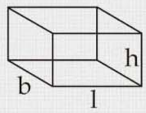
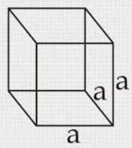
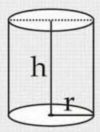
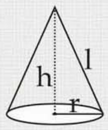
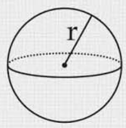
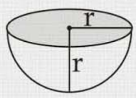
Volume and Surface Area

Exercise 13A

Name of the solid	Figure	Volume	Laterial/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2+\pi r^2$ or $3\pi r^2$

Question 1:

(i) length = 12cm, breadth = 8 cm and height = 4.5 cm

∴ Volume of cuboid = l x b x h

= (12 x 8 x 4.5) cm^3 = 432 cm^3

∴ Lateral surface area of a cuboid = 2(l + b) x h

= [2(12 + 8) x 4.5] cm^2

= (2 x 20 x 4.5) cm^2 = 180 cm^2

∴ Total surface area cuboid = 2(lb + b h + l h)

= 2(12 x 8 + 8 x 4.5 + 12 x 4.5) cm^2

= 2(96 + 36 + 54) cm^2

= (2 x 186) cm^2

= 372 cm^2

(ii) Length 26 m, breadth = 14 m and height = 6.5 m

∴ Volume of a cuboid = l x b x h

= (26 x 14 x 6.5) m^3

= 2366 m^3

∴ Lateral surface area of a cuboid = 2 (l + b) x h

= [2(26+14) x 6.5] m^2

= (2 x 40 x 6.5) m^2

$= 520\ m^2$

∴ Total surface area = 2(lb+ bh + lh)

= 2(26 x 14+14 x6.5 +26 x6.5)

$= 2\ (364+91+169)\ m^2$

$= (2 \times 624)$ m2$= 1248\ m^2$.

(iii) Length = 15 m, breadth = 6m and height = 5 dm = 0.5 m

∴ Volume of a cuboid = l x b x h

$= (15 \times 6 \times 0.5)$ m3$=45\ m^3$.

∴ Lateral surface area = 2(l + b) x h

$= [2(15 + 6) \times 0.5]\ m^2$

$= (2 \times 21 \times 0.5)$ m2$=21\ m^2$

∴ Total surface area =2(lb+ bh + lh)

$= 2(15 \times 6 + 6 \times 0.5 + 15 \times 0.5)\ m^2$

$= 2(90+3+7.5)\ m^2$

$= (2 \times 100.5)\ m^2$

$=201\ m^2$

(iv) Length = 24 m, breadth = 25 cm =0.25 m, height = 6m.

∴ Volume of cuboid = l x b x h

$= (24 \times 0.25 \times 6)\ m^3$.

$= 36\ m^3$.

∴ Lateral surface area = 2(l + b) x h

$= [2(24 + 0.25) \times 6]\ m^2$

$= (2 \times 24.25 \times 6)\ m^2$

$= 291\ m^2$.

∴ Total surface area =2(lb+ bh + lh)

$=2(24 \times 0.25 + 0.25 \times 6 + 24 \times 6)\ m^2$

$= 2(6+1.5+144)\ m^2$

$= (2 \times 151.5)\ m^2$

$=303\ m^2$.

Question 2:

Length of Cistern = 8 m

Breadth of Cistern = 6 m

And Height (depth) of Cistern =2.5 m

∴ Capacity of the Cistern = Volume of cistern

∴ Volume of Cistern = (l x b x h)

$= (8 \times 6 \times 2.5)\ m^3$

$=120\ m^3$

Area of the iron sheet required = Total surface area of the cistem.

∴ Total surface area = 2(lb +bh +lh)

$= 2(8 \times 6 + 6 \times 2.5 + 2.5 \times 8)\ m^2$

$= 2(48 + 15 + 20)\ m^2$

$= (2 \times 83)$ m2$=166\ m^2$

Question 3:

Length of a room =9m,

Breadth of a room = 8m

And height of room = 6.5 m

∴ Area of 4 walls = Lateral surface area

= 2 (l+ b) x h

= [2 (9+8) x 6.5] m^2

= (2 x 17 x 6.5) m^2

=221 m^2

∴ Area not be whitewashed = (area of 1 door) + (area of 2 windows)

= (2 x 1.5) m^2 + (2 x 1.5 x 1) m^2

= 3m^2 + 3m^2 =6m^2

∴ Area to be whitewashed = (221-6) m^2 =215 m^2

∴ Cost of whitewashing the walls at the rate of Rs.6.40 per

Square meter = Rs. (6.40 x 215) = Rs. 1376

Question 4:

Length of plank = 5m =500 cm

Breadth of plank = 25 m

Height of plank = 10 cm

∴ Volume of plank = l x b x h

= (500 x 25 x 10) cm^3

Now,

Length of pit = 20 m = 2000 cm

Breadth of pit = 6m = 600cm

Height of pit = 80 cm

∴ Volume of one pit = (2000 x 600 800) cm^3

$$\therefore \text{Number of planks that can be stored} = \frac{\text{Volume of pit}}{\text{Volume of plank}}$$

$$= \frac{(2000 \times 600 \times 80)}{(500 \times 25 \times 10)} = 768$$

Question 5:

Length of wall = 8m = 800cm

Breadth of wall = 6m = 600 cm

Height of wall = 22.5 cm

∴ Volume of wall = l x b x h

= (800 x 600 x22.5) cm^3

Length of brick = 25cm

Breadth of brick = 11.25cm

Height of brick = 6cm

∴ Volume of brick = (25 x 11.25 x 6)cm^3

$$\therefore \text{Number of bricks required} = \frac{\text{Volume of the wall}}{\text{Volume of brick}}$$

$$= \frac{(800 \times 600 \times 22.5)}{(25 \times 11.25 \times 6)} = 6400$$

Question 6:

Length of wall = 15m
Breadth of wall = 0.3m
Height of wall = 4m
$\therefore$ Volume of the wall = $(15 \times 0.3 \times 4)\ m^3 = 18 m^3$

$$\text{Volume of mortar} = \left(\frac{1}{12} \times 18\right) = 1.5\ m^3$$

$$\text{Volume of wall} = (18 - 1.5) m^3 = 16.5 = \frac{33}{2}\ m^3$$

Length of brick = 22 cm
Breadth of brick = 12.5 cm
Height of brick = 7.5 cm

$$\therefore \text{Volume of 1 brick} = \left(\frac{22}{100} \times \frac{12.5}{100} \times \frac{7.5}{100}\right) m^3$$

$$= \left(\frac{33}{16000}\right) m^3$$

$$\therefore \text{Number of bricks} = \frac{\text{Volume of bricks}}{\text{Volume of 1brick}}$$

$$= \left(\frac{33}{2} \times \frac{16000}{33}\right) = 8000$$

Question 7:

External length of cistern = 1.35 m = 135 cm
External breadth of cistern = 1.08 m =108 cm
External height of cistern = 90cm
$\therefore$ External volume of cistern = $(135 \times 108 \times 90)\ cm^3$
$= 1312200\ cm^3$
Internal length of cistern = $(135 - 2 \times 2.5)$ cm
$= (135 - 5)$ cm = 130 cm
Internal breadth of cistern = $(108 - 2 \times 2.5)$ cm
$= (108 - 5)$ cm = 103 cm
Internal height of cistern = $(90 - 2.5)$ cm = 87.5 cm
$\therefore$ Capacity of the cistern = Internal volume of cistern
$= (130 \times 103 \times 87.5)\ cm^3$
$= 1171625\ cm^3$
Volume of the iron used = External volume of the cistern
-Internal volume of the cistern
$= (1312200 - 1171625)\ cm^3$
$= 140575\ cm^3$

Question 8:

Depth of the river = 2 m
Breadth of the river = 45 m

Length of the river = 3 K M /h $= \left(\frac{3 \times 1000}{60}\right)$ m/min

= 50m /min.

∴ Volume of water running into the sea per minute = (50 x 45 x 2) m^3

= 4500 m^3

Question 9:

Total cost of sheet = Rs. 1620
Cost of metal sheet per square meter = Rs.30

∴ Area of the sheet required = $\left(\frac{\text{Total cost}}{\text{rate}/m^2}\right)$ sq.m.

$= \left(\frac{1620}{30}\right)$ sq.m = 54 sq.m.

Length of box = 5m
Breadth of box = 3m

Now, Let the height of the box be x meters.
∴ Area of the sheet = Total surface area of the box.

$= 2(lb + bh + lh)$

$54 = 2(5 \times 3 + 3 \times x + 5 \times x)$

$54 = 2(15 + 3x + 5x)$

$54 = 2(15 + 8x)$

∴ $2(15 + 8x) = 54$

⇒ $30 + 16x = 54$

⇒ $16x = 54 - 30$

⇒ $x = \frac{24}{16} = 1.5\text{m}$

∴ The height of the box = 1.5 m.

Question 10:

Length of room = 10 m
Breadth of room = 10 m
Height of room = 5 m

∴ Length of the longest pole = length of diagonal

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{10^2 + 10^2 + 5^2}$

$= \sqrt{100 + 100 + 25} = \sqrt{225} = 15\text{ m}$

∴ The length of the longest pole that can be put in a room with given
Dimensions = 15 m.

Question 11:

Length of hall = 20 m
Breadth of hall = 16 m
Height of hall = 4.5 m

∴ Volume of hall = $l \times b \times h$

= (20 x 16 x 4.5) m^3

Volume of air needed per person = 5 m^3

∴ Number of persons = $\left(\frac{\text{Volume of the hall}}{\text{Volume of air needed per person}}\right)$

$= \left(\frac{20 \times 16 \times 4.5}{5}\right) = 288.$

Question 12:

Length of classroom $= 10m$
Breadth of classroom $= 6.4\ m$
Height of classroom $= 5\ m$
Each student is given $1.6\ m^2$ of the floor area.

$$\text{Number of students} = \frac{(\text{area of the room})}{1.6}$$

$$= \frac{(10 \times 6.4)}{1.6} = \frac{64}{1.6} = 40$$

$\therefore$ Number of students = 40

$$\therefore \text{Air required by each student} = \left(\frac{\text{Volume of the room}}{\text{number of students}}\right) m^3$$

$$= \left(\frac{10 \times 6.4 \times 5}{40}\right) m^3 \left(\frac{320}{40}\right) m^3$$

$$= 8m^3$$

Question 13:

Volume of a cuboid $= 1536\ m^3$
Length of the cuboid $= 16\ m$
Let the breadth and height of the cuboid be 3x and 2x.
$\therefore$ Volume of cuboid $= l \times b \times h$

$\Rightarrow \quad 1536 = (16 \times 3x \times 2x)$

$\Rightarrow \quad 1536 = 96x^2$

$$\Rightarrow \quad x^2 = \frac{1536}{96} = 16$$

$\therefore \quad x = \sqrt{16} = 4\ m.$

$\therefore$ Breadth of the cuboid $= 3x = 3 \times 4 = 12m$

And height of the cuboid $= 2x = 2x\ 4 = 8\ m$

Question 14:

Surface area of a cuboid $= 758\ cm^2$
Length $= 14\ cm$
Breadth $= 11\ cm$
Let the height of the cuboid $= h\ cm$
$\therefore$ Surface area of cuboid $= 2(lb + bh + lh)$

$\Rightarrow \quad 758 = 2(14 \times 11 + 11 \times h + 14 \times h)$

$\Rightarrow \quad 758 = 2(154 + 11h + 14h)$

$\Rightarrow \quad 758 = 2(154 + 25h)$

$\Rightarrow \quad 758 = 308 + 50h$

$\Rightarrow \quad 50\ h = 758 - 308$

$$\therefore \quad h = \frac{450}{50} = 9cm.$$

$\therefore$ The height of the cuboid = 9 cm

Question 15:

(a) Each edge of a cube = 9m

$\therefore$ Volume of a cube $= a^3$

$= (9)^3\ m^3 = 729\ m^3$

$\therefore$ Lateral surface area of cube $= 4a^2$

$= 4 \times (9)^2$

$= (4 \times 81)\ m^2$

$= 324\ m^2$

$\therefore$ Total surface area of a cube $= 6a^2$

$= 6 \times (9)^2$

$= (6 \times 81)\ m^2$

$= 486\ m^2$

$\therefore$ Diagonal of cube $= \sqrt{3}\ a$

$= \sqrt{3} \times 9$

$= (1.73 \times 9)\ m = 15.57\ m$

(b) $\therefore$ Each edge of a cube = 6.5 cm

Volume of a cube $= a^3 = (6.5)^3\ cm^3$

$= 274.625\ cm^3$

$\therefore$ Lateral surface area of a cube $= 4a^2$

$= 4 \times (6.5)^2\ cm^2$

$= (4 \times 42.25)\ cm^2$

$= 169\ cm^2$

Total surface area of a cube $= 6a^2$

$= 6 \times (6.5)^2 cm^2$

$= (6 \times 42.25)\ cm^2$

$= 253.5\ m^2$

$\therefore$ Diagonal of cube $= \sqrt{3}\ a$

$= \sqrt{3} \times 6.5$

$= (1.73 \times 6.5)\ cm$

$= 11.245\ cm.$

Question 16:

Let each side of the cube be a cm.

Then, the total surface area of the cube $= (6a^2)\ cm^2$

$\therefore \quad 6a^2 = 1176$

$\Rightarrow \quad a^2 = \frac{1176}{6} = 196$

$\Rightarrow \quad a = \sqrt{196} = 14\ cm$

$\therefore$ Volume of the cube $= a^3$

$= (14)^3 = (14 \times 14 \times 14)\ cm^3$

$= 2744\ cm^3.$

Question 17:

Let each side of the cube be a cm

Then, the lateral surface area of the cube $= (4a^2)\ cm^2$

$\therefore \quad 4a^2 = 900$

$\Rightarrow \quad a^2 = \frac{900}{4} = 225$

$\therefore \quad a = \sqrt{225} = 15\ cm$

$\therefore$ Volume of the cube $= a^3$

$= (15)^3 = (15 \times 15 \times 15) cm^3$

$= 3375\ cm^3.$

Question 18:

Volume of the cube $= 512\ cm^3$ [Volume $= a^3$]

$\therefore$ Each edge of the cube $= \sqrt[3]{512} = 8$ cm.

$\therefore$ Surface area of cube $= 6a^2$

$= 6 \times (8)^2\ cm^2$

$= (6 \times 64)\ cm^2$

$= 384\ cm^2$

Question 19:

Volume of the new cube = [$(3)^3 + (4)^3 + (5)^3$] cm
= (27 + 64 + 125) cm^2
= 216 cm^2
Now edge of this cube = a cm
And, a^3 = 216
∴ a = 6cm
Lateral surface area of the new cube = $4a^2$ cm^2.
= 4 x $(6)^2$ cm^2
= (4 x 36) cm^2
= 144 cm^2
∴ The lateral surface area of the new cube formed =144 cm^2.

Question 20:

1 hectare = 10000 m^2
Area = 2 hectares = 2× 10000 m^2
Depth of the ground = 5 cm = $\frac{5}{100}$ m
Volume of water = (area x depth)
$$= \left(2 \times 10000 \times \frac{5}{100}\right) m^3$$
$= 1000\ m^3$
∴ Volume of water that falls =1000 m^3

Exercise 13B

Question 1:

Here, r = 5cm and h=21 cm

∴ Volume of the cylinder = $(\Pi r^2 h)$
$$= \left(\frac{22}{7} \times 5^2 \times 21\right) cm^3$$
$$= \left(\frac{22}{7} \times 25 \times 21\right) cm^3$$
$= 1650\ cm^3$.

∴ Curved surface area of a cylinder = $(2\Pi rh)$
$$= 2 \times \left(\frac{22}{7} \times 5 \times 21\right) cm^2$$
= 660 cm^2

Question 2:

Here, diameter = 28 cm

$$\text{Radius} = \left(\frac{28}{2}\right)\text{cm} = 14 \text{ cm and}$$

height = 40 cm

$\therefore$ Curved surface area = $(2\Pi rh)$

$$= \left(2 \times \frac{22}{7} \times 14 \times 40\right)\text{cm}^2$$

$$= 3520\text{cm}^2$$

$\therefore$ Total surface area $= (2\Pi rh + 2\Pi r^2)$

$$= \left(2 \times \frac{22}{7} \times 14 \times 40 + 2 \times \frac{22}{7} \times 14^2\right)$$

$$= (3520 + 1232) = 4752 \text{ cm}^2$$

$\therefore$ Volume of the cylinder $= (\Pi r^2 h)$

$$= \left(\frac{22}{7} \times 14^2 \times 40\right)\text{cm}^3$$

$$= \left(\frac{22}{7} \times 14 \times 14 \times 40\right)\text{cm}^3$$

$$= 24640 \text{ cm}^3.$$

Question 3:

Here, radius (r) = 10.5 cm and height = 60 cm.

$\therefore$ Volume of the cylinder = $(\Pi r^2 h)$

$$= \left(\frac{22}{7} \times 10.5 \times 10.5 \times 60\right)\text{cm}^3$$

$$= 20790 \text{ cm}^3$$

$\therefore$ Weight of the solid cylinder if the material of the cylinder

Weighs 5 g per cm^3 = (20790 x 5) = 103950 g

$$= \frac{103950}{1000} \qquad [\because 1000\text{g} = 1 \text{ kg}]$$

$$= 103.95 \text{ kg}$$

Question 4:

Here, curved surface area = 1210 cm^2

$$\text{Diameter} = 20\text{cm} \Rightarrow \text{radius} = \frac{20}{2} = 10\text{cm}$$

$\therefore$ Curved surface area of the cylinder $= 2\Pi rh$

$$\Rightarrow \quad 1210 = 2 \times \frac{22}{7} \times 10 \times h$$

$$\Rightarrow \quad h = \left(\frac{1210 \times 7}{2 \times 22 \times 10}\right)\text{cm} = 19.25\text{cm}$$

$\therefore$ Height = 19.25 cm

$\therefore$ Volume of the cylinder = $(\Pi r^2 h)$

$$= \left(\frac{22}{7} \times 10^2 \times 19.25\right)\text{cm}^3$$

$$= \left(\frac{22}{7} \times 10 \times 10 \times 19.25\right)\text{cm}^3$$

$$= 6050\text{cm}^3$$

$\therefore$ Volume of the cylinder $= 6050\text{cm}^3$.

Question 5:

Let base radius be r and height be h

Then, $2\Pi rh = 4400\ cm^2$

And $2\Pi r = 110\ cm$

$$\Rightarrow \quad \frac{2\Pi rh}{2\Pi r} = \frac{4400}{110}$$

$$\Rightarrow \quad h = 40cm$$

$$\therefore \quad 2 \times \frac{22}{7} \times r \times h \times 40 = 4400cm.$$

$$\Rightarrow \quad r = \left(\frac{4400 \times 7}{44 \times 40}\right)cm = \frac{35}{2}\ cm.$$

$\therefore$ Volume of the cylinder $= \Pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40\right)cm^3$$

$$= 38500cm^3.$$

Question 6:

Let the radius (r) = 2x cm and height (h) =3x cm

Then, Volume of cylinder $= (\Pi r^2 h)$

$$\text{Volume} = \left[\frac{22}{7} \times (2x)^2 \times 3x\right]$$

$$\text{Volume} = \left[\frac{22}{7} \times 4x^2 \times 3x\right]$$

$$\text{Volume} = \frac{22}{7} \times 12x^3$$

$$\Rightarrow \quad 1617 = \frac{22}{7} \times 12x^3$$

$$[\because \text{volume given} = 1617cm^3]$$

$$\Rightarrow \quad 12x^3 = \frac{1617 \times 7}{22}$$

$$\Rightarrow \quad x^3 = \frac{1617 \times 7}{22 \times 12} = \left(\frac{7}{2}\right)^3$$

$$\Rightarrow \quad x = \frac{7}{2}$$

$$\therefore \quad \text{radius} = 2x = 2 \times \frac{7}{2} = 7cm$$

$$\text{and height} = 3x = 3 \times \frac{7}{2} = \frac{21}{2}\ cm]$$

$$\text{T otal surface area} = 2\Pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 7\left(\frac{21}{2} + 7\right)cm^2$$

$$= 44 \times \left(\frac{21 + 14}{2}\right)cm^2$$

$$= (22 \times 35)cm^2 = 770cm^2$$

Question 7:

$$\text{Curved surface area} = \frac{1}{3} \times (\text{total surface area})$$

$$= \left(\frac{1}{3} \times 462\right) cm^2 = 154 cm^2$$

(Total surface area)- (Curved surface area)

$$= (462\text{-}154)\ cm^2 = 308\ cm^2$$

$$\Rightarrow \quad 2\Pi r^2 = 308$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 308$$

$$\Rightarrow \quad r^2 = \frac{308 \times 7}{44} = 49$$

$$\Rightarrow \quad r = \sqrt{49} = 7cm$$

Now, curved surface area $= 2\Pi rh = 154\ cm^2$

$$= 2 \times \frac{22}{7} \times 7 \times h = 154\ cm^2$$

$$= h = \frac{154}{44} = 3.5cm$$

Now, r = 7 cm and h = 3.5 cm

Volume of the cylinder $= (\Pi r^2 h)$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 3.5\right) cm^3$$

$$= 539 cm^3$$

$\therefore$ The volume of the cylinder = 539 cm^3.

Question 8:

$$\text{Curved surface area} = \frac{2}{3} \times (\text{total surface area})$$

$$= \left(\frac{2}{3} \times 231\right) cm^2 = 154 cm^2$$

(Total surface area) - (Curved surface area)

$$= (231 - 154)\ cm^2 = 77\ cm^2$$

$$2\pi r^2 = 77\ cm^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 77$$

$$\Rightarrow \quad r^2 = \frac{77 \times 7}{44} = \frac{49}{4}$$

$$\Rightarrow \quad r = \sqrt{\frac{49}{4}} = \frac{7}{2} cm$$

Now, $\quad 2\pi rh = 154 cm^2$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times \frac{7}{2} \times h = 154 cm^2$$

$$\Rightarrow \quad h = \frac{154}{22} = 7 cm$$

Now, $r = \frac{7}{2}$ cm and h = 7 cm

Volume of the cylinder $= \pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7\right) cm^3$$

$$= 269.5\ cm^3$$

Volume of the cylinder = 269.5 cm^3

Question 9:

Here, $(r+h) = 37$ m $[\because \text{given}]$

And, total surface area $= 2\pi r(r+h) = 1628 m^2$

$$\Rightarrow 2\pi r \times 37 = 1628 m^2$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 37 = 1628$$

$$\Rightarrow r = \frac{1628 \times 7}{44 \times 37} = 7 m$$

And $(r+h) = 37 m$

$$\Rightarrow (7+h) = 37$$

$$\Rightarrow h = 37 - 7 = 30 m$$

$$\text{Volume} = \pi r^2 h$$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 30\right) m^3 = 4620 m^3.$$

Question 10:

Curved surface area $= 2\pi rh$

Total surface area $= 2\pi r(h+r)$

Since they are in the ratio of 1: 2

$$\therefore \frac{2\pi rh}{2\pi r(h+r)} = \frac{1}{2}$$

$$\Rightarrow \frac{h}{h+r} = \frac{1}{2}$$

$$\Rightarrow 2h = h+r$$

$$\Rightarrow 2h - h = r$$

$$\Rightarrow h = r$$

$$2\pi r(h+r) = 616 \text{ cm}^2$$

$$\Rightarrow 4\pi r^2 = 616 \text{ cm}^2 \quad [\text{Putting } h = r]$$

$$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 616$$

$$\Rightarrow r^2 = \frac{616 \times 7}{88} = 49$$

$$\Rightarrow r = \sqrt{49} = 7 \text{ cm}$$

Then, $r = 7$ cm and $h = 7$ cm

$$\therefore \text{Volume} = (\pi r^2 h)$$

$$= \left(\frac{22}{7} \times 7 \times 7 \times 7\right) \text{cm}^3 = 1078 \text{cm}^3$$

$\therefore$ the volume of the cylinder $= 1078 \text{ cm}^3$.

Question 11:

$1 \text{cm}^3 = 1 \text{ cm} \times 1 \text{cm} \times 1 \text{cm}$ and $1 \text{cm} = 0.01 \text{m}$

Therefore,

Volume of the

gold $= 0.01 \text{m} \times 0.01 \text{m} \times 0.01 \text{m} = 0.000001 \text{m}^3$......(1)

Diameter of the wire drawn $= 0.1$ mm

Radius of the wire drawn $= \frac{0.1}{2} \text{mm} = 0.05 \text{mm}$

$r = 0.00005$ m ....(2)

Length of the wire $= h$ m ....(3)

Volume of the wire drawn = Volume of the gold

$\Rightarrow \pi r^2 h = 0.000001$

$\Rightarrow \pi \times 0.00005 \times 0.00005 \times h = 0.000001$ [from equations (1), (2) and (3)]

$$h = \frac{0.000001 \times 7}{0.00005 \times 0.00005 \times 22}$$

$\therefore$ $h = 127.27$m

$\therefore$ the length of the wire is 127.27m

Question 12:

Let the radii of the two cylinders be 2R and 3R.

And their heights be 5H and 3H.

$$\text{Then, } \frac{V_1}{V_2} = \frac{\pi \times (2R)^2 \times 5H}{\pi \times (3R)^2 \times 3H} = \frac{\pi \times 4R^2 \times 5H}{\pi \times 9R^2 \times 3H} = \frac{20}{27}$$

$\therefore$ the ratio of their volumes = 20 : 27

$$\text{Now, } \frac{S_1}{S_2} = \frac{2\pi (2R)(5H)}{2\pi (3R)(3H)} = \frac{10}{9}$$

$\therefore$ the ratio of their curved surface = 10 : 9

Question 13:

For the tin having square base,

side = 12 cm and height = 17.5 cm.

$\therefore$ Volume = $(12 \times 12 \times 17.5)\,cm^3 = 2520\,cm^3$

Now, diameter of tin with cylindrical base = 12 cm

$\therefore$ radius = $\left(\frac{12}{2}\right)$ cm = 6 cm and height = 17.5 cm

$$\therefore \text{ Volume} = \left(\frac{22}{7} \times 6 \times 6 \times 17.5\right) cm^3 = 1980\,cm^3$$

Tin with square base has more capacity by $(2520 - 1980)\,cm^3$

$= 540\,cm^3$.

Question 14:

Here, cylindrical bucket has diameter = 28 cm.

$\therefore$ radius $= \left(\frac{28}{2}\right)$ cm = 14 cm and height = 72 cm.

Length of the tank = 66 cm

Breadth of the tank = 28 cm

$\therefore$ Volume of tank = Volume of cylindrical bucket

$$\Rightarrow \quad l \times b \times h = \pi r^2 h$$

$$\Rightarrow \quad 66 \times 28 \times h = \frac{22}{7} \times 14 \times 14 \times 72$$

$$\Rightarrow \quad h = \left(\frac{22 \times 2 \times 14 \times 72}{66 \times 28}\right) cm$$

$$\Rightarrow \quad h = 24\ cm.$$

$\therefore$ The height of the water level in the tank = 24 cm.

Question 15:

Internal radius $= \left(\frac{3}{2}\right)$ cm = 1.5 cm

And, external radius = (1.5 + 1) cm = 2.5 cm

$$\text{Volume of cast iron} = \left[\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100\right] cm^3$$

$$= \pi \times 100 \times \left[(2.5)^2 - (1.5)^2\right] cm^3$$

$$= \frac{22}{7} \times 100 \times [6.25 - 2.25]\, cm^3$$

$$= \left(\frac{22}{7} \times 100 \times 4\right) cm^3$$

$$\therefore \quad \text{Weight} = \left(\frac{22}{7} \times 100 \times 4 \times \frac{21}{1000}\right) kg$$

$[\because 1 kg = 1000 g]$

= 26.4 kg.

$\therefore$ the weight of the iron pipe = 26.4 kg.

Question 16:

Internal diameter of the tube $= 10.4\,cm$

$$\text{internal radius} = \left(\frac{10.4}{2}\right)cm = 5.2\,cm$$

$$\text{and length} = 25\,cm$$

$$\text{and external radius} = (5.2+0.8)\,cm = 6\,cm$$

$$\text{Required volume} = \left[\pi\times(6)^2\times25-\pi\times(5.2)^2\times25\right]cm^3$$

$$= \pi\times25\left[(6)^2-(5.2)^2\right]cm^3$$

$$= \frac{22}{7}\times25\,[36-27.04]\,cm^3$$

$$= \left(\frac{22}{7}\times25\times8.96\right)cm^3$$

$$= 704\,cm^3$$

$\therefore$ the volume of the metal $= 704\,cm^3$

Question 17:

Length $= 7\,cm = $ (height)

Diameter $= 5mm \Rightarrow$ radius $= \left(\frac{5}{2}\right)mm = 2.5mm$

$$= 0.25\,cm$$

$$\therefore \text{ Volume of the barrel} = \pi r^2 h$$

$$= \left(\frac{22}{7}\times0.25\times0.25\times7\right)cm^3$$

$$= \frac{11}{8}\,cm^3$$

$\frac{11}{8}\,cm^3$ is used for writing 330 words.

So, $\left(\frac{1}{5}\times1000\right)cm^3$ will be used for writing

$$\left(330\times\frac{8}{11}\times\frac{1}{5}\times1000\right)\text{ words}$$

$$= 48000 \text{ words}$$

Question 18:

$$\text{Weight of the graphite} = \left[\frac{22}{7}\times(0.05)^2\times10\times2.1\right]g$$

$$= \frac{33}{200}\,g$$

$$\text{Weight of wood} = \left[\frac{22}{7}\times10\left\{(0.35)^2-(0.05)^2\right\}\times0.7\right]$$

$$= \left[\frac{22}{7}\times10(0.1225-0.0025)\times0.7\right]$$

$$= \frac{66}{25}\,g$$

$$\therefore \text{ Total weight of the pencil} = \left(\frac{33}{200}+\frac{66}{25}\right)g$$

$$= \left(\frac{33+528}{200}\right)g = \frac{561}{200} = 2.805\,g$$

$\therefore$ Weight of the whole pencil $= 2.805\,g$

Exercise 13C

Question 1:

Here, $r = 35$ cm and $h = 84$ cm

$$\therefore \text{ Volume of the cone } = \frac{1}{3}\pi r^2 h$$
$$= \left(\frac{1}{3}\times\frac{22}{7}\times 35\times 35\times 84\right)\text{cm}^3$$
$$= 107800 \text{ cm}^3$$

$$\therefore \text{ Curved surface area } = \left(\pi r\sqrt{h^2+r^2}\right) \quad \left[\because l = \sqrt{h^2+r^2}\right]$$
$$= \pi r\sqrt{84^2+35^2}$$
$$= \pi r\sqrt{8281}$$
$$= \frac{22}{7}\times 35\times 91$$
$$= 10010 \text{ cm}^2$$

$$\therefore \text{ Total surface area } = \pi r(l+r)$$

Now,
$$l = \sqrt{h^2+r^2}$$
$$= \sqrt{84^2+35^2}$$
$$= \sqrt{7056+1225} = \sqrt{8281} = 91 \text{ cm}$$

$$\therefore \text{ Total surface area } = \frac{22}{7}\times 35(91+35)$$
$$= (22\times 5\times 126)\text{cm}^2 = 13860 \text{ cm}^2$$

Question 2:

Here, height $(h) = 6$ cm and slant height $(\ell) = 10$ cm

$$\therefore \text{ radius}(r) = \sqrt{\ell^2 - h^2}$$
$$= \sqrt{10^2 - 6^2} = \sqrt{100-36}$$
$$= \sqrt{64} = 8 \text{ cm}$$

$$\therefore \text{ Volume of cone } = \frac{1}{3}\pi r^2 h$$
$$= \left(\frac{1}{3}\times 3.14\times 8\times 8\times 6\right)\text{cm}^3$$
$$= 401.92 \text{ cm}^3$$

$$\therefore \text{ Curved surface area } = \pi r\ell$$
$$= (3.14\times 8\times 10)\text{ cm}^2$$
$$= 251.2 \text{ cm}^2$$

$$\therefore \text{ Total surface area } = \pi r(\ell + r)$$
$$= \pi r(10+8)$$
$$= (3.14\times 8\times 18)\text{cm}^2$$
$$= 452.16 \text{ cm}^2$$

Question 3:

Here, Volume $= (100\pi)\text{cm}^3$, height$(h) = 12$ cm

$$\text{Volume of the cone } = \frac{1}{3}\pi r^2 h$$
$$\Rightarrow 100\pi = \frac{1}{3}\pi\times r^2\times 12$$
$$\Rightarrow r^2 = \frac{100\pi\times 3}{\pi\times 12}$$
$$\Rightarrow r^2 = 25$$
$$\Rightarrow r = \sqrt{25} = 5 \text{ cm.}$$

$$\text{Slant height}(\ell) = \sqrt{h^2+r^2}$$
$$= \sqrt{12^2+5^2}$$
$$\ell = \sqrt{144+25} = \sqrt{169} = 13 \text{ cm}$$

$$\therefore \text{ Slant height, } \ell = 13 \text{ cm}$$
$$\therefore \text{ Curved surface area} = \pi r\ell$$
$$= \pi\times 5\times 13 \text{cm}^2$$
$$= 65\pi \text{ cm}^2$$

Question 5:

Here, curved surface area $= 550\,cm^2$ and

slant height $(\ell) = 25$ cm

$\therefore$ Curved surface area $= \pi r\ell$

$$\Rightarrow \quad 550 = \frac{22}{7} \times r \times 25$$

$$\Rightarrow \quad r = \left(\frac{550 \times 7}{22 \times 25}\right) cm = 7\,cm$$

Now, height (h) $= \sqrt{\ell^2 - r^2}$

$$= \sqrt{(25)^2 - (7)^2}$$

$$= \sqrt{625 - 49}$$

$$= \sqrt{576} = 24\,cm$$

$\therefore$ height of the cone $= 24$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24\right) cm^3$$

$$= 1232\,cm^3$$

$\therefore$ Volume of the cone $= 1232\,cm^3$

Question 6:

Here, radius, $r = 35$ cm and slant height, $\ell = 37$ cm

$$\therefore \quad h = \sqrt{\ell^2 - r^2}$$

$$= \sqrt{(37)^2 - (35)^2}$$

$$= \sqrt{1369 - 1225} = \sqrt{144} = 12\,cm$$

$\therefore$ height(h) $= 12$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12\right) cm^3$$

$$= 15400\,cm^3$$

$\therefore$ Volume of the cone $= 15400\,cm^3$

Question 7:

Here, curved surface area $= 4070\,cm^2$

Diameter $= 70$ cm $\Rightarrow$ radius $= \left(\frac{70}{2}\right)$ cm $= 35$ cm

$\therefore$ Curved surface area $= \pi r\ell$

$$\Rightarrow \quad 4070 = \frac{22}{7} \times 35 \times \ell$$

$$\Rightarrow \quad \ell = \left(\frac{4070}{110}\right) cm = 37\,cm$$

$\therefore$ slant height $= 37$ cm.

Question 8:

Here, radius $= 7$ m and height(h) $= 24$ m

$\therefore$ slant height$(\ell) = \sqrt{h^2 + r^2}$

$$= \sqrt{(24)^2 + (7)^2}$$

$$\ell = \sqrt{576 + 49} = \sqrt{625} = 25\,m$$

Now, area of cloth $= \pi r\ell$

$$= \left(\frac{22}{7} \times 7 \times 25\right) m^2 = 550\,m^2$$

$$\therefore \text{length of cloth} = \frac{\text{area of cloth}}{\text{width of cloth}} = \left(\frac{550}{2.5}\right) m$$

$$= 220\,m$$

$\therefore$ Length of cloth required to make a conical tent $= 220$ m

Question 9:

Here, height of cone = 3.6 cm and radius = 1.6 cm

After melting, its radius = 1.2 cm

Volume of original cone = Volume of cone after melting

$$\therefore \quad \frac{1}{3}\pi \times 1.6 \times 1.6 \times 3.6 = \frac{1}{3}\pi \times 1.2 \times 1.2 \times h$$

$$\Rightarrow \quad h = \frac{\frac{1}{3}\pi \times 1.6 \times 1.6 \times 3.6}{\frac{1}{3}\pi \times 1.2 \times 1.2} = 6.4 \text{ cm}$$

$\therefore$ height of new cone = 6.4 cm

Question 10:

Let their heights be h and 3h

And, their radii be 3r and r.

$$\text{Then,} \quad V_1 = \frac{1}{3}\pi(3r)^2 \times h$$

$$\text{and,} \quad V_2 = \frac{1}{3}\pi r^2 \times 3h$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi(3r)^2 \times h}{\frac{1}{3}\pi r^2 \times 3h} = \frac{3}{1}$$

$$\therefore \quad V_1 : V_2 = 3 : 1$$

Question 11:

Radius of the cylinder, $R = \left(\frac{105}{2}\right)$m and its height, H = 3m

Slant height (ℓ) = 53m

$\therefore$ area of canvas = $(2\pi RH + \pi R\ell)$

$$= \left[\left(2 \times \frac{22}{7} \times \frac{105}{2} \times 3\right) + \left(\frac{22}{7} \times \frac{105}{2} \times 53\right)\right] \text{m}^2$$

$$= (990 + 8745)\ \text{m}^2$$

$$= 9735\ \text{m}^2$$

$$\therefore \quad \text{length of canvas} = \left(\frac{\text{area of canvas}}{\text{width of canvas}}\right) \text{m}$$

$$= \left(\frac{9735}{5}\right) = 1947 \text{ m.}$$

Question 12:

Let the radius be r metres and height be h metres.

Area of the base $= (11 \times 4)\ \text{m}^2 = 44\ \text{m}^2$

$$\therefore \quad \pi r^2 = 44$$

$$\Rightarrow \quad r^2 = \left(44 \times \frac{7}{22}\right) = 14 \text{m}$$

$$\Rightarrow \quad r^2 = 14 \text{m}$$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$\therefore \text{ Volume of the cone} = (11 \times 20)\ \text{m}^3 = 220\ \text{m}^3$$

$$\Rightarrow \quad 220 = \frac{1}{3} \times \frac{22}{7} \times 14 \times h$$

$$\Rightarrow \quad h = \frac{220 \times 3}{22 \times 2} = 15 \text{ m}$$

$\therefore$ the height of the cone = 15m.

Question 13:

Here, height of the cylindrical bucket = 32 m and radius = 18 cm.
Now, let the radius of the heap be R cm and its slant height be ℓ cm

$$\text{Then,} \quad \pi \times (18)^2 \times 32 = \frac{1}{3}\pi \times R^2 \times 24$$

$$\Rightarrow \quad R^2 = \frac{\pi \times 18 \times 18 \times 32 \times 3}{\pi \times 24} = 1296$$

$$\Rightarrow \quad R = \sqrt{1296} = 36 \text{ cm.}$$

$\therefore$ Radius of the heap = 36 cm

$$\text{Slant height}(\ell) = \sqrt{h^2 + R^2}$$
$$= \sqrt{(24)^2 + (36)^2}$$
$$= \sqrt{576 + 1296}$$
$$= \sqrt{1872} = 43.27 \text{cm}$$

$\therefore$ Slant height of the heap = 43.27 cm.

Question 14:

Let the curved surface areas of cylinder and cone be 8x and 5x.

$$\text{Then, } 2\pi rh = 8x \ .....(i)$$

$$\text{and,} \quad \pi r\sqrt{h^2 + r^2} = 5x \ .....(ii)$$

Squaring both sides of equation (i), we have

$$(2\pi rh)^2 = (8x)^2$$
$$4\pi^2 r^2 h^2 = 64x^2 \ .....(iii)$$

From (ii) we have,

$$\pi r\sqrt{h^2 + r^2} = 5x$$

Squaring both sides,

$$\Rightarrow \quad \pi^2 r^2 (h^2 + r^2) = 25x^2 \ .....(iv)$$

$$\Rightarrow \quad \frac{4\pi^2 r^2 h^2}{\pi^2 r^2 (h^2 + r^2)} = \frac{64}{25} \quad [\text{Divide (iii) by (iv)}]$$

$$\Rightarrow \quad \frac{h^2}{(h^2 + r^2)} = \frac{16}{25}$$

$$\Rightarrow \quad 9h^2 = 16r^2$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{r}{h} = \frac{3}{4}$$

$\therefore$ The ratio of radius and height = 3 : 4

Question 15:

Here, height(h) of cylinder = 2.8 m = 280 cm
and diameter = 20 cm

$$\Rightarrow \text{ radius} = \left(\frac{20}{2}\right) = 10 \text{ cm}$$

height(H) of the cone = 42 cm

$$\therefore \text{ Volume of the pillar} = (\pi r^2 h + \frac{1}{3}\pi r^2 H) \text{ cm}^3$$
$$= \pi r^2 (h + \frac{1}{3}H) \text{ cm}^3$$
$$= \frac{22}{7} \times 10 \times 10 (280 + \frac{1}{3} \times 42) \text{ cm}^3$$
$$= \frac{2200}{7} \times [280 + 14]$$
$$= 92400 \text{ cm}^3$$

$$\therefore \text{ Weight of pillar} = \left(\frac{92400 \times 7.5}{1000}\right) \text{kg} = 693 \text{kg}$$

Question 16:

Let the smaller cone have radius = r cm and height = h cm
And, let the radius of the given original cone be R cm
Since the two triangles, ΔOCD and ΔOAB
are similar to each other, we have

Then, $\frac{r}{R} = \frac{h}{30}$ $[\because \Delta OCD \sim \Delta OAB]$

$\Rightarrow$ $r = \frac{Rh}{30}$ ....(1)

Given that the volume of the small cone is
$\frac{1}{27}$ of the volume of the given cone.

$$\therefore \quad \frac{1}{3}\pi r^2 h = \frac{1}{27}\times\frac{1}{3}\pi R^2\times 30 \quad \text{[given]}$$

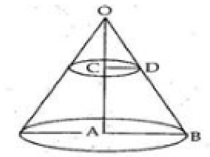
$$\Rightarrow \quad \frac{1}{3}\pi\left(\frac{hR}{30}\right)^2 h = \frac{1}{81}\pi R^2\times 30 \text{ [from (1)]}$$

$$\Rightarrow \quad \frac{1}{3}\pi\frac{h^3R^2}{900} = \frac{1}{81}\pi R^2\times 30$$

$$\Rightarrow \quad h^3 = \frac{1\times 30\times 900\times 3}{81}$$

$$\Rightarrow \quad h^3 = 1000 \text{ cm}^3$$

$$\Rightarrow \quad h = 10\text{cm}$$



From the figure,
$AC = (OA - OC)$
$= (30 - 10)\text{cm} = 20\text{cm}$
$\therefore$ the required height = 20 cm

Question 17:

Here, height(h) = 10 cm and radius = 6 cm

$$\therefore \text{Volume of the remaining solid} = (\pi r^2 h) - (\frac{1}{3}\pi r^2 h)$$

$$= (\pi\times 6\times 6\times 10)\text{ cm}^3 - \left(\frac{1}{3}\pi\times 6\times 6\times 10\right)\text{cm}^3$$

$$= \frac{2}{3}\pi\times 6\times 6\times 10 \text{ cm}^3$$

$$= \left(\frac{2}{3}\times 3.14\times 360\right)\text{ cm}^3 = 753.6 \text{ cm}^3$$

$$\therefore \text{Volume of the remaining solid} = 753.6 \text{ cm}^3$$

Question 18:

Diameter of the pipe = 5mm = 0.5cm

Radius of the pipe = $\frac{0.5}{2}$ = 0.25cm

Length of the pipe = 10 metres = 1000 cm

Volume that flows in 1 min = $\left[\pi\times(0.25)^2\times 1000\right]$ cm^3

$\therefore$ Volume of the conical vessel = $\left[\frac{1}{3}\pi\times(20)^2\times 24\right]$ cm^3

$$\therefore \quad \text{Required time} = \left[\frac{\frac{1}{3}\pi\times(20)^2\times 24}{\pi\times(0.25)^2\times 1000}\right]\text{min}$$

$$= \left[\frac{\frac{1}{3}\pi\times 400\times 24}{\pi\times 0.0625\times 1000}\right]\text{min}$$

$= 51.2$ min

$=$ 51 min 12 sec

Exercise 13D

Question 1:

(i) Radius of sphere = 3.5 cm

$\therefore$ Volume of the sphere $= \left(\frac{4}{3}\pi r^3\right)$

$= \left(\frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5\right)\ cm^3$

$= 179.67\ cm^3$

$\therefore$ Surface area of the sphere $= \left(4\pi r^2\right)$

$= \left(4 \times \frac{22}{7} \times 3.5 \times 3.5\right)\ cm^2$

$= 154\ cm^2$

(ii) Radius of the sphere = 4.2 cm

$\therefore$ Volume of the sphere $= \left(\frac{4}{3}\pi r^3\right)$

$= \left(\frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2\right) cm^3$

$= 310.464\ cm^3$

$\therefore$ Surface area of the sphere $= \left(4\pi r^2\right)$

$= \left(4 \times \frac{22}{7} \times 4.2 \times 4.2\right) cm^2$

$= 221.76\ cm^2$

(iii) Radius of sphere = 5 m

$\therefore$ Volume of the sphere $= \left(\frac{4}{3}\pi r^3\right)$

$= \left(\frac{4}{3} \times \frac{22}{7} \times 5 \times 5 \times 5\right) m^3$

$= 523.81\ m^3$

$\therefore$ Surface area of the sphere $= \left(4\pi r^2\right)$

$= \left(4 \times \frac{22}{7} \times 5 \times 5\right) m^2$

$= 314.28\ m^2$

Question 2:

$$\text{Volume of the sphere} = \left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad 38808 = \frac{4}{3} \times \frac{22}{7} \times r^3 \quad [\because \text{Volume} = 38808\,cm^3]$$

$$\Rightarrow \quad r^3 = \frac{38808 \times 3 \times 7}{88} = 9261$$

$$\Rightarrow \quad r = 21\,cm$$

$$\therefore \text{ Surface area of the sphere} = 4\pi r^2$$

$$= \left(4 \times \frac{22}{7} \times 21 \times 21\right) cm^2$$

$$= 5544\,cm^2$$

Question 3:

$$\text{Volume of the sphere} = 606.375 m^3 \quad \ldots.(1)$$

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad 606.375 = \frac{4}{3} \times \frac{22}{7} \times r^3 \quad [\text{from } (1)]$$

$$\Rightarrow \quad r^3 = \frac{606.375 \times 3 \times 7}{4 \times 22}$$

$$= 144.703125$$

$$\Rightarrow \quad r = 5.25\,m$$

$$\text{Surface area of the sphere} = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 5.25 \times 5.25 m^2$$

$$= 346.5 m^2$$

Question 4:

Let the radius of the sphere be r m.

$$\text{Then, its surface area} = (4\pi r^2)$$

$$\therefore \quad (4\pi r^2) = 394.24$$

$$[\text{Surface area} = 394.24 m^2]$$

$$4 \times \frac{22}{7} \times r^2 = 394.24$$

$$r^2 = \left(\frac{394.24 \times 7}{4 \times 22}\right) = 31.36$$

$$r = \sqrt{31.36} = 5.6 m$$

$$\therefore \text{ radius of the sphere} = 5.6 m$$

$$\therefore \text{Volume of the sphere} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 5.6 \times 5.6 \times 5.6\right) m^3$$

$$= 735.91 m^3$$

$$\therefore \text{Volume of the sphere} = 735.91 m^3$$

Question 5:

Surface area of sphere $=(4\pi r^2)$

$$\therefore \quad (4\pi r^2) = (576\pi)$$

$$\left[\text{Surface area} = 576\pi\ cm^2\right]$$

$$\Rightarrow \quad r^2 = \frac{(576\pi)}{(4\pi)}$$

$$\Rightarrow \quad r = \sqrt{144} = 12\,cm$$

$$\therefore \text{Volume of the sphere} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3}\times\pi\times 12\times 12\times 12\right) cm^3$$

$$= (2304\pi)\,cm^3$$

$$\therefore \text{Volume of the sphere} = (2304\pi)\,cm^3$$

Question 6:

Outer diameter of spherical shell = 12 cm

$$\text{radius} = 6\,cm \qquad \left[\text{radius} = \frac{D}{2}\right]$$

Outer diameter of spherical shell = 8 cm

radius = 4 cm

$$\text{Now, Volume of the outer shell} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3}\times\frac{22}{7}\times 6\times 6\times 6\right) cm^3$$

$$= 905.15\,cm^3$$

$$\therefore \text{Volume of the inner shell} = \left(\frac{4}{3}\pi r^3\right)$$

$$= \left(\frac{4}{3}\times\frac{22}{7}\times 4\times 4\times 4\right) cm^3$$

$$= 268.20\,cm^3$$

$$\therefore \text{Volume of metal contained in the shell} = (\text{Volume of outer}) - (\text{Volume of inner})$$

$$= (905.15 - 268.20)\,cm^3$$

$$= 636.95\,cm^3$$

$$\therefore \text{Outer surface area} = 4\pi r^2$$

$$= \left(4\times\frac{22}{7}\times 6\times 6\right) cm^2$$

$$= 452.57\,cm^2$$

Question 7:

Here, diameter of the lead shot = 3mm

$$\therefore \quad \text{radius} = \left(\frac{3}{2}\right) mm = \left(\frac{0.3}{2}\right) cm$$

$$[1mm = 10\,cm]$$

$$\text{Now, number of lead shots} = \frac{\text{Volume of the cuboid}}{\text{Volume of 1 lead shot}}$$

$$= \left\{\frac{(12\times 11\times 9)}{\frac{4}{3}\times\frac{22}{7}\times\left(\frac{0.3}{2}\right)^3}\right\}$$

$$= \left\{\frac{(12\times 11\times 9)}{\frac{4}{3}\times\frac{22}{7}\times\frac{0.027}{8}}\right\}$$

$$= \left\{\frac{12\times 11\times 9\times 3\times 7\times 8}{4\times 22\times 0.027}\right\} = 84000$$

$\therefore$ number of lead shots = 84000.

Question 8:

Here, radius of 1 lead ball = 1 cm

and radius of sphere = 8 cm

$$\therefore \quad \text{Number of lead balls} = \frac{\text{Volume of the sphere}}{\text{Volume of 1 lead ball}}$$

$$= \frac{\left(\frac{4}{3}\pi R^3\right) \text{ cm}^3}{\left(\frac{4}{3}\pi r^3\right) \text{ cm}^3}$$

$$= \left\{\frac{\frac{4}{3} \times \frac{22}{7} \times 8^3}{\frac{4}{3} \times \frac{22}{7} \times 1^3}\right\}$$

$$= \left\{\frac{\frac{4}{3} \times \frac{22}{7} \times 512}{\frac{4}{3} \times \frac{22}{7} \times 1}\right\} = 512$$

$\therefore$ number of lead balls = 512.

Question 9:

Here, radius of sphere = 3 cm

Diameter of spherical ball = 0.6 cm $\quad \left[\because \text{radius} = \frac{D}{2}\right]$

Radius of spherical ball = 0.3 cm

$$\therefore \quad \text{Number of balls} = \frac{\text{Volume of the sphere}}{\text{Volume of 1 small ball}}$$

$$= \left\{\frac{\frac{4}{3} \times \frac{22}{7} \times 3^3 \quad \text{cm}^3}{\frac{4}{3} \times \frac{22}{7} \times (0.3)^3 \quad \text{cm}^3}\right\}$$

$$= \left\{\frac{\frac{4}{3} \times \frac{22}{7} \times 27}{\frac{4}{3} \times \frac{22}{7} \times 0.027}\right\} = 1000$$

$\therefore$ number of small balls obtained = 1000.

Question 10:

Here, radius of sphere = 10.5 cm = $\left(\frac{21}{2}\right)$ cm

Radius of smaller cone = 3.5 cm = $\left(\frac{7}{2}\right)$ cm and height = 3 cm

$$\text{Now number of cones} = \frac{\text{Volume of the sphere}}{\text{Volume of 1 small cone}}$$

$$= \frac{\left\{\frac{4}{3} \pi \times \left(\frac{21}{2}\right)^3 \text{ cm}^3\right\}}{\left\{\frac{1}{3} \pi \times \left(\frac{7}{2}\right)^2 \times 3 \text{ cm}^3\right\}}$$

$$= \left(\frac{\frac{4}{3} \times \frac{9261}{8}}{\frac{1}{3} \times \frac{49}{4} \times 3}\right) = \frac{\frac{9261}{6}}{\frac{49}{4}}$$

$$= \frac{9261}{6} \times \frac{4}{49} = 126$$

$\therefore$ Number of cones obtained = 126.

Question 11:

$$\text{Diameter of a sphere} = 12\,\text{cm}$$

$$\text{radius} = \frac{\text{Diameter}}{2}$$

$$= \frac{12}{2}$$

$$= 6\,\text{cm}$$

$$\therefore \text{ Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 \quad \text{(i)}$$

$$\text{Diameter of cylinder} = 8\,\text{cm}$$

$$\text{Radius of cylinder} = \frac{\text{Diameter}}{2}$$

$$\text{Radius of cylinder} = \frac{8}{2}$$

$$\text{Radius of cylinder} = 4\,\text{cm}$$

$$\text{Height of the cylinder} = 90\,\text{cm}$$

$$\therefore \text{ Volume of the cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 4 \times 4 \times 90 \qquad \text{(ii)}$$

$$\text{Number of spheres} = \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$\text{Number of spheres} = \frac{\frac{22}{7} \times 4 \times 4 \times 90 \ \ \text{cm}^3}{\frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 \ \ \text{cm}^3} \left[\text{(ii)} \div \text{(i)}\right]$$

$$\text{Number of spheres} = 5.$$

Question 12:

$$\text{Here, Diameter of a sphere} = 6\,\text{cm}$$

$$\therefore \qquad \text{radius}(R) = \left(\frac{6}{2}\right)\text{cm} = 3\,\text{cm}$$

$$\text{Diameter of wire} = 2\,\text{mm}$$

$$\therefore \qquad \text{radius}(r) = 1\,\text{mm} = 0.1\,\text{cm}$$

Let the required length of wire be h cm.

Then,

$$\pi \times (r)^2 \times h = \frac{4}{3} \times \pi \times (R)^3$$

$$\Rightarrow \qquad \pi \times (0.1)^2 \times h = \frac{4}{3} \times \pi \times (3)^3$$

$$\Rightarrow \qquad h = \frac{\frac{4}{3} \times \pi \times 27}{\pi \times (0.1)^2}$$

$$= \left(\frac{4 \times 9}{0.01}\right)\text{cm} = \frac{36}{0.01}$$

$$= 3600\,\text{cm} = 36\,\text{m}$$

$$\therefore \quad \text{the length of the wire} = 36\,\text{m}.$$

Question 13:

$$\text{Here, diameter of sphere} = 18\text{cm}$$

$$\therefore \quad \text{radius of sphere} = \left(\frac{18}{2}\right)\text{cm} = 9\text{cm}$$

$$\text{Length of the wire} = 108\text{m} = 10800\text{cm}$$

Then,

$$\frac{4}{3}\pi \times (r)^3 = \pi \times r^2 \times 10800$$

$$\Rightarrow \quad \frac{4}{3}\pi \times (9)^3 = \pi \times r^2 \times 10800$$

$$\Rightarrow \quad r^2 = \frac{\frac{4}{3} \times \pi \times 729}{\pi \times 10800}$$

$$= \frac{4 \times 243}{10800} = \frac{972}{10800} = \frac{9}{100}$$

$$\Rightarrow \quad r = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$$

$$\therefore \quad r = 0.3\text{cm}$$

$$\text{So,} \quad \text{Diameter} = (2 \times 0.3)\text{cm} = 0.6\text{cm.}$$

Question 14:

$$\text{Here, diameter of sphere} = 15.6\text{cm}$$

$$\therefore \text{Radius of sphere} = \left(\frac{15.6}{2}\right)\text{cm} = 7.8\text{cm}$$

$$\text{and, height of cone} = 31.2\text{cm}$$

Then,

$$\frac{4}{3}\pi \times R^3 = \frac{1}{3}\pi \times r^2 \times h$$

$$\Rightarrow \quad \frac{4}{3}\pi \times (7.8)^3 = \frac{1}{3}\pi \times r^2 \times 31.2$$

$$\Rightarrow \quad r^2 = \frac{\frac{4}{3} \times \pi \times (7.8)^3}{\frac{1}{3} \times \pi \times 31.2}$$

$$r^2 = \left(\frac{4 \times 474.552}{31.2}\right) = (60.84) = (7.8)^2$$

$$\Rightarrow \quad r = 7.8\text{cm}$$

$$\therefore \text{Diameter of cone} = (2 \times 7.8)\text{cm} = 15.6\text{cm.}$$

Question 15:

$$\text{Here, diameter of sphere} = 28\text{cm}$$

$$\therefore \text{radius of sphere} = \left(\frac{28}{2}\right)\text{cm} = 14\text{cm}$$

$$\text{Diameter of cone} = 35$$

$$\therefore \text{radius of cone} = \left(\frac{35}{2}\right)\text{cm} = 17.5\text{cm}$$

$$\therefore \frac{4}{3} \times \pi \times R^3 = \frac{1}{3}\pi \times (r)^2 \times h$$

$$\Rightarrow \quad h = \frac{\frac{4}{3} \times \pi \times (14)^3}{\frac{1}{3} \times \pi \times (17.5)^2}$$

$$= \left(\frac{4 \times 2744}{306.25}\right)\text{cm}$$

$$= \left(\frac{10976}{306.25}\right)\text{cm} = 35.84\text{cm}$$

$$\therefore \quad \text{Height of the cone} = 35.84\text{cm}$$

Question 16:

Let the radius of the third ball be r cm

Then,

$$\frac{4}{3}\times\pi\times(3)^3=\frac{4}{3}\pi\left(\frac{3}{2}\right)^3+\frac{4}{3}\times\pi(2)^3+\frac{4}{3}\pi\times(r)^3$$

$$\Rightarrow \quad \frac{4}{3}\times\pi\times27=\frac{4}{3}\pi\times\frac{27}{8}+\frac{4}{3}\times\pi\times8+\frac{4}{3}\pi\times(r)^3$$

$$\Rightarrow \quad 27=\frac{27}{8}+8+(r)^3$$

$$\Rightarrow \quad r^3=\left\{27-\left(\frac{27}{8}+8\right)\right\}$$

$$\Rightarrow \quad r^3=\left\{27-\left(\frac{27+64}{8}\right)\right\}$$

$$\Rightarrow \quad r^3=\left\{27-\frac{91}{8}\right\}$$

$$\Rightarrow \quad r^3=\left\{\frac{216-91}{8}\right\}$$

$$\Rightarrow \quad r^3=\frac{125}{8} \quad \Rightarrow \quad r^3=\left(\frac{5}{2}\right)^3$$

$$\Rightarrow \quad r=\frac{5}{2}=2.5\text{cm}$$

$\therefore$ radius of the third ball $=2.5$cm

Question 17:

Let the radii of two spheres be x and 2x and their respective surface areas be S_1 and S_2.

Then, $$\frac{S_1}{S_2}=\frac{4\pi x^2}{4\pi(2x)^2}$$

$$=\frac{x^2}{4x^2}=\frac{1}{4}$$

$\therefore$ the ratio of their surface areas $=1:4$.

Question 18:

Let the radii of two spheres be r and R

Then,

$$\frac{4\pi r^2}{4\pi R^2}=\frac{1}{4}$$

$$\Rightarrow \quad \left(\frac{r}{R}\right)^2=\left(\frac{1}{2}\right)^2 \Rightarrow \frac{r}{R}=\frac{1}{2}$$

Let V_1 and V_2 be the volumes of the respective spheres whose radii are r and R

$$\therefore \quad \frac{V_1}{V_2}=\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}=\left(\frac{r}{R}\right)^3=\left(\frac{1}{2}\right)^3=\frac{1}{8}$$

$\therefore$ the ratio of their volume $=1:8$.

Question 19:

Let the radius of ball be r cm and R be the radius of the cylindrical tub.

Then,

$$\frac{4}{3}\times\pi\times(r)^3=\pi\times R^2\times h$$

$$\Rightarrow\quad \frac{4}{3}\times\pi\times(r)^3=\pi\times(12)^2\times 6.75$$

$$\Rightarrow\quad (r)^3=\frac{\pi\times 144\times 6.75}{\frac{4}{3}\times\pi}=\frac{144\times 6.75}{\frac{4}{3}}$$

$$r^3=\frac{972\times 3}{4}=\frac{2916}{4}=729$$

$$\Rightarrow\quad r=9\text{cm}$$

$\therefore$ the radius of the ball $=9$cm.

Question 20:

Radius of the cylindrical bucket $=15$cm

Height of the cylindrical bucket $=20$cm

Volume of the water in the bucket $=\pi\times 15\times 15\times 20\ \text{cm}^3$

Radius of spherical ball $=9$cm

Volume of the spherical ball $=\frac{4}{3}\times\pi\times 9\times 9\times 9\ \text{cm}^3$.....(1)

Increase in the water level $=$ h cm

Volume of the increased water level $=\pi\times 15\times 15\times h\ \text{cm}^3$.....(2)

Equating (1) and (2), we have

$$\pi\times 15\times 15\times h=\frac{4}{3}\times\pi\times 9\times 9\times 9$$

$$h=\frac{\frac{4}{3}\times\pi\times 9\times 9\times 9}{\pi\times 15\times 15}$$

$$h=4.32cm$$

Question 21:

Radius of hemisphere $=9$cm

Height of cone $=72$ cm

Let the radius of the base of cone be r cm.

Then,

$$\frac{1}{3}\times\pi\times r^2\times h=\frac{2}{3}\times\pi\times R^3$$

$$\Rightarrow\quad \frac{1}{3}\pi\times r^2\times 72=\frac{2}{3}\times\pi\times(9)^3$$

$$\Rightarrow\quad r^2=\frac{\frac{2}{3}\times\pi\times 729}{\frac{1}{3}\times\pi\times 72}=\frac{2\times 729}{72}$$

$$r^2=\frac{1458}{72}=20.25$$

$$\Rightarrow\quad r=4.5\text{cm}$$

$\therefore$ the radius of the base of the cone $=4.5$cm.

Question 22:

Here, internal radius of hemisphere bowl (R) = 9 cm

Diameter of bottle = 3 cm

$\Rightarrow$ radius (r) = $\left(\frac{3}{2}\right)$ cm

and, height of bottle = 4 cm

$\therefore$ Number of bottles = $\frac{\text{Volume of the bowl}}{\text{Volume of each bottle}}$

$$= \left\{\frac{\frac{2}{3}\pi \times R^3}{\pi \times (r)^2 \times h}\right\}$$

$$= \left\{\frac{\frac{2}{3}\pi \times (9)^3}{\pi \times \left(\frac{3}{2}\right)^2 \times 4}\right\}$$

$$= \frac{\left\{\frac{2}{3} \times 9 \times 9 \times 9\right\}}{\frac{9}{4} \times 4}$$

$$= \frac{2 \times 3 \times 81}{9} = 54$$

$\therefore$ the number of bottle required = 54.

Question 23:

Internal radius(r) = 8cm

External radius(R) = 9cm

Density of metal = 4.5g per cm^3

$\therefore$ weight of the shell = $\left[\frac{4}{3}\pi \times \{(R)^3 - (r)^3\} \times \text{density}\right]$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times \{(9)^3 - (8)^3\} \times \frac{4.5}{1000}\right]\text{kg}$$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times \{729 - 512\} \times \frac{4.5}{1000}\right]\text{kg}$$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times 217 \times \frac{4.5}{1000}\right]\text{kg}$$

$$= \left(\frac{85932}{21000}\right)\text{kg} = 4.092\text{kg}$$

$\therefore$ weight of the shell = 4.092 kg.