

# KISHORE VAIGYANIK PRO TSAHAN YOJANA - 2010

Date : 31-10-2010

Duration : 3 Hours

Max. Marks : 64

## STREAM - SA

### GENERAL INSTRUCTIONS

- The Test Booklet consists of **80** questions.
- There are Two parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

### MARKING SCHEME :

#### PART-I :

##### MATHEMATICS

Question No. **1 to 10** consist of **ONE (1)** mark for each correct response.

##### PHYSICS

Question No. **11 to 20** consist of **ONE (1)** mark for each correct response.

##### CHEMISTRY

Question No. **21 to 30** consist of **ONE (1)** mark for each correct response.

##### BIOLOGY

Question No. **31 to 40** consist of **ONE (1)** mark for each correct response.

#### PART-II :

##### MATHEMATICS

Question No. **1 to 3** consist of **TWO (2)** marks for each correct response.

##### PHYSICS

Question No. **4 to 6** consist of **TWO (2)** marks for each correct response.

##### CHEMISTRY

Question No. **7 to 9** consist of **TWO (2)** marks for each correct response.

##### BIOLOGY

Question No. **10 to 12** consist of **TWO (2)** marks for each correct response.

# Part-I

## One-Mark Questions

### MATHEMATICS

- 1 A student notices that the roots of the equation  $x^2 + bx + a = 0$  are each 1 less than the roots of the equation  $x^2 + ax + b = 0$ . Then  $a + b$  is

- A. possibly any real number
- B. -2
- C. -4
- D. -5

- 2 If  $x, y$  are real numbers such that

$$3^{x+1} - 3^{y-1} = 24,$$

then the value of  $(x+y)(x-y)$  is

- A. 0
  - B. 1
  - C. 2
  - D. 3
- 3 The number of positive integers  $n$  in the set  $\{1, 2, 3, \dots, 100\}$

for which the number  $\frac{1^2 + 2^2 + 3^2 + \dots + n^2}{1 + 2 + 3 + \dots + n}$  is an integer is

- A. 33
- B. 34
- C. 50
- D. 100

- 4 The three different face diagonals of a cuboid (rectangular parallelepiped) have lengths 39, 40, 41. The length of the main diagonal of the cuboid which joins a pair of opposite corners is

A. 49      B.  $49\sqrt{2}$       C. 60      D.  $60\sqrt{2}$

- 5 The sides of a triangle  $ABC$  are positive integers. The smallest side has length 1. Which of the following statements is true?

A. The area of  $ABC$  is always a rational number  
 B. The area of  $ABC$  is always an irrational number  
 C. The perimeter of  $ABC$  is an even integer  
 D. The information provided is not sufficient to conclude any of the statements A, B or C above

- 6 Consider a square  $ABCD$  of side 12 and let  $M, N$  be the midpoints of  $AB, CD$  respectively. Take a point  $P$  on  $MN$  and let  $AP=r$ ,  $PC=s$ . Then the area of the triangle whose sides are  $r, s, 12$  is

A. 72      B. 36      C.  $\frac{rs}{2}$       D.  $\frac{rs}{4}$

- 7 A cow is tied to a corner (vertex) of a regular hexagonal fenced area of side  $a$  metres by a rope of length  $5a/2$  metres in a grass field. (The cow cannot graze inside the fenced area.) What is the maximum possible area of the grass field to which the cow has access to graze?

A.  $5\pi a^2$       B.  $\frac{5}{2}\pi a^2$       C.  $6\pi a^2$       D.  $3\pi a^2$

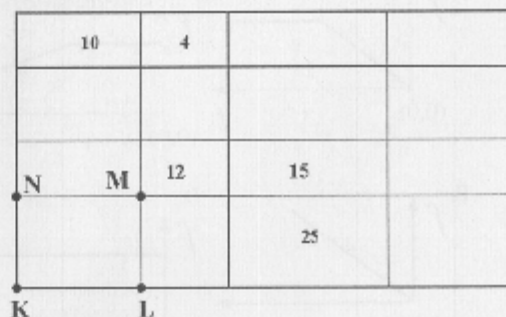
- 8 A closed conical vessel is filled with water fully and is placed with its vertex down. The water is let out at a constant speed. After 21 minutes, it was found that the height of the water column is half of the original height. How much more time in minutes does it require to empty the vessel?

A. 21      B. 14      C. 7      D. 3

- 9 I carried 1000 kg of watermelon in summer by train. In the beginning, the water content was 99%. By the time I reached the destination, the water content had dropped to 98%. The reduction in the weight of the watermelon was

A. 10 kg      B. 50 kg      C. 100 kg      D. 500 kg

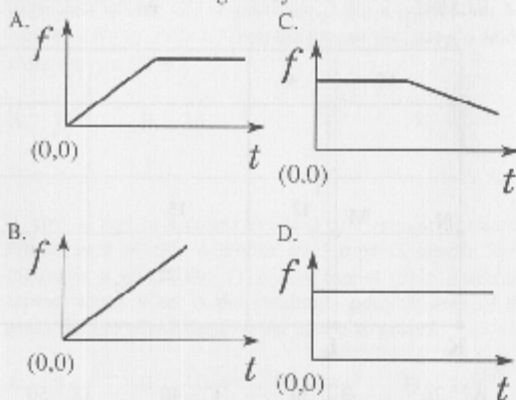
- 10 A rectangle is divided into 16 sub-rectangles as in the figure; the number in each sub-rectangle represents the area of that sub-rectangle. What is the area of the rectangle  $KLMN$ ?



A. 20      B. 30      C. 40      D. 50

## PHYSICS

- 11 A hollow pendulum bob filled with water has a small hole at the bottom through which water escapes at a constant rate. Which of the following statements describes the variation of the time period ( $T$ ) of the pendulum as the water flows out?
- A.  $T$  decreases first and then increases  
 B.  $T$  increases first and then decreases  
 C.  $T$  increases throughout  
 D.  $T$  does not change
- 12 A block of mass  $M$  rests on a rough horizontal table. A steadily increasing horizontal force is applied such that the block starts to slide on the table without toppling. The force is continued even after sliding has started. Assume the coefficients of static and kinetic friction between the table and the block to be equal. The correct representation of the variation of the frictional force,  $f$ , exerted by the table on the block with time  $t$  is given by



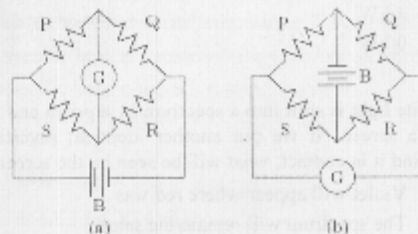
- 13 A soldier with a machine gun, falling from an airplane gets detached from his parachute. He is able to resist the downward acceleration if he shoots 40 bullets a second at the speed of 500 m/s. If the weight of a bullet is 49 gm, what is the weight of the man with the gun? Ignore resistance due to air and assume the acceleration due to gravity  $g = 9.8 \text{ ms}^{-2}$ .

A. 50 kg B. 75 kg C. 100 kg D. 125 kg

- 14 A planet of mass  $m$  is moving around a star of mass  $M$  and radius  $R$  in a circular orbit of radius  $r$ . The star abruptly shrinks to half its radius without any loss of mass. What change will be there in the orbit of the planet?

A. The planet will escape from the star  
 B. The radius of the orbit will increase  
 C. The radius of the orbit will decrease  
 D. The radius of the orbit will not change

- 15 Figure (a) below shows a Wheatstone bridge in which P, Q, R, S are fixed resistances, G is a galvanometer and B is a battery. For this particular case the galvanometer shows zero deflection. Now, only the positions of B and G are interchanged, as shown in figure (b). The new deflection of the galvanometer

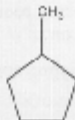



- A. is to the left  
 B. is to the right  
 C. is zero  
 D. depends on the values of P, Q, R, S

- 16 12 positive charges of magnitude  $q$  are placed on a circle of radius  $R$  in a manner that they are equally spaced. A charge  $+Q$  is placed at the centre. If one of the charges  $q$  is removed, then the force on  $Q$  is
- zero
  - $\frac{qQ}{4\pi\epsilon_0 R^2}$  away from the position of the removed charge
  - $\frac{11qQ}{4\pi\epsilon_0 R^2}$  away from the position of the removed charge
  - $\frac{qQ}{4\pi\epsilon_0 R^2}$  towards the position of the removed charge
- 17 An electric heater consists of a nichrome coil and runs under 220 V, consuming 1 kW power. Part of its coil burned out and it was reconnected after cutting off the burnt portion. The power it will consume now is
- more than 1 kW
  - less than 1 kW, but not zero
  - 1 kW
  - 0 kW
- 18 White light is split into a spectrum by a prism and it is seen on a screen. If we put another identical inverted prism behind it in contact, what will be seen on the screen?
- Violet will appear where red was
  - The spectrum will remain the same
  - There will be no spectrum, but only the original light with no deviation
  - There will be no spectrum, but the original light will be laterally displaced
- 19 Two identical blocks of metal are at  $20^\circ\text{C}$  and  $80^\circ\text{C}$ , respectively. The specific heat of the material of the two blocks increases with temperature. Which of the following is true about the final temperature  $T_f$  when the two blocks are brought into contact (assuming that no heat is lost to the surroundings)?
- $T_f$  will be  $50^\circ\text{C}$
  - $T_f$  will be more than  $50^\circ\text{C}$
  - $T_f$  will be less than  $50^\circ\text{C}$
  - $T_f$  can be either more than or less than  $50^\circ\text{C}$  depending on the precise variation of the specific heat with temperature
- 20 A new temperature scale uses  $X$  as a unit of temperature, where the numerical value of the temperature  $t_X$  in this scale is related to the absolute temperature  $T$  by  $t_X = 3T + 300$ . If the specific heat of a material using this unit is  $1400 \text{ J kg}^{-1}\text{X}^{-1}$  its specific heat in the S.I. system of units is
- $4200 \text{ J kg}^{-1}\text{K}^{-1}$
  - $1400 \text{ J kg}^{-1}\text{K}^{-1}$
  - $466.7 \text{ J kg}^{-1}\text{K}^{-1}$
  - impossible to determine from the information provided

## CHEMISTRY

- 21 The boiling points of 0.01 M aqueous solutions of sucrose, NaCl and  $\text{CaCl}_2$  would be  
 A. the same  
 B. highest for sucrose solution  
 C. highest for NaCl solution  
 D. highest for  $\text{CaCl}_2$  solution
- 22 The correct electronic configuration for the ground state of silicon (atomic number 14) is  
 A.  $1s^2 2s^2 2p^6 3s^2 3p^2$   
 B.  $1s^2 2s^2 2p^6 3p^4$   
 C.  $1s^2 2s^2 2p^4 3s^2 3p^2$   
 D.  $1s^2 2s^2 2p^6 3s^1 3p^2$
- 23 The molar mass of  $\text{CaCO}_3$  is 100 g. The maximum amount of carbon dioxide that can be liberated on heating 25 g of  $\text{CaCO}_3$  is  
 A. 11 g    B. 5.5 g    C. 22 g    D. 2.2 g
- 24 The atomic radii of the elements across the second period of the periodic table  
 A. decrease due to increase in atomic number  
 B. decrease due to increase in effective nuclear charge  
 C. decrease due to increase in atomic weights  
 D. increase due to increase in the effective nuclear charge
- 25 Among  $\text{NH}_3$ ,  $\text{BCl}_3$ ,  $\text{Cl}_2$  and  $\text{N}_2$ , the compound that does NOT satisfy the octet rule is  
 A.  $\text{NH}_3$     B.  $\text{BCl}_3$     C.  $\text{Cl}_2$     D.  $\text{N}_2$
- 26 The gas produced on heating  $\text{MnO}_2$  with conc. HCl is  
 A.  $\text{Cl}_2$     B.  $\text{H}_2$     C.  $\text{O}_2$     D.  $\text{O}_3$

- 27 The number of covalent bonds in  $\text{C}_6\text{H}_7\text{Br}$ , is  
 A. 12    B. 10    C. 13    D. 11
- 28 An aqueous solution of HCl has a pH of 2.0. When water is added to increase the pH to 5.0, the hydrogen ion concentration  
 A. remains the same  
 B. decreases three-fold  
 C. increases three-fold  
 D. decreases thousand-fold
- 29 Consider two sealed jars of equal volume. One contains 2 g of hydrogen at 200 K and the other contains 28 g of nitrogen at 400 K. The gases in the two jars will have  
 A. the same pressure  
 B. the same average kinetic energy  
 C. the same number of molecules  
 D. the same average molecular speed
- 30 Identify the stereoisomeric pair from the following choices.  
 A.  $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$  and  $\text{CH}_3\text{CH}_2\text{OCH}_3$   
 B.  $\text{CH}_3\text{CH}_2\text{CH}_2\text{Cl}$  and  $\text{CH}_3\text{CHClCH}_3$   
 C.  $\text{CH}_3-\text{C}(\text{H})=\text{C}(\text{H})-\text{CH}_3$  and  $\text{CH}_3-\text{C}(\text{H})(\text{CH}_3)-\text{C}(\text{H})-\text{CH}_3$   
 D.  and 

## BIOLOGY

- 31 Which of the following is a water-borne disease?
- A. Tuberculosis                      C. Chickenpox  
B. Malaria                              D. Cholera
- 32 In his seminal work on genetics, Gregor Mendel described the physical traits in the pea plant as being controlled by two 'factors'. What term is used to define these factors today?
- A. Chromosomes                      C. Alleles  
B. Genes                                  D. Hybrids
- 33 A majority of the tree species of peninsular Indian origin fruit in the months of
- A. April - May                          C. December - January  
B. August - September              D. All months of the year
- 34 In frogs, body proportions do not change with their growth. A frog that is twice as long as another will be heavier by approximately
- A. Two-fold                              C. Six-fold  
B. Four-fold                              D. Eight-fold
- 35 Which of the following has the widest angle of binocular vision?
- A. Rat              B. Duck              C. Eagle              D. Owl
- 36 The two alleles of a locus which an offspring receives from the male and female gametes are situated on
- A. Two different homologs of the same chromosome  
B. Two different chromosomes  
C. Sex chromosomes  
D. A single chromosome
- 37 Ants locate sucrose by
- A. Using a strong sense of smell  
B. Using a keen sense of vision  
C. Physical contact with sucrose  
D. Sensing the particular wavelength of light emitted/reflected by sucrose
- 38 The interior of a cow-dung pile kept for a few days is quite warm. This is mostly because
- A. Cellulose present in the dung is a good insulator  
B. Bacterial metabolism inside the dung releases heat  
C. Undigested material releases heat due to oxidation by air  
D. Dung is dark and absorbs a lot of heat
- 39 Which one of these is the correct path for a reflex action?
- A. Receptor-Motor Neuron-Spinal Cord-Sensory Neuron-Effector  
B. Effector-Sensory Neuron-Spinal Cord-Motor Neuron-Receptor  
C. Receptor- Sensory Neuron-Spinal Cord-Motor Neuron-Effector  
D. Sensory Neuron- Receptor-Motor Neuron-Spinal Cord-Effector
- 40 Insectivorous plants digest insects to get an essential nutrient. Other plants generally get this nutrient from the soil. What is this nutrient?
- A. Oxygen                                  C. Carbon dioxide  
B. Nitrogen                                  D. Phosphates
-

## Part-II

### MATHEMATICS

- 1 In a triangle  $ABC$ ,  $D$  and  $E$  are points on  $AB$ ,  $AC$  respectively such that  $DE$  is parallel to  $BC$ . Suppose  $BE$ ,  $CD$  intersect at  $O$ . If the areas of the triangles  $ADE$  and  $ODE$  are 3 and 1 respectively, find the area of the triangle  $ABC$ , with justification. (5 Marks)

Answer:



- 2 Leela and Madan pooled their music CD's and sold them. They got as many rupees for each CD as the total number of CD's they sold. They share the money as follows: Leela first takes 10 rupees, then Madan takes 10 rupees and they continue taking 10 rupees alternately till Madan is left out with less than 10 rupees to take. Find the amount that is left out for Madan at the end, with justification. (5 Marks)

Answer:

- 3 (a) Show that for every natural number  $n$  relatively prime to 10, there is another natural number  $m$  all of whose digits are 1's such that  $n$  divides  $m$ . (3 Marks)
- (b) Hence or otherwise show that every positive rational number can be expressed in the form  $\frac{a}{10^b(10^c-1)}$  for some natural numbers  $a, b, c$ . (2 Marks)

Answer:

# PHYSICS

- 4 Consider the two circuits P and Q, shown below, which are used to measure the unknown resistance  $R$ .



In each case, the resistance is estimated by using Ohm's law:  $R_{est} = V/I$ , where  $V$  and  $I$  are the readings of the voltmeter and the ammeter respectively. The meter resistances,  $R_V$  and  $R_A$  are such that  $R_A \ll R \ll R_V$ . The internal resistance of the battery may be ignored. The absolute error in the estimate of the resistance is denoted by  $\delta R = |R - R_{est}|$ .

- Express  $\delta R_P$  in terms of the given resistance values. (2 marks)
- Express  $\delta R_Q$  in terms of the given resistance values. (2 marks)
- For what value of  $R$  will  $\delta R_P \approx \delta R_Q$ ? (1 mark)

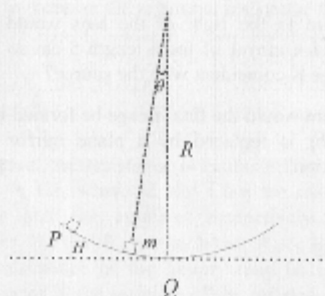
**Answer:**

- 5 A point source is placed 20 cm to the left of a concave lens of focal length 10 cm.

- Where is the image formed? (1 mark)
- Where to the right of the lens would you place a concave mirror of focal length 5 cm so that the final image is coincident with the source? (2 marks)
- Where would the final image be formed if the concave mirror is replaced by a plane mirror at the same position? (2 marks)

**Answer:**

- 6 A block of mass  $m$  is sliding on a fixed frictionless concave surface of radius  $R$ . It is released from rest at point P which is at a height of  $H \ll R$  from the lowest point Q.



- (a) What is the potential energy as a function of  $\theta$ , taking the lowest point Q as the reference level for potential energy? (1 mark)
- (b) What is the kinetic energy as a function of  $\theta$ ? (1 mark)
- (c) What is the time taken for the particle to reach from point P to the lowest point Q? (2 marks)
- (d) How much force is exerted by the block on the concave surface at the point Q? (1 mark)

**Answer:**

## CHEMISTRY

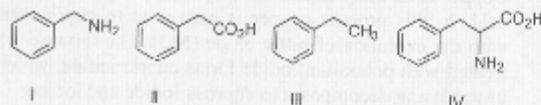
- 7 Copper in an alloy is estimated by dissolving in conc. nitric acid. In this process copper is converted to cupric nitrate with the evolution of nitric oxide (NO). The mixture when treated with potassium iodide forms cupric iodide, which is unstable and decomposes to cuprous iodide and iodine. The amount of copper in the alloy is estimated by titrating the liberated iodine with sodium thiosulfate. The reactions are:
- a  $\text{Cu} + b \text{HNO}_3 \rightarrow$  c  $\text{Cu}(\text{NO}_3)_2 + d \text{NO} + e \text{H}_2\text{O}$
- f  $\text{CuI}_2 \rightarrow$  g  $\text{Cu}_2\text{I}_2 + h \text{I}_2$
- i  $\text{Na}_2\text{S}_2\text{O}_3 + j \text{I}_2 \rightarrow$  k  $\text{Na}_2\text{S}_4\text{O}_6 + l \text{NaI}$

(fill up the blanks)

- (a) The coefficients are: a = \_\_\_\_, b = \_\_\_\_, c = \_\_\_\_, d = \_\_\_\_ and e = \_\_\_\_\_. (1 mark, no partial marking)
- (b) The coefficients are: f = \_\_\_\_, g = \_\_\_\_ and h = \_\_\_\_\_. (1 mark, no partial marking)
- (c) The coefficients are: i = \_\_\_\_, j = \_\_\_\_, k = \_\_\_\_ and l = \_\_\_\_\_. (1 mark, no partial marking)
- (d) If 2.54 g of  $\text{I}_2$  is evolved from a 2.0 g sample of the alloy, what is the percentage of copper in the alloy? (atomic weights of iodine and copper are 127 and 63.5, respectively). (2 marks, no partial marking)

**Answer:**

- 8 You have been given four bottles marked A, B, C and D each containing one of the organic compounds given below.



The following observations were made.

- (i) The compound in the bottle A did not dissolve in either 1 N NaOH or 1 N HCl.
- (ii) The compound in the bottle B dissolved in 1 N NaOH but not in 1 N HCl.
- (iii) The compound in the bottle C dissolved in both 1 N NaOH and 1 N HCl.
- (iv) The compound in the bottle D did not dissolve in 1 N NaOH but dissolved in 1 N HCl.

(fill up the blanks)

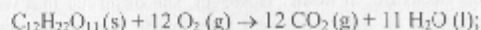
- (a) Indicate the compounds in: bottle A = \_\_\_\_\_, bottle B = \_\_\_\_\_, bottle C = \_\_\_\_\_ and bottle D = \_\_\_\_\_.

(4 × 1 mark = 4 marks, no partial marking)

- (b) The compound with the highest solubility in distilled water is \_\_\_\_\_. (1 mark, no partial marking)

(indicate the answers by the compound numbers)

- 9 Assume that a human body requires 2500 kcal of energy each day for metabolic activity and sucrose is the only source of energy, as per the equation



$$\Delta H = -5.6 \times 10^6 \text{ J.}$$

(fill up the blanks)

- (a) The energy requirement of the human body per day is \_\_\_\_\_ kJ. (1 mark, no partial marking)

- (b) The mass of sucrose required to provide this energy is \_\_\_\_\_ g and the volume of  $CO_2$  (at STP) produced is \_\_\_\_\_ litres.

(2 + 2 marks, no partial marking)

## BIOLOGY

- 10 Mohini, a resident of Chandigarh went to Shimla with her parents. There she found the same plant that they have in their backyard, at home. However, she observed that while the plants in their backyard bore white flowers, those in Shimla had pink flowers. She brought home some seeds of the plant from Shimla and planted them in Chandigarh. Upon performing self-breeding for several generations she found that the plant from Shimla produced only white flowers.

(a) According to you what might be the reason for this observation – genetic or environmental factors?

(1 mark)

(b) Suggest a simple experiment to determine whether this variation is genetic in nature.

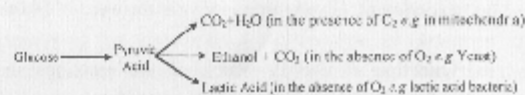
(2 marks)

(c) Suggest another experiment to check whether this variation in flower color is due to environmental factors.

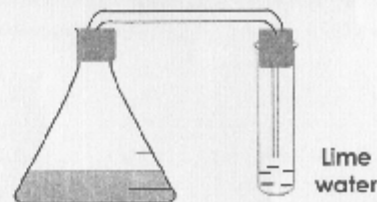
(2 marks)

**Answer:**

- 11 The break-down of glucose in a cell occurs in any of the following pathways:



Three experiments (A, B, C) have been set up. In each experiment, a flask contains the organism in growth medium, glucose and a brown dye that changes its colour to



**Organism in Culture Medium  
+ Glucose + Dye**

yellow when the pH decreases. The mouth of the flask is attached to a test tube containing lime water (Calcium Hydroxide; as shown in the figure). In C, but not in A and B, air is removed from the flask before beginning the experiment.

After a period of growth, the following observations were made:

- A: Lime water turns milky; the dye colour remains the same.  
B: The dye colour changes; lime water does not turn milky.  
C: Lime water turns milky; the dye colour remains the same.

(a) **Question :** Identify which of the reactions in the pathways depicted above is taking place in each experiment. Give reasons for your answer. (4 marks)

(b) **Question :** Identify which of the reactions in the pathways depicted above is expected to occur in Red Blood Cells (RBCs). (1 mark)

**Answer:**

12 A scientist has a house just beside a busy highway. He collects leaves from some plants growing in his garden to do radio-carbon dating (to estimate the age of the plant by estimating the amount of a radioisotope of carbon in its tissues). Surprisingly the radio-carbon dating shows that the plant is a few thousand years old!

(a) Was the result of the radio-carbon dating wrong or can you propose a reason for such an observation? (3 marks)

(b) What simple experiment can be done to test the reason that you have proposed? (2 marks)

**Answer:**

# Answer Keys

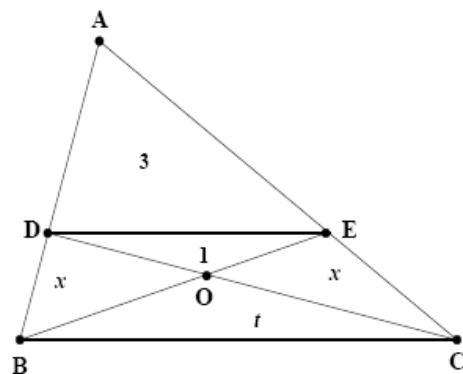
SA answer key for Part "A"	
Q_No	Ans_key
1	C
2	D
3	B
4	A
5	B
6	B
7	A
8	D
9	D
10	D
11	B
12	A
13	C
14	D
15	C
16	D
17	A
18	D
19	B
20	A
21	D
22	A
23	A
24	B
25	B
26	A
27	A
28	D
29	C
30	C
31	D
32	C
33	A
34	D
35	D
36	A
37	C
38	B
39	C
40	B



## Part B - Solution

Q.1

**Solution:**



We denote the area of triangle  $PQR$  by  $[PQR]$ . We see that  $[BOD]$  and  $[COE]$  are equal. Let the common value be  $x$ , and let  $[BOC] = t$ . Using the fact that the ratio of areas of two triangles having equal altitudes is the same as the ratio of their respective bases, we obtain

$$\frac{x}{1} = \frac{BO}{OE} = \frac{t}{x}.$$

This gives  $t = x^2$ . Now  $ADE$  and  $ABC$  are similar so that

$$\frac{[ADE]}{[ABC]} = \frac{DE^2}{BC^2} = \frac{[ODE]}{[OBC]},$$

since  $ODE$  and  $OCB$  are also similar. This implies that

$$\frac{3}{4 + 2x + t} = \frac{1}{t},$$

which simplifies to  $t = 2 + x$ . Using  $t = x^2$ , we get a quadratic in  $x$ :  $x^2 - x - 2 = 0$ . Its solutions are  $x = 2$  and  $x = -1$ . Since  $x$  cannot be negative,  $x = 2$  and  $t = 4$ . Thus  $[ABC] = 4 + 2x + t = 4 + 4 + 4 = 12$ .

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Q.2

**Solution:** Let  $t$  be the total number of CD's that Leela and Madan together sold. Then they obtain  $t^2$  rupees together. Since Leela is the first one to take 10 rupees and also the last one to take 10 rupees, we must have

$$t^2 = 10(\text{an odd number}) + (\text{a number less than } 10).$$

Suppose  $t = 10q + r$ , where  $r$  is the remainder when  $t$  is divided by 10. Then  $t^2 = 100q^2 + 20qr + r^2$ . Comparing, we conclude that

$$r^2 = 10(\text{an odd number}) + (\text{a number less than } 10).$$

But we know that  $0 \leq r < 10$ . Taking  $r = 0, 1, 2, \dots, 9$ , we see that  $r = 4$  or  $6$  (for other values of  $r$ , tens place in  $r^2$  is even). But then  $r^2 = 16$  or  $36$ . Hence the amount left for Madan at the end is 6 rupees.

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### Q.3

(a) Divide the  $n+1$  numbers  $1, 11, 111, \dots, 111 \dots 1$  (all having only 1 as digits) by  $n$ . Among the  $n+1$  remainders so obtained, two must be equal as the possibilities for remainders are  $0, 1, 2, \dots, n-1$  which are  $n$  in number. Thus there must be two numbers  $x = 11 \dots 1$  and  $y = 11 \dots 1$  having say  $j$  digits and  $k$  digits respectively which leave the same remainders after division by  $n$ . We may take  $j < k$ . Now we see that  $y - x$  is divisible by  $n$ . But  $y - x = 11 \dots 100 \dots 0$  where there are  $k - j$  number of 1's and remaining zeros. Since  $n$  is coprime to 10, we see that  $n$  divides  $m = 11 \dots 1$ , a number having only 1's as its digits.

(b) If  $p/q$  is any rational number ( $p > 0, q > 0$ ), then we may write  $q = 2^r 5^s t$ , where  $t$  is coprime to 10. Choose a number  $m$  having only 1's as its digits and is divisible by  $t$ . Consider  $9m$ , which has only 9 as its digits and is still divisible by  $t$ . Let  $k = 9m/t$ . We see that

$$qk = 9m2^r 5^s = (10^c - 1)2^r 5^s,$$

where  $c$  is the number of digits in  $m$ . Hence we can find  $d$  such that  $qd = 10^b(10^c - 1)$  (multiply by a suitable power of 2 if  $s > r$  and by a suitable power of 5 if  $r > s$ ). Then

$$\frac{p}{q} = \frac{pd}{qd} = \frac{a}{10^b(10^c - 1)},$$

where  $a = pd$ .

### Q. 4

**Solution:** For P:  $I = I_R + I_V = V/R + V/R_V$

$$\begin{aligned} R &= \frac{V}{I} \left[ \frac{R_V}{R_V - V/I} \right] \\ &= R_{\text{est}} \left[ \frac{1}{1 - R_{\text{est}}/R_V} \right] \\ &\approx R_{\text{est}} [1 + R_{\text{est}}/R_V] \quad (\text{neglecting higher order terms in } R_{\text{est}}/R_V) \end{aligned}$$

$$\delta R_P = |R_{\text{est}} - R| = R_{\text{est}}^2/R_V \approx \frac{R_{\text{est}}^2}{R_V}$$

Alternatively,

$$\begin{aligned} R_{\text{est}} &= \frac{V}{I} = \frac{R_V R}{R_V + R} \\ \delta R_P &= |R_{\text{est}} - R| = R \left[ \frac{R_V}{R_V + R} - 1 \right] \approx \frac{R^2}{R_V} \end{aligned}$$

For Q:  $V = I(R + R_A)$

$$R = V/I - R_A = R_{\text{est}} - R_A$$

$$\delta R_Q = |R_{\text{est}} - R| = R_A$$

If  $R = \sqrt{R_A R_V}$ , then  $\delta R_P / \delta R_Q = R_{\text{est}}^2 / (R_A R_V) = R_{\text{est}}^2 / R^2 \approx 1$

Q. 5

**Solution:** (a) Object is at  $2f$ , so the image is formed at the same distance from the lens (20 cm) to the right. (b) Since light has to retrace its path, the mirror should be placed so that the previous image is at its center of curvature. Thus the mirror must be placed 30 cm to the right of the lens. (c) For the plane mirror, reflection forms an image 40 cm to the right of the lens. Using the lens formula, we see that the final image is formed at a distance of  $40/3$  cm to the left of the lens.

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Q.6

**Solution:** (a)  $V(\theta) = mgR(1 - \cos \theta)$ , (b)  $mgH - mgR(1 - \cos \theta)$ , (c) For  $H \ll R$  the body executes SHM with a time period of  $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$  - the time taken for it to travel from  $P$  to  $Q$  will be a quarter of this, *i.e.*  $\frac{1}{8\pi}\sqrt{\frac{R}{g}}$ . (d) At the lowest point, the speed is given by  $\frac{1}{2}mv^2 = mgH$ . So,  $T - mg = \frac{mv^2}{R} = \frac{2mgH}{R}$ , and thus  $T = mg\left(1 + \frac{2H}{R}\right)$ .

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Q. 7

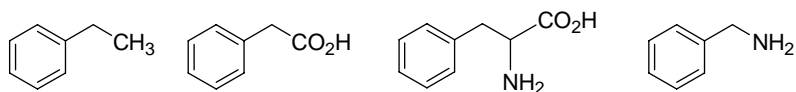
Answers:

- (a) a = 3, b = 8, c = 3, d = 2 and e = 4.
  - (b) f = 2, g = 1, h = 1.
  - (c) i = 2, j = 1, k = 1, l = 2
  - (d) 2.54 g of  $I_2 = 1/100$  mole of  $I_2$   
= 2/100 gm atom of Cu  
% Cu =  $(2/100) \times (63.5/2) = 63.5\%$
-

Q. 8

Answers:

Bottle A = III, Bottle B = II, Bottle C = IV, Bottle D = I



Compound with the highest solubility in distilled water: IV

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Q. 9

Answers:

(a)  $2500 \times 4.184 \text{ kJ} = 10460 \text{ kJ}$

(b) 342 g of sucrose produces 5600 kJ of energy. To provide 10460 kJ we need  $10460 \times 342 / 5600 \text{ g} = 638 \text{ g}$

$638 \text{ g} / 342 \text{ g} \times 12 \times 22.4 \text{ L} = 501 \text{ L}$

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Q.10

- Answers:
- (a) Difference in flower color is most likely due to environmental factors
  - (b) Perform cross breeding between the plants from Chandigarh and those from Shimla to find out whether we get any pink flower or flowers with any shade of color between pink and white in the F1 generation
  - (c) Grow the plants from Chandigarh in Shimla and check whether they still produce white flowers or bear pink flowers
-

Q.11

Answers:

(a) In experiment A, ethanol fermentation occurs producing  $\text{CO}_2$ , turning lime water milky. Since acid is not produced the dye colour does not change.

In experiment B, lactic acid fermentation takes place, which produces acid but does not produce  $\text{CO}_2$ . Hence dye colour changes to yellow but the lime water does not turn milky .

In experiment C, since the lime water turns milky, ethanol fermentation is occurring.

In addition, since removal of air did not affect the reaction, the fermentation is anaerobic and yeast must be the organism in the flask.

(b) In RBCs, lactic acid fermentation occurs.

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Q. 12

Answers:

(a) The result of the radio-carbon dating was correct.

Reason: Vehicles running on the highway beside the house emitted carbon dioxide from the combustion of petrol or diesel, which are fossil fuels. The carbon in this carbon dioxide, coming from living material that has been converted into petroleum millions of years ago, would get assimilated into the tissues of the plant as it uses carbon dioxide from the surrounding atmosphere for photosynthesis. Therefore tissues of the plant, when used for radio-carbon dating, would show the age of the plant to be many thousands of years old.

(b) A simple experiment to test the validity of this explanation would be to collect seeds from the plant and grow them in a plot of land away from the highway or other sources of carbon dioxide coming from the burning of fossil fuels. Radio-carbon dating of plants growing from these seeds should show them as young plants.