrt{p}\right)^2$$

$\Rightarrow$ $m + n + 2\sqrt{m}\sqrt{n} = p$

$\Rightarrow$ $m + n - p = -2\sqrt{mn}$

Again squaring both the sides, we get $(m + n - p)^2 = 4mn$

Que 9. Expand: $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

Sol.

$$\left[\frac{1}{x} + \frac{y}{3}\right]^3 = \left(\frac{1}{x}\right)^3 + 3\left(\frac{1}{x}\right)^2 \frac{y}{3} + 3\frac{1}{x}\left(\frac{y}{3}\right)^2 + \left(\frac{y}{3}\right)^3$$

$$= \left(\frac{1}{x}\right)^3 + 3.\left(\frac{1^2}{x^2}\right)\frac{y}{3} + 3.\frac{1}{x}\frac{y^2}{3^2} + \frac{y^2}{3^3}$$

$$= \frac{1}{x^3} + \frac{y}{x^2} + \frac{y^2}{3x} + \frac{y^3}{27}$$

Que 10. Evaluate: (104)^3 using a suitable identity.

Sol. $(104)^3 = (100+4)^3$

Using identity $(x+y)^3 = x^3 + 3xy\,(x+y) + y^3$

We get,

$$(100+4)^3 = (100)^3 + 3 \times 100 \times 4(100+4) + 4^3$$

$$= 10{,}00{,}000 + 1{,}200 \times 104 + 64$$

$$= 10{,}00{,}000 + 1{,}24{,}800 + 64$$

$$= 11{,}24{,}864$$

Que 11. Evaluate 105×108 without multiplying directly.

Sol. $105 \times 108 = (100 + 5)\,(100 + 8)$

Using identity $(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

$$We\ get,\ \ 105 \times 108 = 100^2 + (5+8)\,100 + 5 \times 8$$

$$= 10000 + 1300 + 40 = 11340$$

Que 12. Find the value of $x^2 + \frac{1}{x^2}, if\ x - \frac{1}{x} = \sqrt{3}$.

Sol. $x - \frac{1}{x} = \sqrt{3}$

Squaring both the sides, we get $\left(x - \frac{1}{x}\right)^2 = \left(\sqrt{3}\right)^2$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 3$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 3 + 2 \Rightarrow x^2 + \frac{1}{x^2} = 5$$

Que 13. Factorise: $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$ by splitting the middle term.

Sol. $5\sqrt{5}x^2 + 30x + 8\sqrt{5} = 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5}$

$$= 5x(\sqrt{5}x + 4) + 2\sqrt{5}\,(\sqrt{5}x + 4)$$

$$= (\sqrt{5}x + 4)\,(\sqrt{5}x + 2\sqrt{5}) = \sqrt{5}(\sqrt{5}x + 2)(\sqrt{5}x + 4)$$

Que 14. Factorise: $2x^5 + 432x^2\ y^3$.

Sol. We have, $2x^5 + 432x^2y^3 = 2x^2\ (x^3 + 216y^3)$

$$= 2x^2\ (x^3 + 6^3y^3) = 2x^2\ [x^3 + (6y)^3]$$

$$= 2x^2(x + 6y)[x^2 - x.6y + (6y)^3]$$

$$= 2x^2(x + 6y)(x^2 - 6xy + 36y^2)$$

Que 15. Factorise: $125x^3 + 27y^3 + 8z^3 - 90xyz$.

Sol. $125x^3 + 27y^3 + 8z^3 - 90xyz$

$$= 5^3x^3 + 3^3y^3 + 2^3z^3 - 90xyz$$

$$= (5x)^3 + (3y)^3 + (2z)^3 - 3 \times 5x \times 3y \times 2z$$

$$= (5x + 3y + 2z)[(5x)^2 + (3y)^2 + (2z)^2 - (5x)(3y) - (3y)(2z) - (2z)(5x)]$$

$$= (5x + 3y + 2z)(25x^2 + 9y^2 + 4z^2 - 15xy - 6yz - 10zx)$$

Que 16. Factorise: $\frac{r^3}{8} - \frac{s^3}{343} - \frac{t^3}{216} - \frac{1}{28}rst$.

Sol.

$$\frac{r^3}{8} - \frac{s^3}{343} - \frac{t^3}{216} - \frac{1}{28}rst.$$

$$\left(\frac{r}{8}\right)^3 + \left(\frac{-s}{7}\right)^3 + \left(\frac{-t}{6}\right)^3 - 3\left(\frac{r}{2}\right)\left(\frac{-s}{7}\right)\left(\frac{-t}{6}\right)$$

$$= \left[\frac{r}{2} + \left(\frac{-s}{7}\right) + \left(\frac{-t}{6}\right)\right]\left[\left(\frac{r}{2}\right)^2 + \left(\frac{-s}{7}\right) + \left(\frac{-t}{6}\right)^2 - \frac{r}{2}\left(\frac{-s}{7}\right) - \left(\frac{-s}{7}\right)\left(\frac{-t}{6}\right) - \left(\frac{-t}{6}\right)\left(\frac{r}{2}\right)\right]$$

$$= \left(\frac{r}{2} - \frac{s}{7} - \frac{t}{6}\right)\left(\frac{r^2}{4} + \frac{s^2}{49} + \frac{t^2}{36} + \frac{rs}{14} - \frac{st}{42} + \frac{tr}{12}\right)$$

Que 17. Factorise: $2x^2 - 7x - 15$ by splitting the middle term.

Sol. $2x^2 - 7x - 15$

$$= 2x^2 - 10x + 3x - 15$$

$$= 2x(x - 5) + 3(x - 5)$$

$$= (x - 5)(2x + 3)$$

Que 18. Factorise: $125x^3 – 343y^3$.

Sol. $125x^3 – 343y^3$.

$$= 5^3x^3 - 7^3x^3 = (5x)^3 - (7y)^3$$

$$= (5x - 7y)[(5x)^2 + 5x.7y + (7y)^2]$$

$$= (5x - 7y)(25x^2 + 35xy + 49y^2)$$

Que 19. Factorise: $3x^2 + 4y^2 + 25z^2 - 4\sqrt{3xy} - 20yz + 10\sqrt{3}zx$.

Sol. $3x^2 + 4y^2 + 25z^2 - 4\sqrt{3xy} - 20yz + 10\sqrt{3}zx$.

$$= \left(\sqrt{3}x\right)^2 + (-2y)^2 + (5z)^2 + 2\left(\sqrt{3x}\right)(-2y) + 2(-2y)(5z) + 2(5z)\left(\sqrt{5}x\right)$$

$$= (\sqrt{3}x - 2y + 5z)^2 = \left(\sqrt{3}x - 2y + 5z\right)\left(\sqrt{3}x - 2y + 5z\right)$$

Que 20. Factorise: $\left(5r + \frac{2}{3}\right)^2 - \left(2r - \frac{1}{3}\right)^2$.

Sol.

$$\left(5r + \frac{2}{3}\right)^2 - \left(2r - \frac{1}{3}\right)^2$$

$$= \left(5r + \frac{2}{3} + 2r - \frac{1}{3}\right)\left[5r + \frac{2}{3} - \left(2r - \frac{1}{3}\right)\right]$$

$$= \left(7r + \frac{2}{3} - \frac{1}{3}\right)\left(5r - 2r + \frac{2}{3} + \frac{1}{3}\right)$$

$$= \left(7r + \frac{1}{3}\right)(3r + 1).$$

Que 21. Without actually calculating the cubes, find the value of:

$$\left(\frac{-3}{4}\right)^3 + \left(\frac{-5}{8}\right)^3 + \left(\frac{11}{8}\right)^3.$$

Sol. Let

$$a = \frac{-3}{4}, b = \frac{-5}{8}, c = \frac{11}{8}$$

$$\therefore\ a + b + c = \frac{-3}{4} - \frac{5}{8} + \frac{11}{8}$$

$$= \frac{-6 - 5 + 11}{8} = 0$$

$$If\ a + b = 0, then\ a^3 + b^3 + c^3 = 3abc$$

$$\therefore \left(\frac{-3}{4}\right)^3 + \left(\frac{-5}{8}\right)^3 + \left(\frac{11}{8}\right)^3 = 3\left(\frac{-3}{4}\right)\left(\frac{-5}{8}\right)\left(\frac{11}{8}\right) = \frac{495}{256}$$

Que 22. Without finding the cubes, factorise: $(\mathbf{2r - 3s})^3 + (\mathbf{3s - 5t})^3 + (\mathbf{5t - 2r})^3$.

Sol. Let $a = 2r - 3s, b = 3s - 5t + 5t - 2r = 0$

$$\therefore\ a + b + c = 2r - 3s + 3s - 5t + 5t - 2r = 0$$

$$If\ a + b + c = 0, then\ a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow (2r - 3s)^3 + (3s - 5t)^3 + (5t - 2r)^3 = 3(2r - 3s)(3s - 5t)(5t - 2r)$$

Que 23. Find the value of $x^3 - 8y^3 - 36xy - 216\ When\ x = 2y + 6$.

Sol.

$$x^3 - 8y^3 - 216 - 36xy = x^3 + (-2y)^3 + (-6)^3 - 3.x(-2y)(-6)$$

$$= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$$

$$\therefore = 0 \times (x^2 + 4y^2 + 36 + 2xy - 12y + 6x)[\because\ x = 2y + 6 \Rightarrow x - 2y - 6 = 0]$$

$$= 0$$

Que 24. If a, b, c are all non-zero and a + b + c = 0, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

Sol. We have,

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$$

$$LHS \qquad \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc} = 3(if\ a + b + c = 0, then\ a^3 + b^3 + c^3 = 3abc) = RHS$$

Que 25. Simplify: $(\mathbf{2x - 5y})^3 - (\mathbf{2x + 5y})^3$.

Sol. $(2x - 5y)^3 - (2x + 5y)^3$

$$= [8x^3 - 125y^3 - 3 \times 2x \times 5y(2x - 5y)] - [8x^3 + 125y^3 + 3 \times 2x \times 5y(2x + 5y)]$$

$$[using\ the\ identity\ (a + b)^3 = a^3 + b^3 + 3ab(a + b)\ and\ (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$$

$$= (8x^3 - 125y^3 - 60x^2y + 150xy^2) - (8x^3 + 125y^3 + 60x^2y + 150xy^2)$$

$$= -250y^3 - 120x^2y$$

Que 26. Factorise $\left(9x-\frac{1}{5}\right)^2-\left(x+\frac{1}{3}\right)^2$.

Sol. We have, $\left(9x-\frac{1}{5}\right)^2-\left(x+\frac{1}{3}\right)^2$.

$$=\left[\left(9x-\frac{1}{5}\right)-\left(x+\frac{1}{3}\right)\right]\left[\left(9x-\frac{1}{5}\right)+\left(x\frac{1}{3}\right)\right] \qquad [\because a^2-b^2 = (a-b)(a+b)]$$

$$=\left(9x-\frac{1}{5}-x-\frac{1}{3}\right)\left(9x-\frac{1}{5}+x+\frac{1}{3}\right)$$

$$=\left(8x-\frac{1}{5}-\frac{1}{3}\right)\left(10x-\frac{1}{5}+\frac{1}{3}\right)$$

$$=\left(\frac{120x-3-5}{15}\right)\left(\frac{150x-3+5}{15}\right)$$

$$=\left(\frac{120x-8}{15}\right)\left(\frac{150x+2}{15}\right)$$