CBSE Test Paper 03

Chapter 4 Quadratic Equation

1. The sum S of first n even natural numbers is given by the relation S = n(n +1). If the sum is 420, then the value of 'n' is **(1)**

   a. 20
   b. 21
   c. 24
   d. 22

2. The discriminant of the equation (2a + b) x = x^2 + 2ab is _____ **(1)**

   a. (2a + b2)
   b. (2a – b)2
   c. (2a + b)2
   d. (2a –b2)

3. A cyclist takes 2 hours less to cover a distance of 200 km, if he increases his speed by 5 km/hr. Then his original speed is **(1)**

   a. 26 km/hr
   b. 20 km/hr
   c. 24 km/hr
   d. 25 km/hr

4. If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + c^2 + b^2 = 0$ are equal, then **(1)**

   a. $b = ac$
   b. $b^2 = ac$
   c. $b = \frac{2ac}{a+c}$
   d. $2b = a + c$

5. If α and β are the roots of $2x^2 - 3x - 1 = 0$, then the value of $\alpha^2 + \beta^2$ is **(1)**

a. $\frac{13}{2}$
b. $\frac{13}{4}$
c. $\frac{-13}{2}$
d. $\frac{-13}{4}$

6. Show that x = - 3 is a solution of $x^2 + 6x + 9 = 0$. **(1)**

7. If - 2 is a root of the equation $3x^2 - 5x + 2k = 0$, find the value of k. **(1)**

8. Check whether $x^3 - 3x^2 + 5x = (x - 2)^3$ is quadratic equation or not. **(1)**

9. If p, q and r are rational numbers and $p \neq q \neq r$, then find the roots of the equation $(p^2 - q^2)x^2 - (q^2 - r^2)x + r^2 - p^2 = 0$. **(1)**

10. Solve the quadratic equation by factorization:
    $x^2 - x - a(a + 1) = 0$ **(1)**

11. The product of Tanvy's age (in years) 5 years ago and her age 8 years later is 30. Find her present age. **(2)**

12. Solve: $4x^2 - 12x + 9 = 0$. **(2)**

13. Show that the equation $x^2 + 6x + 6 = 0$ has real roots and solve it. **(2)**

14. If –4 is a root of the quadratic equation $x^2 + px – 4 = 0$ and the quadratic equation $x^2 + px + k = 0$ has equal roots. Find the value of k. **(3)**

15. Solve the quadratic equations by factorization method
    $\frac{4}{x} - 3 = \frac{5}{2x+3} x \neq 0, \frac{-3}{2}$ **(3)**

16. Some students planned a picnic. The total budget for hiring a bus was Rs. 1440. Later on, eight of these refused to go and instead paid their total share of money towards the fee of one economically weaker student of their class, and thus, the cost for each member who went for picnic, increased by Rs. 30.
    i. How many students attended the picnic?
    ii. How much money in total was paid towards the fee? Which value is reflected in

this question? **(3)**

17. Find the values of k for which the given equation has real and equal roots:
    $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ **(3)**

18. There is a square field whose side is 44 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at Rs.2.75 and Rs. 1.50 per square metre, respectively, is Rs.4904. Find the width of the gravel path. **(4)**

19. If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, find the values of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal. **(4)**

20. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal. **(4)**

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Solution

1. a. 20

   Explanation: Given: $n\left(n+1\right)=420$

   $\Rightarrow n^2+n=420$

   $\Rightarrow n^2+n-420=0$

   $\Rightarrow n^2+21n-20n-420=0$

   $\Rightarrow n\left(n+21\right)-20\left(n+21\right)=0$

   $\Rightarrow \left(n-20\right)\left(n+21\right)=0$

   $\Rightarrow n-20=0, n+21=0$

   $\Rightarrow n=20$ and $n=-21$ [$n=-21$ is not possible]

   Therefore, the value of n is 20.

2. b. $(2a – b)^2$

   Explanation: $(2a + b)\ x = x^2 + 2ab$

   $x^2 - (2a + b)\ x + 2ab = 0$

   $D = b^2 - 4ac$

   $D = (2a + b)^2 - 4\times1\times2ab$

   $D = 4a^2 + b^2 + 4ab - 8ab$

   $D = 4a^2 + b^2 - 4ab$

   $D = (2a - b)^2$

3. b. 20 km/hr

   Explanation: Let the original speed be x km/h

   $\therefore$ Time taken to cover 200 km at the rate of x km/h = $\frac{200}{x}$ hrs

   New rate = $\left(x+5\right)$ km/h

   $\therefore$ Time taken to cover 200 km at new rate = $\frac{200}{x+5}$ hrs

   According to question, $\frac{200}{x}-\frac{200}{x+5}=2$

   $\Rightarrow 200\left[\frac{1}{x}-\frac{1}{x+5}\right]=2$

   $\Rightarrow 200\left[\frac{x+5-x}{x(x+5)}\right]=2$

$\Rightarrow \frac{1000}{x^2+5x} = 2$

$\Rightarrow x^2 + 5x - 500 = 0$

$\Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x\left(x + 25\right) - 20\left(x + 25\right) = 0$

$\Rightarrow \left(x + 25\right)\left(x - 20\right) = 0$

$\Rightarrow x + 25 = 0$ and $x - 20 = 0$

$\Rightarrow x = -25$ and $x = 20$

Therefore, the original speed is 20 km/h.

4. b. $b^2 = ac$

Explanation: If the roots of the equation

$(a^2 + b^2)x^2 - 2b(a + c)x + c^2 + b^2 = 0$ are equal, then $b^2 - 4ac = 0$

$\Rightarrow \left[-2b\left(a + c\right)\right]^2 - 4 \times \left(a^2 + b^2\right) \times \left(c^2 + b^2\right) = 0$

$\Rightarrow 4b^2\left[a^2 + c^2 + 2ac\right] - 4\left[a^2c^2 + a^2b^2 + b^2c^2 + b^4\right] = 0$

$\Rightarrow 4\left[a^2b^2 + b^2c^2 + 2ab^2c - a^2c^2 - a^2b^2 - b^2c^2 - b^4\right] = 0$

$\Rightarrow 2ab^2c - a^2c^2 - b^4 = 0$

$\Rightarrow \left(b^2 - ac\right)^2 = 0$

$\Rightarrow b^2 - ac = 0$

$\Rightarrow b^2 = ac$

5. b. $\frac{13}{4}$

Explanation: Here $a = 2, b = -3, c = -1$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{(-3)^2 - 2\times 2\times (-1)}{(2)^2} = \frac{9+4}{4} = \frac{13}{4}$

6. $x^2 + 6x + 9 = 0$.

Put x = -3 in the equation

$\Rightarrow (-3)^2 + 6(-3) + 9$

$\Rightarrow 9 - 18 + 9 = 0$

Hence, it is a solution of the given equation.

7. We have the following equation,

$3x^2 - 5x + 2k = 0$

Putting x = -2

$3(-2)^2 - 5(-2) + 2k = 0$

$\Rightarrow 12 + 10 + 2k = 0$

$\Rightarrow 22 + 2k = 0$

$\Rightarrow k = \frac{-22}{2}$

$\Rightarrow k = -11$

8. Here given equation is

   $x^3 - 3x^2 + 5x = (x - 2)^3$

   $\Rightarrow x^3 - 3x^2 + 5x = x^3 - 6x^2 + 12x - 8$

   $\Rightarrow 3x^2 - 7x + 8 = 0$

   which is of the form $ax^2 + bx + c = 0$

   Hence, given equation is a quadratic equation.

9. In quadratic equation $ax^2 + bx + c = 0$, if $a + c = b$, then roots are -1 and $\frac{-c}{a}$.

   Here $p^2 - q^2 + r^2 - p^2 = -(q^2 - r^2)$

   $\therefore$ roots are -1 and $\frac{-(r^2 - p^2)}{p^2 - q^2}$.

10. According to the question, $x^2 - x - a(a + 1) = 0$

    $\Rightarrow x^2 - (a + 1)x + ax - a(a + 1) = 0$

    $\Rightarrow x(x - a - 1) + a(x - a - 1) = 0$

    $\Rightarrow (x - a - 1)(x + a) = 0$

    $\Rightarrow x - a - 1 = 0$ or $x + a = 0$

    $\Rightarrow x = a + 1$ or $x = -a$

11. Let the present age of tanvy be x years

    Tanvy's age five years ago = (x - 5)

    Tanvy's age eight years later = (x + 8)

    According to question,

    $(x - 5)(x + 8) = 30$

    $\Rightarrow x^2 + 8x - 5x - 40 = 30$

    $\Rightarrow x^2 + 3x - 40 - 30 = 0$

$\Rightarrow x^2 + 3x - 70 = 0$

$\Rightarrow x^2 + 10x - 7x - 70 = 0$

$\Rightarrow x(x + 10) - 7(x + 10) = 0$

$\Rightarrow x + 10 = 0$ or $x - 7 = 0$

$\Rightarrow x = -10$ or $x = 7$

$\Rightarrow x = 7$ ($\because$ age cannot be negative)

Therefore, the present age of tanvy is 7 years.

12. We have,

$4x^2 - 12x + 9 = 0$

Here, $4 \times 9 = 36$ so to factor the middle term in given equation we have $(-6) \times (-6) = 36$, and $(-6) + (-6) = -12$.

$\Rightarrow 4x^2 - 6x - 6x + 9 = 0 \Rightarrow 2x(2x - 3) - 3(2x - 3) = 0$

$\Rightarrow (2x - 3)(2x - 3) = 0 \Rightarrow (2x - 3)^2 = 0$

$\Rightarrow 2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Hence, $x = \frac{3}{2}$ is the repeated root of the given equation.

13. We have the following equation,

$x^2 + 6x + 6 = 0$ where

$a = 1$, $b = 6$ and $c = 6$.

$\therefore D = (b^2 - 4ac) = (36 - 4 \times 1 \times 6) = 12 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{12} = 2\sqrt{3}$

$\therefore \quad \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{(-6+2\sqrt{3})}{2\times 1} = \frac{(-6+2\sqrt{3})}{2}$ = (-3 + $\sqrt{3}$),

$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{(-6-2\sqrt{3})}{2\times 1} = \frac{(-6-2\sqrt{3})}{2} = (-3 - \sqrt{3})$

Therefore, (-3 + $\sqrt{3}$) and (-3 - $\sqrt{3}$) are the roots of the equation.

14. We have, $x^2 + px - 4 = 0$

$\because$ -4 is the root of the given equation

Substitute $x = -4$ in the given equation, we get

$(-4)^2 + p(-4) - 4 = 0$

$\Longrightarrow 16 - 4p - 4 = 0$

$\Longrightarrow 4p = 12$ or $p = 3$

The equation becomes $x^2 + 3x - 4 = 0$

Substituting the value of p = 3 in the equation $x^2 + px + k = 0$, we get

$x^2 + 3x + k = 0$

Here a = 1, b = 3, c= k

$\therefore$ D = $b^2 - 4ac = (3)^2 - 4\ (1)\ (k)$

= 9 - 4k

For equal roots, D = 0

$\implies$ 9 - 4k = 0 or $k = \frac{9}{4}$

15. Given,

$\frac{4}{x} - 3 = \frac{5}{2x+3}$

Taking LCM, we get

$\Rightarrow \quad \frac{4-3x}{x} = \frac{5}{2x+3}$

After cross multiplication,

$\Rightarrow$ (4 - 3x) (2x + 3) = 5x

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 12 - x - 6x^2 = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow$ x (x + 2) - 1(x+ 2) = 0

$\Rightarrow$ (x + 2)(x - 1) = 0 $\Rightarrow$ x + 2 = 0 or, x - l = 0 $\Rightarrow$ x = - 2 or, x = 1

16. Let x students planned the picnic.

Then, (x - 8) students attended the picnic.

Total bus charges = Rs. 1440

$\therefore \quad \frac{1440}{(x-8)} - \frac{1440}{x} = 30$

$\Rightarrow \frac{1}{(x-8)} - \frac{1}{x} = \frac{30}{1440} \Rightarrow \frac{x-(x-8)}{(x-8)x} = \frac{1}{48}$

$\Rightarrow \quad \frac{8}{(x^2-8x)} = \frac{1}{48} \Rightarrow x^2 - 8x = 384$ [by cross multiplication]

$\Rightarrow x^2 - 8x - 384 = 0 \Rightarrow x^2 - 24x + 16x - 384 = 0$

$\Rightarrow$ x(x - 24) + 16(x - 24) = 0 $\Rightarrow$ (x - 24)(x + 16) = 0

$\Rightarrow$ x - 24 = 0 or x + 16 = 0 $\Rightarrow$ x = 24 or x = -16

$\Rightarrow$ x = 24 [$\because$ number of students cannot be negative]

Thus, 24 students planned the picnic.

i. Number of students who attended the picnic = (24 - 8) = 16

ii. Share of 24 students = Rs. 1440

Share of 8 students = Rs. $\left(\frac{1440}{24} \times 8\right)$ = Rs. 480

$\therefore$ money paid towards the fee = Rs. 480

The value reflected in the given question is 'charity'.

17. Given, x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0

Here, a = 1, b = -2(1 + 3k) and c = 7(3 + 2k)

The given equation will have equal roots, if

D = 0

$\Rightarrow$ b^2 - 4ac = 0

$\Rightarrow$4(1 + 3k)^2 - 4 $\times$ 1 $\times$ 7(3 + 2k) = 0

$\Rightarrow$4(3k + 1)^2 - 4 $\times$ 1 $\times$ 7(3 + 2k) = 0

$\Rightarrow$ 4(9k^2 + 1 +6k) - 4(21 + 14k) = 0

$\Rightarrow$ 4(9k^2 +1 +6k) - 84 - 56k = 0

$\Rightarrow 36k^2 + 4 + 24k - 84 - 56k = 0$

$\Rightarrow$ 4(9k^2 - 8k- 20) = 0

$\Rightarrow$ 9k^2 - 8k -20 = 0

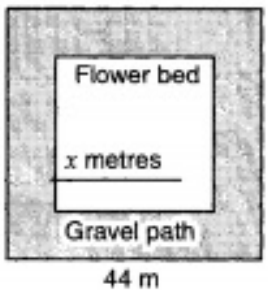
$\Rightarrow$ 9k^2 - 18k + 10k - 20 = 0

$\Rightarrow$ (k - 2) (9k + 10) = 0 $\Rightarrow$ k - 2 = 0 or, 9k + 10 = 0 $\Rightarrow k = 2 \text{ or, } k = -\frac{10}{9}$

18. Let width of the gravel path be x m. Then,

Each side of the square flower bed = (44 - 2x) m.

Area of the square field = 44 $\times$ 44 = 1936 m^2



Area of the flower bed = (44 - 2x)(44 - 2x) = $(44 - 2x)^2$ m^2

$\therefore$ Area of the gravel path = Area of field - Area of flower bed

= 1936 - $(44 - 2x)^2$

= 1936 - $(1936 - 176x + 4x^2)$

= 1936 - 1936 + 176x - $4x^2$

= $(176x - 4x^2)$ m^2

Cost of laying the flower bed = (Area of flower bed) (Rate per sq. m)

$=(44-2x)^2 \times \frac{275}{100} = \frac{11}{4}(44-2x)^2 = 11(22-x)^2$

Cost of gravelling the path = (Area of the path) $\times$ (Rate per sq. m)

$=(176x-4x^2)\frac{150}{100} = 6(44x-x^2)$

According to question,

$\therefore 11(22-x)^2+6(44x-x^2)=4904$

$\Rightarrow$ $11(484 - 44x + x^2) + (264x - 6x^2) = 4904$

$\Rightarrow$ $5x^2 - 220x + 5324 = 4908$

$\Rightarrow$ $5x^2 - 220x + 420 = 0$

$\Rightarrow$ $x^2 - 44x + 84 = 0$

$\Rightarrow$ $x^2 - 42x - 2x + 84 = 0$

$\Rightarrow x(x-42)-2(-42)=0$

$\Rightarrow$ $(x-2)(x-42)=0$

$\Rightarrow$ $x=2$ or $x=42$

But, $x \neq 42$, as the side of the square is 44 m. Therefore, x = 2.

Hence, the width of the gravel path is 2 metres.

19. Here, x = - 2 is a root of $3x^2 + 7x + p = 0$

$\Rightarrow 3(-2)^2 + 7\times(-2) + p = 0$

$\Rightarrow p = 2$

$\therefore x^2 + k(4x + k - 1) + p = 0$ becomes

$x^2 + 4kx + k^2 - k + 2 = 0$....(i)

Comparing eq. (i) with $ax^2 + bx + c = 0$, we get

a = 1 , b = 4k and c = k^2 - k + 2

Roots of eq (i) are equal

So, D =b^2 - 4ac =0

$\Rightarrow (4k)^2 - 4 \times 1 \times (k^2 - k + 2) = 0$

$\Rightarrow 16k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow 12k^2 + 4k - 8 = 0$

$\Rightarrow 12k^2 + 4k - 8 = 0$

$\Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow 3k(k + 1) - 2(k + 1) = 0$

$\Rightarrow (k + 1)(3k - 2) = 0$

$\Rightarrow k = -1, k = \frac{2}{3}$

20. For equal roots of $x^2 + 2px + mn = 0, 4p^2 - 4mn = 0$

or, $p^2 = mn$ .....(i)

For equal roots of

$x^2 - 2(m + n)x + \left(m^2 + n^2 + 2p^2\right) = 0$

$4(m + n)^2 - 4\left(m^2 + n^2 + 2p^2\right) = 0$

$m^2 + n^2 + 2mn - m^2 - n^2 - 2(mn) = 0$ {From (i))

$\therefore$ If root of $x^2 + 2px + mn = 0$ are equal then those of

$x^2 - 2a(m + n)x + \left(m^2 + n^2 + 2p^2\right) = 0$ are equal.