Chapter

1

Set Theory and Relations

Set Theory

Introduction

A set is well defined class or collection of objects.

A set is often described in the following two ways.

(1) **Roster method or Listing method :** In this method a set is described by listing elements, separated by commas, within braces {}. The set of vowels of English alphabet may be described as $\{a, e, i, o, u\}$.

(2) **Set-builder method or Rule method :** In this method, a set is described by a characterizing property $P(x)$ of its elements x. In such a case the set is described by $\{x : P(x)$ holds$\}$ or $\{x \mid P(x)$ holds$\}$, which is read as 'the set of all x such that $P(x)$ holds'. The symbol '|' or ':' is read as 'such that'.

The set $A = \{0, 1, 4, 9, 16, ...\}$ can be written as $A = \{x^2 \mid x \in Z\}$.

❑ Symbols

Symbol	Meaning
$\Rightarrow$	Implies
$\in$	Belongs to
$A \subset B$	A is a subset of B
$\Leftrightarrow$	Implies and is implied by
$\notin$	Does not belong to
s.t.(: or |)	Such that
$\forall$	For every
$\exists$	There exists
iff	If and only if
&	And
$a \mid b$	a is a divisor of b
N	Set of natural numbers
I or Z	Set of integers
R	Set of real numbers
C	Set of complex numbers
Q	Set of rational numbers

Types of sets

(1) **Null set or Empty set :** The set which contains no element at all is called the null set. This set is sometimes also called the 'empty set' or the 'void set'. It is denoted by the symbol ϕ or {}.

(2) **Singleton set :** A set consisting of a single element is called a singleton set. The set {5} is a singleton set.

(3) **Finite set :** A set is called a finite set if it is either void set or its elements can be listed (counted, labelled) by natural number 1, 2, 3, ... and the process of listing terminates at a certain natural number n (say).

Cardinal number of a finite set **:** The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by $n(A)$ or $O(A)$.

(4) **Infinite set :** A set whose elements cannot be listed by the natural numbers 1, 2, 3, ..., n, for any natural number n is called an infinite set.

(5) **Equivalent set :** Two finite sets A and B are equivalent if their cardinal numbers are same *i.e.* $n(A) = n(B)$.

Example **:** $A = \{1, 3, 5, 7\}$; $B = \{10, 12, 14, 16\}$ are equivalent sets, $[\because O(A) = O(B) = 4]$.

(6) **Equal set :** Two sets A and B are said to be equal *iff* every element of A is an element of B and also every element of B is an element of A. Symbolically, $A = B$ if $x \in A \Leftrightarrow x \in B$.

Example **:** If $A = \{2, 3, 5, 6\}$ and $B = \{6, 5, 3, 2\}$. Then $A = B$, because each element of A is an element of B and vice-versa.

(7) **Universal set :** A set that contains all sets in a given context is called the universal set.

It should be noted that universal set is not unique. It may differ in problem to problem.

(8) **Power set :** If S is any set, then the family of all the subsets of S is called the power set of S.

The power set of S is denoted by $P(S)$. Symbolically, $P(S) = \{T : T \subseteq S\}$. Obviously ϕ and S are both elements of $P(S)$.

***Example* :** Let $S = \{a, b, c\}$, then $P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

Power set of a given set is always non-empty.

(9) **Subsets (Set inclusion) :** Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B.

If A is subset of B, we write $A \subseteq B$, which is read as "A is a subset of B" or "A is contained in B".

Thus, $A \subseteq B \Rightarrow a \in A \Rightarrow a \in B$.

***Proper and improper subsets* :** If A is a subset of B and $A \neq B$, then A is a proper subset of B. We write this as $A \subset B$.

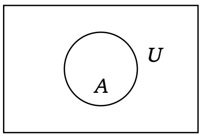
The null set ϕ is subset of every set and every set is subset of itself, *i.e.,* $\phi \subset A$ and $A \subseteq A$ for every set A. They are called improper subsets of A. Thus every non-empty set has two improper subsets. It should be noted that ϕ has only one subset ϕ which is improper.

All other subsets of A are called its proper subsets. Thus, if $A \subset B$, $A \neq B$, $A \neq \phi$, then A is said to be proper subset of B.

***Example* :** Let $A = \{1, 2\}$. Then A has $\phi; \{1\}, \{2\}, \{1, 2\}$ as its subsets out of which ϕ and $\{1, 2\}$ are improper and $\{1\}$ and $\{2\}$ are proper subsets.

Venn-Euler diagrams

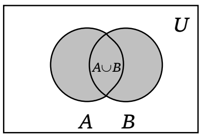
The combination of rectangles and circles are called *Venn-Euler diagrams* or simply **Venn-diagrams**.



If A and B are not equal but they have some common elements, then to represent A and B we draw two intersecting circles. Two disjoints sets are represented by two non-intersecting circles.

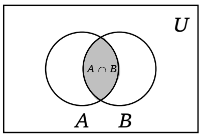
Operations on sets

(1) **Union of sets :** Let A and B be two sets. The union of A and B is the set of all elements which are in set A or in B. We denote the union of A and B by $A \cup B$, which is usually read as "A union B".



Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

(2) **Intersection of sets :** Let A and B be two sets. The intersection of A and B is the set of all those elements that belong to both A and B.



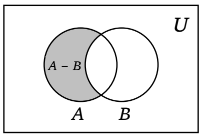
The intersection of A and B is denoted by $A \cap B$ (read as "A intersection B").

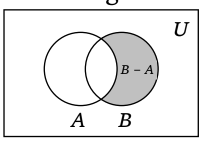
Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

(3) **Disjoint sets :** Two sets A and B are said to be disjoint, if $A \cap B = \phi$. If $A \cap B \neq \phi$, then A and B are said to be non-intersecting or non-overlapping sets.

***Example* :** Sets {1, 2}; {3, 4} are disjoint sets.

(4) **Difference of sets :** Let A and B be two sets. The difference of A and B written as $A - B$, is the set of all those elements of A which do not belong to B.





Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, the difference $B - A$ is the set of all those elements of B that do not belong to A *i.e.,* $B - A = \{x \in B : x \notin A\}$.

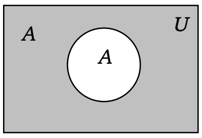
***Example* :** Consider the sets $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A - B = \{1, 2\}; B - A = \{4, 5\}$.

(5) **Symmetric difference of two sets :** Let A and B be two sets. The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$. Thus, $A \Delta B$ = $(A - B) \cup (B - A) = \{x : x \notin A \cap B\}$.

(6) **Complement of a set :** Let U be the universal set and let A be a set such that $A \subset U$. Then, the complement of A with respect to U is denoted by A' or A^c or $C(A)$ or $U - A$ and is defined the set of all those elements of U which are not in A.

Thus, $A' = \{x \in U : x \notin A\}$.

Clearly, $x \in A' \Leftrightarrow x \notin A$



***Example* :** Consider $U = \{1, 2, ..., 10\}$ and $A = \{1, 3, 5, 7, 9\}$.

Then $A' = \{2, 4, 6, 8, 10\}$

Some important results on number of elements in sets

If A, B and C are finite sets and U be the finite universal set, then (1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(2) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets.

(3) $n(A - B) = n(A) - n(A \cap B)$ *i.e.,* $n(A - B) + n(A \cap B) = n(A)$

(4) $n(A \Delta B)$ = Number of elements which belong to exactly one of A or B $= n((A - B) \cup (B - A)) = n(A - B) + n(B - A)$

[$\because$ $(A - B)$ and $(B - A)$ are disjoint]

$= n(A) - n(A \cap B) + n(B) - n(A \cap B) = n(A) + n(B) - 2n(A \cap B)$

(5) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(6) n (Number of elements in exactly two of the sets A, B, C) $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(7) n(Number of elements in exactly one of the sets A, B, C) $= n(A) + n(B) + n(C)$

$- 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(8) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(9) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

Laws of algebra of sets

(1) **Idempotent laws:** For any set A, we have

(i) $A \cup A = A$ (ii) $A \cap A = A$

(2) **Identity laws:** For any set A, we have

(i) $A \cup \phi = A$ (ii) $A \cap U = A$

i.e., ϕ and U are identity elements for union and intersection respectively.

(3) **Commutative laws:** For any two sets A and B, we have

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

(iii) $A \Delta B = B \Delta A$

i.e., union, intersection and symmetric difference of two sets are commutative.

(iv) $A - B \neq B - A$ (v) $A \times B \neq B \times A$

i.e., difference and cartesian product of two sets are not commutative

(4) **Associative laws:** If A, B and C are any three sets, then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

(iii) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

i.e., union, intersection and symmetric difference of two sets are associative.

(iv) $(A - B) - C \neq A - (B - C)$ (v) $(A \times B) \times C \neq A \times (B \times C)$

i.e., difference and cartesian product of two sets are not associative.

(5) **Distributive law:** If A, B and C are any three sets, then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e., union and intersection are distributive over intersection and union respectively.

(iii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iv) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(v) $A \times (B - C) = (A \times B) - (A \times C)$

(6) **De-Morgan's law :** If A, B and C are any three sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $A - (B \cap C) = (A - B) \cup (A - C)$

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$

(7) If A and B are any two sets, then

(i) $A - B = A \cap B'$ (ii) $B - A = B \cap A'$

(iii) $A - B = A \Leftrightarrow A \cap B = \phi$ (iv) $(A - B) \cup B = A \cup B$

(v) $(A - B) \cap B = \phi$ (vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

(vii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

(8) If A, B and C are any three sets, then

(i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Cartesian product of sets

Cartesian product of sets : Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = [(a, b) : a \in A \text{ and } b \in B]$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$.

Example **:** Let $A = \{a, b, c\}$ and $B = \{p, q\}$.

Then $A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$

Also $B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$

Important theorems on cartesian product of sets:

Theorem 1 : For any three sets A, B, C

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Theorem 2 : For any three sets A, B, C

$A \times (B - C) = (A \times B) - (A \times C)$

Theorem 3 : If A and B are any two non-empty sets, then

$A \times B = B \times A \Leftrightarrow A = B$

Theorem 4 : If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

Theorem 5 : If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

Theorem 6 : If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

Theorem 7 : For any sets A, B, C, D

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Theorem 8 : For any three sets A, B, C

(i) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

(ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

Relations

Definition

Let A and B be two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that a is related to b by the relation R and write it as aRb. If $(a, b) \in R$, we write it as aRb.

(1) **Total number of relations :** Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subset of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines relation from A to B, so total number of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation $A \times B$ are trivial relations from A to B.

(2) **Domain and range of a relation :** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Dom $(R) = \{a : (a, b) \in R\}$ and Range $(R) = \{b : (a, b) \in R\}$.

Inverse relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Clearly $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$. Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1})

Example **:** Let $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $R = \{(a, 1), (a, 3), (b, 3), (c, 3)\}$.

Then, (i) $R^{-1} = \{(1, a), (3, a), (3, b), (3, c)\}$

(ii) Dom $(R) = \{a, b, c\}$ = Range (R^{-1})

(iii) Range $(R) = \{1, 3\}$ = Dom (R^{-1})

Types of relations

(1) **Reflexive relation :** A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$.

Example **:** Let $A = \{1, 2, 3\}$ and $R = \{(1, 1); (1, 3)\}$

Then R is not reflexive since $3 \in A$ but $(3, 3) \notin R$

A reflexive relation on A is not necessarily the identity relation on A.

The universal relation on a non-void set A is reflexive.

(2) **Symmetric relation :** A relation R on a set A is said to be a symmetric relation *iff* $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

i.e., $aRb \Rightarrow bRa$ for all $a, b \in A$.

it should be noted that R is symmetric iff $R^{-1} = R$

The identity and the universal relations on a non-void set are symmetric relations.

A reflexive relation on a set A is not necessarily symmetric.

(3) **Anti-symmetric relation :** Let A be any set. A relation R on set A is said to be an anti-symmetric relation *iff* $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$.

Thus, if $a \neq b$ then a may be related to b or b may be related to a, but never both.

(4) **Transitive relation :** Let A be any set. A relation R on set A is said to be a transitive relation *iff*

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$ *i.e.*, aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in A$.

Transitivity fails only when there exists a, b, c such that $a R b$, $b R c$ but $a \not R c$.

Example **:** Consider the set $A = \{1, 2, 3\}$ and the relations

$R_1 = \{(1, 2), (1, 3)\}$; $R_2 = \{(1, 2)\}$; $R_3 = \{(1, 1)\}$;

$R_4 = \{(1, 2), (2, 1), (1, 1)\}$

Then R_1, R_2, R_3 are transitive while R_4 is not transitive since in $R_4, (2, 1) \in R_4; (1, 2) \in R_4$ but $(2, 2) \notin R_4$.

The identity and the universal relations on a non-void sets are transitive.

(5) **Identity relation :** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only. Every identity relation will be reflexive, symmetric and transitive.

Example **:** On the set = {1, 2, 3}, $R = \{(1, 1), (2, 2), (3, 3)\}$ is the identity relation on A.

It is interesting to note that every identity relation is reflexive but every reflexive relation need not be an identity relation.

(6) **Equivalence relation :** A relation R on a set A is said to be an equivalence relation on A *iff*

(i) It is reflexive *i.e.* $(a, a) \in R$ for all $a \in A$

(ii) It is symmetric *i.e.* $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$

(iii) It is transitive *i.e.* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Congruence modulo **(*m*) :** Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m if $a - b$ is divisible by m and we write $a \equiv b \pmod{m}$.

Thus $a \equiv b \pmod{m} \Leftrightarrow a - b$ is divisible by m. For example, $18 \equiv 3 \pmod{5}$ because 18 – 3 = 15 which is divisible by 5. Similarly, $3 \equiv 13 \pmod{2}$ because 3 – 13 = –10 which is divisible by 2. But $25 \neq 2 \pmod{4}$ because 4 is not a divisor of 25 – 3 = 22.

The relation "Congruence modulo m" is an equivalence relation.

Equivalence classes of an equivalence relation

Let R be equivalence relation in $A (\neq \phi)$. Let $a \in A$. Then the equivalence class of a, denoted by $[a]$ or $\{\bar{a}\}$ is defined as the set of all those points of A which are related to a under the relation R. Thus $[a] = \{x \in A : x R a\}$.

It is easy to see that

(1) $b \in [a] \Rightarrow a \in [b]$

(2) $b \in [a] \Rightarrow [a] = [b]$

(3) Two equivalence classes are either disjoint or identical.

Composition of relations

Let R and S be two relations from sets A to B and B to C respectively. Then we can define a relation SoR from A to C such that $(a, c) \in SoR \Leftrightarrow \exists\, b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

This relation is called the composition of R and S.

For example, if $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$, $C=\{p, q, r, s\}$ be three sets such that $R = \{(1, a), (2, b), (1, c), (2, d)\}$ is a relation from A to B and $S = \{(a, s), (b, r), (c, r)\}$ is a relation from B to C. Then SoR is a relation from A to C given by $SoR = \{(1, s)\ (2, r)\ (1, r)\}$

In this case RoS does not exist.

In general $RoS \neq SoR$. Also $(SoR)^{-1} = R^{-1}oS^{-1}$.

Tips & Tricks

- Equal sets are always equivalent but equivalent sets may need not be equal set.
- If A has n elements, then $P(A)$ has 2^n elements.
- The total number of subset of a finite set containing n elements is 2^n.
- If $A_1, A_2, \ldots\ldots, A_n$ is a finite family of sets, then their union is denoted by $\bigcup_{i=1}^{n} A_i$ or $A_1 \cup A_2 \cup A_3 \ldots\ldots \cup A_n$.
- If $A_1, A_2, A_3 \ldots\ldots, A_n$ is a finite family of sets, then their intersection is denoted by $\bigcap_{i=1}^{n} A_i$ or $A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n$.
- $R - Q$ is the set of all irrational numbers.
- Let A and B two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2

elements in common.

✍ The identity relation on a set A is an anti-symmetric relation.

✍ The universal relation on a set A containing at least two elements is not anti-symmetric, because if $a \neq b$ are in A, then a is related to b and b is related to a under the universal relation will imply that $a = b$ but $a \neq b$.

✍ The set $\{(a, a): a \in A\} = D$ is called the diagonal line of $A \times A$. Then "the relation R in A is antisymmetric iff $R \cap R^{-1} \subseteq D$".

✍ The relation 'is congruent to' on the set T of all triangles in a plane is a transitive relation.

✍ If R and S are two equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.

✍ The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.

✍ The inverse of an equivalence relation is an equivalence relation.

Ordinary Thinking

Objective Questions

1. The set of intelligent students in a class is **[AMU 1998]**
(a) A null set
(b) A singleton set
(c) A finite set
(d) Not a well defined collection

2. Which of the following is the empty set **[Karnataka CET 1990]**
(a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$
(b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$
(c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$
(d) $\{x : x$ is a real number and $x^2 = x + 2\}$

3. The set $A = \{x : x \in R, x^2 = 16$ and $2x = 6\}$ equals **[Karnataka CET 1995]**
(a) ϕ (b) {14, 3, 4}
(c) {3} (d) {4}

4. If a set A has n elements, then the total number of subsets of A is **[Roorkee 1991; Karnataka CET 1992, 2000]**
(a) n (b) n^2
(c) 2^n (d) $2n$

5. The number of proper subsets of the set {1, 2, 3} is **[JMIEE 2000]**
(a) 8 (b) 7
(c) 6 (d) 5

6. Given the sets $A = \{1, 2, 3\}, B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is **[MNR 1988; Kurukshetra CEE 1996]**
(a) {3} (b) {1, 2, 3, 4}
(c) {1, 2, 4, 5} (d) {1, 2, 3, 4, 5, 6}

7. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to **[Karnataka CET 1996]**
(a) A (b) B
(c) A^c (d) B^c

8. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to **[AMU 1998; Kurukshetra CEE 1999]**
(a) A (b) B
(c) ϕ (d) $A \cap B^c$

9. If the sets A and B are defined as
$A = \{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\}$
$B = \{(x, y) : y = -x, x \in R\}$, then
(a) $A \cap B = A$ (b) $A \cap B = B$
(c) $A \cap B = \phi$ (d) None of these

10. Let $A = [x : x \in R, |x| < 1]$; $B = [x : x \in R, |x - 1| \geq 1]$ and $A \cup B = R - D$, then the set D is
(a) $[x : 1 < x \leq 2]$ (b) $[x : 1 \leq x < 2]$
(c) $[x : 1 \leq x \leq 2]$ (d) None of these

11. If the sets A and B are defined as $A = \{(x, y) : y = e^x, x \in R\}$; $B = \{(x, y) : y = x, x \in R\}$, then 33333 **[UPSEAT 1994, 99, 2002]**
(a) $B \subseteq A$ (b) $A \subseteq B$
(c) $A \cap B = \phi$ (d) $A \cup B = A$

12. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, then $X \cup Y$ is equal to **[Karnataka CET 1997]**
(a) X (b) Y
(c) N (d) None of these

13. Let $n(U) = 700, n(A) = 200, n(B) = 300$ and $n(A \cap B) = 100$, then $n(A^c \cap B^c) =$ **[Kurukshetra CEE 1999]**
(a) 400 (b) 600
(c) 300 (d) 200

14. In a town of 10,000 families it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is **[Roorkee 1997]**
(a) 3100 (b) 3300
(c) 2900 (d) 1400

15. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is **[Kerala (Engg.) 2002]**
(a) 80 percent (b) 40 percent
(c) 60 percent (d) 70 percent

16. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is **[UPSEAT 1990]**

(a) 6 (b) 9
(c) 7 (d) All of these

17. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to **[Pb. CET 2001]**
(a) $(A \times B) \cup (A \times C)$ (b) $(A \cup B) \times (A \cup C)$
(c) $(A \times B) \cap (A \times C)$ (d) None of these

18. If A, B and C are any three sets, then $A - (B \cup C)$ is equal to
(a) $(A - B) \cup (A - C)$ (b) $(A - B) \cap (A - C)$
(c) $(A - B) \cup C$ (d) $(A - B) \cap C$

19. If A, B and C are non-empty sets, then $(A - B) \cup (B - A)$ equals **[AMU 1992, 1998; DCE 1998]**
(a) $(A \cup B) - B$ (b) $A - (A \cap B)$
(c) $(A \cup B) - (A \cap B)$ (d) $(A \cap B) \cup (A \cup B)$

20. If $A = \{2, 4, 5\}$, $B = \{7, 8, 9\}$, then $n(A \times B)$ is equal to
(a) 6 (b) 9
(c) 3 (d) 0

21. If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is **[Karnataka CET 1999]**
(a) $p + q$ (b) $p + q + 1$
(c) pq (d) p^2

22. If $A = \{a, b\}, B = \{c, d\}, C = \{d, e\}$, then $\{(a,c),(a,d),(a,e),(b,c),(b,d),(b,e)\}$ is equal to **[AMU 1999; Him. CET 2002]**
(a) $A \cap (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \times (B \cup C)$ (d) $A \times (B \cap C)$

23. If P, Q and R are subsets of a set A, then $R \times (P^c \cup Q^c)^c =$ **[Karnataka CET 1993]**
(a) $(R \times P) \cap (R \times Q)$ (b) $(R \times Q) \cap (R \times P)$
(c) $(R \times P) \cup (R \times Q)$ (d) None of these

24. In rule method the null set is represented by **[Karnataka CET 1998]**
(a) $\{\}$ (b) ϕ
(c) $\{x : x = x\}$ (d) $\{x : x \ne x\}$

25. $A = \{x : x \ne x\}$ represents **[Kurukshetra CEE 1998]**
(a) {0} (b) {}
(c) {1} (d) $\{x\}$

26. If $Q = \left\{x : x = \frac{1}{y}, \text{where } y \in N\right\}$, then
(a) $0 \in Q$ (b) $1 \in Q$
(c) $2 \in Q$ (d) $\frac{2}{3} \in Q$

27. Which set is the subset of all given sets
(a) {1, 2, 3, 4,......} (b) {1}
(c) {0} (d) {}

28. Let $S = \{0, 1, 5, 4, 7\}$. Then the total number of subsets of S is
(a) 64 (b) 32
(c) 40 (d) 20

29. The number of non-empty subsets of the set {1, 2, 3, 4} is **[Karnataka CET 1997; AMU 1998]**
(a) 15 (b) 14
(c) 16 (d) 17

30. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(a) {2, 3, 5} (b) {3, 5, 9}
(c) {1, 2, 5, 9} (d) None of these

31. If $A \cap B = B$, then **[JMIEE 2000]**
(a) $A \subset B$ (b) $B \subset A$
(c) $A = \phi$ (d) $B = \phi$

32. If A and B are two sets, then $A \cup B = A \cap B$ iff
(a) $A \subseteq B$ (b) $B \subseteq A$
(c) $A = B$ (d) None of these

33. Let A and B be two sets. Then
(a) $A \cup B \subseteq A \cap B$ (b) $A \cap B \subseteq A \cup B$
(c) $A \cap B = A \cup B$ (d) None of these

34. Let $A = \{(x, y) : y = e^x, x \in R\}$, $B = \{(x, y) : y = e^{-x}, x \in R\}$. Then
(a) $A \cap B = \phi$ (b) $A \cap B \ne \phi$
(c) $A \cup B = R^2$ (d) None of these

35. If $A = \{2, 3, 4, 8, 10\}$, $B = \{3, 4, 5, 10, 12\}$, $C = \{4, 5, 6, 12, 14\}$ then $(A \cap B) \cup (A \cap C)$ is equal to
(a) {3, 4, 10} (b) {2, 8, 10}
(c) {4, 5, 6} (d) {3, 5, 14}

36. If A and B are any two sets, then $A \cap (A \cup B)$ is equal to
(a) A (b) B
(c) A^c (d) B^c

37. If A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then **[Roorkee 1991]**
(a) $A = B$ (b) $B = C$
(c) $A = C$ (d) $A = B = C$

38. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{a, b, d, e\}$, then $A \cap (B \cup C)$ is **[Kurukshetra CEE 1997]**
(a) $\{a, b, c\}$ (b) $\{b, c, d\}$
(c) $\{a, b, d, e\}$ (d) $\{e\}$

39. If A and B are sets, then $A \cap (B - A)$ is
(a) ϕ (b) A
(c) B (d) None of these

40. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to
(a) A (b) B
(c) ϕ (d) None of these

41. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 5\}, B = \{6, 7\}$, then $A \cap B'$ is
(a) B' (b) A
(c) A' (d) B

42. If A is any set, then
(a) $A \cup A' = \phi$ (b) $A \cup A' = U$
(c) $A \cap A' = U$ (d) None of these

43. If $N_a = [an : n \in N]$, then $N_5 \cap N_7 =$ **[Kerala(Engg.) 2005]**
(a) N_7 (b) N
(c) N_{35} (d) N_5
(e) N_{12}

44. If $aN = \{ax : x \in N\}$, then the set $3N \cap 7N$ is
(a) 21 N (b) 10 N

(c) 4 N (d) None of these

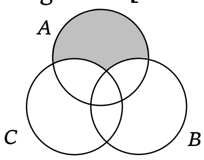
45. The shaded region in the given figure is **[NDA 2000]**

(a) $A \cap (B \cup C)$

(b) $A \cup (B \cap C)$

(c) $A \cap (B - C)$

(d) $A - (B \cup C)$



46. If A and B are two sets then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to

(a) $A \cup B$ (b) $A \cap B$

(c) A (d) B'

47. Let A and B be two sets then $(A \cup B)' \cup (A' \cap B)$ is equal to

(a) A' (b) A

(c) B' (d) None of these

48. Let U be the universal set and $A \cup B \cup C = U$. Then $\{(A-B)\cup(B-C)\cup(C-A)\}'$ is equal to

(a) $A \cup B \cup C$ (b) $A \cup (B \cap C)$

(c) $A \cap B \cap C$ (d) $A \cap (B \cup C)$

49. If $n(A) = 3$, $n(B) = 6$ and $A \subseteq B$. Then the number of elements in $A \cup B$ is equal to

(a) 3 (b) 9

(c) 6 (d) None of these

50. Let A and B be two sets such that $n(A) = 0.16, n(B) = 0.14, n(A \cup B) = 0.25$. Then $n(A \cap B)$ is equal to **[JMIEE 2001]**

(a) 0.3 (b) 0.5

(c) 0.05 (d) None of these

51. If A and B are disjoint, then $n(A \cup B)$ is equal to

(a) $n(A)$ (b) $n(B)$

(c) $n(A) + n(B)$ (d) $n(A).n(B)$

52. If A and B are not disjoint sets, then $n(A \cup B)$ is equal to **[Kerala (Engg.) 2001]**

(a) $n(A) + n(B)$ (b) $n(A) + n(B) - n(A \cap B)$

(c) $n(A) + n(B) + n(A \cap B)$ (d) $n(A)n(B)$

(e) $n(A) - n(B)$

53. In a battle 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. The minimum value of x is

(a) 10 (b) 12

(c) 15 (d) None of these

54. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is **[DCE 1995; MP PET 1996]**

(a) 128 (b) 216

(c) 240 (d) 160

55. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, then

(a) $x = 39$ (b) $x = 63$

(c) $39 \le x \le 63$ (d) None of these

56. 20 teachers of a school either teach mathematics or physics. 12 of them teach mathematics while 4 teach both the subjects. Then the number of teachers teaching physics only is

(a) 12 (b) 8

(c) 16 (d) None of these

57. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. The total number of members in the three athletic teams is

(a) 43 (b) 76

(c) 49 (d) None of these

58. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then the number of students who have passed in Physics only is **[DCE 1993; ISM Dhanbad 1994]**

(a) 22 (b) 33

(c) 10 (d) 45

59. If A and B are two sets, then $A \times B = B \times A$ if

(a) $A \subseteq B$ (b) $B \subseteq A$

(c) $A = B$ (d) None of these

60. If A and B be any two sets, then $(A \cap B)'$ is equal to

(a) $A' \cap B'$ (b) $A' \cup B'$

(c) $A \cap B$ (d) $A \cup B$

61. Let A and B be subsets of a set X. Then

(a) $A - B = A \cup B$ (b) $A - B = A \cap B$

(c) $A - B = A^c \cap B$ (d) $A - B = A \cap B^c$

62. Let A and B be two sets in the universal set. Then $A - B$ equals

(a) $A \cap B^c$ (b) $A^c \cap B$

(c) $A \cap B$ (d) None of these

63. If A, B and C are any three sets, then $A - (B \cap C)$ is equal to

(a) $(A - B) \cup (A - C)$ (b) $(A - B) \cap (A - C)$

(c) $(A - B) \cup C$ (d) $(A - B) \cap C$

64. If A, B, C are three sets, then $A \cap (B \cup C)$ is equal to

(a) $(A \cup B) \cap (A \cup C)$ (b) $(A \cap B) \cup (A \cap C)$

(c) $(A \cup B) \cup (A \cup C)$ (d) None of these

65. If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$, then $(A - B) \times (B - C)$ is

(a) $\{(1, 2), (1, 5), (2, 5)\}$ (b) $\{(1, 4)\}$

(c) $(1, 4)$ (d) None of these

66. If (1, 3), (2, 5) and (3, 3) are three elements of $A \times B$ and the total number of elements in $A \times B$ is 6, then the remaining elements of $A \times B$ are

(a) (1, 5); (2, 3); (3, 5) (b) (5, 1); (3, 2); (5, 3)

(c) (1, 5); (2, 3); (5, 3) (d) None of these

67. $A = \{1, 2, 3\}$ and $B = \{3, 8\}$, then $(A \cup B) \times (A \cap B)$ is
(a) {(3, 1), (3, 2), (3, 3), (3, 8)}
(b) {(1, 3), (2, 3), (3, 3), (8, 3)}
(c) {(1, 2), (2, 2), (3, 3), (8, 8)}
(d) {(8, 3), (8, 2), (8, 1), (8, 8)}

68. If $A = \{2, 3, 5\}$, $B = \{2, 5, 6\}$, then $(A - B) \times (A \cap B)$ is
(a) {(3, 2), (3, 3), (3, 5)} (b) {(3, 2), (3, 5), (3, 6)}
(c) {(3, 2), (3, 5)} (d) None of these

69. In a class of 30 pupils, 12 take needle work, 16 take physics and 18 take history. If all the 30 students take at least one subject and no one takes all three then the number of pupils taking 2 subjects is **[J & K 2005]**
(a) 16 (b) 6
(c) 8 (d) 20

70. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C) =$ **[Kerala (Engg.) 2005]**
(a) 288 (b) 1
(c) 12 (d) 17
(e) 2

71. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$, where Z is the set of all integers, is **[Kerala (Engg.) 2005]**
(a) 2 (b) 4
(c) 8 (d) 12
(e) 16

72. If $A = \{1, 2, 3, 4\}$; $B = \{a, b\}$ and f is a mapping such that $f : A \to B$, then $A \times B$ is **[DCE 2005]**
(a) {(a, 1), (3, b)}
(b) {(a, 2), (4, b)}
(c) {(1, a),(1, b),(2, a),(2, b),(3, a),(3, b),(4, a), (4, b)}
(d) None of these

73. If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$, $C = \{3, 4, 6\}$, then $(A \cup B) \cap C$ is **[Orissa JEE 2004]**
(a) {3, 4, 6} (b) {1, 2, 3}
(c) {1, 4, 3} (d) None of these

74. If $A = \{x, y\}$ then the power set of A is **[Pb. CET 2004, UPSEAT 2000]**
(a) $\{x^x, y^y\}$
(b) $\{\phi, x, y\}$
(c) $\{\phi, \{x\}, \{2y\}\}$
(d) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

75. A set contains $2n + 1$ elements. The number of subsets of this set containing more than n elements is equal to **[UPSEAT 2001, 04]**
(a) 2^{n-1} (b) 2^n
(c) 2^{n+1} (d) 2^{2n}

76. Which of the following is a true statement **[UPSEAT 2005]**
(a) $\{a\} \in \{a, b, c\}$ (b) $\{a\} \subseteq \{a, b, c\}$
(c) $\phi \in \{a, b, c\}$ (d) None of these

77. If $A = \{x : x$ is a multiple of 4$\}$ and $B = \{x : x$ is a multiple of 6$\}$ then $A \subset B$ consists of all multiples of **[UPSEAT 2000]**
(a) 16 (b) 12
(c) 8 (d) 4

78. A class has 175 students. The following data shows the number of students obtaining one or more subjects. Mathematics 100, Physics 70, Chemistry 40; Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone **[Kerala (Engg.) 2003]**
(a) 35 (b) 48
(c) 60 (d) 22
(e) 30

79. Consider the following relations :
(1) $A - B = A - (A \cap B)$
(2) $A = (A \cap B) \cup (A - B)$
(3) $A - (B \cup C) = (A - B) \cup (A - C)$
which of these is/are correct **[NDA 2003]**
(a) 1 and 3 (b) 2 only
(c) 2 and 3 (d) 1 and 2

80. If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are **[Kerala (Engg.) 2004]**
(a) 2^{99} (b) 99^2
(c) 100 (d) 18
(e) 9

81. Given $n(U) = 20$, $n(A) = 12$, $n(B) = 9$, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n((A \cup B)^C) =$ **[Kerala (Engg.) 2004]**
(a) 17 (b) 9
(c) 11 (d) 3
(e) 16

Relations

1. Let $A = \{1, 2, 3\}$. The total number of distinct relations that can be defined over A is
(a) 2^9 (b) 6
(c) 8 (d) None of these

2. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are relations from X to Y

(a) $R_1 = \{(x, y) | \; y = 2 + x, x \in X, y \in Y\}$

(b) $R_2 = \{(1,1),(2,1),(3,3),(4,3),(5,5)\}$

(c) $R_3 = \{(1,1),(1,3)(3,5),(3,7),(5,7)\}$

(d) $R_4 = \{(1,3),(2,5),(2,4),(7,9)\}$

3. Given two finite sets A and B such that $n(A)$ = 2, $n(B)$ = 3. Then total number of relations from A to B is

(a) 4 (b) 8

(c) 64 (d) None of these

4. The relation R defined on the set of natural numbers as {(a, b) : a differs from b by 3}, is given by

(a) {(1, 4), (2, 5), (3, 6),.....} (b) {(4,1),(5,2),(6, 3),.....}

(c) {(1, 3), (2, 6), (3, 9),..} (d) None of these

5. The relation R is defined on the set of natural numbers as {(a, b) : a = 2b}. Then R^{-1} is given by

(a) {(2, 1), (4, 2), (6, 3).....} (b){(1,2),(2,4), (3,6)....}

(c) R^{-1} is not defined (d) None of these

6. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is

(a) Reflexive but not symmetric

(b) Reflexive but not transitive

(c) Symmetric and Transitive

(d) Neither symmetric nor transitive

7. The relation "less than" in the set of natural numbers is **[UPSEAT 1994, 98, 99; AMU 1999]**

(a) Only symmetric (b) Only transitive

(c) Only reflexive (d) Equivalence relation

8. Let $P = \{(x, y) | \; x^2 + y^2 = 1, x, y \in R\}$. Then P is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Anti-symmetric

9. Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is

(a) Less than n

(b) Greater than or equal to n

(c) Less than or equal to n

(d) None of these

10. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

11. Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is

(a) Reflexive (b) Symmetric

(c) Anti-symmetric (d) Transitive

12. If R is a relation from a set A to a set B and S is a relation from B to a set C, then the relation SoR

(a) Is from A to C (b) Is from C to A

(c) Does not exist (d) None of these

13. If $R \subset A \times B$ and $S \subset B \times C$ be two relations, then $(SoR)^{-1} =$

(a) $S^{-1}oR^{-1}$ (b) $R^{-1}oS^{-1}$

(c) SoR (d) RoS

14. If R be a relation < from A = {1,2, 3, 4} to B = {1, 3, 5} *i.e.,* $(a, b) \in R \Leftrightarrow a < b$, then RoR^{-1} is

(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}

(b) {(3, 1) (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}

(c) {(3, 3), (3, 5), (5, 3), (5, 5)}

(d) {(3, 3) (3, 4), (4, 5)}

15. A relation from P to Q is

(a) A universal set of $P \times Q$ (b) $P \times Q$

(c) An equivalent set of $P \times Q$ (d) A subset of $P \times Q$

16. Let A = {a, b, c} and B = {1, 2}. Consider a relation R defined from set A to set B. Then R is equal to set

[Kurukshetra CEE 1995]

(a) A (b) B

(c) $A \times B$ (d) $B \times A$

17. Let $n(A) = n$. Then the number of all relations on A is

(a) 2^n (b) $2^{(n)!}$

(c) 2^{n^2} (d) None of these

18. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is

(a) 2^{mn} (b) $2^{mn} - 1$

(c) $2mn$ (d) m^n

19. Let R be a reflexive relation on a finite set A having n-elements, and let there be m ordered pairs in R. Then

(a) $m \ge n$ (b) $m \le n$

(c) $m = n$ (d) None of these

20. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = {(x, y) : $|x^2 - y^2| < 16$} is given by

(a) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)}

(b) {(2, 2), (3, 2), (4, 2), (2, 4)}

(c) {(3, 3), (3, 4), (5, 4), (4, 3), (3, 1)}

(d) None of these

21. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is

(a) {2, 3, 5} (b) {3, 5}

(c) {2, 3, 4} (d) {2, 3, 4, 5}

22. Let R be a relation on N defined by $x + 2y = 8$. The domain of R is

(a) {2, 4, 8} (b) {2, 4, 6, 8}

(c) {2, 4, 6} (d) {1, 2, 3, 4}

23. If $R = \{(x, y) | \; x, y \in Z, x^2 + y^2 \le 4\}$ is a relation in Z, then domain of R is

(a) {0, 1, 2} (b) {0, – 1, – 2}

(c) {– 2, – 1, 0, 1, 2} (d) None of these

24. R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$. Then R^{-1} is

(a) {(8, 11), (10, 13)} (b) {(11, 18), (13, 10)}

(c) {(10, 13), (8, 11)} (d) None of these

25. Let A = {1, 2, 3}, B = {1, 3, 5}. If relation R from A to B is given by R ={(1, 3), (2, 5), (3, 3)}. Then R^{-1} is

(a) {(3, 3), (3, 1), (5, 2)} (b) {(1, 3), (2, 5), (3, 3)}

(c) {(1, 3), (5, 2)} (d) None of these

26. Let R be a reflexive relation on a set A and I be the identity relation on A. Then

(a) $R \subset I$ (b) $I \subset R$

(c) $R = I$ (d) None of these

27. Let A = {1, 2, 3, 4} and R be a relation in A given by R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)}.

Then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) An equivalence relation

28. An integer m is said to be related to another integer n if m is a multiple of n. Then the relation is

(a) Reflexive and symmetric

(b) Reflexive and transitive

(c) Symmetric and transitive (d)Equivalence relation

29. The relation R defined in N as $aRb \Leftrightarrow b$ is divisible by a is

(a) Reflexive but not symmetric

(b) Symmetric but not transitive

(c) Symmetric and transitive

(d) None of these

30. Let R be a relation on a set A such that $R = R^{-1}$, then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

31. Let R = {(a, a)} be a relation on a set A. Then R is

(a) Symmetric

(b) Antisymmetric

(c) Symmetric and antisymmetric

(d) Neither symmetric nor anti-symmetric

32. The relation "is subset of" on the power set $P(A)$ of a set A is

(a) Symmetric (b) Anti-symmetric

(c) Equivalency relation (d) None of these

33. The relation R defined on a set A is antisymmetric if $(a,b) \in R \Rightarrow (b,a) \in R$ for

(a) Every (a, b) $\in R$ (b) No $(a,b) \in R$

(c) No $(a,b), a \neq b, \in R$ (d) None of these

34. In the set A = {1, 2, 3, 4, 5}, a relation R is defined by R = {(x, y)| x, y $\in$ A and $x < y$}. Then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

35. Let A be the non-void set of the children in a family. The relation 'x is a brother of y' on A is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

36. Let A = {1, 2, 3, 4} and let R= {(2, 2), (3, 3), (4, 4), (1, 2)} be a relation on A. Then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

37. The void relation on a set A is

(a) Reflexive (b) Symmetric and transitive

(c) Reflexive and symmetric (d)Reflective &transitive

38. Let R_1 be a relation defined by $R_1 = \{(a,b) | a \geq b, a,b \in R\}$. Then R_1 is **[UPSEAT 1999]**

(a) An equivalence relation on R

(b) Reflexive, transitive but not symmetric

(c) Symmetric, Transitive but not reflexive

(d) Neither transitive not reflexive but symmetric

39. Which one of the following relations on R is an equivalence relation

(a) $aR_1 b \Leftrightarrow |a| = |b|$ (b) $aR_2b \Leftrightarrow a \geq b$

(c) $aR_3b \Leftrightarrow a$ divides b (d) $aR_4b \Leftrightarrow a < b$

40. If R is an equivalence relation on a set A, then R^{-1} is

(a) Reflexive only

(b) Symmetric but not transitive

(c) Equivalence

(d) None of these

41. R is a relation over the set of real numbers and it is given by $nm \geq 0$. Then R is

(a) Symmetric and transitive

(b) Reflexive and symmetric

(c) A partial order relation

(d) An equivalence relation

42. In order that a relation R defined on a non-empty set A is an equivalence relation, it is sufficient, if R **[Karnataka CET 1990]**

(a) Is reflexive (b) Is symmetric

(c) Is transitive

(d) Possesses all the above three properties

43. The relation "congruence modulo m" is

(a) Reflexive only (b) Transitive only

(c) Symmetric only (d) An equivalence relation

44. Solution set of $x \equiv 3$ (mod 7), $p \in Z$, is given by

(a) {3} (b) $\{7p - 3 : p \in Z\}$

(c) $\{7p + 3 : p \in Z\}$ (d) None of these

45. Let R and S be two equivalence relations on a set A. Then

(a) $R \cup S$ is an equivalence relation on A

(b) $R \cap S$ is an equivalence relation on A

(c) $R - S$ is an equivalence relation on A

(d) None of these

46. Let R and S be two relations on a set A. Then

(a) R and S are transitive, then $R \cup S$ is also transitive

(b) R and S are transitive, then $R \cap S$ is also transitive

(c) R and S are reflexive, then $R \cap S$ is also reflexive

(d) R and S are symmetric then $R \cup S$ is also symmetric

47. Let R = {(1, 3), (2, 2), (3, 2)} and S = {(2, 1), (3, 2), (2, 3)} be two relations on set A = {1, 2, 3}. Then RoS =

(a) {(1, 3), (2, 2), (3, 2), (2, 1), (2, 3)}

(b) {(3, 2), (1, 3)}

(c) {(2, 3), (3, 2), (2, 2)}

(d) {(2, 3), (3, 2)}

48. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R\beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

49. Let R be a relation over the set $N \times N$ and it is defined by $(a,b)R(c,d) \Rightarrow a+d=b+c.$ Then R is

(a) Reflexive only (b) Symmetric only

(c) Transitive only (d) An equivalence relation

50. Let n be a fixed positive integer. Define a relation R on the set Z of integers by, $aRb \Leftrightarrow n|\ a-b\ |$. Then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

51. Let $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A = \{3,6,9,12\}$. The relation is **[AIEEE 2005]**

(a) An equivalence relation

(b) Reflexive and symmetric only

(c) Reflexive and transitive only

(d) Reflexive only

52. $x^2 = xy$ is a relation which is **[Orissa JEE 2005]**

(a) Symmetric (b) Reflexive

(c) Transitive (d) None of these

53. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is **[AIEEE 2004]**

(a) Reflexive (b) Transitive

(c) Not symmetric (d) A function

54. The number of reflexive relations of a set with four elements is equal to **[UPSEAT 2004]**

(a) 2^{16} (b) 2^{12}

(c) 2^8 (d) 2^4

55. Let S be the set of all real numbers. Then the relation R = {(a, b) : 1 + ab > 0} on S is **[NDA 2003]**

(a) Reflexive and symmetric but not transitive

(b) Reflexive and transitive but not symmetric

(c) Symmetric, transitive but not reflexive

(d) Reflexive, transitive and symmetric

(e) None of the above is true

56. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is **[NDA 2003]**

(a) 2^9 (b) 9^2

(c) 3^2 (d) 2^{9-1}

Critical Thinking

Objective Questions

1. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n-1) : n \in N\}$, then

(a) $X \subseteq Y$ (b) $Y \subseteq X$

(c) $X = Y$ (d) None of these

2. If $N_a = \{an : n \in N\}$, then $N_3 \cap N_4 =$

(a) N_7 (b) N_{12}

(c) N_3 (d) N_4

3. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$ **[MNR 1987; Karnataka CET 1996]**

(a) 3 (b) 6

(c) 9 (d) 18

4. If $A = [(x,y) : x^2 + y^2 = 25]$ and B = $[(x,y) : x^2 + 9y^2 = 144]$, then $A \cap B$ contains **[AMU 1996; Pb. CET 2002]**

(a) One point (b) Three points

(c) Two points (d) Four points

5. If $A = [x : x$ is a multiple of 3] and $B = [x : x$ is a multiple of 5], then A – B is ($\bar{A}$ means complement of A) **[AMU 1998]**

(a) $\bar{A} \cap B$ (b) $A \cap \bar{B}$

(c) $\bar{A} \cap \bar{B}$ (d) $\overline{A \cap B}$

6. If $A = \{x : x^2 - 5x + 6 = 0\}, B = \{2,4\}, C = \{4,5\}$, then $A \times (B \cap C)$ is **[Kerala (Engg.) 2002]**

(a) {(2, 4), (3, 4)} (b) {(4, 2), (4, 3)}

(c) {(2, 4), (3, 4), (4, 4)} (d) {(2,2),(3,3),(4,4),(5,5)}

7. In a college of 300 students, every student reads 5 newspaper and every newspaper is read by 60 students. The no. of newspaper is **[IIT 1998]**

(a) At least 30 (b) At most 20

(c) Exactly 25 (d) None of these

8. Let A = {1, 2, 3, 4, 5}; B = {2, 3, 6, 7}. Then the number of elements in ($A \times B$) $\cap$ ($B \times A$) is

(a) 18 (b) 6

(c) 4 (d) 0

9. Let A = {1, 2, 3}, B = {1, 3, 5}. A relation $R : A \rightarrow B$ is defined by R = {(1, 3), (1, 5), (2, 1)}. Then R^{-1} is defined by

(a) {(1,2), (3,1), (1,3), (1,5)} (b){(1, 2), (3, 1), (2, 1)}

(c) {(1, 2), (5, 1), (3, 1)} (d) None of these

10. Let R be the relation on the set R of all real numbers defined by $a\ R\ b$ iff $|a-b|\le 1$. Then R is

(a) Reflexive and Symmetric (b) Symmetric only

(c) Transitive only (d) Anti-symmetric only

11. With reference to a universal set, the inclusion of a subset in another, is relation, which is **[CET 1995]**

(a) Symmetric only (b) Equivalence relation

(c) Reflexive only (d) None of these

12. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (*i.e.,* $n|m$). Then R is

(a) Reflexive and symmetric

(b) Transitive and symmetric

(c) Equivalence

(d) Reflexive, transitive but not symmetric

13. Let R and S be two non-void relations on a set A. Which of the following statements is false

(a) R and S are transitive $\Rightarrow R \cup S$ is transitive

(b) R and S are transitive $\Rightarrow R \cap S$ is transitive

(c) R and S are symmetric $\Rightarrow R \cup S$ is symmetric

(d) R and S are reflexive $\Rightarrow R \cap S$ is reflexive

14. Let a relation R be defined by R = {(4, 5); (1, 4); (4, 6); (7, 6); (3, 7)} then $R^{-1}oR$ is

(a) {(1, 1), (4, 4), (4, 7), (7, 4), (7, 7), (3, 3)}

(b) {(1, 1), (4, 4), (7, 7), (3, 3)}

(c) {(1, 5), (1, 6), (3, 6)}

(d) None of these

15. Let R be a relation on the set N be defined by $\{(x, y) |\ x, y \in N, 2x + y = 41\}$. Then R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

Answers

Set theory

1	d	2	b	3	a	4	c	5	c
6	b	7	a	8	d	9	c	10	b
11	c	12	b	13	c	14	b	15	c
16	d	17	a	18	b	19	c	20	b
21	c	22	c	23	a,b	24	d	25	b
26	b	27	d	28	b	29	a	30	b
31	b	32	c	33	b	34	b	35	a
36	a	37	b	38	a	39	a	40	c
41	b	42	b	43	c	44	a	45	d
46	a	47	a	48	c	49	c	50	c
51	c	52	b	53	a	54	d	55	c
56	a	57	a	58	d	59	c	60	b
61	d	62	a	63	a	64	b	65	b
66	a	67	b	68	c	69	a	70	e
71	c	72	c	73	a	74	d	75	d
76	a	77	b	78	c	79	d	80	b
81	d								

Relations

1	a	2	a,b,c	3	c	4	b	5	b
6	a	7	b	8	b	9	b	10	a
11	b	12	a	13	b	14	c	15	d
16	c	17	c	18	a	19	a	20	d
21	d	22	c	23	c	24	a	25	a
26	b	27	a,b	28	b	29	a	30	b
31	c	32	b	33	c	34	c	35	bc
36	c	37	b	38	b	39	a	40	c
41	d	42	d	43	d	44	c	45	b
46	b,c,d	47	c	48	b	49	d	50	a,b,c,d
51	c	52	b	53	c	54	d	55	a
56	a								

Critical Thinking Questions

1	a	2	b	3	b	4	d	5	b
6	a	7	c	8	c	9	c	10	a
11	d	12	d	13	a	14	a	15	d

Answers and Solutions

Set theory

1. (d) Since, intelligency is not defined for students in a class *i.e.,* Not a well defined collection.

2. (b) Since $x^2+1=0$, gives $x^2=-1 \Rightarrow x=\pm i$

$\therefore$ x is not real but x is real (given)

$\therefore$ No value of x is possible.

3. (a) $x^2=16 \Rightarrow x=\pm 4$ $\qquad 2x=6 \Rightarrow x=3$

There is no value of x which satisfies both the above equations. Thus, $A=\phi$.

4. (c) $A={}^nC_0+{}^nC_1+........+{}^nC_n=2^n$.

5. (c) Number of proper subsets of the set {1, 2, 3) $=2^3-2=6$.

6. (b) $B\cap C=\{4\}$, $\therefore A\cup(B\cap C)$ = {1, 2, 3, 4}.

7. (a) $A\cap B\subseteq A$. Hence $A\cup(A\cap B)=A$.

8. (d) $A\cap(A\cap B)^c=A\cap(A^c\cup B^c)$ = $(A\cap A^c)\cup(A\cap B^c)$ = $\phi\cup(A\cap B^c)=A\cap B^c$.

9. (c) Since $y=\frac{1}{x}, y=-x$ meet when $-x=\frac{1}{x}$ $\Rightarrow$ $x^2=-1$, which does not give any real value of x. Hence, $A\cap B=\phi$.

10. (b) $A=[x: x\in R, -1<x<1]$

$B=[x: x\in R: x-1\le -1$ or $x-1\ge 1]$

= $[x: x\in R: x\le 0 \text{ or } x\ge 2]$

$\therefore A\cup B=R-D$, where D = $[x: x\in R, 1\le x<2]$.

11. (c) Since, $y=e^x$ and $y=x$ do not meet for any $x\in R$ $\therefore$ $A\cap B=\phi$.

12. (b) Since, $4^n-3n-1=(3+1)^n-3n-1$

$=3^n+{}^nC_1 3^{n-1}+{}^nC_2 3^{n-2}+.....+{}^nC_{n-1}3+{}^nC_n-3n-1$

$={}^nC_2 3^2+{}^nC_3.3^3+...+{}^nC_n 3^n$, $({}^nC_0={}^nC_n, {}^nC_1={}^nC_{n-1}$ etc.)

$=9[{}^nC_2+{}^nC_3(3)+.....+{}^nC_n 3^{n-1}]$

$\therefore 4^n-3n-1$ is a multiple of 9 for $n\ge 2$.

For $n=1$, 4^n-3n-1 = $4-3-1=0$,

For $n=2$, 4^n-3n-1 = $16-6-1=9$

$\therefore 4^n-3n-1$ is a multiple of 9 for all $n\in N$

$\therefore$ X contains elements, which are multiples of 9, and clearly Y contains all multiples of 9.

$\therefore X\subseteq Y$ *i.e.,* $X\cup Y=Y$.

13. (c) $n(A^c\cap B^c)$ = $n[(A\cup B)^c]$ = $n(U)-n(A\cup B)$

= $n(U)-[n(A)+n(B)-n(A\cap B)]$

= 700 – [200 + 300 – 100] = 300.

14. (b) $n(A)$ = 40% of 10,000 = 4,000

$n(B)$ = 20% of 10,000 = 2,000

$n(C)$ = 10% of 10,000 = 1,000

$n(A\cap B)$ = 5% of 10,000 = 500

$n(B\cap C)$ = 3% of 10,000 = 300

$n(C\cap A)$ = 4% of 10,000 = 400

$n(A\cap B\cap C)$ = 2% of 10,000 = 200

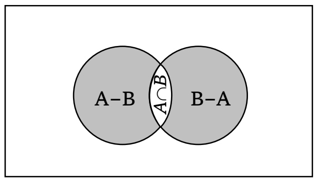
We want to find $n(A\cap B^c\cap C^c)$ = $n[A\cap(B\cup C)^c]$

= $n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
= $n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
= 4000 – [500 + 400 – 200] = 4000 – 700 = 3300.

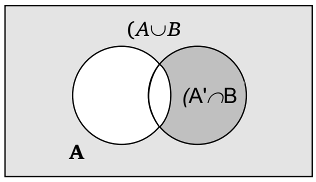
15. (c) $n(C) = 20$, $n(B) = 50$, $n(C \cap B) = 10$
Now $n(C \cup B) = n(C) + n(B) - n(C \cap B)$
= 20 + 50 – 10 = 60.
Hence, required number of persons = 60%.

16. (d) $n(M) = 23$, $n(P) = 24$, $n(C) = 19$
$n(M \cap P) = 12$, $n(M \cap C) = 9$, $n(P \cap C) = 7$
$n(M \cap P \cap C) = 4$
We have to find $n(M \cap P' \cap C')$, $n(P \cap M' \cap C')$, $n(C \cap M' \cap P')$
Now $n(M \cap P' \cap C') = n[M \cap (P \cup C)']$
$= n(M) - n(M \cap (P \cup C))$
$= n(M) - n[(M \cap P) \cup (M \cap C)]$
$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$
= 23 – 12 – 9 + 4 = 27 – 21 = 6
$n(P \cap M' \cap C') = n[P \cap (M \cup C)']$
$= n(P) - n[P \cap (M \cup C)] = n(P) - n[(P \cap M) \cup (P \cap C)]$
$= n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$
= 24 – 12 – 7 + 4 = 9
$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$
= 19 – 7 – 9 + 4 = 23 – 16 = 7.

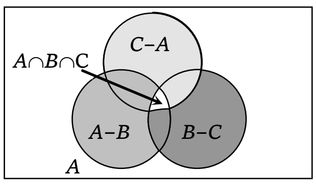
17. (a) It is distributive law.
18. (b) It is De' Morgan law.
19. (c) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.
20. (b) $A \times B = \{(2, 7), (2, 8), (2, 9), (4, 7), (4, 8), (4, 9), (5, 7), (5, 8), (5, 9)\}$
$n(A \times B) = n(A) \,.\, n(B) = 3 \times 3 = 9$.
21. (c) $n(A \times B) = pq$.
22. (c) $B \cup C = \{c, d\} \cup (d, e\} = \{c, d, e\}$
$\therefore A \times (B \cup C) = \{a, b\} \times \{c, d, e\}$
$= \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$.
23. (a,b) $R \times (P^c \cup Q^c)^c = R \times [(P^c)^c \cap (Q^c)^c]$
$= R \times (P \cap Q) = (R \times P) \cap (R \times Q) = (R \times Q) \cap (R \times P)$.
24. (d) It is fundamental concept.
25. (b) It is fundamental concept.
26. (b) Since $\frac{1}{y} \neq 0, \frac{1}{y} \neq 2, \frac{1}{y} \neq \frac{-2}{3}$, $[\because y \in N]$
$\therefore \frac{1}{y}$ can be 1, $[\because y$ can be 1].
27. (d) Null set is the subset of all given sets.
28. (b) $S = \{0, 1, 5, 4, 7\}$,
then, total number of subsets of S is 2^n.
Hence, $2^5 = 32$.
29. (a) The number of non- empty subsets $= 2^n - 1$
$2^4 - 1 = 16 - 1 = 15$.
30. (b) Given $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$. Hence, $A = \{3, 5, 9\}$.
31. (b) Since $A \cap B = B$, $\therefore B \subset A$.
32. (c) Let $x \in A \Rightarrow x \in A \cup B$, $[\because A \subseteq A \cup B]$
$\Rightarrow x \in A \cap B$, $[\because A \cup B = A \cap B]$
$\Rightarrow x \in A$ and $x \in B \Rightarrow x \in B$, $\therefore A \subseteq B$
Similarly, $x \in B \Rightarrow x \in A$, $\therefore B \subseteq A$
Now $A \subseteq B, B \subseteq A \Rightarrow A = B$.
33. (b) $A \cap B \subseteq A \subseteq A \cup B$, $\therefore A \cap B \subseteq A \cup B$.
34. (b) $\because y = e^x$, $y = e^{-x}$ will meet, when $e^x = e^{-x}$
$\Rightarrow e^{2x} = 1, \therefore x = 0, y = 1$
$\therefore A$ and B meet on (0, 1), $\therefore A \cap B = \phi$.
35. (a) $A \cap B = \{2, 3, 4, 8, 10\} \cap \{3, 4, 5, 10, 12\}$
$= \{3, 4, 10\}$, $A \cap C = \{4\}$.
$\therefore (A \cap B) \cup (A \cap C) = \{3, 4, 10\}$.
36. (a) $A \cap (A \cup B) = A$, $[\because A \subseteq B \cup A]$.
37. (b) It is obvious.
38. (a) $B \cup C = \{a, b, c, d, e\}$
$\therefore A \cap (B \cup C) = \{a, b, c\} \cap \{a, b, c, d, e\} = \{a, b, c\}$.
39. (a) $A \cap (B - A) = \phi$, $[\because x \in B - A \Rightarrow x \notin A]$.
40. (c) $A \cap (A \cup B)' = A \cap (A' \cap B')$, $(\because (A \cup B)' = A' \cap B')$
$= (A \cap A') \cap B'$, (by associative law)
$= \phi \cap B'$, $(\because A \cap A' = \phi)$
$= \phi$.
41. (b) $B' = \{1, 2, 3, 4, 5, 8, 9, 10\}$
$\therefore A \cap B' = \{1, 2, 5\} \cap \{1, 2, 3, 4, 5, 8, 9, 10\} = \{1, 2, 5\} = A$
42. (b) It is obvious.
43. (c) $N_5 \cap N_7 = N_{35}$,
[$\because$5 and 7 are relatively prime numbers].
44. (a) $3N = \{x \in N : x$ is a multiple of 3}
$7N = \{x \in N : x$ is a multiple of 7}
$\therefore 3N \cap 7N = \{x \in$ is a multiple of 3 and 7}
$= \{x \in N : x$ is a multiple of 3 and 7}
$= \{x \in N : x$ is a multiple of 21}=21N.
45. (d) It is obvious.
46. (a) From *Venn-Euler's* diagram,



$\therefore (A - B) \cup (B - A) \cup (A \cap B) = A \cup B$.
47. (a) From *Venn-Euler's* Diagram,



$\therefore (A \cup B)' \cup (A' \cap B) = A'$.
48. (c) From *Venn-Euler's* Diagram,



Clearly, $\{(A - B) \cup (B - C) \cup (C - A)\}' = A \cap B \cap C$.
49. (c) Since $A \subseteq B$, $\therefore A \cup B = B$.

So, $n(A \cup B) = n(B) = 6$.

50. (c) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$0.25 = 0.16 + 0.14 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 0.30 - 0.25 = 0.05$.

51. (c) Since A and B are disjoint, $\therefore A \cap B = \phi$

$n(A \cap B) = 0$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(A) + n(B) - 0 = n(A) + n(B)$.

52. (b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

53. (a) Minimum value of $n = 100 - (30 + 20 + 25 + 15))$

$= 100 - 90 = 10$.

54. (d) $n(C) = 224, n(H) = 240, n(B) = 336$

$n(H \cap B) = 64$, $n(B \cap C) = 80$

$n(H \cap C) = 40$, $n(C \cap H \cap B) = 24$

$n(C^c \cap H^c \cap B^C) = n[(C \cup H \cup B)^c]$

$= n(\cup) - n(C \cup H \cup B)$

$= 800 - [n(C) + n(H) + n(B) - n(H \cap C)$

$-n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

$= 800 - [224 + 240 + 336 - 64 - 80 - 40 + 24]$

$= 800 - 640 = 160$.

55. (c) Let A denote the set of Americans who like cheese and let B denote the set of Americans who like apples.

Let Population of American be 100.

Then $n(A) = 63, n(B) = 76$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 63 + 76 - n(A \cap B)$

$\therefore n(A \cup B) + n(A \cap B) = 139$

$\Rightarrow n(A \cap B) = 139 - n(A \cup B)$

But $n(A \cup B) \le 100$

$\therefore -n(A \cup B) \ge -100$

$\therefore 139 - n(A \cup B) \ge 139 - 100 = 39$

$\therefore n(A \cap B) \ge 39$ *i.e.*, $39 \le n(A \cap B)$ .....(i)

Again, $A \cap B \subseteq A, A \cap B \subseteq B$

$\therefore n(A \cap B) \le n(A) = 63$ and $n(A \cap B) \le n(B) = 76$

$\therefore n(A \cap B) \le 63$ .....(ii)

Then, $39 \le n(A \cap B) \le 63 \Rightarrow 39 \le x \le 63$.

56. (a) Let $n(P)$ = Number of teachers in Physics.

$n(M)$ = Number of teachers in Maths

$n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$20 = n(P) + 12 - 4 \Rightarrow n(P) = 12$.

57. (a) Let B, H, F denote the sets of members who are on the basketball team, hockey team and football team respectively.

Then we are given $n(B) = 21, n(H) = 26, n(F) = 29$

$n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8$.

We have to find $n(B \cup H \cup F)$.

To find this, we use the formula

$n(B \cup H \cup F) = n(B) + n(H) + n(F)$

$-n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

Hence, $n(B \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8 = 43$

Thus these are 43 members in all.

58. (d) $n(M) = 55, n(P) = 67, n(M \cup P) = 100$

Now, $n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$100 = 55 + 67 - n(M \cap P)$

$\therefore n(M \cap P) = 122 - 100 = 22$

Now n (P only) $= n(P) - n(M \cap P) = 67 - 22 = 45$.

59. (c) In general, $A \times B \ne B \times A$

$A \times B = B \times A$ is true, if $A = B$.

60. (b) From De' morgan's law, $(A \cap B)' = A' \cup B'$.

61. (d) $A - B = \{x : x \in A \text{ and } x \notin B\}$

$= \{x : x \in A \text{ and } x \in B^c\} = A \cap B^c$.

62. (a) It is obvious.

63. (a) From De' morgan's law, $A - (B \cap C) = (A - B) \cup (A - C)$.

64. (b) From Distributive law, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

65. (b) $A - B = \{1\}$ and $B - C = \{4\}$

$(A - B) \times (B - C) = \{(1, 4)\}$.

66. (a) It is obvious.

67. (b) $A \cup B = \{1, 2, 3, 8\}$; $A \cap B = \{3\}$

$(A \cup B) \times (A \cap B) = \{(1,3), (2,3), (3,3), (8,3)\}$.

68. (c) $A - B = \{3\}, A \cap B = \{2,5\}$

$(A - B) \times (A \cap B) = \{(3, 2); (3, 5)\}$.

69. (a) Given $n(N) = 12$, $n(P) = 16$, $n(H) = 18$, $n(N \cup P \cup H) = 30$

From, $n(N \cup P \cup H) = n(N) + n(P) + n(H) - n(N \cap P)$

$-n(P \cap H) - n(N \cap H) + n(N \cap P \cap H)$

$\therefore n(N \cap P) + n(P \cap H) + n(N \cap H) = 16$

Now, number of pupils taking two subjects

$= n(N \cap P) + n(P \cap H) + n(N \cap H) - 3n(N \cap P \cap H)$

$= 16 - 0 = 16$.

70. (e) $n(A) = 4$, $n(B) = 3$

$n(A) \times n(B) \times n(C) = n(A \times B \times C)$

$4 \times 3 \times n(C) = 24 \Rightarrow n(C) = \frac{24}{12} = 2$.

71. (c) Given set is $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$

We can see that, $2(\pm 2)^2 + 3(\pm 3)^2 = 35$

and $2(\pm 4)^2 + 3(\pm 1)^2 = 35$

$\therefore$ (2, 3), (2, –3), (–2, –3), (–2, 3), (4, 1), (4, –1), (–4, –1), (–4, 1) are 8 elements of the set. $\therefore$ $n = 8$.

72. (c) It is obvious.

73. (a) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

$\therefore (A \cup B) \cap C = \{3, 4, 6\}$.

74. (d) It is obvious.

75. (d) Let the original set contains $(2n+1)$ elements, then subsets of this set containing more than n elements, *i.e.*, subsets containing $(n+1)$ elements, $(n+2)$ elements, ... $(2n+1)$ elements.

$\therefore$ Required number of subsets

$= {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$

$= {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + ... + {}^{2n+1}C_1 + {}^{2n+1}C_0$

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_{n-1} + {}^{2n+1}C_n$

$= \frac{1}{2}\left[(1+1)^{2n+1}\right] = \frac{1}{2}[2^{2n+1}] = 2^{2n}$.

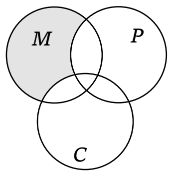
76. (a) It is obvious.

77. (b) $A = \{4, 8, 12, 16, 20, 24, ...\}$

$B = \{6, 12, 18, 24, 30, ...\}$

$\therefore A \subset B = \{12, 24, ...\}$ = {x : x is a multiple of 12}.

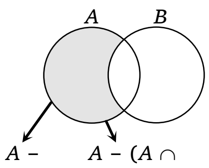
78. (c) n(M alone) $= n(M) - n(M \cap C) - n(M \cap P) + n(M \cap P \cap C)$



$= 100 - 28 - 30 + 18 = 60$.

79. (d) $A - B = A - (A \cap B)$ is correct.

$A = (A \cap B) \cup (A - B)$ is correct.



(3) is false.

$\therefore$ (1) and (2) are true.

80. (b) $n((A \times B) \cap (B \times A))$

$= n((A \cap B) \times (B \cap A)) = n(A \cap B).n(B \cap A)$

$= n(A \cap B).n(A \cap B) = (99)(99) = 99^2$.

81. (d) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 12 + 9 - 4 = 17$

Now, $n((A \cup B)^C) = n(U) - n(A \cup B) = 20 - 17 = 3$.

Relations

1. (a) $n(A \times A) = n(A).n(A) = 3^2 = 9$

So, the total number of subsets of $A \times A$ is 2^9 and a subset of $A \times A$ is a relation over the set A.

2. (a,b,c) R_4 is not a relation from X to Y, because (7, 9) $\in R_4$ but (7, 9) $\notin X \times Y$.

3. (c) Here $n(A \times B)$ = 2 × 3 = 6

Since every subset of A × B defines a relation from A to B, number of relation from A to B is equal to number of subsets of $A \times B = 2^6 = 64$.

4. (b) $R = \{(a, b) : a, b \in N, a - b = 3\} = \{((n+3), n) : n \in N\}$

$= \{(4, 1), (5, 2), (6, 3), ...\}$.

5. (b) R = {(2, 1), (4, 2), (6, 3),......}.

So, R^{-1} = {(1, 2), (2, 4), (3, 6),.....}.

6. (a) Since (1, 1); (2, 2); (3, 3) $\in R$ therefore R is reflexive. (1, 2) $\in R$ but (2, 1) $\notin R$, therefore R is not symmetric. It can be easily seen that R is transitive.

7. (b) Since $x < y, y < z \Rightarrow x < z \forall x, y, z \in N$

$\therefore x R y, y R z \Rightarrow x R z$, $\therefore$ Relation is transitive,

$\therefore x < y$ does not give $y < x$,

$\therefore$ Relation is not symmetric.

Since $x < x$ does not hold, hence relation is not reflexive.

8. (b) Obviously, the relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$.

9. (b) Since R is an equivalence relation on set A, therefore (a, a) $\in R$ for all $a \in A$. Hence, R has at least n ordered pairs.

10. (a) For any $x \in R,$ we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number.

$\Rightarrow xRx$ for all x. So, R is reflexive.

R is not symmetric, because $\sqrt{2}R1$ but $1 \not R \sqrt{2}$, R is not transitive also because $\sqrt{2}$ R1 and $1R2\sqrt{2}$ but $\sqrt{2} \not R 2\sqrt{2}$.

11. (b) Clearly, the relation is symmetric but it is neither reflexive nor transitive.

12. (a) It is obvious.

13. (b) It is obvious.

14. (c) We have, R = {(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4, 5)}

R^{-1} = {(3, 1), (5, 1), (3, 2), (5, 2); (5, 3); (5, 4)}

Hence RoR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}.

15. (d) A relation from P to Q is a subset of $P \times Q$.

16. (c) $R = A \times B$.

17. (c) Number of relations on the set $A =$ Number of subsets of $A \times A = 2^{n^2}$, $[\because n(A \times A) = n^2]$.

18. (a) It is obvious.

19. (a) Since R is reflexive relation on A, therefore $(a, a) \in R$ for all $a \in A$.

The minimum number of ordered pairs in R is n.

Hence, $m \geq n$.

20. (d) Here $R = \{(x, y) : |x^2 - y^2| < 16\}$

and given $A = \{1, 2, 3, 4, 5\}$

$\therefore R = \{(1,2)(1,3)(1,4);(2,1)(2,2)(2,3)(2,4);(3,1)(3,2)$

$(3,3)(3,4);(4,1)(4,2)(4,3);(4,4)(4,5),(5,4)(5,5)\}$.

21. (d) Given, $xRy \Rightarrow x$ is relatively prime to y.

$\therefore$ Domain of $R = \{2, 3, 4, 5\}$.

22. (c) R be a relation on N defined by $x + 2y = 8$.

$\therefore$ $R\{(2,3); (4,2); (6,1)\}$

Hence, Domain of $R = \{2, 4, 6\}$.

23. (c) $\because R = \{(x, y) | \ x, y \in Z, x^2 + y^2 \leq 4\}$

$\therefore$ $R = \{(-2,0),(-1,0),(-1,1),(0,-1)(0,1),(0,2),(0,-2)$

$(1,0),(1,1),(2,0)\}$

Hence, Domain of $R = \{-2, -1, 0, 1, 2\}$.

24. (a) R is a relation from {11, 12, 13}to {8, 10, 12} defined by $y = x - 3 \Rightarrow x - y = 3$

$\therefore$ R = {11, 8},{13, 10}.

Hence, $R^{-1} = \{8, 11\}; \{10, 13\}$.

25. (a) It is obvious.

26. (b) It is obvious.

27. (a,b) (1, 1)(2, 2)(3, 3)(4, 4) $\in R$; $\therefore$ R is reflexive.

$\because$ (1, 2) (3, 1)$\in R$ and also (2, 1)(1, 3) $\in R$.

Hence, R is symmetric. But clearly R is not transitive.

28. (b) For any integer n, we have $n|\ n \Rightarrow nRn$

So, nRn for all $n \in Z \Rightarrow R$ is reflexive

Now 2|6 but 6∤2, $\Rightarrow (2,6) \in R$ but $(6, 2) \notin R$

So, R is not symmetric.

Let $(m,n) \in R$ and $(n,p) \in R$.

Then $\left.\begin{matrix}(m,n) \in R \Rightarrow m|\ n \\ (n,p) \in R \Rightarrow n|\ p\end{matrix}\right] \Rightarrow m|\ p \Rightarrow (m,p) \in R$

So, R is transitive.

Hence, R is reflexive and transitive but it is not symmetric.

29. (a) For any $a \in N$, we find that $a|a$, therefore R is reflexive but R is not transitive, because aRb does not imply that bRa.

30. (b) Let $(a,b) \in R$

Then, $(a,b) \in R \Rightarrow (b,a) \in R^{-1}$, [By def. of R^{-1}]

$\Rightarrow (b,a) \in R$, $[\because R = R^{-1}]$

So, R is symmetric.

31. (c) It is obvious.

32. (b) The relation is not symmetric, because $A \subset B$ does not imply that $B \subset A$. But it is anti-symmetric because $A \subset B$ and $B \subset A \Rightarrow A = B$.

33. (c) It is obvious.

34. (c) Since $x \not< x$, therefore R is not reflexive. Also $x < y$ does not imply that $y < x$, So R is not symmetric. Let xRy and yRz. Then, $x < y$ and $y < z \Rightarrow x < z$ *i.e.*, xRz. Hence R is transitive.

35. (b,c) x is a brother of y, y is also brother of x.

So, it is symmetric. Clearly it is transitive.

36. (c) Since $(1, 1) \notin R$ so, is not reflexive.

Now $(1, 2) \in R$ but $(2,1) \notin R$, therefore R is not symmetric Clearly R is transitive.

37. (b) The void relation R on A is not reflexive as $(a, a) \notin R$ for any $a \in A$. The void relation is symmetric and transitive.

38. (b) For any $a \in R$, we have $a \geq a$, Therefore the relation R is reflexive but it is not symmetric as $(2, 1) \in R$ but $(1, 2) \notin R$. The relation R is transitive also, because $(a,b) \in R, (b,c) \in R$ imply that $a \geq b$ and $b \geq c$ which is turn imply that $a \geq c$.

39. (a) It is obvious.

40. (c) It is obvious.

41. (d) It is obvious.

42. (d) It is obvious.

43. (d) It is obvious.

44. (c) $x \equiv 3 \pmod 7 \Rightarrow x - 3 = 7p, (p \in z)$

$\Rightarrow x = 7p + 3, p \in z$ *i.e.*, $\{7p + 3 : p \in z\}$.

45. (b) Given, R and S are relations on set A.

$\therefore R \subseteq A \times A$ and $S \subseteq A \times A \Rightarrow R \cap C \subseteq A \times A$

$\Rightarrow R \cap S$ is also a relation on A.

Reflexivity : Let a be an arbitrary element of A.

Then, $a \in A \Rightarrow (a,a) \in R$ and $(a,a) \in S$,

[$\because R$ and S are reflexive]

$\Rightarrow (a,a) \in R \cap S$

Thus, $(a,a) \in R \cap S$ for all $a \in A$.

So, $R \cap S$ is a reflexive relation on A.

Symmetry : Let $a, b \in A$ such that $(a,b) \in R \cap S$.

Then, $(a,b) \in R \cap S \Rightarrow (a,b) \in R$ and $(a,b) \in S$

$\Rightarrow (b,a) \in R$ and $(b,a) \in S$,

[$\because R$ and S are symmetric]

$\Rightarrow (b,a) \in R \cap S$

Thus, $(a,b) \in R \cap S$

$\Rightarrow (b,a) \in R \cap S$ for all $(a,b) \in R \cap S$.

So, $R \cap S$ is symmetric on A.

Transitivity : Let $a,b,c \in A$ such that $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$. Then, $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$

$\Rightarrow \{((a,b) \in R \text{ and } (a,b) \in S)\}$

and $\{((b,c) \in R \text{ and } (b,c) \in S\}$

$\Rightarrow \{(a,b) \in R, (b,c) \in R\}$ and $\{(a,b) \in S, (b,c) \in S\}$

$\Rightarrow (a,c) \in R$ and $(a,c) \in S$

$\left[\begin{matrix}\because R \text{ and } S \text{ are transitive So} \\ (a,b) \in R \text{ and } (b,c) \in R \Rightarrow (a,c) \in R \\ (a,b) \in S \text{ and } (b,c) \in S \Rightarrow (a,c) \in S\end{matrix}\right.$

$\Rightarrow (a,c) \in R \cap S$

Thus, $(a,b) \in R \cap S$ and $(b,c) \in R \cap S \Rightarrow (a,c) \in R \cap S$.

So, $R \cap S$ is transitive on A.

Hence, R is an equivalence relation on A.

46. (b, c, d) These are fundamental concepts.

47. (c) Here $R = \{(1,3),(2,2);(3,2)\}$, $S = \{(2,1);(3,2);(2,3)\}$

Then $RoS = \{(2,3);(3,2);(2,2)\}$.

48. (b) Here $\alpha R \beta \Leftrightarrow \alpha \perp \beta \ \therefore \ \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$

Hence, R is symmetric.

49. (d) We have $(a,b)R(a,b)$ for all $(a,b) \in N \times N$

Since $a + b = b + a$. Hence, R is reflexive.

R is symmetric for we have $(a,b)R(c,d) \Rightarrow a + d = b + c$

$\Rightarrow d + a = c + b \Rightarrow c + b = d + a \Rightarrow (c,d)R(e,f)$.

Then by definition of R, we have

$a + d = b + c$ and $c + f = d + e$,

whence by addition, we get

$a + d + c + f = b + c + d + e$ or $a + f = b + e$

Hence, $(a,b)\ R(e,f)$

Thus, $(a, b)\ R(c,d)$ and $(c,d)R(e,f) \Rightarrow (a,b)R(e,f)$.

50. (a,b,c,d) It is obvious.

51. (c) Here (3, 3), (6, 6), (9, 9), (12, 12), [Reflexive]; (3, 6), (6, 12), (3, 12), [Transitive].

Hence, reflexive and transitive only.

52. (b) It is obvious.

53. (c) Given $A = \{1, 2, 3, 4\}$

$R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$

$(2, 3) \in R$ but $(3, 2) \notin R$. Hence R is not symmetric.

R is not reflexive as $(1, 1) \notin R$.

R is not a function as $(2, 4) \in R$ and $(2, 3) \in R$.

R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

54. (d) Total number of reflexive relations in a set with n elements $= 2^n$.

Therefore, total number of reflexive relation set with 4 elements $= 2^4$.

55. (a) Since $1 + a.a = 1 + a^2 > 0, \forall a \in S$, $\therefore (a,a) \in R$

$\therefore R$ is reflexive.

Also $(a,b)\in R \Rightarrow 1+ab>0 \Rightarrow 1+ba>0 \Rightarrow (b,a)\in R$,

$\therefore$ R is symmetric.

$\because (a,b)\in R$ and $(b,c)\in R$ need not imply $(a,c)\in R$.

Hence, R is not transitive.

56. (a) $A=\{2,4,6\}$; $B=\{2,3,5\}$

$\therefore A\times B$ contains $3\times 3=9$ elements.

Hence, number of relations from A to $B=2^9$.

Critical Thinking Questions

1. (a) Since $8^n-7n-1=(7+1)^n-7n-1$

$=7^n+{}^nC_1 7^{n-1}+{}^nC_2 7^{n-2}+.....+{}^nC_{n-1}7+{}^nC_n-7n-1$

$={}^nC_2 7^2+{}^nC_3 7^3+..+{}^nC_n 7^n$, $({}^nC_0={}^nC_n, {}^nC_1={}^nC_{n-1}$ etc.)

$=49[{}^nC_2+{}^nC_3(7)+......+{}^nC_n 7^{n-2}]$

$\therefore$ 8^n-7n-1 is a multiple of 49 for $n\ge 2$

For $n=1$, $8^n-7n-1=8-7-1=0$;

For $n=2$, $8^n-7n-1=64-14-1=49$

$\therefore$ 8^n-7n-1 is a multiple of 49 for all $n\in N$.

$\therefore$ X contains elements which are multiples of 49 and clearly Y contains all multiplies of 49. $\therefore$ $X\subseteq Y$.

2. (b) $N_3\cap N_4=\{3,6,9,12,15......\}\cap\{4,8,12,16,20,.....\}$

$=\{12, 24, 36......\}=N_{12}$.

Trick : $N_3\cap N_4=N_{12}$

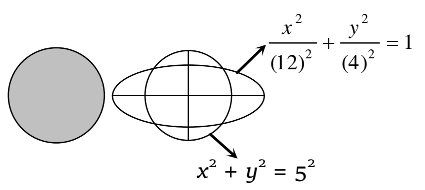
[$\because$ 3, 4 are relatively prime numbers].

3. (b) $n(A\cup B)=n(A)+n(B)-n(A\cap B)=3+6-n(A\cap B)$

Since, maximum number of elements in $A\cap B=3$

$\therefore$ Minimum number of elements in $A\cup B=9-3=6$.

4. (d) A = Set of all values (x, y) : $x^2+y^2=25=5^2$



$B=\frac{x^2}{144}+\frac{y^2}{16}=1$ *i.e.*, $\frac{x^2}{(12)^2}+\frac{y^2}{(4)^2}=1$.

Clearly, $A\cap B$ consists of four points.

5. (b) $A-B=A\cap B^c=A\cap\overline{B}$.

6. (a) Clearly, $A=\{2,3\}$, $B=\{2,4\}$, $C=\{4,5\}$

$B\cap C=\{4\}$

$\therefore A\times(B\cap C)=\{(2,4);(3,4)\}$.

7. (c) Let number of newspapers be x. If every students reads one newspaper, the number of students would be $x(60)=60x$

Since, every students reads 5 newspapers

$\therefore$ Numbers of students $=\frac{x\times 60}{5}=300$, $x=25$.

8. (c) Here A and B sets having 2 elements in common, so $A\times B$ and $B\times A$ have 2^2 *i.e.*, 4 elements in common.

Hence, $n[(A\times B)\cap(B\times A)]=4$.

9. (c) $(x,y)\in R\Leftrightarrow(y,x)\in R^{-1}$, $\therefore$ $R^{-1}=\{(3,1),(5,1),(1,2)\}$.

10. (a) $|a-a|=0<1$ $\therefore a\,R\,a\;\forall a\in R$

$\therefore$ R is reflexive.

Again $a\,R\,b\Rightarrow|a-b|\le 1\Rightarrow|b-a|\le 1\Rightarrow bRa$

$\therefore$ R is symmetric, Again $1R\frac{1}{2}$ and $\frac{1}{2}R1$ but $\frac{1}{2}\ne 1$

$\therefore$ R is not anti-symmetric.

Further, 1 R 2 and 2 R 3 but $1\not R 3$, [$\because |1-3|=2>1$]

$\therefore$ R is not transitive.

11. (d) Since $A\subseteq A$. $\therefore$ Relation '$\subseteq$' is reflexive.

Since $A\subseteq B$, $B\subseteq C\Rightarrow A\subseteq C$

$\therefore$ Relation '$\subseteq$' is transitive.

But $A\subseteq B$, $\Rightarrow B\subseteq A$, $\therefore$ Relation is not symmetric.

12. (d) Since $n\mid n$ for all $n\in N$, therefore R is reflexive. Since $2\mid 6$ but $6\nmid 2$, therefore R is not symmetric.

Let $n\;R\;m$ and $m\;R\;p\Rightarrow n|m$ and $m|p\Rightarrow n|p\Rightarrow nRp$. So, R is transitive.

13. (a) Let $A=\{1,2,3\}$ and $R=\{(1,1),(1,2)\}$, $S=\{(2,2)(2,3)\}$ be transitive relations on A.

Then $R\cup S=\{(1,1);(1,2);(2,2);(2,3)\}$

Obviously, $R\cup S$ is not transitive. Since $(1,2)\in R\cup S$ and $(2,3)\in R\cup S$ but $(1,3)\notin R\cup S$.

14. (a) We first find R^{-1}, we have

$R^{-1}=\{(5,4);(4,1);(6,4);(6,7);(7,3)\}$. We now obtain the elements of $R^{-1}oR$ we first pick the element of R and then of R^{-1}. Since $(4,5)\in R$ and $(5,4)\in R^{-1}$, we have $(4,4)\in R^{-1}oR$

Similarly, $(1,4)\in R,(4,1)\in R^{-1}\Rightarrow(1,1)\in R^{-1}oR$

$(4,6)\in R,(6,4)\in R^{-1}\Rightarrow(4,4)\in R^{-1}oR$,

$(4,6)\in R,(6,7)\in R^{-1}\Rightarrow(4,7)\in R^{-1}oR$

$(7,6)\in R,(6,4)\in R^{-1}\Rightarrow(7,4)\in R^{-1}oR$,

$(7,6)\in R,(6,7)\in R^{-1}\Rightarrow(7,7)\in R^{-1}oR$

$(3,7)\in R,(7,3)\in R^{-1}\Rightarrow(3,3)\in R^{-1}oR$,

Hence, $R^{-1}oR=\{(1,1);(4,4);(4,7);(7,4),(7,7);(3,3)\}$.

15. (d) On the set N of natural numbers,

$R=\{(x,y):x,y\in N, 2x+y=41\}$.

Since $(1,1)\notin R$ as $2.1+1=3\ne 41$. So, R is not reflexive.

$(1,39)\in R$ but $(39,1)\notin R$. So R is not symmetric (20, 1)

(1, 39 $\in R$. But $(20,39)\notin R$, So R is not transitive.

Set Theory and Relations

SET Self Evaluation Test - 1

1. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are **[MNR 1998, 91; UPSEAT 1999, 2000; Kerala (Engg.) 2003, 05]**

(a) 7, 6 (b) 6, 3

(c) 5, 1 (d) 8, 7

2. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where b, $c \in N$ are relatively prime, then **[DCE 1999]**

(a) $d = bc$ (b) $c = bd$

(c) $b = cd$ (d) None of these

3. Suppose $A_1, A_2, A_3, ..., A_{30}$ are thirty sets each having 5 elements and B_1, B_2, ..., B_n are n sets each with 3 elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each elements of S belongs to exactly 10 of the $A_i's$ and exactly 9 of the $B_j's$. Then n is equal to **[IIT 1981; MNR 1987]**

(a) 15 (b) 3

(c) 45 (d) None of these

4. If $A = \{1, 2, 3, 4, 5\}$, then the number of proper subsets of A is **[Pb. CET 2001; Karnataka CET 1997]**

(a) 120 (b) 30

(c) 31 (d) 32

5. Let A and B be two non-empty subsets of a set X such that A is not a subset of B, then **[UPSEAT 1995, 99]**

(a) A is always a subset of the complement of B

(b) B is always a subset of A

(c) A and B are always disjoint

(d) A and the complement of B are always non-disjoint

6. If $A = [x : f(x) = 0]$ and $B = [x : g(x) = 0]$, then $A \cap B$ will be

(a) $[f(x)]^2 + [g(x)]^2 = 0$ (b) $\frac{f(x)}{g(x)}$

(c) $\frac{g(x)}{f(x)}$ (d) None of these

7. If $n(A) = 3$ and $n(B) = 6$ and $A \subseteq B$. Then the number of elements in $A \cap B$ is equal to

(a) 3 (b) 9

(c) 6 (d) None of these

8. In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statements in this regard:

1. 10% families own both a car and a phone

2. 35% families own either a car or a phone

3. 40,000 families live in the town

Which of the above statements are correct

(a) 1 and 2 (b) 1 and 3

(c) 2 and 3 (d) 1, 2 and 3

9. Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, the minimum number of ordered pairs which when added to R make it an equivalence relation is

(a) 5 (b) 6

(c) 7 (d) 8

10. Let $A = \{2, 4, 6, 8\}$. A relation R on A is defined by $R = \{(2,4), (4,2), (4,6), (6,4)\}$. Then R is **[CET 1995]**

(a) Anti-symmetric (b) Reflexive

(c) Symmetric (d) Transitive

11. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b+c) = bc(a+d)$, then R is **[Roorkee 1995]**

(a) Symmetric only (b) Reflexive only

(c) Transitive only (d) An equivalence relation

12. The solution set of $8x \equiv 6 (\text{mod } 14), x \in Z$, are

(a) [8] ∪ [6] (b) [8] ∪ [14]

(c) [6] ∪ [13] (d) [8] ∪ [6] ∪ [13]

13. If A = {1, 2, 3} , B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x is greater than y'. The range of R is

(a) {1, 4, 6, 9} (b) {4, 6, 9}

(c) {1} (d) None of these

14. Let A = {p, q, r}. Which of the following is an equivalence relation on A

(a) $R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$

(b) $R_2 = \{(r, q), (r, p), (r, r), (q, q)\}$

(c) $R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$

(d) None of these

15. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then the relation R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

Answers and Solutions

(SET - 1)

1. (b) Since $2^m - 2^n = 56 = 8\times 7 = 2^3\times 7$

$\Rightarrow 2^n(2^{m-n}-1) = 2^3\times 7$, $\therefore n = 3$ and $2^{m-n} = 8 = 2^3$

$\Rightarrow m-n=3 \Rightarrow m-3=3 \Rightarrow m=6$; $\therefore m=6, n=3$.

2. (a) $bN =$ the set of positive integral multiples of b, $cN =$ the set of positive integral multiplies of c.

$\therefore$ $bN\cap cN =$ the set of positive integral multiples of bc

$= b\subset N$, [$\because b,c$ are prime]

$\therefore$ $d = bc$.

3. (c) $O(S) = O\left(\bigcup_{i=1}^{30} A_i\right) = \frac{1}{10}(5\times 30) = 15$

Since, element in the union S belongs to 10 of $A_i{}'s$

Also, $O(S) = O\left(\bigcup_{j=1}^{n} B_j\right) = \frac{3n}{9} = \frac{n}{3}$,

$\therefore \frac{n}{3} = 15 \Rightarrow n = 45$.

4. (c) The number of proper subset $= 2^n - 1$

$= 2^5 - 1 = 32 - 1 = 31$.

5. (d) $\because$ A is not a subset of B

$\therefore$ Some point of A will not be a point of B, So that point will being to B^c. Hence A and complement of B are always non-disjoint.

6. (a) $A\cap B = \{x : x\in A$ and $x\in B\}$

$= [x : f(x) = 0$ and $g(x) = 0] = [f(x)]^2 + [g(x)]^2 = 0$.

7. (a) Since $A\subseteq B$, $\therefore$ $A\cap B = A$

$\therefore$ $n(A\cap B) = n(A) = 3$.

8. (c) $n(P) = 25\%$, $n(C) = 15\%$

$n(P^c\cap C^c) = 65\%$, $n(P\cap C) = 2000$

Since, $n(P^c\cap C^c) = 65\%$

$\therefore n(P\cup C)^c = 65\%$ and $n(P\cup C) = 35\%$

Now, $n(P\cup C) = n(P) + n(C) - n(P\cap C)$

$35 = 25 + 15 - n(P\cap C)$

$\therefore n(P\cap C) = 40 - 35 = 5$. Thus $n(P\cap C) = 5\%$

But $n(P\cap C) = 2000$

$\therefore$ Total number of families $= \frac{2000\times 100}{5} = 40{,}000$

Since, $n(P\cup C) = 35\%$

and total number of families = 40,000

and $n(P\cap C) = 5\%$. $\therefore$ (2) and (3) are correct.

9. (c) R is reflexive if it contains (1, 1), (2, 2), (3, 3)

$\because$ $(1,2)\in R, (2,3)\in R$

$\therefore$ R is symmetric if $(2, 1), (3, 2) \in R$.

Now, $R = \{(1,1),(2,2),(3,3),(2,1),(3,2),(2,3),(1,2)\}$

R will be transitive if $(3, 1); (1, 3) \in R$. Thus, R becomes an equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1) (3,2) (1, 3) (3, 1). Hence, the total number of ordered pairs is 7.

10. (c) $A = \{2, 4, 6, 8\}$; $R = \{(2, 4)(4, 2)(4, 6)(6, 4)\}$

$(a, b)\in R \Rightarrow (b, a)\in R$ and also $R^{-1} = R$.

Hence, R is symmetric.

11. (d) For $(a, b), (c, d) \in N\times N$

$(a,b)R(c,d) \Rightarrow ad(b+c) = bc(a+d)$

Reflexive : Since $ab(b+a) = ba(a+b)\forall ab\in N$,

$\therefore (a,b)R(a,b)$, $\therefore$ R is reflexive.

Symmetric : For $(a,b),(c,d)\in N\times N$, let $(a,b)R(c,d)$

$\therefore$ $ad(b+c) = bc(a+d) \Rightarrow bc(a+d) = ad(b+c)$

$\Rightarrow cb(d+a) = da(c+b) \Rightarrow (c,d)R(a,b)$

$\therefore$ R is symmetric

Transitive : For $(a,b),(c,d),(e,f)\in N\times N$,

Let $(a,b)R(c,d),(c,d)R(e,f)$

$\therefore$ $ad(b+c) = bc(a+d)$, $cf(d+e) = de(c+f)$

$\Rightarrow adb + adc = bca + bcd$ .....(i)

and $cfd + cfe = dec + def$ .....(ii)

(i) $\times$ ef + (ii) $\times$ ab gives,

$adbef + adcef + cfdab + cfeab$

$= bcaef + bcdef + decab + defab$

$\Rightarrow adcf(b+e) = bcde(a+f) \Rightarrow af(b+e) = be(a+f)$

$\Rightarrow (a,b)R(e,f)$.

$\therefore$ R is transitive. R is an equivalence relation.

12. (c) $8x - 6 = 14P, (x\in Z)$

$\Rightarrow x = \frac{1}{8}[14P + 6]$, $(x\in Z)$

$\Rightarrow x = \frac{1}{4}(7P+3) \Rightarrow x = 6, 13, 20, 27, 34, 41, 48, \ldots$

$\therefore$ Sol set = {6, 20, 34, 48,...} $\cup$ {13, 27, 41, ...}

= [6] $\cup$ [13],

[6], [13] are equivalence classes of 6 and 13.

13. (c) Here R is a relation A to B defined by 'x is greater than y' $\therefore$ $R = \{(2,1);(3,1)\}$ Hence, range of $R = \{1\}$.

14. (d) Here $A = \{p, q, r\}$

R_1 is not symmetric because $(p,q) \in R_1$ but $(q,p) \notin R_1$

R_2 is not symmetric because $(r,q) \in R_2$ but $(q,r) \notin R_2$

R_3 is not symmetric because $(p,q) \in R_3$ but $(q,p) \notin R_3$.

Hence, R_1, R_2, R_3 are not equivalence relation.

15. (a,b,c,d) Here $l_1 R l_2$

l_1 is parallel l_2 and also l_2 is parallel to l_1, so it is symmetric.

Clearly, it is also reflexive and transitive. Hence it is equivalence relation.

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