CHAPTER 4

Individual and Market Demand

Chapter 3 laid the foundation for the theory of consumer demand. We discussed the nature of consumers' preferences and saw how, given a budget constraint, consumers choose a consumption basket that maximizes their satisfaction. From here it's a short step to analyzing demand itself and how the demand for a good depends on its price, the prices of other goods, and income.

We begin by examining the demands of individual consumers. Since we know how changes in price and income affect a person's budget line, we can determine how they affect consumption choice. In this way we can also determine a person's demand curve for a good. Next, we will see' how individual demand curves can be aggregated to determine the market demand curve. We will also study the characteristics of demand and see why the demands for some kinds of goods differ considerably from the demands for others. In addition, we will show how demand curves can be used to measure the benefits that people receive when they consume a product, above and beyond the expenditure they make. Finally, we will briefly describe some of the methods that can be used to obtain useful empirical information about demand.

4.1 *Individual Demand*

This section shows how the demand curve of an individual consumer follows from the consumption choices that a person makes when faced with a budget constraint. To illustrate the concepts graphically, we will limit the available goods to food and clothing, as in Chapter 3.

Price Changes

We begin by examining how a person's consumption of food and clothing changes when the price of food changes. Figures 4.1a and 4.1b show the consumption choices that one would make when allocating a fixed amount of income between the two goods as the price of food changes.

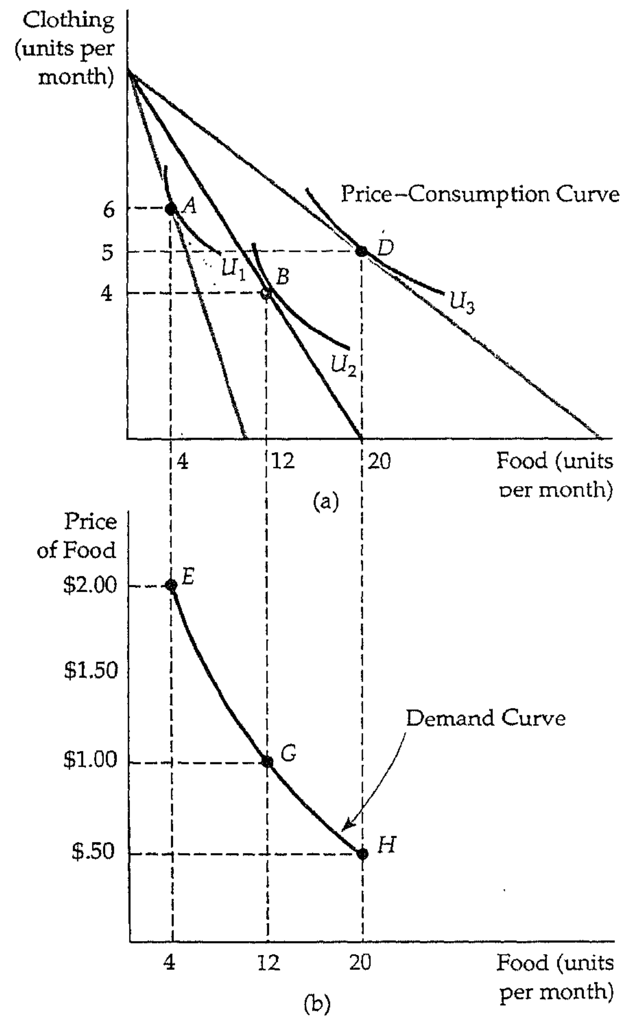

FIGURE 4.1a and b Effect of Price Changes. A reduction in the price of food, with income and the price of clothing fixed, causes this consumer to choose a different market basket. In (a) the market baskets that maximize utility for various prices of food (point *A*, \$2; *B*, \$1; *D*, \$0.50) trace out the price-consumption curve. Part (b) gives the demand curve, which relates the price of food to the quantity demanded. (Points *E, G,* and *H* correspond to points *A, B,* and *D*, respectively.)

Initially, the price of food is $1, the price of clothing is $2, and the consumer's income is $20. The utility-maximizing consumption choice is at point *B* in Figure 4.1a. Here, the consumer buys 12 units of food and 4 units of clothing, which achieves the level of utility associated with indifference curve U_2.

Now look at Figure 4.1b, which shows the relationship between the price of food and the quantity demanded. The horizontal axis measures the quantity of food consumed, just as in Figure 4.1a, but the vertical axis now measures the price of food. Point *G* in Figure 4.1b corresponds to point *B* in Figure 4.1a. At *G* the price of food is $1, and the consumer purchases 12 units of food.

Suppose the price of food increases to $2. As we saw in Chapter 3, the budget line in Figure 4.1a rotates inward about the vertical intercept, becoming twice as steep as before. The higher relative price of food has increased the magnitude of the slope of the budget line. The consumer now achieves maximum utility at *A*, which is on a lower indifference curve U_1. (Because the price of food has risen, the consumer's purchasing power, and hence attainable utility, has fallen.) At A, the consumer chooses 4 units of food and 6 units of clothing. In Figure 4.1b, this modified consumption choice is at *E*, which shows that at a price of $2,4 units of food are demanded. Finally, what will happen if the price of food *decreases* to 50 cents? Now the budget line rotates outward, so the consumer can achieve the higher level of utility associated with indifference curve U_3 in Figure 4.1a by selecting *D*, with 20 units of food and 5 units of clothing. Point *H* in Figure 4.1b shows the price of 50 cents and the quantity demanded of 20 units of food.

The Demand Curve

We can go on to include all possible changes in the price of food. In Figure 4.1a, the *price-consumption curve* traces the utility-maximizing combinations of food and clothing associated with each and every price of food. Note that as the price of food falls, attainable utility increases, and the consumer buys more food. This pattern of increasing consumption of a good in response to a decrease in price almost always holds. But what happens to the consumption of clothing as the price of food falls? As Figure 4.1a shows, the consumption of clothing may either increase or decrease. Both food *and* clothing consumption can increase because the decrease in the price of food has increased the consumer's ability to purchase both goods.

The *demand curve* shown in Figure 4.1b relates the quantity of food that the consumer will buy to the price of food. The demand curve has two important properties. First, the level of utility that can be attained changes as we move along the curve. The lower the price of the product, the higher the level of utility. (Note from Figure 4.1a that a higher indifference curve is reached as the price falls.) Again, this simply reflects the fact that as the price of a product falls, the consumer's purchasing power increases.

Second, at every point on the demand curve, the consumer is maximizing utility by satisfying the condition that the marginal rate of substitution of food for clothing equals the ratio of the prices of food and clothing. As the price of food falls, the price ratio and the MRS also fall. In Figure 4.1 the price ratio falls from 1 ($2/$2) at *E* (because the curve U_1 is tangent to a budget line with a slope of -1 at *A*) to ½ ($1/$2) at *G*, to ¼ ($0.50/$2) at *H*. Because the consumer is maximizing utility, the MRS of food for clothing decreases as we move down the demand curve. This makes intuitive sense because it tells us that the relative value of food falls as the consumer buys more of it.

The fact that the marginal rate of substitution varies along the individual's demand curve tells us something about how consumers value the consumption of a good or service. Suppose we were to ask a consumer how much she would be willing to pay for an additional unit of food when she is currently consuming 4 units of it. Point E on the demand curve in Figure 4.1b provides the answer: $2. Why? As we pointed out above, since the MRS of food for clothing is 1 at *E*, one additional unit of food is worth one additional unit of clothing. But a unit of clothing costs $2, which is, therefore, the value (or marginal benefit) obtained by consuming an additional unit of food. Thus, as we move down the demand curve in Figure 4.1b, the MRS falls, and the value the consumer places on an additional unit of food falls from $2 to $1 to $0.50.

Income Changes

We have seen what happens to the consumption of food and clothing when the price of food changes. Now let's see what happens when income changes.

The effects of a change in income can be analyzed in much the same way as a price change. Figure 4.2a shows the consumption choices that a consumer would make when allocating a fixed income to food and clothing, when the price of food is $1 and the price of clothing is $2. Initially the consumer's income is $10. The utility-maximizing consumption choice is then at A, at which she buys 4 units of food and 3 units of clothing.

This choice of 4 units of food is also shown in Figure 4.2b as E on demand curve D_1. Demand curve D_1 is the curve that would be traced out if we held income fixed at $10 *but varied the price of food.* Because we are holding the price of food constant, we will observe only a single point E on this demand curve.

What happens if the consumer's income is increased to $20? Her budget line then shifts outward parallel to the original budget line, allowing her to attain the utility level associated with indifference curve U_2. Her optimal consumption choice is now at *B*, where she buys 10 units of food and 5 units of clothing. In Figure 4.2b, her consumption of food is shown as *G* on demand curve D_2. (D_2 is the demand curve that would be traced out if we held income fixed at $20 but varied the price of food.) Finally, note that if her income increases to $30, she chooses *D*, with a market basket containing 16 units of food (and 7 units of clothing), represented by *H* in Figure 4.2b.

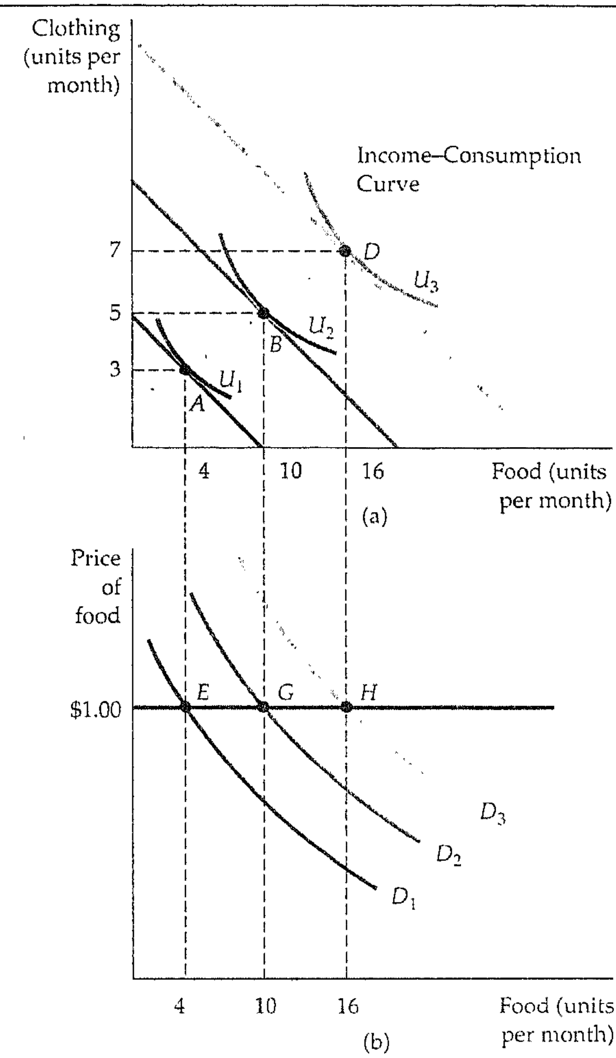

FIGURE 4.2a and b Effect of Income Changes. An increase in their income, with the prices of all goods fixed, causes consumers to alter their choice of market basket. In part (a) the market baskets that maximize consumer satisfaction for various incomes (point A, $10; B, $20; D, $30) trace out the income-consumption curve. The shift to the right of the demand curve in response to the increases in income is shown in part (b). (Points *E*, *G*, and *H* correspond to points *A*, *B*, and *D*, respectively.)

We could go on to include all possible changes in income. In Figure 4.2a, the *income-consumption curve* traces out the utility-maximizmg combinations of food and clothing associated with each and every income level. This income-consumption curve is upward sloping because the consumption of both food and clothing increases as income increases. Previously, we saw that a change in the price of a good corresponded to a movement along a demand curve. Here, the story is different. Because each demand curve is measured for a particular level of income, any change in income must lead to a shift in the demand curve itself. Thus, *A* on the income-consumption curve in Figure 4.2a corresponds to *E* on demand curve D_1 in Figure 4.2b, and *B* corresponds to *F* on a different demand curve D_2. The upward-sloping income consumption curve implies that an increase in income causes a shift to the right in the demand curve, in this case from D_1 to D_2 to D_3.

When the income-consumption curve has a positive slope, the quantity demanded increases with income and the income elasticity of demand is positive. The greater the shifts to the right of the demand curve, the larger the income elasticity. In this case the goods are described as *normal:* Consumers want to buy more of them as their income increases. In some cases, quantity demanded falls as income increases, and the income elasticity of demand is negative. We then describe the good as *inferior.* The term *inferior* is not pejorative-it simply means that consumption falls when income rises. For example, hamburger is inferior for some people because as their income increases they want to buy less hamburger and more steak.

Figure 4.3 shows the income-consumption curve for an inferior good. For relatively low levels of income, both hamburger and steak are normal goods.

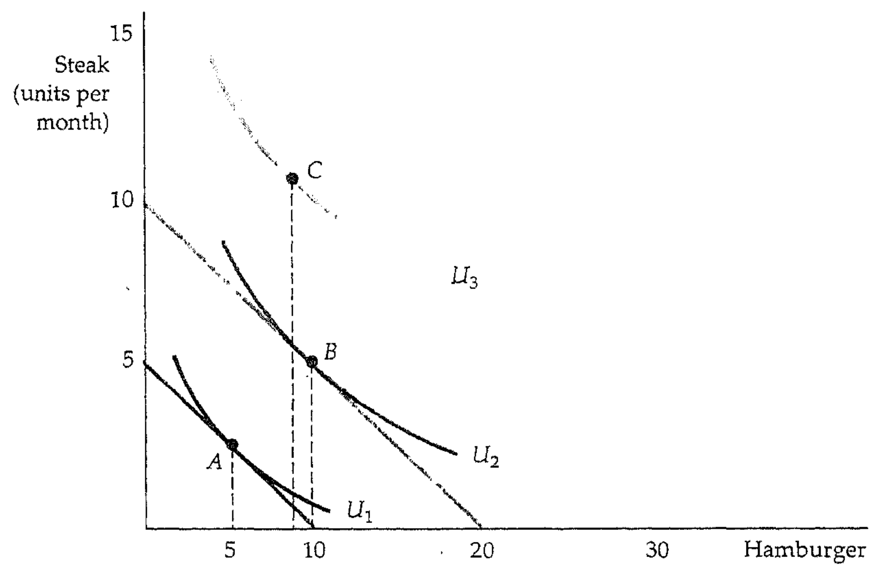

FIGURE 4.3 An Inferior Good. An increase in a person's income can lead to less consumption of one of the two goods being purchased. Here, hamburger is a normal good between *A* and *B*, but becomes an inferior good when the income-consumption curve bends backward between *B* and *C*.

However, as income rises, the income-consumption curve bends backward (from point *B* to *C*). This occurs because hamburger has become an inferior good-its consumption has fallen as income has increased;

Engel Curves

Income-consumption curves can be used to construct *Engel curves,* which relate the quantity of a good consumed to income. Figure 4.4 shows how such curves are constructed for two different goods. Figure 4.4a, which shows an upward-sloping Engel curve, is derived directly from Figure 4.2a. In both figures, as income increases from $10 to $20 to $30, the consumption of food increases from 4 to 10 to 16 units. The upward-sloping Engel curve (like the upward-sloping income-consumption curve in Figure 4.2a) applies to all normal goods. Note that an Engel curve for clothing would have a similar shape (clothing consumption increases from 3 to 5 to 7 units as income increases).

Figure 4.4b shows the Engel curve for hamburgers, derived from Figure 4.3. We see that hamburger consumption increases from 5 to 10 units as income increases from $10 to $20. As income increases further, from $20 to $30, consumption falls to 8 units. The portion of the Engel curve that is downward sloping is the income range in which hamburger is an inferior good.

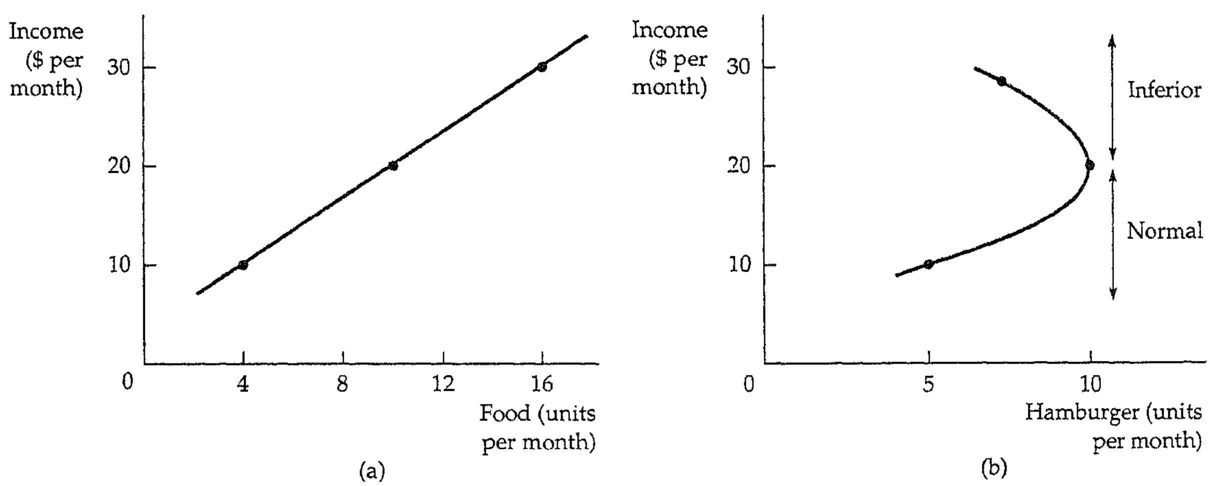

FIGURE 4.4a and b Engel Curves. Engel curves relate the quantity of a good consumed to income. In (a) food is a normal good, and the Engel curve is upward sloping. In (b), however, hamburger is a normal good for income less than $20 per month and an inferior good for income greater than $20 per month.

EXAMPLE 4.1 CONSUMER EXPENDITURES IN THE UNITED STATES

Engel curves explain how consumer spending varies among income groups. Table 4.1 illustrates this for some items taken from a recent survey by the U.S. Bureau of Labor Statistics. The data are averaged over many households, but they can easily be interpreted as describing the expenditures of a typical family

Note that the data relate *expenditures* on a particular item, rather than the *quantity* of the item, to income. The first two items, entertainment and owner-occupied housing, are consumption goods for which the income elasticity of demand is very high. Average family expenditures on entertainment increase almost sixfold when we move from the lowest to highest income group. The same pattern applies to the sales of homes; here, there is a sevenfold increase in expenditures from the lowest to the highest category.

In contrast, the third item, rental housing expenditures, actually falls with income. This pattern reflects the fact that most higher income individuals own, rather than rent, homes. Finally, health care is a consumption item for which the income elasticity is positive, but quite low. Here, average family expenditures increase modestly with income.

TABLE 4.1 Annual U.S. Household Consumer Expenditures

	Income Group (1991$)					
Expenditures ($) on:	0–$9,999	$10,000–$19,999	$20,000–$29,999	$30,000–$39,999	$40,000–$49,999	$50,000+
Entertainment	545	661	1158	1280	1528	3072
Owned dwellings	1172	1526	2156	3164	4494	7800
Rented dwellings	1493	1790	2078	1897	1401	971
Health care	932	1250	1499	1522	1627	1707

SOURCE: U.S. Department of Labor, Bureau of Labor Statistics, "Consumer Expenditure Interview Survey: 1984-1991."

Substitutes and Complements

The demand curves that we graphed in Chapter 2 showed the relationship between the price of a good and the quantity demanded, with preferences, income, and the prices of all other goods held constant. For many goods, demand is related to the consumption and to the prices of other goods. Baseball bats and baseballs, hot dogs and mustard, and computer hardware and software are all examples of goods whose consumption is complementary-when the consumption of one increases, the consumption of the other is likely to increase as well. Other goods, such as cola and diet" cola, owner-occupied houses and rental apartments, and movie tickets and videpcassette rentals, are substitutable-when the consumption of one increases, the consumption of the other is likely to decrease.

We call two goods *substitutes* if an increase (decrease) in the price of one leads to an increase (decrease) in the quantity demanded of the other. If the price of a movie ticket rises, we would expect individuals to rent more videos, since movie tickets and videos are substitutes. Two goods are *complements* if an increase (decrease) in the price of one good leads to a decrease (increase) in the quantity demanded of the other. If the price of gasoline goes up, causing gasoline consumption to fall, we would expect the consumption of motor oil to fall as well, since gasoline and motor oil are used together. (Two goods are *independent* if a change in the price of one good has no effect on the quantity demanded of the other.)

One way to see whether two goods are complements or substitutes is to examine the price-consumption curve. Thus, in the downward-sloping portion of the price-consumption curve of Figure 4.1, food and clothing are substitutes-the lower price of food leads to a lower consumption of clothing (perhaps because as food expenditures increase, less income is available to spend on clothing). Similarly, food and clothing are complements in the upward-sloping portion of the curve; here, the lower price of food leads to higher clothing consumption (perhaps because the consumer eats more meals at restaurants and must be suitably dressed).

The fact that goods can be complements or substitutes suggests that when studying the effects of price changes in one market, it may be important to look at the consequences in related markets. The interrelationships among markets are discussed in more detail in Chapter 16.

4.2 Income and Substitution Effects

A fall in the price of a good has two effects. First, consumers enjoy an increase in real purchasing power; they are better off because they can buy the same amount of the good for less money and thus have money left over for additional purchases. Second, they will tend to consume more of the good that has become cheaper, and less of those goods that are now relatively more expensive. These two effects normally occur simultaneously, but it will be useful to distinguish between them in our analysis. The specifics are illustrated in Figure 4.5, where the initial budget line is *RS* and there are two goods, food and clothing. Here, the consumer maximizes utility by choosing the market basket at *A*, thereby obtaining the level of utility associated with the indifference curve U_1.

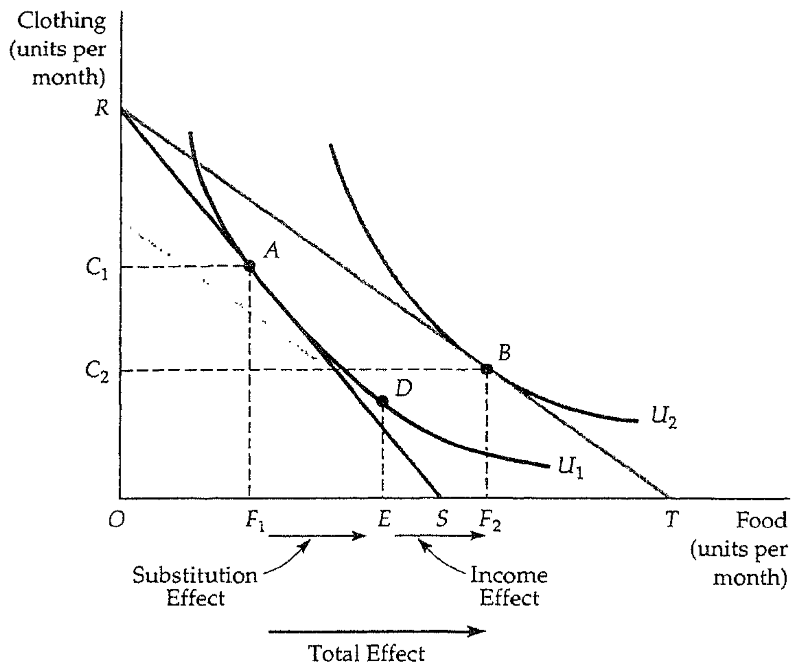

FIGURE 4.5 Income and Substitution Effects-Normal Good. A decrease in the price of food has an income effect and a substitution effect The consumer is initially at *A* on budget line *RS*. When the price of food falls, consumption increases by F_1F_2 as the consumer moves to *B*. The substitution effect, F_1E (associated with a move from A to D) changes the relative prices of food and clothing but keeps real income (satisfaction) constant. The income effect EF_2 (associated with a move from *D* to *B*) keeps relative prices constant but increases purchasing power. Food is a normal good because the income effect EF_2 is positive.

Now, let's see what happens if the price of food falls, causing the budget line to rotate outward to line *RT*. The consumer now chooses the market basket at *B* on indifference curve U_2 Since market basket *B* was chosen even though market basket *A* was feasible, we know by revealed preference that *B*. is preferred to *A*. Thus, the reduction in the price of food allows the consumer to increase her level of satisfaction-her purchasing power has increased. The total change in the consumption of food caused by the lower price is given by F_1F_2. Initially, the consumer purchased OF_1 units of food, but after the price change, food consumption has increased to OF_2. Line segment F_1F_2, therefore, represents the increase in desired food purchases. What has happened to the consumption of clothing? It has fallen from OC_1 to OC_2, a drop represented by line segment C_1C_2. Remember, food is now relatively inexpensive while clothing is now relatively costly.

Substitution Effect

The drop in price has a substitution effect and an income effect. The *substitution effect is* the change in food consumption associated with a change in the price of food, *with the level of utility held constant.* The substitution effect captures the change in food consumption that occurs as a result of the price change that makes food relatively cheaper than clothing. This substitution is marked by a movement along an indifference curve. In Figure 4.5, the substitution effect can be obtained by drawing a budget line parallel to the new budget line *RT* (reflecting the lower relative price of food), but that is just tangent to the original indifference curve U_1 (holding the level of satisfaction constant). The new lower budget line reflects the fact that nominal income was subtracted in order to isolate the substitution effect. Given that budget line, the consumer chooses market basket *D* and consumes *OE* units of food. The line segment F_1E thus represents the substitution effect.

Figure 4.5 makes it clear that when the price of food declines, the substitution effect always leads to an increase in the quantity of food demanded. The

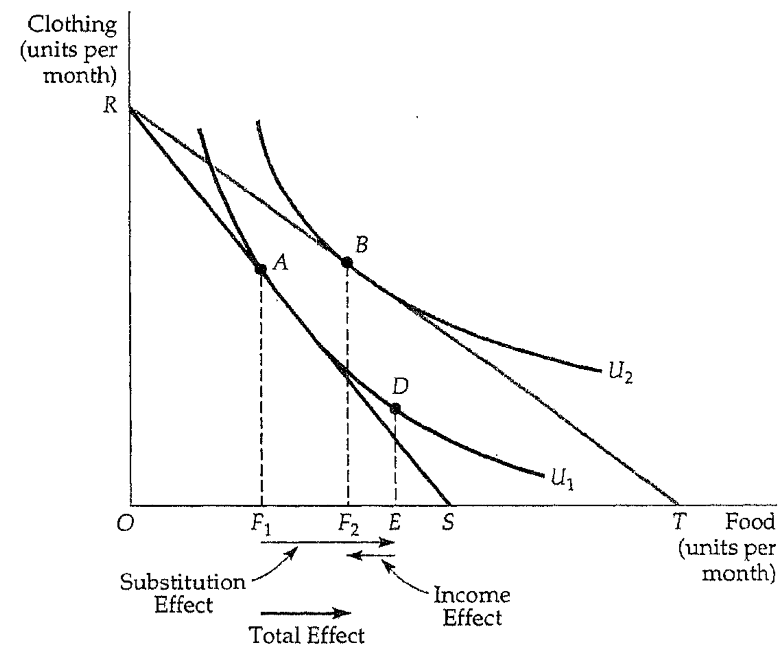

FIGURE 4.6 Income and Substitution Effects-Inferior Good. The effect of a decrease in the price of food is again broken down into a substitution effect F_1E and an income effect EF_2. In this case, food is an inferior good, because the income effect is negative. However, the substitution effect exceeds the income effect, so the decrease in the price of food leads to an increase in the quantity of food demanded.

explanation lies in our assumption that preferences are convex. Thus, with the convex indifference curves shown in the figure, the point that maximizes satisfaction on the new budget line *RT* must lie below and to the right of the original point of tangency.

Income Effect

Now consider the *income effect*-the change in food consumption brought about by the increase in purchasing power, with the price of food held constant. In Figure 4.5, the income effect occurs when the consumer's nominal income has been restored-the budget line passing through *D* shifts outward to budget line *RT*. The consumer chooses market basket *B* rather than market basket *D* on indifference curve U_2 (because the lower price of food has increased the consumer's level of utility). The increase in food consumption from *OE* to OF_2 is the measure of the income effect,which is positive, because food is a normal good. Because it reflects a movement from one indifference curve to another, the income effect measures the change in the consumer's purchasing power.

When a good is inferior, the income effect is negative-as income rises, consumption falls. Figure 4.6 shows income and substitution effects for an inferior good. The negative income effect is measured by line segment EF_2. Even with inferior goods, the income effect is rarely large enough to outweigh the substitution effect. As a result, when the price of an inferior good falls, its consumption almost always increases.

A Special Case-The Giffen Good

The income effect may theoretically be large enough to cause the demand curve for a good to slope upward. We call such a good a *Giffen good,* and Figure 4.7 shows the income and substitution effects. Initially, the consumer is at *A*, consuming relatively little clothing and much food. Now the price of food declines. The decline in the price of food frees enough income sc that the consumer desires to buy more clothing and fewer units of food, as illustrated by *B*. By revealed preference, the consumer is better off at *B* rathel than *A* even though less food is consumed. Perhaps the better-dressed consumer is likely to receive more dinner invitations and have less need to cool at home!

Although intriguing, the Giffen good is rarely of practical interest. It requires a large negative income effect. But the income effect is usually small-most goods individually account for only a small part of the consumer's budget And large income effects are often associated with normal rather than inferion goods (e.g., for housing, food, or transportation).

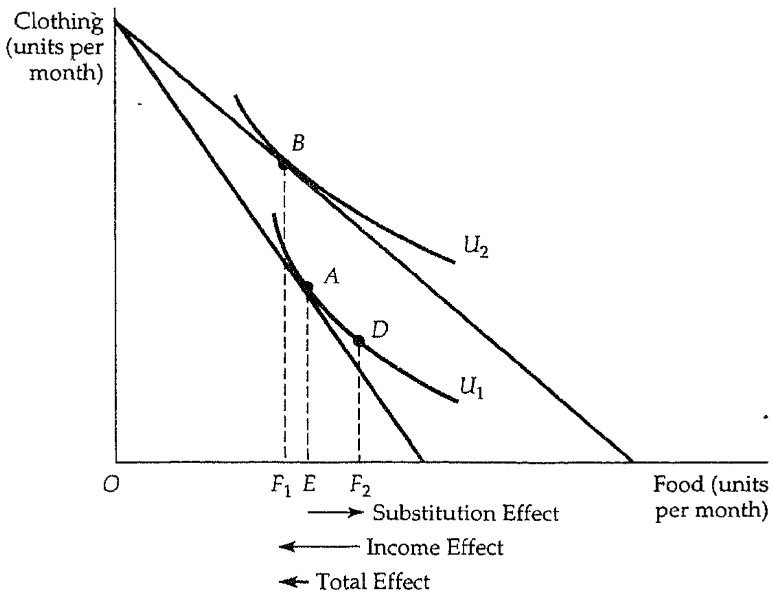

FIGURE 4.7 Upward-Sloping Demand Curve: The Giffen Good. When food is an inferior good, and the income effect is large enough to dominate the substitution effect, the demand curve will be upward-sloping. The consumer is initially at point *A*. But after the price of food falls, the consumer moves to *B* and consumes less food. The income effect F_2F_1 is larger than the substitution effect EF_2, so that the decrease in the price of food leads to a lower quantity of food demanded.

EXAMPLE 4.2 THE EFFECTS OF A GASOLINE TAX

For years, following the Arab oil crisis of 1973-1974, the U.S. government considered increasing the federal tax on gasoline. In 1993, a modest 7}6 cent tax increase was enacted as part of a larger budget reform package. This increase was much less than the $1 to $2 tax increase that would have been necessary to put U.S. gasoline prices on a par with those in Europe. Because an important goal of higher gasoline taxes has been to discourage the consumption of gasoline (and not to raise revenue), the government has also considered ways of passing the resulting income back to consumers. One popular suggestion is a rebate program in which the tax revenues would be returned to households on an equal per capita basis. What would be the effect of such a program?

To begin, lets focus on the effect of the program over a period of five years. The relevant price elasticity of demand is about -0.5.[1] Suppose that a low-

[1] We saw in Chapter 2 that the price elasticity of demand for gasoline varied substantially from the short run to the long run, ranging from -0.11 in the short run to -1.17 in the long run.

income consumer uses about 1200 gallons of gasoline a year, that gasoline costs $1 per gallon, and that the consumer's annual income is $9000.

Figure 4.8 shows the effect of the gasoline tax. (The graph has not been drawn to scale, so the effects we are discussing can be seen more clearly.) The original budget line is *AB*, and the consumer maximizes utility (on indifference curve U_2) by consuming the market basket at *C*, buying 1200 gallons of gasoline and spending $7800 on other goods. If the tax is 50 cents per gallon, price will increase by 50 percent, shifting the new budget line to AD.[2] (Recall that when price changes and income stays fixed, the budget line rotates around a pivotal point on the unchanged axis.) With a price elasticity of -0.5, consumption will decline 25 percent from 1200 to 900 gallons, as shown by the

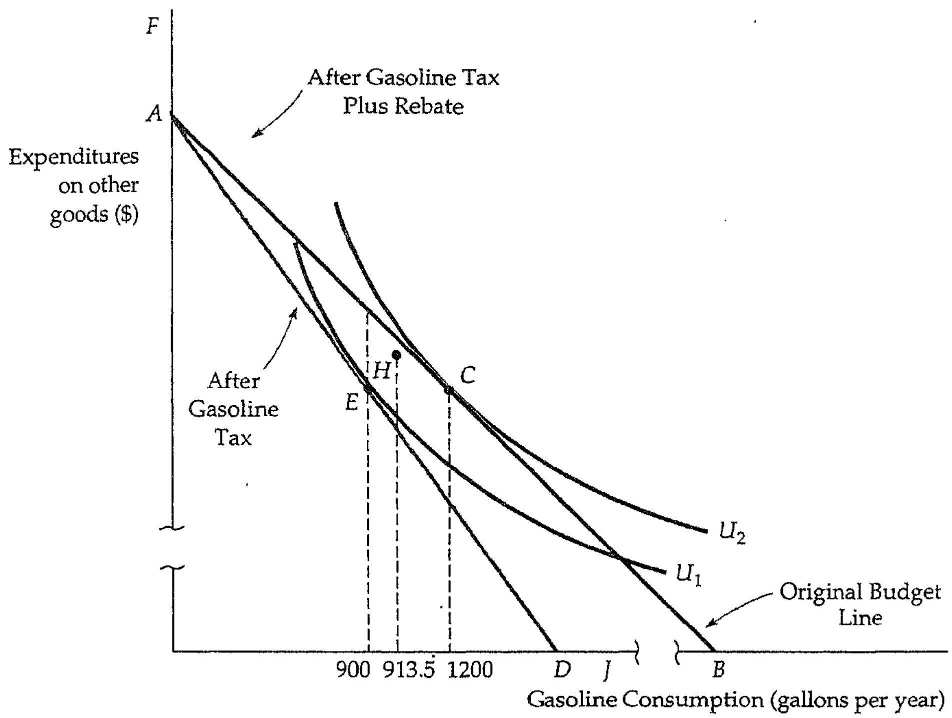

FIGURE 4.8 Effect of a Gasoline Tax with a Rebate. A gasoline tax is imposed when the consumer is initially buying 1200 gallons of gasoline at point *C*. After the tax the budget line shifts from *AB* to *AD* and the consumer maximizes his preferences by choosing *E*, with a gasoline consumption of 900 gallons. However, when the proceeds of the tax are rebated to the consumer, his consumption increases somewhat to 913.5 gallons at *H*. Despite the rebate program, the consumer's gasoline consumption has fallen, as has his level of satisfaction.

[2] To simplify the example, we have assumed that all of the gasoline tax is paid by consumers in the form of a higher price. A more general analysis of tax shifting is presented in Chapter 9.

utility-maximizing point *E* on indifference curve U_1 (because for every 1 percent increase in the price of gasoline, quantity demanded drops by ½ percent).

The rebate program, however, partially counters this effect. Suppose that the tax revenue per person is about $450 (900 gallons times 50 cents per gallon), so that each consumer receives a $450 rebate. How does this increased income affect gasoline consumption? The effect can be shown graphically by shifting the budget line upward by $450 to line *FJ*, which is parallel to *AD*. How much gasoline does our consumer buy now? In Chapter 2 we saw that the income elasticity of demand for gasoline is approximately 0.3. Because the $450 represents a 5 percent increase in income ($450/$9000 = 0.05), we would expect the rebate to increase consumption by 1.5 percent (0.3 times 5 percent) of 900 gallons, or 13.5 gallons. The new utility-maximizing consumption choice at *H* illustrates this. Despite the rebate program, the tax would reduce gasoline consumption by 286.5 gallons, from 1200 to 913.5. Because the income elasticity of demand for gasoline is relatively low, the income effect of the rebate program is dominated by the substitution effect, and the program with a rebate does reduce consumption.

Figure 4.8 reveals that a gasoline tax program with a rebate makes the average low-income consumer slightly worse off, because *H* lies just below in-difference curve U_2.[3] Why introduce such a program? Those who support gasoline taxes argue that they promote national security (by reducing dependence on foreign oil) and encourage conservation, thus helping to weaken OPEC.

4.3 *Market Demand*

So far we have discussed the demand curve for an individual consumer. But where do *market demand* curves come from? In this section we show how market demand curves can be derived as the sum of the individual demand curves of all consumers in a particular market.

From Individual to Market Demand

To keep things simple, let's assume that only three consumers (*A, B,* and *C*) are in the market for coffee. Table 4.2 tabulates several points on each of these

[3] Of course, some consumers (those who spend little on gasoline) will be better off after receiving the rebate, while others (those who spend a lot on gasoline) will be worse off.

TABLE 4.2 Determining the Market Demand Curve

(1) Price ($)	(2) Individual *A* (units)	(3) Individual *B* (units)	(4) Individual *C* (units)	(5) Market (units)
1	6	10	16	32
2	4	8	13	25
3	2	6	10	18
4	0	4	7	11
5	0	2	4	6

consumers' demand curves. The market demand, column (5), is found by adding columns (2), (3), and (4) to determine the total quantity demanded at every price. For example, when the price is $3, the total quantity demanded is 2 + 6 + 10, or 18.

Figure 4.9 shows these same three consumers' demand curves for coffee (labeled D_A, D_B, and D_C). In the graph, the market demand curve is the *horizontal summation* of the demands of each of the consumers. We sum horizontally to find the total amount that the three consumers will demand at any given price. For example, when the price is $4, the quantity demanded by the market (11 units) is the sum of the quantity demanded by *A* (no units), by *B* (4 units), and by *C* (7 units). Because all the individual demand curves slope downward, the market demand curve will also slope downward. However, the market demand curve need not be a straight line, even though each of the individual demand curves is. In Figure 4.9, for example, the market demand curve is *kinked* because one consumer makes no purchases at prices the other consumers find inviting (those above $4).

Two points should be noted. First, the market demand curve will shift to the right as more consumers enter the market. Second, factors that influence the demands of many consumers will also affect the market demand. Suppose, for example, that most consumers in a particular market earn more income, and as a result, increase their demands for coffee. Because each consumer's demand curve shifts to the right, so will the market demand curve.

The aggregation of individual demands into market demands is not just a theoretical exercise. It becomes important in practice when market demands are built up *from* the demands of different demographic groups or from consumers located in different areas. For example, we might obtain information about the demand for home computers by adding independently obtained information about the demands of (i) households with children, (ii) households without children, and (iii) single individuals. Or we might determine the U.S. demand for natural gas by aggregating -the demands for natural gas of. the major regions (East, South, Midwest, Mountain, and West, for example).

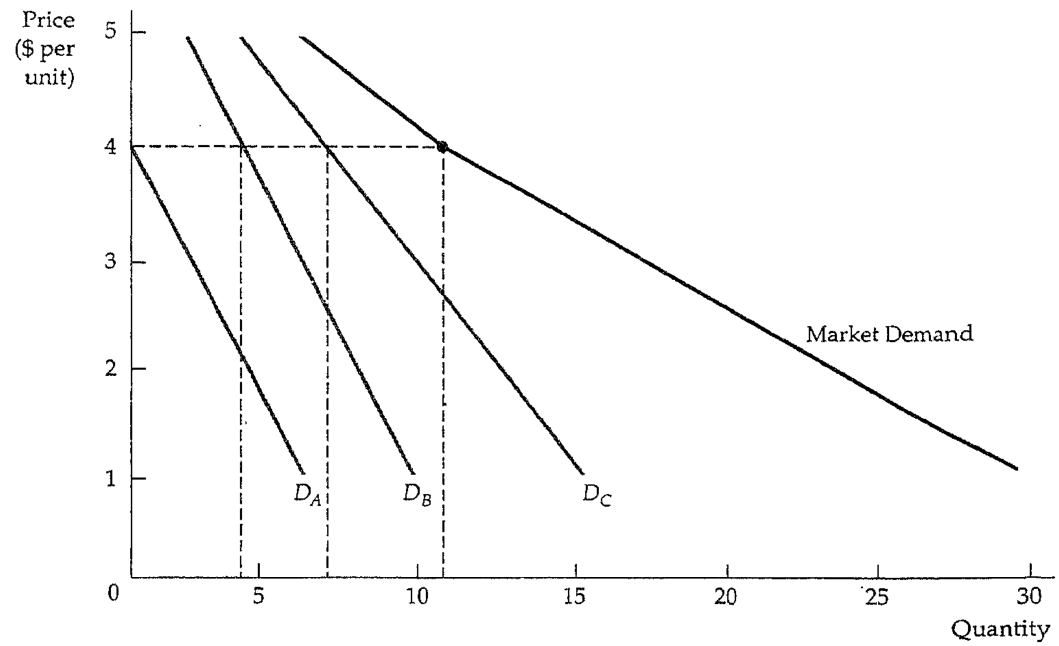

FIGURE 4.9 Summing to Obtain a Market Demand Curve. The market demand curve is obtained by summing the consumers' demand curves D_A, D_B, and D_C. At each price, the quantity of food demanded by the market is the sum of the quantity demanded by each consumer. For example, at a price of $4, the quantity demanded by the market (11 units) is the sum of the quantity demanded by *A* (no units), by *B* (4 units), and by *C* (7 units).

Point and Arc Elasticities of Demand

Recall from Chapter 2 that the price elasticity of demand measures the percentage change in the quantity demanded resulting from a percentage change in price at any point on a demand curve. Denoting the quantity of a good by *Q* and its price by *P*, the *point elasticity of demand* is:

$$E_P = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q/\Delta P}{Q/P} \tag{4.1}$$

(Here A means "a change in" so *AQ/Q* is the percentage change in *Q*.)

When demand is inelastic (i.e., E_P is less than 1 in magnitude), the quantity demanded is relatively unresponsive to changes in price. As a result, the total expenditure on the product increases when the price increases. Suppose, for example, that a family currently uses 1000 gallons of gasoline a year when the price is $1 per gallon. Suppose, in addition, that the family's price elasticity of demand for gasoline is -0.5. Then if the price of gasoline increases to $1.10 (a 10 percent increase), the consumption of gasoline falls to 950 gallons (a 5 percent decrease). Total expenditures on gasoline, however, will increase from $1000 (1000 gallons X $1 per gallon) to $1045 (950 gallons X $1.10 per gallon).

In contrast, when demand is elastic (E_P is greater than 1 in magnitude), the total expenditure on the product decreases as the price goes up. Suppose that a family buys 100 pounds of chicken a year, at a price of $2 per pound, and that the price elasticity of demand for chicken is -1.5. Then if the price of chicken increases to $2.20 (a 10 percent increase), the family's consumption of chicken falls to 85 pounds a year (a 15 percent decrease). Total expenditures on chicken will fall as well, from $200 (100 pounds X $2 per pound) to $187 (85 pounds X $2.20 per pound).

If the elasticity of demand is -1 (the *unit elastic* case), total expenditure remains the same after a price change. Then a price increase leads to a decrease in quantity demanded that is just sufficient to leave the total expenditure on the good unchanged.

Table 4.3 summarizes the relationship between elasticity and expenditure. It is useful to review the table from the point of view of the seller of the good rather than the buyer. When demand is inelastic, a price increase leads only to a small decrease in quantity demanded, so that the total revenue received by the seller increases. But when demand is elastic, a price increase leads to a large decline in quantity demanded, and total revenue falls.

There are times when we want to calculate a price elasticity over some portion of the demand curve rather than at a point. Suppose, for example, that we are concerned with a portion of a demand curve in which the price of a product increases from $10 to $11, while the quantity demanded falls from 100 to 95. How should we calculate the price elasticity of demand? We can calculate that AQ = -5, and AP = 1, but what values do we use for P and Q in the formula $E_P = (AQ/AP)(P/Q)$?

If we use the lower price of $10, we find that $E_P = (-5)(^{10}/_{100}) = -0.50$ However, if we use the higher price, we find that $E_P = (-5)(^{11}/_{95}) = -0.58$. The difference between the two elasticities is not large, but it is discomforting to have two choices, neither of which is obviously preferable to the other. To solve this problem when we are dealing with large price changes, we use the *arc elasticity* of demand:

$$E_P = (\Delta Q/P)(\overline{P}/\overline{Q}) \quad (4.2)$$

TABLE 4.3 Price Elasticity and Consumer Expenditures

Demand	If Price Increases, Expenditures	If Price Decreases, Expenditures
Inelastic	Increase	Decrease
Unit elastic	Are unchanged	Are unchanged
Elastic	Decrease	Increase

where $\overline{P}$ is the *average* of the two prices and $\overline{Q}$ is the *average* of the two quantities.

In our example, the average price is $10.50 and the average quantity is 97.5 so the arc elasticity is $E_P = (-5)(10.5/97.5) = -0.54$. The arc elasticity will always lie somewhere (but not necessarily halfway) between the two point elasticities calculated at the lower and the higher prices.

EXAMPLE 4.3 THE AGGREGATE DEMAND FOR WHEAT

In Chapter 2 (Example 2.2), we discussed the two components of the demand for wheat-domestic demand (by U.S. consumers) and export demand (by foreign consumers). Let us see how the world demand for wheat in 1981 can be obtained by aggregating the domestic and foreign demands. The domestic demand for wheat is given by the equation $Q_{DD} = 1000 - 46P$, where Q_{DD} is the number of bushels (in millions) demanded domestically, and P is the price in dollars per bushel. Export demand is given by $Q_{DE} = 2550 - 220P$, where Q_{DE} is the number of bushels (in millions) demanded from abroad. As shown in Figure 4.10, the domestic demand for wheat, given by *AB*, is relatively price

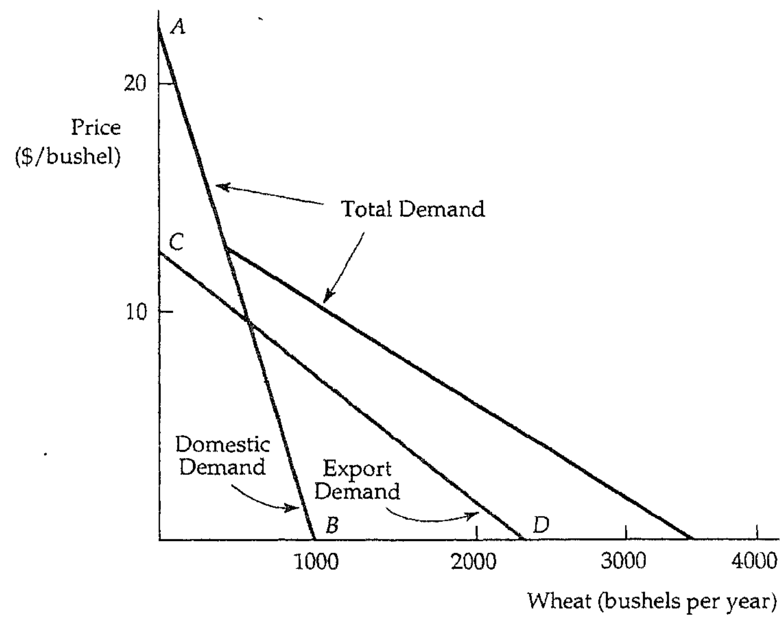

FIGURE 4.10 The Aggregate Demand for Wheat. The total world demand for wheat is the horizontal sum of the domestic demand *AB* and the export demand *CD*. Even though each individual demand curve is linear, the market demand curve is kinked, reflecting that there is no export demand when the price of wheat is greater than $12 per bushel.

inelastic. In fact, statistical studies have shown that the price elasticity of domestic demand is about -0.2. However, export demand, given by *CD*, is more price elastic, with an elasticity of demand of -0.4 to -0.5. Export demand is more elastic than domestic demand because many poorer countries that import U.S. wheat turn to other grains and foodstuffs if wheat prices rise.[4]

To obtain the world demand for wheat, we set the left-hand side of each demand equation equal to the quantity of wheat (the variable on the horizontal axis). Then we add the right-hand side of the equations. Therefore, $Q_D = Q_{DD} + Q_{DE} = (1000 - 46P) + (2550 - 220P) = 3550 - 266P$.

At all prices above *C*, there is no export demand, so world demand and domestic demand are identical. However, below *C*, there is both domestic and export demand. As a result, demand is obtained by adding the quantity demanded of domestic wheat and export wheat at each price level. As the figure shows, the world demand for wheat is kinked. The kink occurs at the price level C above which there is no export demand.

EXAMPLE 4.4 THE DEMAND FOR HOUSING

A family's demand for housing depends on the age and status of the household making the purchasing decision. One approach to housing demand is to relate the number of rooms per house for each household (the quantity demanded) to an estimate of the price of an additional room in a house and to the household's family income.[5] (Prices of rooms vary across the United States

TABLE 4.4 Price and Income Elasticities of the Demand for Rooms

Group	Price Elasticity	Income Elasticity
Single individuals	−0.14	0.19
Married, Head of household age less than 30, 1 child	−0.22	0.07
Married, Head age 30–39, 2 or more children	0	0.11
Married, Head age 50 or older, 1 child	−0.08	0.18

[4] For a survey of statistical studies of demand and supply elasticities and an analysis of the U.S. wheat market, see Larry Salathe and Sudchada Langley, "An Empirical Analysis of Alternative Export Subsidy Programs for U.S. Wheat," *Agricultural Economics Research* 38, No. 1 (Winter 1986).

[5] See Mahlon Strazheim, *An Econometric Analysis of the Urban Housing Market* (New York: National Bureau of Economic Research, 1975), Chapter 4.

because of differences in construction costs.) Table 4.4 lists some of the price and income elasticities obtained for different demographic groups.

In general, the elasticities show that the size of houses that consumers demand (as measured by the number of rooms) is relatively insensitive to differences in either income or price. However, differences among subgroups of the population are important. For example, married families with young heads of households have a price elasticity of -0.22, substantially greater than married households with older household heads. Presumably, families buying houses are more price sensitive when the parents and their children are younger and the parents may plan on having more children. Among married households, the income elasticity of demand for rooms also increases with age, which tells us that older households buy larger houses than younger households.

Price and income elasticities of demand for housing also depend on where people live.[6] Demand in the central cities is substantially more price elastic than the suburban elasticities. Income elasticities, however, increase as one moves farther from the central city. Thus, poorer (on average) central city residents (who live where the price of land is relatively high) are more price sensitive in their housing choices than their wealthier suburban counterparts.

4.4 *Consumer Surplus*

Consumers buy goods because the purchase makes them better off. *Consumer surplus* measures how much better off individuals in the aggregate are by being able to buy a good in the market. Because different consumers value consumption of particular goods differently, the maximum amount they are willing to pay for those goods also differs. *Consumer surplus is the difference between what a consumer is willing to pay for a good and what the consumer actually pays when buying it.* Suppose, for example, that a student would have been willing to pay $13 for a rock concert ticket, even though she had to pay only $12. The $1 that she saved is her consumer surplus.[7] When we add the consumer sur-

[6] See Alien C. Goodman and Masahiro Kawai, "Functional Form, Sample Selection/ and Housing Demand," *Journal of Urban Economics* 20 (Sept. 1986): 155-167.

[7] Measuring consumer surplus in dollars involves an implicit assumption about the shape of the consumers' indifference curves-that a consumer's marginal utility associated with increases in income remains constant within the range of income in question. In many cases, this is a reasonable assumption, although it might be suspect when large changes in income are involved. See Robert D. Willing, "Consumer Surplus Without Apology," *American Economic Review* 65 (1976): 589-597.

pluses of all consumers who buy a good, we obtain a measure of the aggregate consumer surplus.

Consumer surplus can be calculated easily if we know the demand curve. To see the relationship between demand and consumer surplus, examine the individual demand curve for concert tickets shown in Figure 4.11.[8] Drawing the demand curve to look more like a stepladder than a straight line allows us to measure the value that this consumer obtains from buying tickets.

When deciding how many tickets to buy, the student might calculate as follows: The first ticket costs $14 but is worth $20. This $20 valuation is obtained by using the demand curve to find the maximum amount that the student will pay for each additional ticket ($20 is the maximum this student will pay for the first ticket). The ticket is worth purchasing because it generates $6 of surplus value above and beyond the cost of the purchase. The second ticket is also worth buying because it generates a surplus of $5 ($19 - $14). The third

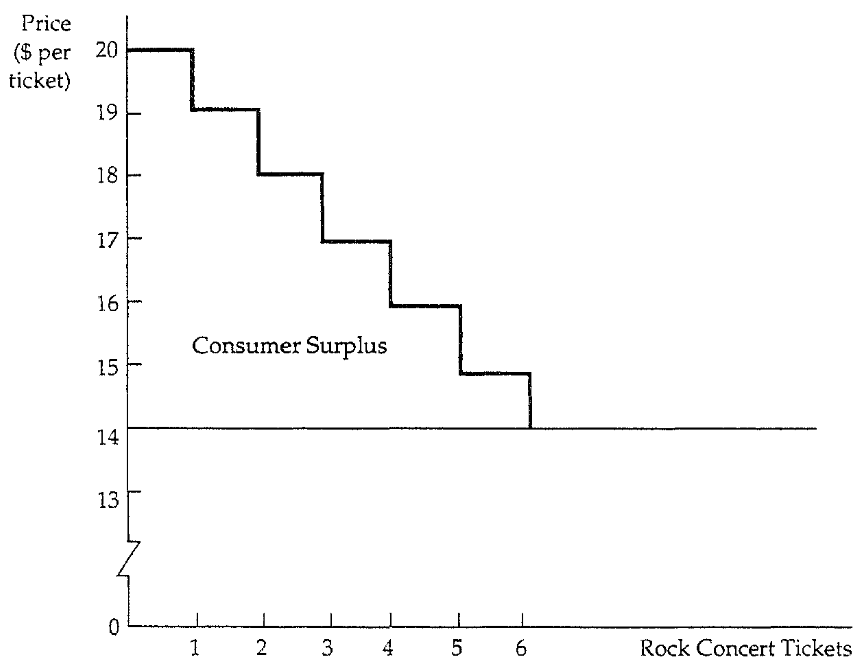

FIGURE 4.11 Consumer Surplus. Consumer surplus is the total benefit from the consumption of a product, net of the total cost of purchasing it. In this figure the consumer surplus associated with 6 concert tickets (purchased at $14 per ticket) is given by the red-shaded area.

[8] The following discussion applies to an individual demand curve, but a similar argument would also apply to a market demand curve.

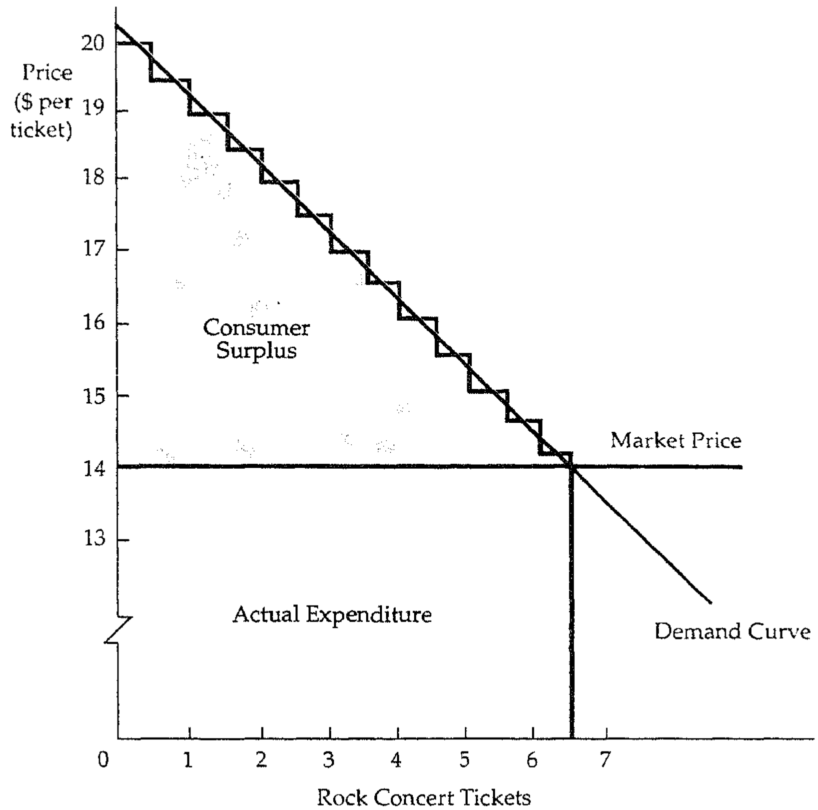

FIGURE 4.12 Consumer Surplus Generalized. When the units of consumption of a good (here, tickets) are small, the consumer surplus can be measured by the area under the demand curve and above the line representing the purchase price of the good. In the figure the consumer surplus is given by the red-shaded triangle.

ticket generates a surplus of \$4. However, the fourth generates a surplus of only \$3, the fifth a surplus of \$2, and the sixth a surplus of just \$1. The student is indifferent about purchasing the seventh ticket (since it generates zero surplus) and prefers not to buy any more than that, as the value of each additional ticket is less than its cost.

In Figure 4.11, consumer surplus is obtained by adding the excess values or surpluses for all units purchased. In this case, consumer surplus = \$6 + \$5 + \$4 + \$3 + \$2 + \$1 = \$21.

In a more general case, the stepladder demand curve can be easily transformed into a straight-line demand curve by making the units of the good smaller and smaller. In Figure 4.12, the stepladder is drawn when half tickets are sold (two students share a ticket), and the stepladder begins to approximate the straight-line demand curve. We use such demand curves as approximations and correspondingly use the triangle in Figure 4.12 to measure consumer surplus. When the demand curve is not a straight line, the consumer surplus is measured by the area below the demand curve and above the price line.[9] To calculate the aggregate consumer surplus in a market, we simply find the area below the *market* demand curve and above the price line.

Consumer surplus has important applications in economics. When added over many individuals, consumer surplus measures the aggregate benefit that consumers obtain from buying goods in a market. When we combine consumer surplus with the aggregate profits that producers obtain, we can evaluate the costs and benefits of alternative market structures and of public policies that alter the behavior of consumers and firms in those markets.

EXAMPLE 4.5 THE VALUE OF CLEAN AIR

Air is free in the sense that one need not pay to breathe it. Yet the absence of a market for air may help explain why the air quality in some cities has been deteriorating for decades. In 1970 Congress amended the Clean Air Act to tighten automobile emissions controls. Were these controls worth it? Were the benefits of cleaning up the air sufficient to outweigh the costs that would be imposed directly on car producers and indirectly on car buyers?

To answer this question. Congress asked the National Academy of Sciences to evaluate these emissions controls in a cost-benefit study.The benefits portion of that study examined how much people value clean air, using empirically determined estimates of the demand for clean air.

Although there is no explicit market for clean air, people do pay more to buy houses where the air is clean than they pay to buy comparable houses in areas with dirtier air. This information was used to estimate the demand for clean air.[10] Detailed data for house prices among neighborhoods of Boston and Los Angeles were compared with the levels of various air pollutants, while the effects of other variables that might affect house value were taken into account statistically. The study determined a demand curve for clean air that looked approximately like that shown in Figure 4.13.

The horizontal axis measures the amount of *air pollution reduction,* and the vertical axis measures the increased value of a home associated with those pollution reductions. For example, consider the demand for cleaner air of a homeowner in a city in which the air is rather dirty, as exemplified by a level of nitrogen oxides (NOX) of 10 parts per 100 million (pphm). If the family were

[9] In the demand curve drawn in Figure 4.12, the consumer surplus is \$21⅛, a close approximation to the \$21 previously determined. This demand curve involves a maximum price of \$20.50 and a quantity sold of 6½. In this case, the triangle has a base of 6½, a height of \$6.50, and an area of \$21⅛ .

[10] The results are summarized in Daniel L. Rubinfeld, "Market Approaches to the Measurement of the Benefits of Air Pollution Abatement,"in Ann Friedlaender, ed., *The Benefits and Costs of Cleaning the Air* (Cambridge, MA: M.I.T Press, 1976): 240-273.

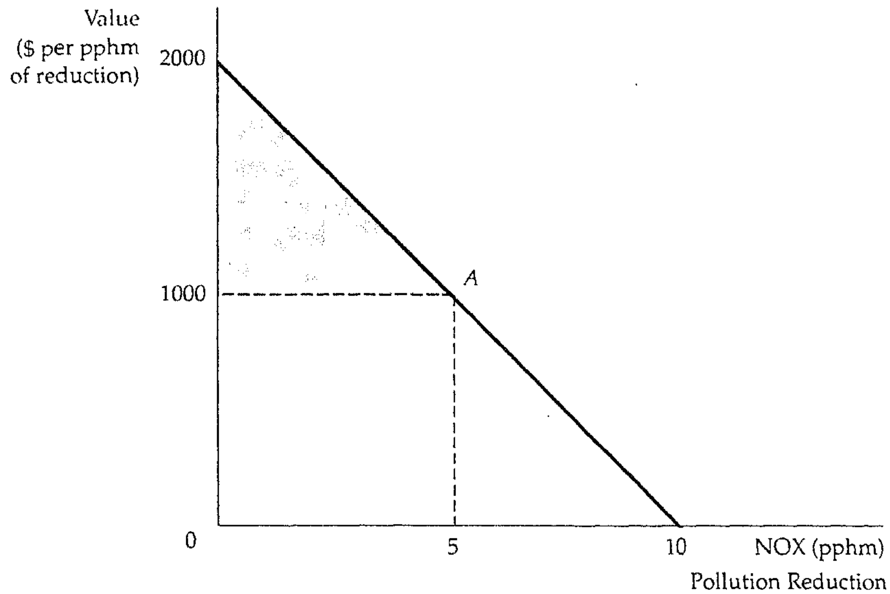

FIGURE 4.13 Valuing Cleaner Air. The shaded area gives the consumer surplus generated when air pollution is reduced by 5 parts per 100 million of nitrogen oxide at a cost of $1000 per part reduced. The surplus is created because most consumers are willing to pay more than $1000 for each unit reduction of nitrogen oxide.

required to pay $1000 for each 1 pphm reduction in air pollution, it would choose *A* on the demand curve to obtain a pollution reduction of 5 pphm.

How much is a 50 percent, or 5 pphm, reduction in pollution worth to the typical family just described? We can measure this value by calculating the consumer surplus associated with reducing air pollution. Since the price for this reduction is $1000 per unit, the family would pay $5000. However, the family values all but the last unit of reduction by more than $1000. As a result, the shaded area in Figure 4.13 gives the value of the cleanup (above and beyond the payment). Since the demand curve is a straight line, the surplus can be calculated from the area of the triangle whose height is $1000 ($2000 - $1000) and whose base is 5 pphm. Therefore, the value to the household of the pollution reduction is $2500.

A complete benefit-cost analysis would use a measure of the total benefit of the cleanup (the benefit per household times the number of households). This could be compared with the total cost of the cleanup to determine whether such a project were worthwhile. We will discuss clean air further in Chapter 18 when we describe the tradeable emissions permits that were introduced by the Clean Air Act of 1990.

4.5 *Network Externalities*

So far we have assumed that people's demands for a good are independent of one another. In other words, Tom's demand for coffee depends on Tom's tastes, his income, the price of coffee, and perhaps the price of tea, but it doesn't depend on Dick's or Harry's demands for coffee. This assumption enabled us to obtain the market demand curve by simply summing individuals' demands.

For some goods, however, a person's demand also depends on the demands of *other* people. In particular, a person's demand may be affected by the number of other people who have purchased the good. If this is the case, there is a *network externality*. Network externalities can be positive or negative. A *positive* network externality exists if the quantity of a good demanded by a typical consumer increases in response to the growth in purchases of other consumers. If the opposite is true, there is a *negative* network externality.

The Bandwagon Effect

One example of a positive network externality is the *bandwagon effect*.[11] This refers to the desire to be in style, to have a good because almost everyone else has it, or to indulge in a fad. The bandwagon effect often arises with children's toys (Barbie dolls, for example). Creating this effect is a major objective in marketing and advertising these toys. Building a bandwagon effect is also often the key to success in selling clothing.

The bandwagon effect is illustrated in Figure 4.14, where the horizontal axis measures the sales of some fashionable good in thousands per month. Suppose consumers think that only 20,000 people have bought the good. This is a small number relative to the U.S. population, so consumers would have little motivation to buy the good to be in style. Some consumers may still buy it (depending on its price), but only for its intrinsic value. In this case, demand is given by the curve D_{20}.

Suppose instead that consumers think that 40,000 people have bought the good. Now they find the good more attractive and want to buy more. The demand curve is D_{40}, which is to the right of D_{20}. Similarly, if consumers thought that 60,000 people had bought the good, the demand curve would be D_{60}, and so on. The more people consumers believe have bought the good, the farther to the right is the demand curve.

Ultimately, consumers would get a good sense of how many people *have* purchased the good. This number would, of course, depend on its price. In Figure 4.14, for example, if the price were $30, 40,000 people would buy the

[11] The bandwagon effect and the snob effect (discussed below) were introduced by Harvey Leibenstein, "Bandwagon, Snob, and Veblen Effects in the Theory of Consumers' Demand," *Quarterly Journal of Economics* 62 (Feb. 1948): 165-201.

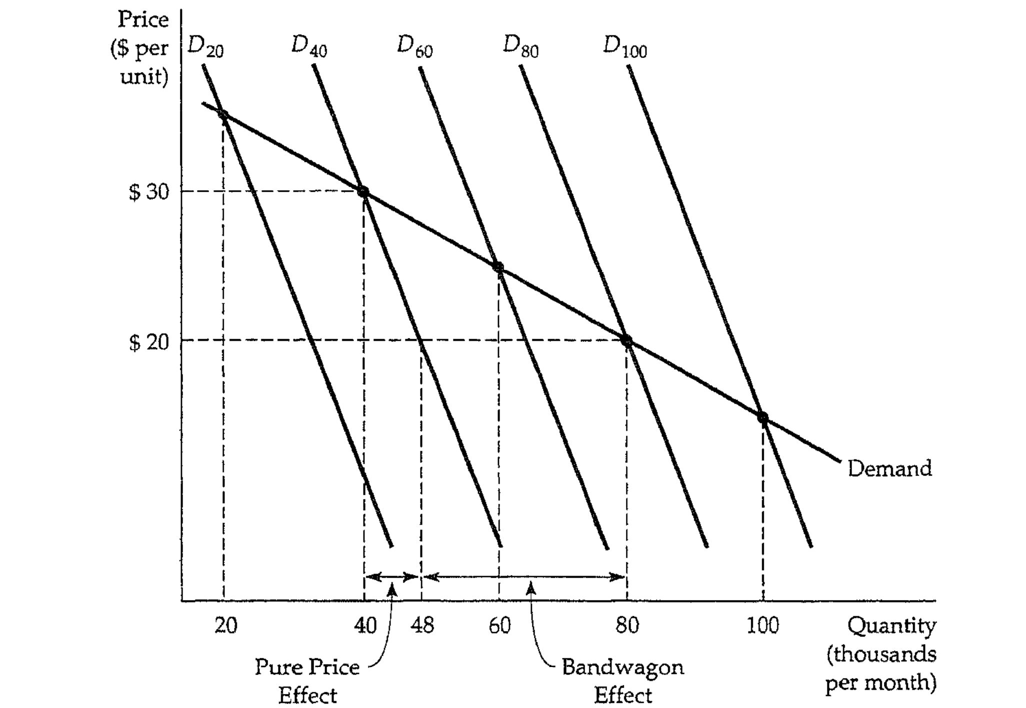

FIGURE 4.14 Positive Network Externality: Bandwagon Effect. A bandwagon effect is an example of a positive network externality, in which the quantity of a good that an individual demands grows in response to the growth of purchases by other individuals. Here the demand for a good shifts to the right from D_{40} to D_{80} due to the bandwagon effect as the price of the product falls from $30 to $20.

good, so the relevant demand curve would be D_{40}. Or if the price were $20, 80,000 people would buy the good, and the relevant demand curve would be D_{80}. *The market demand curve is therefore found by joining the points on the curves D_{20}, D_{40}, D_{60}, D_{80}, and D_{100} that correspond to the quantities 20,000; 40,000; 60,000; 80,000; and 100,000.*

The market demand curve is relatively elastic compared with the curves D_{20}, etc. To see why the bandwagon effect leads to a more elastic demand curve, consider the effect of a drop in price from $30 to $20, with a demand curve of D_{40}. If there were no bandwagon effect, quantity demanded would increase from 40,000 to only 48,000. But as more people buy the good, it becomes stylish to own it, and the bandwagon effect increases quantity demanded further, to 80,000. So the bandwagon effect increases the response of demand to price changes (i.e., makes demand more elastic). As we'll see later, this result has important implications for firms' pricing strategies.

The bandwagon effect is associated with fads and stylishness, but a positive network externality can arise for other reasons. The intrinsic value of some goods to their owners is greater the greater the number of other people who own the goods. For example, if I am the only person to own a compact disc (CD) player, it will not be economical for companies to manufacture compact discs, and without the discs, the CD player will be of little value to me. The more people who own players, the more discs will be manufactured, and the greater will be the value of the player to me. The same is true for personal computers; the more people who own them, the more software will be written, and thus the more useful the computer will be to me.

The Snob Effect

Network externalities are sometimes negative. Consider the *snob effect,* which refers to the desire to own exclusive or unique goods. The quantity demanded of a snob good is higher the *fewer* the people who own it. Rare works of art, specially designed sports cars, and made-to-order clothing are snob goods. Here, the value I get from a painting or sports car is in part the prestige, status, and exclusivity resulting from the fact that few other people own one like it.

Figure 4.15 illustrates the snob effect. D_2 is the demand curve that would apply if consumers believed only 2000 people owned the good. If people believe that 4000 people own the good, it is less exclusive, and its snob value is reduced. Quantity demanded will therefore be lower; the curve D_4 applies. Similarly, if people believe that 6000 people own the good, demand is even smaller, and D_6 applies. Eventually consumers learn how widely owned the good actually is, so the market demand curve is found by joining the points on the curves D_2, D_4 D_6 etc., that actually correspond to the quantities 2000, 4000, 6000, etc.

The snob effect makes market demand less elastic. To see why, suppose the price were initially $30,000, with 2000 people purchasing the good, and was then lowered to $15,000. If there were no snob effect, the quantity purchased would increase to 14,000 (along curve D_2). But as a snob good, its value is greatly reduced if more people own it. The snob effect dampens the increase in quantity demanded, cutting it by 8000 units, so the net increase in sales is only to 6000 units. For many goods, marketing and advertising are geared to creating a snob effect. This means that demand will be less elastic-a result that has important implications for pricing.

Negative network externalities can arise for other reasons. The effect of congestion is a common one. The value I obtain from a lift ticket at a ski resort is lower the more people there are who have bought tickets because I prefer short lines and fewer skiers on the slopes. And likewise for entry to an amusement park, skating rink, or beach.[12]

[12] "Tastes differ, borne people associate a *positive* network externality with skiing or a day on the beach; they enjoy crowds and might find the mountain or beach lonely without them.

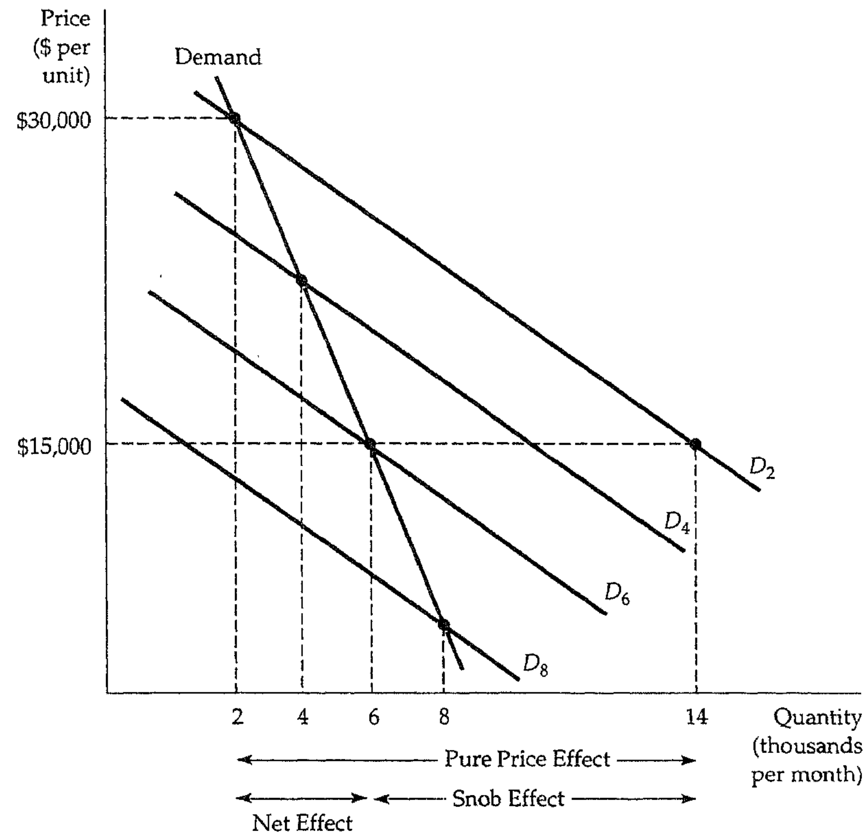

FIGURE 4.15 Negative Network Externality: Snob Effect. A snob effect is an example of a negative network externality, in which the quantity of a good that an individual demands falls in response to the growth of purchases by other individuals. Here the demand for a good shifts to the left from D_2 to D_6 as the price of the product falls from $30,000 to $15,000.

EXAMPLE 4.6 NETWORK EXTERNALITIES AND THE DEMANDS FOR COMPUTERS AND FAX MACHINES

The 1950s and 1960s saw phenomenal growth in the demand for mainframe computers. From 1954 to 1965, for example, annual revenues from the leasing of mainframes increased at the extraordinary rate of 78 percent per year, while prices declined by 20 percent per year. Prices were falling, and the quality of computers was increasing dramatically, but the elasticity of demand would have to have been quite large to account for this kind of growth. IBM, among other computer manufacturers, wanted to know what was going on.

An econometric study by Gregory Chow, then working at IBM, helped provide some answers.[13] Chow found that the demand for computers follows a "saturation curve"-a dynamic process where at first demand is small and grows slowly, but then grows rapidly, until finally nearly everyone likely to buy a computer has done so, and the market is saturated. The rapid growth occurs because of a positive network externality. As more and more organizations own computers, more and better software is written, and more people are trained to use computers, so that the value of having a computer increases. This causes demand to increase, which results in still more software and better trained users, and so on.

This network externality was an important part of the demand for computers. Chow found that it could account for close to half the rapid growth of rentals in 1954-1965. Reductions in the inflation-adjusted price (he found a price elasticity of demand for computers of -1.44) and major increases in the power and quality of computers, which also made them much more useful and effective, accounted for the other half. More recent studies have shown that this process continued through the decades that followed.[14] In fact, this same kind of network externality helped to fuel a rapid rate of growth in the demand for personal computers.

Another product that has experienced explosive growth in recent years is the fax machine. Clearly a strong positive network externality is at work here. Since a fax machine can only transmit to or receive a document from another fax machine, the value of owning one depends crucially on how many other people own them. By 1993, most business offices in the United States had fax machines, and the fax had become a standard means of communicating. This made it essential equipment for almost every business firm.

*4.6 *Empirical Estimation of Demand*

Later in this book, we discuss how demand information is used as an input to firms' economic decision making. For example. General Motors needs to understand automobile demand to decide whether to offer rebates or below-market-interest-rate loans for new cars. Knowledge about demand is also important for public policy decisions-understanding the demand for oil can help Congress decide whether to pass an oil import tax. Here, we briefly ex-

[13] See Gregory Chow, "Technological Change and the Demand for Computers," *American Economic Review* 57, No. 5 (Dec. 1967): 1117-1130.

[14] See Robert J. Gordon, "The Postwar Evolution of Computer Prices," in *Technology and Capital Formation,* Dale W. Jorgenson and Ralph Landau, eds. (Cambridge, MA: M.I.T. Press, 1989).

amine some methods for evaluating and forecasting demand. The more basic statistical tools needed to estimate demand curves and demand elasticities are described' in the Appendix to the book.

Interview and Experimental Approaches to Demand Determination

The most direct way to obtain information about demand is through *interviews* in which consumers are asked how much of a product they might be willing to buy at a given price. Direct approaches such as these, however, are unlikely to succeed because people may lack information or interest, or may want to mislead the interviewer. Therefore, market researchers have designed more successful indirect interview approaches. Consumers might be asked, for example, what their current consumption behavior is and how they would respond if a certain product were available at a 10 percent discount. Or interviewees might be asked how they would expect others to behave. Although indirect survey approaches to demand estimation can be fruitful, the difficulties of the interview approach have forced economists and marketing specialists to look to alternative methods.

In *direct marketing experiments,* actual sales offers are posed to potential customers. An airline, for example, might offer a reduced price on certain flights for six months, partly to learn how this price change affects demand for its flights and how other firms will respond.

Direct experiments are real, not hypothetical, but substantial problems remain. The wrong experiment can be costly, and even if profits and sales rise, the firm cannot be sure that the increase was the result of the experimental change because other factors probably changed at the same time. Also the response to experiments-which consumers often recognize as short-lived-may differ from the response to a permanent change. Finally, a firm can afford to try only a limited number of experiments.

The Statistical Approach to Demand Estimation

Firms often rely on market data based on actual studies of demand. Properly applied, the statistical approach to demand estimation can enable one to sort out the effects of variables, such as income and the prices of other products, on the quantity of a product demanded. Here we outline some of the conceptual issues involved in the statistical approach.

The data in Table 4.5 describe the quantity of raspberries sold in a market each year. Information about the market demand for raspberries might be valuable to an organization representing growers; it would allow them to predict sales on the basis of their own estimates of price and other demand-determining variables. To focus our attention on demand, let's suppose that the quantity of raspberries produced is sensitive to weather conditions but not

TABLE 4.5 Demand Data

Year	Quantity (Q)	Price (P)	Income (I)
1980	4	24	10
1981	7	20	10
1982	8	17	10
1983	13	17	17
1984	16	10	17
1985	15	15	17
1986	19	12	20
1987	20	9	20
1988	22	5	20

to the current price in the market (because farmers make their planting decisions based on last year's price).

The price and quantity data from Table 4.5 are graphed in Figure 4.16. If one believed that price alone determined demand, it would be plausible to describe the demand for the product by drawing a straight line (or other appropriate curve), $Q = a - bP$, which "fit" the points as shown by demand curve *D*. The "least-squares" method of curve-fitting is described in the Appendix to the book.

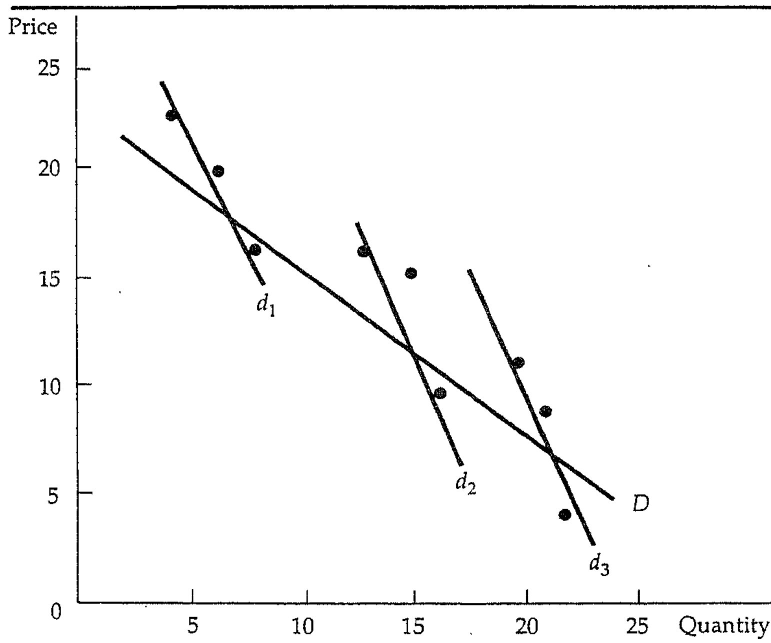

FIGURE 4.16 Estimating Demand. Price and quantity data can be used to determine the form of a demand relationship. But the same data could describe a single demand curve *D*, or three demand curves d_1 d_2, and d_3 that shift over time.

Does curve D (given by the equation $Q = 28.7 - 0.98P$) really represent the demand for the product? The answer is yes, but only if no important factors other than product price affect demand. But in Table 4.5, we have included data for one omitted variable-the average income of purchasers of the product. Note that income (I) has increased twice during the study, suggesting that the demand for agricultural products has shifted twice. Thus, demand curves d_1, d_2, and d_3 in Figure 4.16 give a more likely description of demand. This demand relationship would be described algebraically as

$$Q=a-bP+cI \tag{4.3}$$

The income term in the demand equation allows the demand curve to shift in a parallel fashion as income changes. (The demand relationship, calculated using the least-squares method, is given by $Q = 5.07 - 0.40P + 0.94I$.)

The Form of the Demand Relationship

The demand relationships discussed above are straight lines, so that the -effect of a change in price on quantity demanded is constant. However, the price elasticity of demand varies with the price level. For the demand equation $Q = a - bP$, for example, the price elasticity E_P is:

$$E_P = (\Delta Q/\Delta P)(P/Q) = -b(P/Q) \tag{4.4}$$

Thus, the elasticity increases in magnitude as the price increases (and the quantity demanded falls).

There is no reason to expect elasticities of demand to be constant. Nevertheless, we often find the *isoelastic demand* curve, in which the price elasticity and the income elasticity are constant, useful to work with. When written in its *log-linear form,* it appears as follows:

$$\log(Q) = a - b\log(P) + c\log(I), \tag{4.5}$$

where log () is the logarithmic function, and *a,* b, and c are the constants in the demand equation. The appeal of the log-linear demand relationship is that the slope of the line *-b* is the price elasticity of demand, and the constant c is the income elasticity.[15] Using the data in Table 4.5, for example, we obtained the regression line $\log(Q) = -0.81 - 0.24\log(P) + 1.46\log(I)$.[16] This relation-

[15] The logarithmic function has the property that $\Delta(\log(Q)) = \Delta Q/Q$ for any change in $\log(Q)$. Similarly, $\Delta(\log(P)) = \Delta P/P$ for any change in $\log(P)$. It follows that $\Delta(\log(Q)) = \Delta Q/Q = -b[\Delta \log(P))] = -b(\Delta P/P)$. Therefore, $(\Delta Q/Q)/(\Delta P/P) = -b$, which is the price elasticity of demand. By a similar argument, the income elasticity of demand c is given by $(\Delta Q/Q)/(\Delta I/I)$.

[16] When reporting price and income elasticities of demand, we usually follow one of two procedures. Either we obtain their elasticities from constant elasticity demand equations, or we use other demand relationships and evaluate the price and elasticities-when each of the variables is evaluated at its mean. In the equation $Q = a - bP$, for example, we would calculate the price elasticity using the mean price $\overline{P}$ and the mean quantity solc $\overline{Q}$, so that $E_P = b(\overline{P}/\overline{Q})$.

[redacted]

ship tells us that the price elasticity of demand for raspberries is -0.24 (that is, demand is inelastic), and the income elasticity is 1.46.

The constant elasticity form can also be useful for distinguishing between goods that are complements and goods that are substitutes. Suppose that P_2 represents the price of a second good, which is believed to be related to the product we are studying. Then, we can write the demand function in the following form:

$$\log(Q) = a - b\log(P) + b_2\log(P_2) + c\log(I)$$

When b_2, the cross-price elasticity, is positive, the two goods are substitutes, and when b_2 is negative the two goods are complements.

[redacted]

Summary

1. Individual consumers' demand curves for a commodity can be derived from information about their tastes for all goods and services and from their budget constraints.
2. Engel curves, which describe the relationship between the quantity of a good consumed and income, can be useful for discussions of how consumer expenditures vary with income.
3. Two goods are substitutes if an increase in the price of one good leads to an increase in the quantity demanded of the other. In contrast, two goods are complements if an increase in the price of one leads to a decrease in the quantity demanded of the other.
4. The effect of a price change on the quantity demanded of a good can be broken into two parts-a substitution effect, in which satisfaction remains constant but the price changes, and an income effect, in which the price remains constant but satisfaction changes. Because the income effect can be positive or negative, a price change can have a small or a large effect on quantity demanded. In one unusual but interesting case (that of a Giffen good), the quantity demanded may move in the same direction as the price change (leading to an upward-sloping individual demand curve).
5. The market demand curve is the horizontal summation of the individual demand curves of all consumers in the market for the good. The market demand curve can be used to calculate how much people value the consumption of particular goods and services.
6. Demand is price inelastic when a 1 percent increase in price leads to a less. than 1 percent decrease in quantity demanded, so that the consumer's expenditure increases. Demand is price elastic when a 1 percent increase in price leads to a more than 1 percent decrease in quantity demanded, so that the consumer's expenditure decreases. Demand is unit elastic when a 1 percent increase in price leads to a 1 percent decrease in quantity demanded.
7. The concept of *consumer surplus* can be useful in determining the benefits that people receive from the consumption of a product. Consumer surplus is the difference between what a consumer is willing to pay for a good and what he actually pays when buying it.
8. There is a network externality when one person's demand is affected directly by the purchasing decisions of other consumers. One example of a positive network externality, the

bandwagon effect, occurs when a typical consumer's quantity demanded increases because she considers it stylish to buy a product that others have purchased. An example of a negative network externality, the snob effect, occurs when the quantity demanded by a consumer increases the fewer people who own the good.

9. A number of methods can be used to obtain information about consumer demand. These include interview and experimental approaches, direct marketing experiments, and the more indirect statistical approach. The statistical approach can be very powerful in its application, but it is necessary to determine the appropriate variables that affect demand before the statistical work is done.

Questions for Review

1. How is an individual demand curve different from a market demand curve? Which curve is likely to be more price elastic? (Hint: Assume that there are no network externalities.)

2. Is the demand for a particular brand of product, such as Head skis, likely to be more price elastic or price inelastic than the demand for the aggregate of all brands, such as downhill skis? Explain.

3. Tickets to a rock concert sell for $10. But at that price, the demand is substantially greater than the available number of tickets. Is the value or marginal benefit of an additional ticket greater than, less than, or equal to $10? How might you determine that value?

4. Suppose a person allocates a given budget between two goods, food and clothing. If food is an inferior good, can you tell whether clothing is inferior or normal? Explain.

5. Which of the following combinations of goods are complements and which are substitutes? Could they be either in different circumstances? Discuss.

a. a mathematics class and an economics class
b. tennis balls and a tennis racket
c. steak and lobster
d. a plane trip and a train trip to the same destination
e. bacon and eggs

6. Which of the following events would cause a movement along the demand curve for U.S.-produced clothing, and which would cause a shift in the demand curve?

a. the removal of quotas on the importation of foreign clothes
b. an increase in the income of U.S. citizens
c. a cut in the industry's costs of producing domestic clothes that is passed on to the market in the form of lower clothing prices

7. For which of the following goods is a price increase likely to lead to a substantial income (as well as substitution) effect?

a. salt
b. housing
c. theater tickets
d. food

8. Suppose that the average household in a state consumes 500 gallons of gasoline per year. A ten-cent gasoline tax is introduced, coupled with a $50 annual tax rebate per household. Will the household be better or worse off after the new program is introduced?

9. Which of the following three groups is likely to have the most and which the least price-elastic demand for membership in the Association of Business Economists?

a. students
b. junior executives
c. senior executives

Exercises

1. The ACME corporation determines that at current prices the demand for its computer chips has a price elasticity of-2 in the short run, while the price elasticity for its disc drives is-1.

a. If the corporation decides to raise the price of both products by 10 percent, what will happen to its sales? To its sales revenue? -

b. Can you tell from the available information which product will generate the most revenue for the firm? If yes, why? If not, what additional information would you need?

2. Refer to Example 4.3 on the aggregate demand for wheat. From 1981 to 1990, domestic demand grew in response to growth in U.S. income levels. As a rough approximation, the domestic demand curve in 1990 was $Q_{DD} = 1200 - 55P$. Export demand, however, remained about the same, due to protectionist policies that limited wheat imports. Calculate and draw the aggregate demand curve for wheat in 1990.

3. Vera is shopping for a new videocassette recorder. She hears that the Betamax format is technologically superior to the VHS system. However, she asks her friends and it turns out they all have VHS machines. They agree that the Betamax format provides a better picture, but they add that at the local video store, the Beta format section seems to be getting smaller and smaller. Based on what she observes, Vera buys a VHS machine. Can you explain her decision? Speculate on what would occur if a new 8 mm video format is introduced.

4. Suppose you are in charge of a toll bridge that is essentially cost free. The demand for bridge crossings Q is given by $P = 12 - 2Q$.

a. Draw the demand curve for bridge crossings.

b. How many people would cross the bridge if there were no toll?

c. What is the loss of consumer surplus associated with the charge of a bridge toll of $6?

5. a. Orange juice and apple juice are known to be perfect substitutes. Draw the appropriate price-consumption (for a variable price of orange juice) and income-consumption curves.

b. Left shoes and rights shoes are perfect complements. Draw the appropriate price-consumption and income-consumption curves.

6. Heather's marginal rate of substitution of movie tickets for rental videos is known to be the same no matter how many rental videos she wants. Draw Heather's income consumption curve and her Engel curve for videos.

7. You are managing a $300,000 city budget in which monies are spent on schools and public safety only. You are about to receive aid from the federal government to support a special antidrug law enforcement program. Two programs that are available are (1) a $100,000 grant that must be spent on law enforcement; and (2) a 100 percent matching grant, in which each dollar of local spending-on law enforcement is matched by a dollar of federal money. The federal matching program limits its payment to each city to a maximum of $100,000.

a. Complete the table below with the amounts available for safety:

SCHOOLS	SAFETY no govt. assistance	SAFETY program (1)	SAFETY program (2)
$0			
50/000			
100/000			
150/000			
200,000			
250,000			
300,000			

b. Which program would you (the manager) choose if you wish to maximize the satisfaction of the citizens if you allocate $50,000 of the $300,000 to schools? What about $250,000?

c. Draw the budget constraints for the three options: no aid,program (1), or program (2).

8. By observing an individual's behavior in the situations outlined below, determine the relevant income elasticities of demand for each good (i.e., whether the good is normal or inferior). If you cannot determine the income elasticity, what additional information might you need?

a. Bill spends all his income on books and coffee. He finds $20 while rummaging through a used paperback bin at the bookstore. He immediately buys a new hardcover book of poetry.

b. Bill loses $10 he was going to use to buy a double espresso. He decides to sell his new book at a discount to his friend and use the money to buy coffee.

c. Being bohemian becomes the latest teen fad. As a result, coffee and book prices rise by 25 percent. Bill lowers his consumption of both goods by the same percentage.

d. Bill drops out of art school and gets an M.B.A. instead. He stops reading books and drinking coffee. Now he reads *The Wall Street Journal* and drinks bottled mineral water.

9. Suppose the income elasticity of .demand for food is 0.5, and the price elasticity of demand is -1.0. Suppose also that a woman spends $10,000 a year on food, and that the price of food is $2, and that her income is $25,000.

a. If a $2 sales tax on food were to cause the price of food to double, what would happen to her consumption of food? Hint: Since a large price change is involved, you should assume that the price elasticity measures an arc elasticity, rather than a point elasticity.

b. Suppose that she is given a tax rebate of $5000 to ease the effect of the tax. What would her consumption of food be now?

c. Is she better or worse off when given a rebate equal to the sales tax payments? Discuss.

APPENDIX TO CHAPTER 4

Demand Theory-A Mathematical Treatment

This appendix presents a mathematical treatment of the basics of demand theory. Our goal is to provide a short overview of the theory of demand for students who have some familiarity with the use of calculus. To do this, we will explain and then apply the concept of constrained optimization.

Utility Maximization

Demand theory is based on the premise that consumers maximize utility subject to a budget constraint. Utility is assumed to be an increasing function of the quantities of goods consumed, but marginal utility is assumed to decrease with consumption. The consumer's optimization problem when there are two goods, X and Y, may then be written as

$$\text{Maximize } U(X,Y) \tag{A4.1}$$

subject to the constraint that all income is spent on the two goods:

$$P_xX + P_yY = I \tag{A4.2}$$

Here, $U()$ is the utility function, X and Y are the quantities of the two goods that the consumer purchases, P_x and P_y are the prices of the goods, and I is income.[1]

To determine the individual consumer's demand for the two goods, we choose those values of X and Y that maximize (A4.1) subject to (A4.2). When we know the particular form of the utility function, we can solve to find the consumer's demand for X and Y directly. However, even if we write the utility function in its general form $U(X,Y)$, the technique of constrained optimization can be used to describe the conditions that must hold if the consumer is maximizing utility.

[1] To simplify the mathematics, we assume that the utility function is continuous (with continuous derivatives) and that goods are infinitely divisible.

The Consumer's Optimum

To solve the constrained optimization problem given by equations (A4.1) and (A4.2), we use the method of Lagrange multipliers. Which works as follows. We first write the "Lagrangian" for the problem. To do this, rewrite the constraint (A4.2) as $P_X X + P_Y Y - 7 = 0$. The Lagrangian is then

$$\Phi = U(X,Y) - \lambda(P_X X + P_Y Y - I) \qquad \text{(A43)}$$

The parameter λ's called the *Lagrange multiplier,* we will discuss its interpretation shortly.

If we choose values of *X* and *Y* that satisfy the budget constraint, then the second term in equation (A4.3) will be zero, and maximizing Φ will be equivalent to maximizing $U(X,Y)$. By differentiating Φ with respect to X, Y, and λ and then equating the derivatives to zero, we obtain the necessary conditions for a maximum:[2]

$$\begin{aligned} MU_X(X,Y) - \lambda P_X &= 0 \\ MU_Y(X,Y) - \lambda P_Y &= 0 \\ P_X X + P_Y Y - I &= 0 \end{aligned} \qquad \text{(A4.4)}$$

Here, MU is short for marginal utility (i.e., $MU_X(X,Y) = \partial U(X,Y)/\partial X$, the change in utility from a small increase in the consumption of good X).

The third condition is the original budget constraint. The first two conditions of (A4.4) tell us that each good will be consumed up to the point at which the marginal utility from consumption is a multiple (λ) of the price of the good. To see the implication of this, we combine the first two conditions to obtain the *equal marginal principle:*

$$\lambda = [MU_X(X,Y)/P_X] = MU_Y(X,Y)/P_Y] \qquad \text{(A4.5)}$$

In other words, the marginal utility of each good divided by its price is the same. To be optimizing, *the consumer must be getting the same utility from the last dollar spent by consuming either X or* Y. Were this not the case, consuming more of one good and less of the other would increase utility.

To characterize the individual's optimum in more detail, we can rewrite the information in (A4.5) to obtain

$$MU_X(X,Y)/MU_Y(X,Y) = P_X/P_Y \qquad \text{(A4.6)}$$

Marginal Rate of Substitution

We can use equation (A4.6) to see the link between utility functions and indifference curves. An indifference curve represents all market baskets that give

[2] These conditions are necessary for an "interior" solution in which the consumer consumes positive amounts of both goods. However, the solution could be a "corner" solution in which all of one good and none of the other are consumed.

the consumer the same level of utility. If U^* is a fixed utility level, then the indifference curve that corresponds to that utility level is given by

$$U(X,Y) = U^*$$

As the market baskets are changed by adding X an4 subtracting Y, the total change in utility must equal zero. Therefore

$$MU_X(X,Y)\ dX + MU_Y(X,Y)\ dY = dU^* = 0 \qquad (A4.7)$$

Rearranging,

$$-dY/dX = MU_X(X,Y)/MU_Y(X,Y) = MRS_{XY} \qquad (A4.8)$$

where MRS_{XY} represents the individual's marginal rate of substitution of X for Y. Because the left-hand side of (A4.8) represents the negative of the slope of the indifference curve, it follows that at the point of tangency the individual's marginal rate of substitution (which trades off goods while keeping utility constant) is equal to the individual's ratio of marginal utilities, which in turn is equal to the ratio of the prices of the two goods, from (A4.6).[3]

When the individual indifference curves are convex, the tangency of the indifference curve to the budget line solves the consumer's optimization problem. This was illustrated by Figure 3.11 in Chapter 3.

An Example

In general, the three equations in (A4.4) can be solved to determine the three unknowns X, Y, and λ as a function of the two prices and income. Substitution for λ then allows us to solve for the demands for each of the two goods in terms of income and the prices of the two commodities. This can be most easily seen in terms of an example.

A frequently used utility function is the Cobb-Douglas utility function, which can be represented in two forms:

$$U(X,Y) = a \log(X) + (1 - a) \log(Y)$$

and

$$U(X,Y) = X^a Y^{1-a}$$

The two forms are equivalent for the purposes of demand theory because they both yield the identical demand functions for goods X and Y. We will derive the demand functions for the first form and leave the second as an exercise for the reader.

[3] We are implicitly assuming that the "second-order conditions" for a utility maximum hold, so that the consumer is maximizing rather than minimizing utility. The convexity condition is sufficient for the second-order conditions to be satisfied. In mathematical terms, the condition is that $d(MRS)/dX < 0$, or that $dY^2/dX^2 > 0$, where $-dY/dX$ is the slope of the indifference curve. It is important to note that diminishing marginal utility is not sufficient to ensure that indifference curves are convex.

To find the demand functions tor X and Y, given the usual budget constraint, we first write the Lagrangian:

$$\Phi = a \log(X) + (1 - a)\log(Y) - \lambda(P_X X + P_Y Y - I)$$

Now differentiating with respect to X, Y, and λ, and setting the derivatives equal to zero, we obtain

$$\partial\Phi/\partial X = a/X - \lambda P_X = 0$$

$$\partial\Phi/\partial Y = (1 - a)/Y - \lambda P_Y = 0$$

$$\partial\Phi/\partial\lambda = P_X X + P_Y Y - I = 0$$

The first two conditions imply that

$$P_X X = a/\lambda \quad \text{(A4.9)}$$

$$P_Y Y = (1-a)/\lambda \quad \text{(A4.10)}$$

Combining these with the last condition (the budget constraint) gives: $a/\lambda + (1-a)/\lambda - I = 0$, or $\lambda = 1/I$. Now we can substitute this expression for λ back into (A4,9) and (A4.10) to obtain the demand functions:

$$X = (a/P_X)I$$

$$Y = [(1 - a)/P_Y]I$$

In this example the demand for each good depends only on the price of that good and on income, and not on the price of the other good. Thus, the cross-price elasticities of demand are 0.

Marginal Utility of Income

Whatever the form of the utility function, the Lagrange multiplier λ represents the extra utility generated when the budget constraint is relaxed-in this case by adding one dollar to the budget. To see this, we differentiate the utility function $U(X,Y)$ totally with respect to I:

$$dU/dI = \text{MU}_X(X,Y)(dX/dI) + \text{MU}_Y(X,Y)(dY/dI) \quad \text{(A4.11)}$$

Because any increment in income must be divided between the two goods, it follows that

$$dI = P_X dX + P_Y dY \quad \text{(A4.12)}$$

Substituting from (A4.5) into (A4.11), we get

$$dU/dI = \lambda P_X(dX/dI) + \lambda P_Y(dY/dI) = \lambda(P_X\, dX + P_Y\, dY)/dI \quad \text{(A4.13)}$$

and substituting (A4.12) into (A4.13), we get

$$dU/dI = (\lambda P_X\, dX + \lambda P_Y\, dY)/(P_X\, dX + P_Y\, dY) = \lambda \quad \text{(A4.14)}$$

Thus the Lagrange multiplier is the extra utility that results from an extra dollar of income.

Going back to our original analysis of the conditions for utility maximization, we see from equation (A4.5) that maximization requires that the utility obtained from the consumption of every good, per dollar spent on that good, be equal to the marginal utility of an additional dollar of income. If this were not the case, utility could be increased by spending more on the good with the higher ratio of marginal utility to price, and less on the other good.

To help clarify these results, lets go back to our Cobb-Douglas utility function example. In the Cobb-Douglas example, we saw that when $U = a \log(X) + (1-a)\log Y$, the demand functions were $X = (a/P_X)I$, and $Y = [(1-a)P_Y]I$, and the Lagrange multiplier was $\lambda = 1/I$. Now we can see how the Lagrange multiplier can be interpreted when specific values have been chosen for each of the parameters in the problem. Let $a = 1/2$, $P_X = \$1$, $P_Y = \$2$, and $I = \$100$. Then the choices that maximize utility are $X = 50$ and $Y = 25$. Also note that $\lambda = 1/100$. The Lagrange multiplier tells us that if an additional dollar of income were available to the consumer, the level of .utility achieved would increase by 1/100, This is relatively easy to check. With an income of \$101, the maximizing choices of the two goods are $X = 50.5$ and $Y = 25.25$. A bit of arithmetic tells us that the original level of utility is 3.565, and the'new level of utility is 3.575. As we can see, the additional dollar of income has indeed increased utility by .01, or 1/100.

Duality in Consumer Theory

One important feature of consumer theory is the *dual* nature of the consumer's decision. The optimum choice of X and Y can be analyzed not only as the problem of choosing the highest indifference curve (the maximum value of $U()$) that touches the budget line, but also as the problem of choosing the lowest budget line (the minimum budget expenditure) that touches a given indifference curve. To see this, consider the following dual consumer optimization problem, the problem of minimizing the cost of achieving a particular level of utility:

$$\text{Minimize } P_X X + P_Y Y$$

subject to the constraint that

$$U(X,Y) = U^*$$

The corresponding Lagrangian is given by

$$\Phi = P_X X + P_Y Y - \mu(U(X,Y) - U^*) \qquad \text{(A4.15)}$$

where μ is the Lagrange multiplier. Differentiating Φ with respect to X, Y, and μ, *and* setting the derivatives equal to zero, we find the following necessary conditions for expenditure minimization:

$$P_X - \mu MU_X(X,Y) = 0$$

$$P_Y - \mu MU_Y(X,Y) = 0$$

and

$$U(X,Y) = U^*$$

By solving the first two equations, we see that

$$\mu = [P_X/MU_X(X,Y)] = [P_Y/MU_Y(X,Y)] = 1/\lambda$$

Because it is also true that $MU_X(X,Y)/MU_Y(X,Y) = MRS_{XY} = P_X/P_Y$, the cost-minimizing choice of X and Y must occur at the point of tangency of the budget line and the indifference curve that generates utility U^*. Because this is the same point that maximized utility in our original problem, the dual expenditure minimization problem yields the same demand functions that are obtained from the direct utility maximization problem.

To see how the dual approach works, let's reconsider the Cobb-Douglas example again. The algebra is somewhat easier to follow if we used the exponential form of the Cobb-Douglas utility function, $U(X,Y) = X^aY^{1-a}$, and we will do so here. In this case, the Lagrangian is given by

$$\Phi = P_XX + P_YY - \mu[X^aY^{1-a} - U^*] \quad \text{(A4.16)}$$

Differentiating with respect to X, Y, and μ and equating to zero, we obtain

$$P_X = \mu a U^*/X$$

$$P_Y = \mu(1 - a)U^*/Y$$

Multiplying the first equation by X and the second by Y and adding, we get

$$P_XX + P_YY = \mu U^*$$

If we let I be the cost-minimizing expenditure (the individual must spend all of his income to get utility level U^* or U^* would not have maximized utility in the original problem), then it follows that $\mu = I/U^*$. Substituting in the equations above, we obtain

$$X = aI/P_X \text{ and } Y = (1 - a)I/P_Y$$

These are the same demand functions that we obtained before.

Income and Substitution Effects

The demand function tells us how any individual's utility-maximizing choices respond to changes in income and in the prices of goods. It is important, however, to distinguish that portion of any price change that involves the movemerit along an indifference curve and that portion that involves a movement to a different indifference curve (and therefore a change in purchasing power). To do this, we consider what happens to the demand for good X when the price of X changes. The change in demand can be divided into a substitution effect (the change in quantity demanded when the level of utility is fixed) and an income effect (the change in the quantity demanded with the level of utility changing but the relative price of good X unchanged). We denote the

Slutsky demand-the change in X that results from a unit change in the price of X holding utility constant-by $\partial X/\partial P_X|_{U=U^*}$. Thus the total change in X resulting from a unit change in Px is

$$dX/dP_X = \partial X/\partial P_X|_{U=U^*} + (\partial X/\partial I)(\partial I/\partial P_X) \quad \text{(A4.17)}$$

The first term on the right-hand side of equation (A4.17) is the substitution effect (because utility is fixed), and the second term is the income effect (because income increases).

From the consumer's budget constraint, $I = P_X X + P_Y Y$, we know by differentiation that

$$\partial I/\partial P_X = X \quad \text{(A4.18)}$$

Suppose for the moment that the consumer owned goods X and Y. Then equation (A4.18) would tell us that when the price of good X increases by $1, the amount of income the consumer can obtain by selling the good increases by $$X$. In our theory of the consumer, however,the consumer does not own the good. As a result, equation (A4.18) tells us how much additional income the consumer would need to leave him as well off after the price change as before. For this reason, it is customary to write the income effect as negative (reflecting a loss of purchasing power) rather than positive. Equation (A4.17) then appears as follows:

$$dX/dP_X = \partial X/\partial P_X|_{U=U^*} - X(\partial X/\partial I) \quad \text{(A4.19)}$$

In this new form, called the *Slutsky equation,* the first term represents the substitution effect, the change in demand for good X obtained by keeping utility fixed. The second term is the income effect, the change in purchasing power resulting from the price change times the change in demand resulting from that change in purchasing power.

An alternative way to decompose a price change into substitution and income effects does not involve indifference curves; the approach is usually attributed to Hicks. In Figure 4.17, the consumer initially chooses market basket *A* on budget line *RS.* Suppose that after the price of food falls (and the budget line moves to RT), we take away enough income from the individual, so that he is no better off (and no worse off) than he was before. To do so, we draw a budget line parallel to *RT.* If the budget line passed through *A*, then the consumer would be at least as satisfied as before the price change because he has the option to purchase market basket *A* if he wishes. Therefore, the budget line that leaves him equally as well off must be a line such as R'T', which is parallel to *RT* and which intersects *RS* at a point *B* below and to the right of point *A*.,

Revealed preference tells us that the new market basket that is chosen must lie on line segment *BT*, since all market baskets on line segment *R'B* could have been chosen but were not when the original budget line was *RS.* (Recall that the consumer preferred market basket *A* to any other feasible market basket.) Now note that all points on line segment *R'B* involve more food consumption than does market basket *A*. It follows that the quantity of food de-

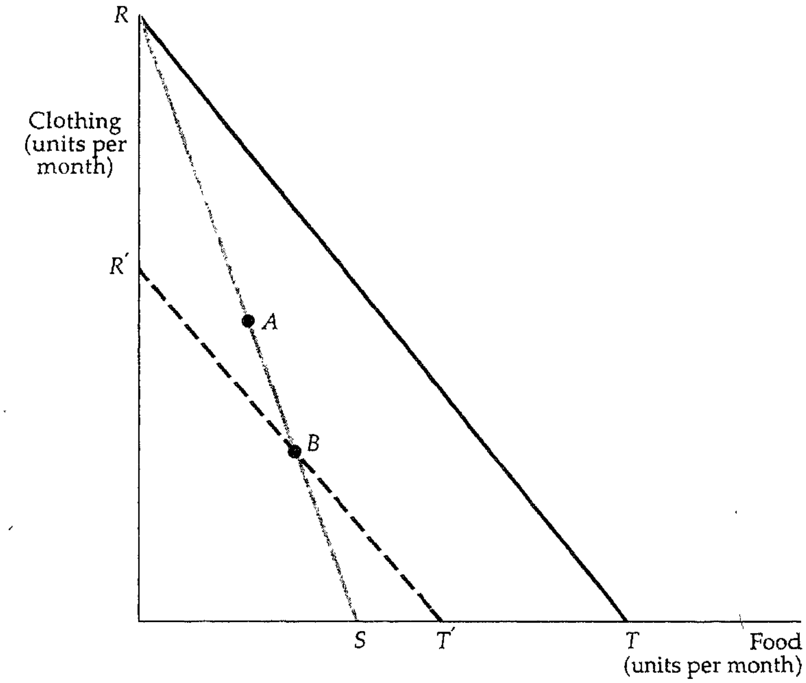

FIGURE 4.17 Hicksian Substitution Effect The individual initially consumes market basket *A*. A decrease in the price of food shifts the budget line from *RS* to *RT*. If a sufficient amount of income is taken away from the individual to make him no better off than he was at *A*, the new market basket chosen must lie on line segment *BT'* of budget line *R'T'* (which intersects *RS to* the right of *A*), and the quantity of food consumed must be greater than at *A*.

manded increases whenever there is a decrease in the price of food, holding utility constant. This negative substitution effect holds for all price changes and does not rely on the assumption of convexity of preferences.

Exercises

1. Which of the following utility functions are consistent with convex indifference curves, and which are not?

a. $U(X,Y) = 2X + 5Y$

b. $U(X,Y)= (XY)$5

c. LJ(X,Y) = Min (X,Y), where Min is the minimum of the two values of X and Y.

2. Show that two utility functions given below generate the identical demand functions for goods X and Y:

a. $U(X,Y) = \log(X) + \log(Y)$

b. $U(X,Y) = (XY)$5

3. Assume that a utility function is given by Min(X,Y), as in Exercise Ic. What is the Slutsky equation that decomposes the change in the demand for X in response to a change in its price? What is the income effect? What is the substitution effect?