Chapter 5: Gravitation

EXERCISES [PAGES 97 - 99]

Exercises | Q 1. (i) | Page 97

Choose the correct option.

The value of acceleration due to gravity is maximum at ________.

1. the equator of the Earth
2. the center of the Earth
3. **the pole of the Earth**
4. slightly above the surface of the Earth

SOLUTION

The value of acceleration due to gravity is maximum at **the pole of the Earth**.

Exercises | Q 1. (ii) | Page 97

Choose the correct option.

The weight of a particle at the center of the Earth is _______.

1. infinite
2. **zero**
3. same as that at other places
4. greater than at the poles

SOLUTION

The weight of a particle at the center of the Earth is **zero**.

Exercises | Q 1. (iii) | Page 97

Choose the correct option.

The gravitational potential due to the Earth is minimum at _______.

1. **the center of the Earth**
2. the surface of the Earth
3. points inside the Earth but not at its center.
4. infinite distance

SOLUTION

The gravitational potential due to the Earth is minimum at **the center of the Earth**.

Exercises | Q 1. (iv) | Page 97

Choose the correct option.

The binding energy of a satellite revolving around the planet in a circular orbit is 3×109 J. Its kinetic energy is ______.

1. 6×10^9 J

2. -3×10^9 J
3. $-6 \times 10^{+9}$ J
4. **$3 \times 10^{+9}$ J**

SOLUTION

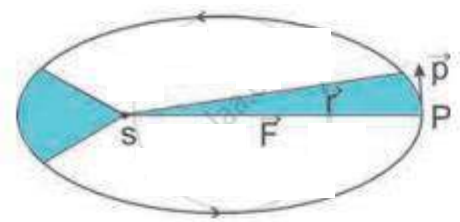
The binding energy of a satellite revolving around the planet in a circular orbit is 3×109 J. Its kinetic energy is **$3 \times 10^{+9}$ J.**

Exercises | Q 2. (i) | Page 97
Answer the following question.
State Kepler's law of equal areas.

SOLUTION

Statement:

The line that joins a planet and the Sun sweeps equal areas in equal intervals of time.



Exercises | Q 2. (ii) | Page 97
Answer the following question.
State Kepler's law of the period.

SOLUTION

Statement:

The square of the time period of revolution of a planet around the Sun is proportional to the cube of the semimajor axis of the ellipse traced by the planet.

Exercises | Q 2. (iii) | Page 97
Answer the following question.
What are the dimensions of the universal gravitational constant?

SOLUTION

The dimensions of universal gravitational constant are: $[L^3M^{-1}T^{-2}]$

Exercises | Q 2. (iv) | Page 97
Answer the following question.
Define the binding energy of a satellite.

SOLUTION

The minimum energy required by a satellite to escape from Earth's gravitational influence is the binding energy of the satellite.

Exercises | Q 2. (v) | Page 97

Answer the following question.
What do you mean by geostationary satellite?

SOLUTION

Some satellites that revolve around the Earth in the equatorial planes have the same sense of rotation as that of the Earth. They also have the same period of rotation as that of the Earth i.e., 24 hours. Due to this, these satellites appear stationary from the Earth's surface and are known as geostationary satellites.

Exercises | Q 2. (vi) | Page 97

Answer the following question.
State Newton's law of gravitation.

SOLUTION

Statement:

Every particle of matter attracts every other particle of matter with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Exercises | Q 2. (vii) | Page 97

Answer the following question.
Define the escape velocity of a satellite.

SOLUTION

The minimum velocity with which a body should be thrown vertically upwards from the surface of the Earth so that it escapes the Earth's gravitational field is called the escape velocity (v_e) of the body.

Exercises | Q 2. (viii) | Page 97

Answer the following question.
What is the variation in acceleration due to gravity with altitude?

SOLUTION

Variation in acceleration due to gravity due to altitude is given by, $g_h = g\left(\frac{R}{R+h}\right)^2$ where,

g_h = acceleration due to gravity of an object placed at h altitude

g = acceleration due to gravity on the surface of the Earth

R = radius of the Earth

h = attitude height of the object from the surface of the Earth.

Hence, acceleration due to gravity decreases with increase in altitude.

Exercises | Q 2. (ix) | Page 97

Answer the following question.

On which factors does the escape speed of a body from the surface of Earth depend?

SOLUTION

The escape speed depends only on the mass and radius of the planet.

Exercises | Q 2. (x) | Page 97

Answer the following question.

As we go from one planet to another planet, how will the mass and weight of a body change?

SOLUTION

1. As we go from one planet to another, mass of a body remains unaffected.
2. However, due to change in mass and radius of planet, acceleration due to gravity acting on the body changes as, $g \propto M/R^2$.

   Hence, weight of the body ∝also changes as, $W \propto M/R^2$.

Exercises | Q 2. (xi) | Page 97

Answer the following question.

What is periodic time of a geostationary satellite?

SOLUTION

The periodic time of a geostationary satellite is same as that of the Earth i.e., one day or 24 hours.

Exercises | Q 2. (xii) | Page 97

Answer the following question.
State Newton’s law of gravitation and express it in vector form.

SOLUTION

1. **Statement:**
   Every particle of matter attracts every other particle of matter with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
2. In vector form, it can be expressed as,
   $$\vec{\mathbf{F}}_{21} = \mathrm{G}\frac{\mathbf{m_1 m_2}}{\mathbf{r^2}}(-\hat{\mathbf{r}}_{21})$$
   where, $\hat{}\ \mathbf{21}$ is the unit vector from m_1 to m_2. The force $\vec{\mathbf{F}}_{21}$ is directed from m_2 to m_1.

Exercises | Q 2. (xiii) | Page 97

Answer the following question.
What do you mean by a gravitational constant? State its SI units.

SOLUTION

1. From Newton’s law of gravitation,
   $$F = G\frac{\mathbf{m_1 m_2}}{\mathbf{r^2}}$$
   where, G = constant called universal gravitational constant. Its value is 6.67×10^{-11} N m^2/kg^2.
2. $$G = \frac{\mathbf{Fr^2}}{\mathbf{m_1 m_2}}$$
   If $m_1 = m_2 = 1$ kg, r = 1 m then F = G.

   Hence, the universal gravitational constant is the force of gravitation between two particles of unit mass separated by unit distance.
3. Unit: N m^2/kg^2 in SI system.

Exercises | Q 2. (xiv) | Page 97

Answer the following question.
Why is a minimum two-stage rocket necessary for launching of a satellite?

SOLUTION

1. For the projection of an artificial satellite, it is necessary for the satellite to have a certain velocity.

2. In a single stage rocket, when the fuel in the first stage of the rocket is ignited on the surface of the Earth, it raises the satellite vertically.

3. The velocity of the projection of the satellite normal to the surface of the Earth is the vertical velocity.

4. If this vertical velocity is less than the escape velocity (v_e), the satellite returns to the Earth's surface. While, if the vertical velocity is greater than or equal to the escape velocity, the satellite will escape from Earth's gravitational influence and go to infinity.

5. Hence, a minimum two-stage rocket, one to raise the satellite to the desired height and another to provide the required horizontal velocity, is necessary for launching of a satellite.

Exercises | Q 2. (xv) | Page 97

Answer the following question.
State the conditions for various possible orbits of satellite depending upon the horizontal speed of projection.

SOLUTION

The path of the satellite depends upon the value of the horizontal speed of projection v_h relative to critical velocity v_c and escape velocity v_e.

Case (I) $v_h < v_c$:

The orbit of the satellite is an ellipse with a point of projection as apogee and Earth at one of the foci. During this elliptical path, if the satellite passes through the Earth's atmosphere, it experiences a nonconservative force of air resistance. As a result, it loses energy and spirals down to the Earth.

Case (II) $v_h = v_c$:

The satellite moves in a stable circular orbit around the Earth.

Case (III) $v_c < v_h < v_e$:

The satellite moves in an elliptical orbit around the Earth with the point of projection as perigee.

Case (IV) $v_h = v_e$:

The satellite travels along the parabolic path and never returns to the point of projection. Its speed will be zero at infinity.

Case (V) $v_h > v_e$:

The satellite escapes from gravitational influence of Earth traversing a hyperbolic path.

Exercises | Q 3. (i) | Page 98

Answer the following question in detail.
Derive an expression for the critical velocity of a satellite.

SOLUTION

The expression for critical velocity:

1. Consider a satellite of mass m revolving round the Earth at height h above its surface. Let M be the mass of the Earth and R be its radius.

2. If the satellite is moving in a circular orbit of the radius (R + h) = r, its speed must be equal to the magnitude of critical velocity v_c.

3. The centripetal force necessary for the circular motion of a satellite is provided by the gravitational force exerted by the satellite on the Earth.
   ∴ Centripetal force = Gravitational force

$$\therefore \frac{mv_c^2}{r} = \frac{GMm}{r^2}$$

$$\therefore v_c^2 = \frac{GM}{r}$$

$$\therefore v_c = \sqrt{\frac{GM}{r}}$$

$$\therefore v_c = \sqrt{\frac{GM}{R+h}} = \sqrt{g_h(R+h)}$$

This is the expression for critical speed at the orbit of radius (R + h).

4. The critical speed of a satellite is independent of the mass of the satellite. It depends upon the mass of the Earth and the height at which the satellite is the revolving or gravitational acceleration at that altitude.

Exercises | Q 3. (ii) | Page 98

Answer the following question in detail.
State any four applications of a communication satellite.

SOLUTION

Applications of communication satellite:

1. For the transmission of television and radio wave signals over large areas of Earth's surface.

2. For broadcasting telecommunication.

3. For military purposes.

4. For navigation surveillance.

Exercises | Q 3. (iii) | Page 98

Answer the following question in detail.

Show that acceleration due to gravity at height h above the Earth's surface is

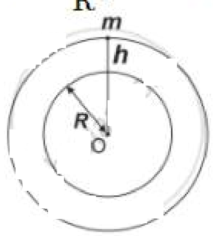
$$g_h = g\left(\frac{R}{R+h}\right)^2$$

Answer the following question in detail.

Discuss the variation of acceleration due to gravity with altitude.

SOLUTION

1. Let, R = radius of the Earth,
   M = mass of the Earth.
   g = acceleration due to gravity at the surface of the Earth.
2. Consider a body of mass m on the surface of the Earth. The acceleration due to gravity on the Earth's surface is given by,
   $g = \frac{GM}{R^2}$ ....(1)



3. The body is taken at height h above the surface of the Earth as shown in the figure. The acceleration due to gravity now changes to,
   $g_h = \frac{GM}{(R+h)^2}$ .....(2)
4. Dividing equation (2) by equation (1), we get,

$$\frac{g_h}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\therefore \frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore g_h = \frac{gR^2}{(R+h)^2}$$

We can rewrite,

$$\therefore g_h = \frac{gR^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\therefore g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

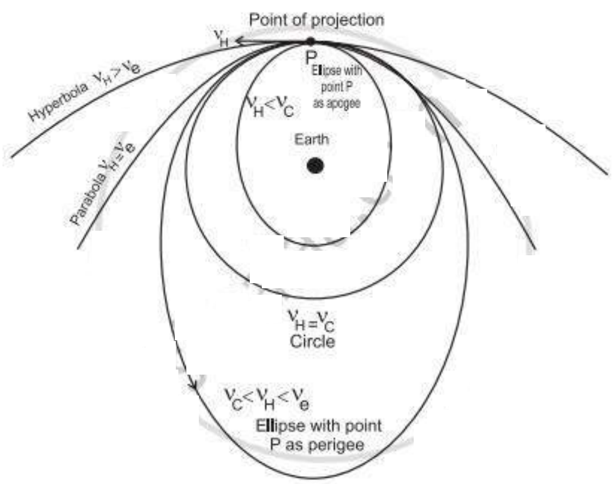
5. For small altitude h, i.e., for $\frac{h}{R} \langle 1$, by neglecting higher power terms of $\frac{h}{R}$, $g_h = g\left(1-\frac{2h}{R}\right)$

This expression can be used to calculate the value of g at height h above the surface of the Earth as long as h << R.

Exercises | Q 3. (iv) | Page 98

Answer the following question in detail.
Draw a well labelled diagram to show different trajectories depending upon the tangential projection speed.

SOLUTION



v_h = horizontal speed of projection

v_c = critical velocity

v_e = escape velocity

Exercises | Q 3. (v) | Page 98

Answer the following question in detail.

Derive an expression for the binding energy of a body at rest on the Earth's surface of a satellite.

SOLUTION

1. Let, M = mass of the Earth
   m = mass of the satellite
   R = radius of the Earth.
2. Since the satellite is at rest on the Earth, v = 0
   ∴ Kinetic energy of satellite,
   K.E. = $\frac{1}{2}mv^2 = 0$
3. Gravitational potential at the Earth's surface = $-\frac{GM}{R}$
   ∴ The potential energy of a satellite
   = Gravitational potential × mass of the satellite
   = $-\frac{GMm}{R}$
4. Total energy of satellite = T.E = P.E + K.E
   ∴ T.E. = $-\frac{GMm}{R} + 0 = -\frac{GMm}{R}$
5. Negative sign in the energy indicates that the satellite is bound to the Earth, due to gravitational force of attraction.
6. For the satellite to be free form Earth's gravitational influence, its total energy should become positive. That energy is the binding energy of the satellite at rest on the surface of the Earth.
   ∴ B.E. = $\frac{GMm}{R}$

Exercises | Q 3. (vi) | Page 98

Answer the following question in detail.
Why an astronaut in an orbiting satellite has a feeling of weightlessness?

SOLUTION

1. For an astronaut, in a satellite, the net force towards the center of the Earth will always be, F = mg – N.
   where, N is the normal reaction.

2. In the case of a revolving satellite, the satellite is performing a circular motion. The acceleration for this motion is centripetal, which is provided by the gravitational acceleration g at the location of the satellite.

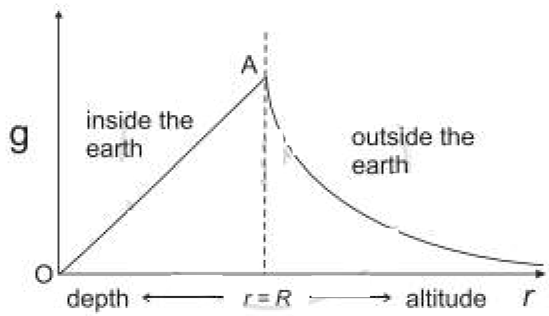
3. In this case, the downward acceleration, a_d = g, or the satellite (along with the astronaut) is in the state of free fall.

4. Thus, the net force acting on astronaut will be, F = mg – ma_d = 0 i.e., the apparent weight will be zero, giving the feeling of total weightlessness.

Exercises | Q 3. (vii) | Page 98

Answer the following question in detail.
Draw a graph showing the variation of gravitational acceleration due to the depth and altitude from the Earth's surface.

SOLUTION



Variation of g due to depth and altitude from the Earth's surface

Exercises | Q 3. (viii) | Page 98

Answer the following question in detail.
At which place on the Earth's surface is the gravitational acceleration maximum? Why?

SOLUTION

1. Gravitational acceleration on the surface of the Earth depends on the latitude of the place as well as the rotation and shape of the Earth.
2. At poles, latitude θ = 90°.
   ∴ g' = g
   i.e., there is no reduction in acceleration due to gravity at poles.
3. Also, shape of the Earth is actually an ellipsoid, bulged at equator. The polar radius of the Earth is 6356 km which minimum. As
   $g \propto \frac{1}{R^2}$, acceleration due to gravity is maximum at poles i.e., 9.8322 m/s^2.

Exercises | Q 3. (ix) | Page 98

Answer the following question in detail.
At which place on the Earth's surface is the gravitational acceleration minimum? Why?

SOLUTION

1. Gravitational acceleration on the surface of the Earth depends on latitude of the place as well as the rotation and shape of the Earth.
2. At equator, latitude θ = 0°.

   ∴ $g' = g - R\omega^2$

   i.e., the acceleration due to gravity is reduced by amount $R\omega^2$ (≈ 0.034 m/s^2) at equator.
3. Also, the shape of the Earth is actually an ellipsoid, bulged at the equator. The equatorial radius of the Earth is 6378 km, which is maximum. As $g \propto \frac{1}{R^2}$, acceleration due to gravity is minimum on the equator i.e., 9.7804 m/s^2.

Exercises | Q 3. (x) | Page 98

Answer the following question in detail.

Derive an expression for variation in gravitational acceleration of the Earth at with latitude.

SOLUTION

1. Latitude is an angle made by the radius vector of any point from the center of the Earth with the equatorial plane.
2. The Earth rotates about its polar axis from west to east with uniform angular velocity ω as shown in the figure. Hence, every point on the surface of the Earth (except the poles) moves in a circle parallel to the equator.
3. The motion of a mass m at point P on the Earth is shown by the dotted circle with the center at O'.
4. Let the latitude of P be θ and the radius of the circle be r.

   ∴ PO' = r

   ∠EOP = θ, E being a point on the equator

   ∴ ∠OPO' = θ

   In Δ OPO', $\cos\theta = \frac{PO'}{PO} = \frac{r}{R}$

   ∴ $r = R\cos\theta$
5. The centripetal acceleration for the mass m, directed along PO' is,

   $a = r\omega^2$

   ∴ $a = r\omega^2 \cos\theta$

   The component of this centripetal acceleration along PO, i.e., towards the centre of the Earth is,

   $a_r = a\cos\theta$

   $\therefore a_r = R\omega^2\cos\theta \times \cos\theta$

   $a_r = R\omega^2\cos^2\theta$

6. Part of the gravitational force of attraction on P acting towards PO is utilized in providing this component of centripetal acceleration. Thus, the effective force of gravitational attraction on m at P can be written as,

   $mg' = mg - mR\omega^2\cos^2\theta$

   Thus, the effective acceleration due to gravity at P is given as,

   $g' = g - R\omega^2\cos^2\theta$

Exercises | Q 3. (xi) | Page 98

Answer the following question.

Define the binding energy of a satellite.

SOLUTION

The minimum energy required by a satellite to escape from Earth's gravitational influence is the binding energy of the satellite.

Exercises | Q 3. (xii) | Page 98

Answer the following question in detail.

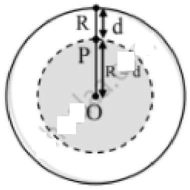
Obtain the formula for the acceleration due to gravity at the depth 'd' below the Earth's surface.

SOLUTION

1. The Earth can be considered to be a sphere made of a large number of concentric uniform spherical shells.
2. When an object is on the surface of the Earth it experiences the gravitational force as if the entire mass of the Earth is concentrated at its center.
3. The acceleration due to gravity on the surface of the Earth is, $g = \frac{GM}{R^2}$
4. Assuming that the density of the Earth is uniform, mass of the Earth is given by

   $M = \text{volume} \times \text{density} = \frac{4}{3}\pi R^3 \rho$

   $\therefore g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = \frac{4}{3}\pi R \rho G$ ....(1)
5. Consider a body at a point P at the depth d below the surface of the Earth as shown in the figure.



Here the force on a body at P due to the outer spherical shell shown by the shaded region, cancel out due to symmetry.

The net force on P is only due to the inner sphere of radius OP = R – d.

6. Acceleration due to gravity because of this sphere is,

$$g_d = \frac{GM'}{(R - d)^2}$$ where,

M' = volume of the inner sphere × density

$$\therefore M' = \frac{4}{3}\pi(R - d)^3 \times \rho$$

$$\therefore g_d = \frac{G \times \frac{4}{3}\pi(R - d)^3\rho}{(R - d)^2}$$

$$\therefore g_d = G \times \frac{4}{3}\pi(R - d)\rho \quad ...(2)$$

7. Dividing equation (2) by equation (1) we get,

$$\frac{g_d}{g} = \frac{R - d}{R}$$

$$\therefore \frac{g_d}{g} = 1 - \frac{d}{R}$$

$$\therefore \frac{g_d}{g} = g\left(1 - \frac{d}{R}\right)$$

This equation gives acceleration due to gravity at depth d below the Earth's surface.

Exercises | Q 3. (xiii) | Page 98

Answer the following question in detail.

State Kepler's three laws of planetary motion.

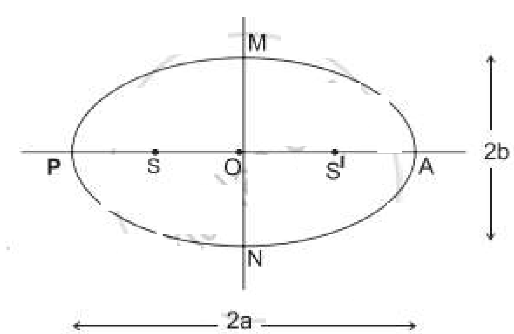
SOLUTION

Kepler's law of orbits:

Statement:

All planets move in elliptical orbits around the Sun with the Sun at one of the foci of the ellipse.

Explanation:



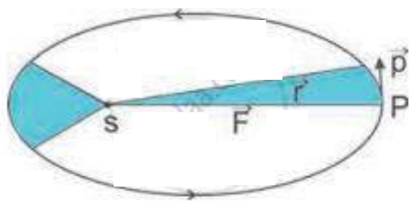
1. The figure shows the orbit of a planet around the Sun.
2. Here, S and S' are the foci of the ellipse and the Sun is situated at S.
3. P is the closest point along the orbit from S and is called perihelion.
4. A is the farthest point from S and is called aphelion.
5. PA is the major axis whose length is 2a. PO and AO are the semimajor axes with lengths ‘a’ each.

   MN is the minor axis whose length is 2b. MO and ON are the semiminor axes with lengths ‘b’ each.

- **Kepler’s law of equal areas:**

Statement:

The line that joins a planet and the Sun sweeps equal areas in equal intervals of time.



Explanation:

1. Kepler observed that planets move faster when they are nearer to the Sun while they move slower when they are farther from the Sun.
2. Suppose the Sun is at the origin. The position of planet is denoted by $\vec{r}$ and its momentum is denoted by $\vec{p}$ (component $\perp$ $\vec{r}$).
3. The area swept by the planet of mass m in given interval Δt is $\overrightarrow{\Delta A}$ which is given by $\overrightarrow{\Delta A} = \frac{1}{2}\left(\vec{r} \times \vec{v}\Delta t\right)$

   .....(1)

   As for small Δt, $\vec{v}$ is perpendicular to $\vec{r}$ and this is the area of the triangle.

   $\therefore \frac{\overrightarrow{\Delta A}}{\Delta t} = \frac{1}{2}\left(\vec{r} \times \vec{v}\right)$ ....(2)
4. Linear momentum $\left(\vec{p}\right)$ is the product of mass and velocity.

   $\vec{p} = m\vec{v}$ ....(3)

   $\therefore$ putting $\vec{v} = \frac{\vec{p}}{m}$ in the equation (2), we get,

   $\frac{\overrightarrow{\Delta A}}{\Delta t} = \frac{1}{2}\left(\vec{r} \times \frac{\vec{p}}{m}\right)$ .....(4)

5. Angular momentum $\vec{L}$ is defined as,

$$\vec{L} = \vec{r} \times \vec{p} \quad ....(5)$$

6. For central force the angular momentum is conserved. Hence, from equations (4) and (5),

$$\frac{\overrightarrow{\Delta A}}{\Delta t} = \frac{\vec{L}}{2\,m} = \text{constant} \quad ......(6)$$

This proves the law of areas.

- **Kepler's law of periods:**

Statement:

The square of the time period of revolution of a planet around the Sun is proportional to the cube of the semimajor axis of the ellipse traced by the planet.

Explanation:

If r is the length of the semimajor axis then, this law states that,

$$T^2 \propto r^3 \text{ or } \frac{T^2}{r^3} = \text{constant}$$

Exercises | Q 3. (xiv) | Page 98

Answer the following question in detail.

State the formula for the acceleration due to gravity at depth 'd' and altitude 'h'. Hence show that their ratio is equal to $\left(\frac{R - d}{R - 2h}\right)$ by assuming that the altitude is very small as compared to the radius of the Earth.

SOLUTION

1. For an object at depth d, acceleration due to gravity of the Earth is given by,

$$g_d = g\left(1 - \frac{d}{R}\right) \quad ....(1)$$

2. Also, the acceleration due to gravity at smaller altitude h is given by,

$$g_h = g\left(1 - \frac{2h}{R}\right) \quad ....(2)$$

3. Hence, dividing equation (1) by equation (2), we get,

$$\frac{g_d}{g_h} = \frac{g\left(1 - \frac{d}{R}\right)}{g\left(1 - \frac{2h}{R}\right)} = \frac{R - d}{R} \times \frac{R}{R - 2h}$$

$$\therefore \frac{g_d}{g_h} = \frac{R - d}{R - 2h}$$

Exercises | Q 3. (xv) | Page 98

Answer the following question in detail.

What is a critical velocity?

SOLUTION

The exact horizontal velocity of projection that must be given to a satellite at a certain height so that it can revolve in a circular orbit round the Earth is called the critical velocity or orbital velocity (v_c).

Exercises | Q 3. (xvi) | Page 98

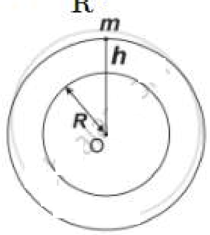
Answer the following question in detail.

Show that acceleration due to gravity at height h above the Earth's surface is

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

SOLUTION

1. Let, R = radius of the Earth,
   M = mass of the Earth.
   g = acceleration due to gravity at the surface of the Earth.
2. Consider a body of mass m on the surface of the Earth. The acceleration due to gravity on the Earth's surface is given by,
   $g = \frac{GM}{R^2}$ ....(1)



3. The body is taken at height h above the surface of the Earth as shown in the figure. The acceleration due to gravity now changes to,
   $g_h = \frac{GM}{(R+h)^2}$ .....(2)

4. Dividing equation (2) by equation (1), we get,

$$\frac{g_h}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\therefore \frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore g_h = \frac{gR^2}{(R+h)^2}$$

We can rewrite,

$$\therefore g_h = \frac{gR^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\therefore g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

5. For small altitude h, i.e., for $\frac{h}{R} \langle 1$, by neglecting higher power terms of $\frac{h}{R}$, $g_h = g\left(1-\frac{2h}{R}\right)$

This expression can be used to calculate the value of g at height h above the surface of the Earth as long as h << R.

Exercises | Q 3. (xvii) | Page 98

Answer the following question in detail.
Define escape speed.

SOLUTION

The minimum velocity with which a body should be thrown vertically upwards from the surface of the Earth so that it escapes the Earth's gravitational field is called the escape velocity (v_e) of the body.

Exercises | Q 3. (xviii) | Page 98

Answer the following question in detail.

Describe how an artificial satellite using a two-stage rocket is launched in an orbit around the Earth.

SOLUTION

1. Launching of a satellite in an orbit around the Earth cannot take place by the use of a single-stage rocket. It requires a minimum of two stage rocket.
2. With the help of the first stage of the rocket, satellite can be taken to the desired height above the surface of the Earth.

3. Then the launcher is rotated in a horizontal direction i.e., through 90° using the remote control and the first stage of the rocket is detached.
4. With the help of the second stage of the rocket, a specific horizontal velocity (v_h) is given to the satellite so that it can revolve in a circular path around the Earth.
5. The satellite follows different paths depending upon the horizontal velocity provided to it.

Exercises | Q 4. (i) | Page 98

Answer the following question in detail.
At what distance below the surface of the Earth, the acceleration due to gravity decreases by 10% of its value at the surface, given the radius of Earth is 6400 km.

SOLUTION

Given: g_d = 90% of g i.e., $\frac{g_d}{g} = 0.9$,

$R = 6400\text{ km} = 6.4 \times 10^6\text{ m}$

To find: Distance below the Earth's surface (d)

Formula: $g_d = g\left(1 - \frac{d}{R}\right)$

Calculation: From formula,

$$\frac{g_d}{g} = \left(1 - \frac{d}{R}\right)$$

$$\therefore \frac{d}{R} = 1 - \frac{g_d}{g}$$

$$\therefore d = R\left(1 - \frac{g_d}{g}\right)$$

$= 6.4 \times 10^6 \times 0.1$

$= 640 \times 10^3$ m

= 640 km

At distance **640 km** below the surface of the Earth, value of acceleration due to gravity decreases by 10%.

Exercises | Q 4. (ii) | Page 98

Answer the following question in detail.
If the Earth were made of wood, the mass of wooden Earth would have been 10% as much as it is now (without change in its diameter). Calculate escape speed from the surface of this Earth.

SOLUTION

Given: M_W = 10% of M = $\frac{M}{10}$, D_W = D or R_W = R,

To find: Escape speed (v_e)

Formula: From formula,

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$\therefore \frac{v_{e_w}}{v_e} = \sqrt{\frac{2GM_w}{R_w} \times \frac{R}{2GM}}$$

$$\therefore \frac{v_{e_w}}{v_e} = \sqrt{\left(\frac{R}{R_w}\right) \times \left(\frac{M_w}{M}\right)} = \sqrt{1 \times \frac{1}{10}} = \sqrt{\frac{1}{10}}$$

$$\therefore \frac{v_{e_w}}{v_e} = v_e \times \frac{1}{\sqrt{10}}$$

As, we know that the escape speed from surface of the Earth is 11.2 km/s,

Substituting value of v_e = 11.2 km/s

$$v_{e_w} = 11.2 \times \frac{1}{\sqrt{10}} = \frac{11.2}{3.162}$$

$$= 11.2 \times \frac{1}{3.1620}$$ ....[Taking square root value]

= antilog{log(11.2) - log(3.162)}

= antilog{1.0492 - 0.5000}

= antilog{0.5492} = 3.542

$\therefore v_{e_w}$ = **3.54 km/s**

The escape velocity from the surface of wooden Earth is 3.54 km/s.

Exercises | Q 4. (iii) | Page 98

Answer the following question in detail.

Calculate the kinetic energy, potential energy, total energy and binding energy of an artificial satellite of mass 2000 kg orbiting at a height of 3600 km above the surface of the Earth.

Given: G = 6.67×10^{-11} Nm^2/kg^2
R = 6400 km, M = 6×10^{24} kg

SOLUTION

Given: m = 2000 kg, h = 3600 km = 3.6×10^6 m,

G = 6.67×10^{-11} Nm^2/kg^2,

R = 6400 km = 6.4×10^6 m,

M = 6×10^{24} kg

To find: 1. Kinetic energy (K.E.)

2. Potential energy (P.E.)

3. Total energy (T.E.)

4. Binding energy (B.E.)

Formulae: 1. K.E. = $\frac{GMm}{2(R + h)}$

2. P.E. = $-\frac{GMm}{R+h}$ = - 2(K.E.)

3. T.E. = K.E. + P.E.

4. B.E. = –T.E.

Calculation: From formula (i),

$$K.E. = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^{3}}{2 \times \left[\left(6.4 \times 10^{6}\right) + \left(3.6 \times 10^{6}\right)\right]}$$

$$= \frac{6.67 \times 6 \times 10^{16}}{10^{7}}$$

= **40.02×10^9 J**

From formula (ii),

P.E. = $-2 \times 40.02 \times 10^9$

= **-80.04×10^9 J**

From formula (iii),

T.E. = $(40.02 \times 10^9) + (-80.02 \times 10^9)$

= **-40.02×10^9 J**

From formula (iv),

B.E. = $-(-40.02 \times 10^9)$

= **40.02×10^9 J**

Exercises | Q 4. (iv) | Page 98

Answer the following question in detail.

Two satellites A and B are revolving round a planet. Their periods of revolution are 1 hour and 8 hour respectively. The radius of orbit of satellite B is 4×10^4 km. Find radius of orbit of satellite A.

SOLUTION

Given: T_A = 1 hour, T_B = 8 hour, $r_B = 4 \times 10^4$ km

To find: Radius of orbit of satellite A (r_A)

Formula: $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Calculation: From formula,

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\therefore T^2 \propto r^3 \quad ...... \left(\because \frac{4\pi^2 r^3}{GM} = \text{constant for a planet}\right)$$

$$\therefore \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

$$\therefore \left(\frac{1}{8}\right)^2 = \left(\frac{r_A}{4 \times 10^4}\right)^3$$

$$\therefore r_A^3 = \frac{1}{(8)^2} \times \left(4 \times 10^4\right)^3$$

$$\therefore r_A^3 = 10^{12}$$

$$\therefore r_A = 1 \times 10^4 \text{ km}$$

Exercises | Q 4. (v) | Page 99

Solve the following problem.

Find the gravitational force between the Sun and the Earth.
Given Mass of the Sun = 1.99×10^{30} kg
Mass of the Earth = 5.98×10^{24} kg
The average distance between the Earth and the Sun = 1.5×10^{11} m.

SOLUTION

Given: $M_S = 1.99 \times 10^{30}$ kg, $M_E = 5.98 \times 10^{24}$ kg, $R = 1.5 \times 10^{11}$ m

To find: Gravitational force between the Sun and the Earth (F)

Formula: $F = \frac{Gm_1m_2}{r^2}$

Calculation: As, we know, $G = 6.67 \times 10^{-11}$ N m^2/kg^2

From formula,

$$F = \frac{GM_sM_E}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 5.98 \times 10^{24}}{(1.5 \times 10^{11})^2}$$

$$\therefore F = \frac{6.67 \times 1.99 \times 5.98}{2.25} \times 10^{21}$$

$= \text{antilog}\{(\log(6.67) + \log(1.99) + \log(5.98) - \log(2.25)\} \times 10^{21}$

$= \text{antilog}\{(0.8241) + (0.2989) + (0.7767) - (0.3522)\} \times 10^{21}$

$= \text{antilog}\{1.5475\} \times 10^{21}$

$= 35.28 \times 10^{21}$

$= 3.5 \times 10^{22}$ N

The gravitational force between the Sun and the Earth is **$= 3.5 \times 10^{22}$ N.**

Exercises | Q 4. (vi) | Page 99

Solve the following problem.

Calculate the acceleration due to gravity at a height of 300 km from the surface of the Earth. ($M = 5.98 \times 10^{24}$ kg, R = 6400 km).

SOLUTION

Given: h = 300 km = 0.3 × 10^6 m, M = 5.98 × 10^{24} kg, R = 6400 km = 6.4 × 10^6 m, G = 6.67 × 10^{-11} Nm^2/kg^2

To find: Acceleration due to gravity at height (g_h)

Formula: $g_h = \frac{GM}{(R+h)^2}$

Calculation: From formula,

$$g_h = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{\left[\left(6.4 \times 10^6\right) + \left(0.3 \times 10^6\right)\right]^2}$$

$$= \frac{6.67 \times 5.98 \times 10^{13}}{(6.7)^2 \times 10^{12}}$$

= antilog{log(6.67) + log(5.98) - 2log(6.7)} × 10

= antilog{0.8241 + 0.7767 - 2(0.8261)} × 10

= antilog{1.6008 - 1.6522} × 10

= antilog{$\bar{1}$.9486} × 10

= 0.8884 × 10 = 8.884 m/s^2

Acceleration due to gravity at 300 km will be **8.884 m/s^2.**

Exercises | Q 4. (vii) | Page 99

Solve the following problem.
Calculate the speed of a satellite in an orbit at a height of 1000 km from the Earth's surface.
(M_E = 5.98 × 10^{24} kg, R = 6.4 × 10^6 m)

SOLUTION

Given: h = 1000 km = 1 × 10^6 m, M_E = 5.98 × 10^{24} kg, R = 6.4 × 10^6 m, G = 6.67 × 10^{-11} N m^2/kg^2

To find: Speed of satellite (v_c)

Formula: $v_c = \sqrt{\frac{GM}{r}}$

Calculation: From formula,

$$v_c = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{[(6.4 \times 10^6) + (1 \times 10^6)]}}$$

$$= \sqrt{\frac{6.67 \times 5.98 \times 10^7}{7.4}}$$

$$= \sqrt{\text{antilog}\{\log(6.67) + \log(5.98) - \log(7.4) \times 10^7\}}$$

$$= \sqrt{\text{antilog}\{0.8241 + 0.7767 - 0.8692\} \times 10^7}$$

$$= \sqrt{\text{antilog}\{0.7316\} \times 10^7}$$

$$= \sqrt{5.391 \times 10^7}$$

$$= \sqrt{53.91 \times 10^6}$$

= 7.343×10^3 .....[Taking square root value]

= 7.343×10^3 m/s

Speed of the satellite at height 1000 km is **7.343×10^3 m/s.**

Exercises | Q 4. (viii) | Page 99

Solve the following problem.

Calculate the value of acceleration due to gravity on the surface of Mars if the radius of Mars = 3.4×10^3 km and its mass is 6.4×10^{23} kg.

SOLUTION

Given: M = 6.4 × 10^{23} kg, R = 3.4 × 10^3 km = 3.4 × 10^6 m,

To find: Acceleration due to gravity on the surface of the Mars (g_M)

Formula: g = $\frac{GM}{R^2}$

Calculation: As, G = 6.67 × 10^{-11} Nm^2/kg^2

From formula,

$$g_M = \frac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{\left(3.4 \times 10^6\right)^2} = \frac{6.67 \times 6.4}{3.4 \times 3.4}$$

= antilog{log(6.67) + log(6.4) - log(3.4) - log(3.4)}

= antilog{(0.8241) + (0.8062) - (0.5315) - (0.5315)}

= antilog {0.5673}

= 3.693 m/s^2

Acceleration due to gravity on the surface of Mars is **3.693 m/s^2**.

Exercises | Q 4. (ix) | Page 99

Solve the following problem.

A planet has mass 6.4 × 10^{24} kg and radius 3.4 × 10^6 m. Calculate the energy required to remove an object of mass 800 kg from the surface of the planet to infinity.

SOLUTION

Given: M = 6.4 × 10^{24} kg, R = 3.4 × 10^6 m, m = 800 kg

To find: Energy required to remove the object from the surface of planet to infinity = B.E.

Formula: B.E. = $\frac{GMm}{R}$

Calculation: We know that,

G = 6.67×10^{-11} N m^2/kg^2

From formula,

B.E. = $\frac{6.67 \times 10^{-11} \times 6.4 \times 10^{24} \times 800}{3.4 \times 10^6}$

∴ B.E. = $\frac{6.67 \times 6.4 \times 8}{3.4} \times 10^9$

$= \frac{6.67 \times 51.2}{3.4} \times 10^9$

= antilog{log(6.67) + log(51.2) – log(3.4)} × 10^9

= antilog{0.8241 + 1.7093 – 0.5315} × 10^9

= antilog{2.0019} × 10^9

= $1.004 \times 10^2 \times 10^9$

= 1.004×10^{11} J

Energy required to remove the object from the surface of the planet is **1.004×10^{11} J.**

Exercises | Q 4. (x) | Page 99

Solve the following problem.

Calculate the value of the universal gravitational constant from the given data. Mass of the Earth = 6×10^{24} kg, Radius of the Earth = 6400 km, and the acceleration due to gravity on the surface = 9.8 m/s^2.

SOLUTION

Given: M = 6 × 10^{24} kg, R = 6400 km = 6.4 × 10^6 m, g = 9.8 m/s^2

To find: Gravitational constant (G)

Formula: $g = \frac{GM}{R^2}$

Calculation: From formula,

$$G = \frac{gR^2}{M}$$

$$G = \frac{9.8 \times \left(6.4 \times 10^6\right)^2}{6 \times 10^{24}} = \frac{401.4 \times 10^{12}}{6 \times 10^{24}}$$

∴ G = 6.69 × 10^{-11} Nm^2/kg^2

The value of gravitational constant is **6.69 × 10^{-11} Nm^2/kg^2** .

Exercises | Q 4. (xi) | Page 99

Solve the following problem.

A body weighs 5.6 kg-wt on the surface of the Earth. How much will be its weight on a planet whose mass is 1/7th mass of the Earth and radius twice that of the Earth's radius?

SOLUTION

Given: W_E = 5.6 kg-wt., $\frac{M_P}{M_E} = \frac{1}{7}, \frac{R_P}{R_E} = 2$

To find: Weight of the body on the surface of the planet (W_p)

Formula: $W = mg = \frac{GMm}{R^2}$

Calculation: From formula,

$$\frac{W_P}{W_E} = \frac{M_P}{M_E} \times \frac{R_E^2}{R_P^2}$$

$$\frac{W_p}{5.6} = \frac{1}{7} \times \left(\frac{1}{2}\right)^2 = \frac{1}{28}$$

$$\therefore W_p = \frac{1}{28} \times 5.6 = 0.2 \text{ kg-wt.}$$

Weight of the body on the surface of a planet will be **0.2 kg-wt.**

Exercises | Q 4. (xii) | Page 99

Solve the following problem.
What is the gravitational potential due to the Earth at a point which is at a height of $2R_E$ above the surface of the Earth?
(Mass of the Earth is 6×10^{24} kg, radius of the Earth = 6400 km and G = 6.67×10^{-11} N m^2 kg^{-2})

SOLUTION

Given: M = 6×10^{24} kg, R_E = 6400 km = 6.4×10^6 m, G = 6.67×10^{-11} N m^2/kg^2, h = $2R_E$

To find: Gravitational potential (V)

Formula: V = $-\frac{GM}{r}$

Calculation: From formula,

$$V = -\frac{GM}{R_E + 2R_E}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{3 \times 6.4 \times 10^6}$$

$$= -\frac{6.67 \times 2}{6.4} \times 10^7$$

= **-2.08×10^7 J kg^{-1}**

Negative sign indicates the attractive nature of gravitational potential.

Gravitational potential due to Earth will be **2.08×10^7 J kg^{-1}** towards the centre of the Earth.