Long Answer Type Questions

[4 Marks]

Que 1. In Fig. 9.24, ABCD is a parallelogram. Point P and Q on BC trisects BC. Prove that ar (ΔAPQ) = (ΔDPQ) = $\frac{1}{6}$ ar ($||^{gm}$ ABCD).

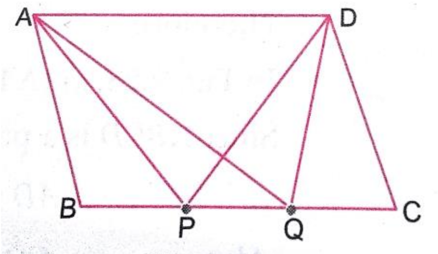

Fig. 9.24

Sol. Through P and Q, draw PR and QS parallel to AB [Fig. 9.25]. Now, PQSR is a parallelogram and its base PQ = $\frac{1}{3}$ BC.

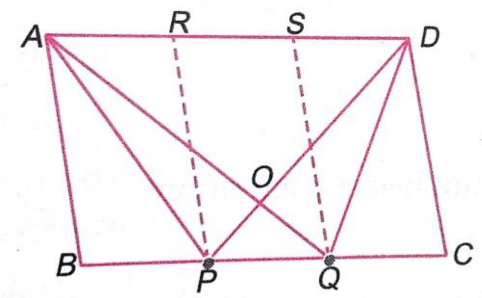

Fig. 9.25

Since ΔAPQ and ΔDPQ are on the same base PQ, and between the same parallel AD and BC.

∴ ar (ΔAPQ) = ar (ΔDPQ) …….(i)

Since ΔAPQ and ΔPQSR are on the same base PQ, and between same parallel PQ and AD.

∴ ar (ΔAPQ) = $\frac{1}{2}$ ar ($||^{gm}$ PQRS) …….(ii)

Now, $$\frac{ar(||^{gm}\ ABCD)}{ar(||^{gm}\ PQRS)} = \frac{BC \times height}{PQ \times height}$$

$$= \frac{3PQ}{PQ}\ (\because\ height\ of\ the\ two\ ||^{gm}\ is\ same)$$

$\Rightarrow\quad ar(||^{gm}\ PQRS) = \frac{1}{3} ar(||^{gm}\ ABCD)$ ……..(iii)

Using equation (ii) and (iii), we have

$$ar\ (\Delta APQ)\ =\ \frac{1}{2} ar(||^{gm}\ PQRS)\ =\ \frac{1}{2} \times \frac{1}{3} ar(||^{gm}\ ABCD)$$

Hence, ar(ΔAPQ) = ar (ΔDPQ) = $\frac{1}{6} ar\ (||^{gm}\ ABCD)$. [Using (i)]

Que 2. ABCD is a quadrilateral [Fig. 9.26]. Aline through D, parallel to AC meets BC produced in P. Prove ar (ΔABP) = ar (quad. ABCD).

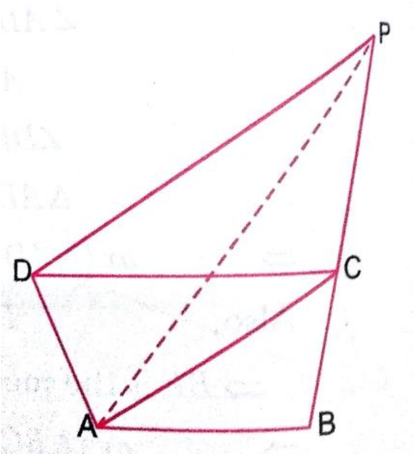

Fig. 9.26

Sol. Given: A quadrilateral ABCD in which DP||AC
To Prove: ar (ΔABP) = ar (quad. ABCD)
Proof: ΔACP and ΔACD are on same base AC and between same parallels AC and DP.
⇒ ar (ΔACP) = ar (ΔACD)
Adding, ar (ΔABC) on both sides,
⇒ ar (ΔABC) + ar (ΔACP) = ar (ΔABC) + ar (ΔACD)
ar (ΔABP) = (quad. ABCD)

Que 3. In Fig. 9.27, ABCD is a parallelogram and BC is produced to point Q such that BC = CQ. If AQ intersects DC at P. Show that ar (ΔBPC) = ar (ΔDPQ).

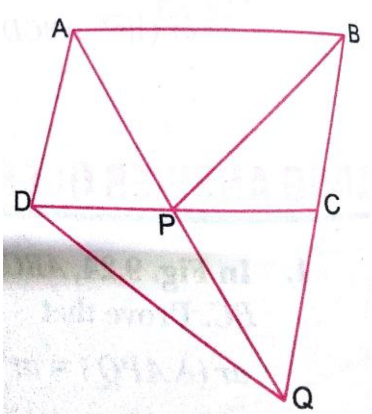

Fig. 9.27

Sol. Join AC. As triangle APC and BPC are on the same base PC and between the same parallels PC and AB.
Therefore,
In Fig. 9.28, ar (ΔAPC) = ar (ΔBPC) ....(i)
Since ABCD is a Parallelogram,

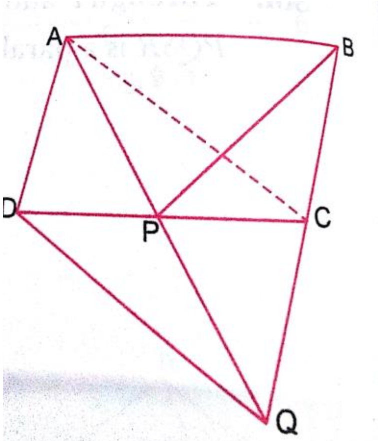

Fig. 9.28

∴ AD = BC (Opposite sides of parallelogram)
Also, CQ = BC (Given)
⇒ AD = CQ
Now, AD||CQ and AD=CQ
∴ ACQD is a parallelogram.
As diagonals of a parallelogram bisect each other.
∴ AP = PQ and CP = DP
In ΔAPC and ΔDPQ, we have
AP = PQ
∠APC = ∠DPQ Vertically opposite angles)
and PC = PD
∴ ΔAPC ≅ ΔDPQ (SAS congruence criterion)
⇒ ar (ΔAPC) = ar (ΔDPQ) ......(ii)
From (i) and (ii), we get
ar (ΔBPC) = ar (ΔDPQ)