Long Answer Questions

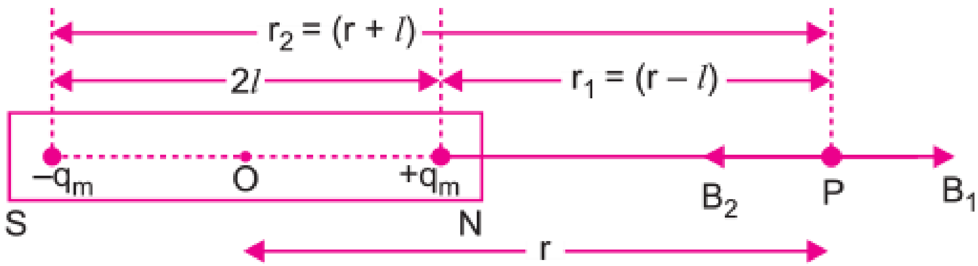
Q. 1. Derive an expression for magnetic field intensity due to a magnetic dipole at a point on its axial line. [5 marks]

Ans. Consider a magnetic dipole (or a bar magnet) SN of length 2l having South Pole at S and North Pole at N. The strength of south and north poles are – q_m and + q_m respectively.

Magnetic moment of magnetic dipole m = q_m 2l, its direction is from S to N.

Consider a point P on the axis of magnetic dipole at a distance r from mid-point O of dipole.

The distance of point P from N-pole,



$r_1 = (r – l)$

The distance of point P from S-pole, $r_2 = (r + l)$

Let B_1 and B_2 be the magnetic field intensities at point P due to north and south poles respectively. The directions of magnetic field due to North Pole is away from N-pole and due to South Pole is towards the S-pole. Therefore,

$$B_1 = \frac{\mu_0}{4\pi} \frac{q_m}{(r - l)^2} \text{ from } N \text{ to } P \text{ and } B_2 = \frac{\mu_0}{4\pi} \frac{q_m}{(r + l)^2} \text{ from } P \text{ to } S$$

Clearly, the directions of magnetic field strengths $\underset{B\,1}{\longrightarrow}$ and $\underset{B\,2}{\longrightarrow}$ are along the same line but opposite to each other and $B_1 > B_2$.

Therefore, the resultant magnetic field intensity due to bar magnet has magnitude equal to the difference of B_1 and B_2 and direction from N to P.

$$i.e., \quad B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{q_m}{(r-l)^2} - \frac{\mu_0}{4\pi}\frac{q_m}{(r+l)^2}$$

$$= \frac{\mu_0}{4\pi} q_m \left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{\mu_0}{4\pi} q_m \left[\frac{(r+l)^2 - (r-l)^2}{(r^2 - l^2)^2}\right]$$

$$= \frac{\mu_0}{4\pi} q_m \left[\frac{4\,\mathrm{rl}}{(r^2 - l^2)^2}\right] = \frac{\mu_0}{4\pi}\frac{2(q_m 2l)r}{(r^2 - l^2)^2}$$

But $q_m\ 2l = m$ (magnetic dipole moment)

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{2m.r}{(r^2 - l^2)^2} \qquad \ldots(1)$$

If the bar magnet is very short and point P is far away from the magnet, the r >> l, therefore, equation (1) takes the form

$$B = \frac{\mu_0}{4\pi}\frac{2\,\mathrm{mr}}{r^4}$$

$$\text{or} \qquad \boxed{B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}} \qquad \ldots(2)$$

This is expression for magnetic field intensity at axial position due to a short bar magnet.

Q. 2. Derive an expression for magnetic field intensity due to a magnetic dipole at a point lies on its equatorial line.

Ans. Consider a point P on equatorial position (or broad side on position) of short bar magnet of length 2l, having north pole (N) and south pole (S) of strength $+q_m$ and $-q_m$ respectively. The distance of point P from the mid-point (O) of magnet is r. Let B_1 and B_2 be the magnetic field intensities due to north and south poles respectively. NP = SP $=\sqrt{r^2} + l^2$.

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{q_m}{r^2+l^2} \text{along } N \text{ to } P$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{q_m}{r^2+l^2} \quad \text{along } P \text{ to } S$$

Clearly, magnitudes of and are equal

$$i.e., |\vec{B}_1| = \vec{B}_2| \qquad \text{or} \qquad B_1 = B_2$$

To find the resultant of $\underset{B1}{\rightarrow}$ and $\underset{B2}{\rightarrow}$ we resolve them along and perpendicular to magnetic axis SN. Components $\underset{B1}{\rightarrow}$ of along and perpendicular to magnetic axis are B_1 cos θ and B_2 sin θ respectively.

Components of $\underset{B2}{\rightarrow}$ along and perpendicular to magnetic axis are B2 cos θ and B2 sin θ respectively. Clearly, components of and perpendicular to axis SN. B1 sin θ and B2 sin θ are equal in magnitude and opposite in direction and hence, cancel each other; while the components of $\underset{B1}{\rightarrow}$ and $\underset{B2}{\rightarrow}$ along the axis are in the same direction and hence, add up to give to resultant magnetic field parallel to the direction $\underset{NS}{\rightarrow}$.

∴ Resultant magnetic field intensity at P.

$B = B_1 \cos\theta + B_2 \cos\theta$

$$\text{But } B_1 = B_2 = \frac{\mu_0}{4\pi}\frac{q_m}{r^2+l^2}$$

$$\text{and } \cos\theta = \frac{\text{ON}}{\text{PN}} = \frac{1}{\sqrt{r^2+l^2}} = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\therefore \quad B = 2B_1\cos\theta = 2\times\frac{\mu_0}{4\pi}\frac{q_m}{(r^2+l^2)}\times\frac{l}{(r^2+l^2)^{1/2}} = \frac{\mu_0}{4\pi}\frac{2q_m l}{(r^2+l^2)^{3/2}}$$

But $q_m.2l = m$, magnetic moment of magnet

$$\therefore \qquad B = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)^{3/2}} \qquad \ldots(3)$$

If the magnet is very short and point P is far away, we have l<<r; so l2 may be neglected as compared to r^2 and so equation (3) takes the form

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{m}{r^3}} \qquad \ldots(4)$$

This is expression for magnetic field intensity at equatorial position of the magnet.

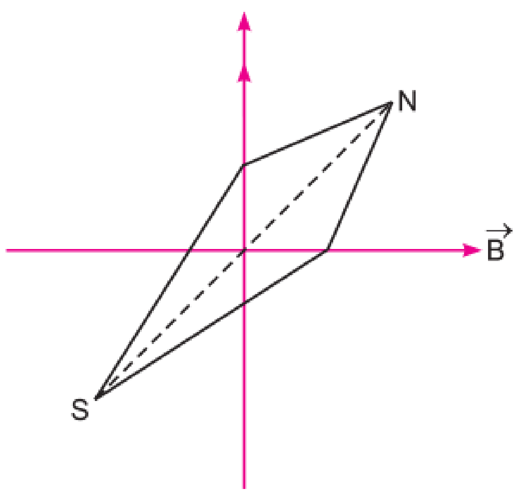
It is clear from equations (2) and (4) that the magnetic field strength due to a short magnetic dipole is inversely proportional to the cube of its distance from the centre of the dipole and the magnetic field intensity at axial position is twice that at equatorial position for the same distance.

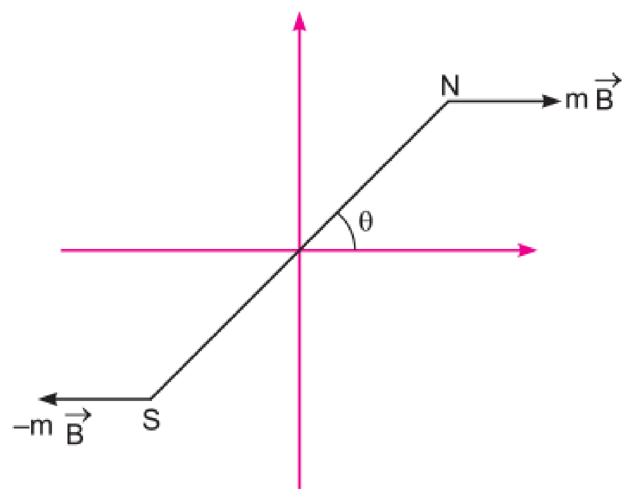
Q. 3. Answer the following the questions.

(i) A small compass needle of magnetic moment 'm' is free to turn about an axis perpendicular to the direction of uniform magnetic field 'B'. The moment of inertia of the needle about the axis is 'I'. The needle is slightly disturbed from its stable position and then released. Prove that it executes simple harmonic motion. Hence deduce the expression for its time period.

(ii) A compass needle, free to turn in a vertical plane orients itself with its axis vertical at a certain place on the earth. Find out the values of (i) horizontal component of earth's magnetic field and (ii) angle of dip at the place.

Ans. (i) If magnetic compass of dipole moment $\vec{m}$ is placed at angle θ in uniform magnetic field, and released it experiences a restoring torque.





$\vec{\tau}$ = – magnetic force × perpendicular distance

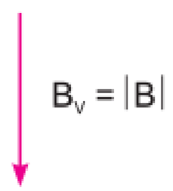
= – $|mB|$. $(2a \sin \theta)$, $\vec{\tau}$ = – $\vec{m} \times \vec{B}$

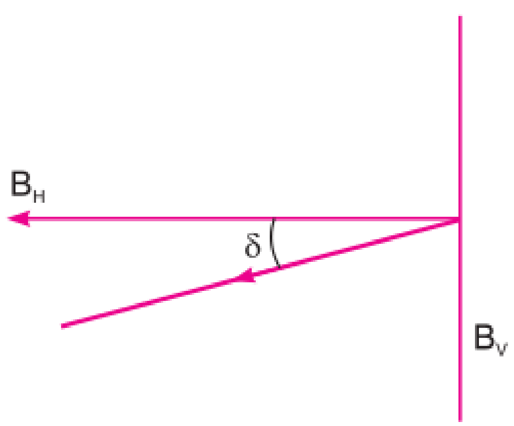
$|\tau|$ = – m $|B|$.sin θ, where m = pole strength

In equilibrium, the equation of motion,

$\Rightarrow I \frac{d^2\theta}{dt^2} = -|m||B|\theta$ (For small angle sin)

$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{|m||B|}{I}\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\left(\frac{mB}{I}\right)\theta$





Since $\frac{d^2\theta}{dt^2} \propto \theta$

It represents the simple harmonic motion with angular frequency

$$\omega^2 = \frac{|m|B|}{I} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mB}}$$