Subject Code: 35 (NS)

MATHEMATICS

(English Version)

Instructions:

1. The question paper has five parts namely A, B, C, D, and E. Answer all the parts.

2. Use the Graph Sheet for the question on Linear Programming Problem on Part –E.

PART-A

Answer all the ten questions: **(10x1=10)**

1. **Let $*$ be the binary operation on N, given by $a*b=LCM$ of a and b. Find** $20*16$.

   Sol.

   20 * 16 = LCM of 20 and 16 = 80

2. **Find the principal value of** $\cos ec^{-1}(\sqrt{2})$.

   Sol.

   $\cos ec^{-1}=-\cos ec^{-1}\sqrt{2}$

   $=-\cos ec^{-1}(\cos ec\frac{\pi}{2})$

   $=-\frac{\pi}{4}$

3. **Construct a** 2×2 **matrix,** $A=\left[a_{ij}\right]$**,Where elements are given by,** $a_{ij}=\frac{i}{j}$.

   Sol.

   $a_{11}=\frac{1}{1}=1$ $\quad$ $a_{12}=\frac{1}{2}$

   $a_{12}=\frac{2}{1}=2$ $\quad$ $a_{22}=\frac{2}{2}=1$

   $\therefore A=\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$

4. If A is a square matrix with $|A| = 8$. Then find the value of $|AA'|$.

Sol.

$|AA'| = |A||A'| = 8 \times 8 = 64$

5. If $y = \cos\sqrt{x}$, find $\frac{dy}{dx}$.

Sol.

$$\frac{dy}{dx} = -\sin(\sqrt{x})\frac{1}{2\sqrt{x}}$$

6. Find $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$.

Sol.

$$\int (x^{\frac{1}{2}} + x^{\frac{-1}{2}})dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

7. Define collinear vectors.

Sol

Two vectors are collinear if they are parallel to the same line, irrespective of their magnitude and direction.

8. Find the direction cosines of a line which makes equal angles with the positive co-ordinate axes.

Sol.

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\alpha = \beta = \gamma$$

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$$3\cos^2\alpha = \frac{1}{3} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

DCS are

$\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

9. Define feasible region in a linear programming problem.

Sol.

The common shaded region of the given constraints LPP is called feasible region.

10. If A and B are independent events,

$P(A)=\frac{3}{5}$

And

P(B)=$\frac{1}{5}$

Then find $P(A\cap B)$.

Sol.

$P(A\cap B)=P(A)\cdot P(B)=\frac{3}{5}\times\frac{1}{5}=\frac{3}{25}$.

PART – B

Answer any ten questions: **(10x2=20)**

11. **If** $f:R\to R$ **defined by** $f(x)=1+x^2$**, then show that f is neither one-one nor onto.**

Sol.

$f(x)=y$

$1+x^2=y\Rightarrow x^2=y-1$

$x=\sqrt{y-1}$

if y = 0 then x = $\sqrt{-1}\notin R$

$\therefore$ 0 has no pre-image

$\therefore$ f is not onto.

12. **Show that** :

$$\sin^{-1}(2x\sqrt{(1-x^2)}=2\cos^{-1}x,\frac{1}{\sqrt{2}}\le x\le 1.$$

Sol.

$LHS=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ put $x=\cos\theta\Rightarrow\theta=\cos^{-1}x$

=$\sin^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$

=$\sin^{-1}\left(2\sin\theta\cos\theta\right)$

=$\sin^{-1}\left(\sin 2\theta\right)$

=2θ

=$2\cos^{-1}x=RHS$.

13. **Solve the equation** $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\left(x>0\right)$

Sol.

Put

$x=\tan\theta$

$\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\frac{1}{2}\tan^{-1}\left(\tan\theta\right)$

$\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)=\frac{1}{2}\theta$

$\frac{\pi}{4}-\theta=\frac{1}{2}\theta\Rightarrow\frac{\pi}{4}=\frac{\theta}{2}+\theta$

$\frac{\pi}{4}=\frac{3}{2}\theta\Rightarrow\theta=\frac{2\pi}{12}=\frac{\pi}{6}$

$\Rightarrow x=\tan\theta=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

14. Find the value of k, if area of triangle is 4 sq, units and verticles are (k,0) , (4,0) and (0,2) using determinant .

Sol.

Area of triangle =

$$\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$4=\frac{1}{2}\begin{vmatrix}k & 0 & 1\\ 4 & 0 & 1\\ 0 & 2 & 1\end{vmatrix}$$

$\Rightarrow k\left(0-2\right)-0+1\left(8-0\right)=8$

$\Rightarrow -2k+8=8$

$\Rightarrow 2k=0\Rightarrow k=0$

On taking $-ve$ sign we get

$-2k+8=-8$

$\Rightarrow 2k=16$

$\Rightarrow k=8$

$\therefore k=0,8$

15. $ax+by^2=\cos y$ **Then find** $\frac{dy}{dx}$.

Sol.

Given that,

$$ax+by^2=\cos y, \frac{dy}{dx}=?$$

Differentiate With respect to 'x'

$$a(1)+b.2y\frac{dy}{dx}=-\sin y\frac{dy}{dx}$$

$$2by\frac{dy}{dx}+\sin y\frac{dy}{dx}=-a$$

$$(2by+\sin y)\frac{dy}{dx}=-a$$

$$\frac{dy}{dx}=\frac{-9}{(2by+\sin y)}$$

16**. Verify Rolle's Theorem for the function** $f(x)=x^2+2x-8, x\in[-4,2]$

Sol.

$f(x)$ is a polynomial in x. Hence is continuous over [-4,2] and differentiable over (-4,2).

Therefore, all the three conditions of the Rolle's Theorem are satisfied.

$\therefore$ There exists a $c\in[-4,2]$ such that

$f'(c)=0$ $\quad\quad$ $f'(x)=2x+2$

$f'(c)=2c+2$

$$\Rightarrow f'(c)=0$$
$$\Rightarrow 2c+2=0$$
$$\Rightarrow 2c=-2$$
$$\Rightarrow c=-1\in[-4,2]$$

Hence proved, Rolle's Theorem.

17. Find the approximate change in the value of a cube of side x meters. Causded by increasing the side by 3%.

Sol.

$v = x^3$

$\frac{dy}{dx} = 3x^2$

$\Delta x = 3\%$ of $x = (0.03)x$

$\Delta v = \frac{dy}{dx}.\Delta x$

$= (3x^2)(0.03)x = 0.09x^3 m^3$

18. Integrate $\frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}$ **with respect to x.**

Sol.

We take,

$\tan\sqrt{x} = t$ and $\frac{\sec^2\sqrt{x}}{2\sqrt{x}}dt$,

Getting,

$I = 2\int t^4 dt$

$= \frac{2}{5}\tan^5\sqrt{x} + c$

19. Evaluate $\int_0^{2/3} \frac{dx}{4+9x^2}$

Sol.

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{x^2+\left(\frac{2}{3}\right)^2}$$

$$= \frac{1}{9}\left[\frac{3}{2}\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{2}{3}}$$

$$= \frac{1}{9}\times\frac{3}{2}\left[\tan^{-1}\frac{3}{2}\times\frac{2}{3}-0\right]$$

$$= \frac{1}{6}\left[\tan^{-1}(1)\right]$$

$$= \frac{1}{6}\times\frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

20. Find the order and degree of the differential equation

Sol.

Order = 1

Degree = 2

21. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ **respectively , in the raio 2:1.**

(1) Internally

(2) Externally

Sol.

The position vector of a point divided the line segment joining two points $P \ and \ Q$ in the ratio $m:n$ is given by

Case I Internally = $\dfrac{m\vec{b}+n\vec{a}}{m+n}$

Case II Externally = $\dfrac{m\vec{b}-n\vec{a}}{m-n}$

Position vectors of P and Q are given as

$OP = \hat{i} + 2j - k \; and \; OQ = -\hat{i} + j + k$

(1) PV of R [dividing (PQ)in the ratio 2:1 externally]

Here m = 2, n = 1

$$= \frac{m\left(PV \text{ of } Q\right) - n\left(PV \text{ of } P\right)}{m-n}$$

$$= \frac{2(PV \text{ of } Q)-1(PV \text{ of } P)}{2-1}$$

$$= 2(-\hat{i} + j + k) + (-1)(\hat{i} + 2j - k)$$

$$= (-3)\hat{i} + 0j + 3k$$

$$= -3\hat{i} + 3k$$

22. Find the area of parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - j + 3k$ **and** $\vec{b} = 2\hat{i} - 7j + k$.

Sol.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & j & k \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}\left[-1+21\right] - j\left[1-6\right] + k\left[-7+2\right] = 20\hat{i} + 5j - 5k$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{400+25+25}$$

$$= \sqrt{450}$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= \sqrt{450} \text{ sq units.}$$

$$= 15\sqrt{2} \text{ sq units.}$$

23. Find the vector and Cartesian equation of the line that passes through the points (3,-2,-5) and

(3,-2, 6).

Sol. Let a and b be the position vectors of points (3,-2,-5), and (3,-2, 6) respectively.

$\therefore \vec{a} = 3\hat{i} - 2\hat{j} - 5\hat{k}$ And $\vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

We know that the vector equation of a line passing through the points having position vectors a and b is

$r = \vec{a} + \lambda(\vec{b} - \vec{a})$

$$\therefore \quad \vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k}\lambda[(3\hat{i} - 2\hat{j} + 6\hat{k}) - (3\hat{i} - 2\hat{j} - 5\hat{k})]$$

$$\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k}\lambda[3\hat{i} - 2\hat{j} + 6\hat{k} - 3\hat{i} + 2\hat{j} + 5\hat{k}]$$

$$\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k}\lambda(11\hat{k})$$

That is vector equation ……(1)

Here putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in equation (1),

We get, $x\hat{i} + y\hat{j} + z\hat{k} = 3\hat{i} - 2\hat{j} + (11\lambda - 5)\hat{k}$

Comparing coefficient of $\hat{i}, \hat{j}$ and $\hat{k}$ on both sides,

$x = 3$, y=-2 and z=11λ-5

$$\Rightarrow \frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11} \quad [\text{where}, \frac{x-3}{d} \neq \infty, \frac{y+2}{0} \neq \infty]$$

Which is Cartesian form of the required line.

24. Find the probability distribution of number of heads in two tosses of a coin.

Sol.

Let X: Number of heads in two tosses of a coin.

Writing $X = 0, 1, 2$

PART – C

Answer any ten questions: **(10x3=30)**

25. Show that the relation R in R (set of real numbers) is defined as $R=\{(a,b): a \le b\}$ is reflexive and transitive but not symmetric.

Sol.

$a \le a$ is always true

Therefore

$a \in R \Rightarrow (a,a) \in R$

$\therefore R$ is reflexive

Let $(a,b) \in R \Rightarrow a \le b$ which does not imply b ≤ a

$\therefore (b,a) \notin R$

$\therefore R$ is not symmetric

Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow a \le b$ and $(b,c) \in R$

$\Rightarrow a \le c$

$\Rightarrow (a,c) \in R$

$\therefore R$ is transitive.

26. Write $\tan^{-1}\left(\frac{\sqrt{1\times x^2}-1}{x}\right), x \ne 0$ **in the simplest form.**

Sol.

Let x=tan$\theta \Rightarrow \theta$=tan$^{-1}x$.

therefore,

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\tan\frac{\theta}{2}\right]$$

$$= \frac{\theta}{2}$$

$$= \frac{\tan^{-1}x}{2} = \frac{1}{2}\tan^{-1}x$$

27. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if AB=BA.

Sol. Let A and B are symmetric

$\therefore A' = A$ and $B' = B$

Let $A\ B$ is symmetric

$\therefore (AB)' = AB,\ B'A' = AB,$ BA =AB

$(AB)' - B'A' = BA = AB$

Conversely Let

$AB = BA$

$\therefore AB$ is symmetric.

28. Differentiate $(\log x)^{\cos x}$ with respect to x.

Sol. $y = (\log x)^{\cos x}$

If taking the log on both sides ,

$$\log y = \log\left[(\log x \cos x)\right]$$
$$\log y = \cos x.\log(\log x)$$

Differentiating both respect to x

$$\frac{1}{y}.\frac{dy}{dx} = \frac{\cos x}{x.\cos x} + \log(\log x)(-\sin x)$$
$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\cos x}{x \log x} - \sin x \log(\log x)\right]$$
$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x \log x} - \sin x \log(\log x)\right]$$

29. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.

Sol:

$y_1 \sin^2 x$ $_2$and y=e$^{\cos x}$

Differentiating both sides then we get,

$$\frac{dy_1}{dx} = 2\sin \cdot \cos x....(i)$$

$And,$

$$\frac{dy_2}{dx} = e^{\cos x}.(-\sin x) \; ....(ii)$$

Now,

Divide eq (i) by eq (ii) then we get,

$$\frac{dy_1}{dy_2} = \frac{2\sin x \cdot \cos x}{e^{\cos x}.(-\sin x)}$$
$$= -\frac{2\cos x}{e^{\cos x}}$$

30. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

Sol:

Let the two number be x, y and $P = xy^3$

We have,

$$x + y = 60$$
$$\Rightarrow x = 60 - y$$

On putting this value in $P = xy^3$

$$P = (60 - y)\,y^3$$
$$\Rightarrow P = 60y^3 - y^4$$

Now, differentiate w.r.t y,

$$\frac{dP}{dy} = 180y^2 - 4y^3$$
$$\Rightarrow \frac{d^2P}{dy^2} = 360y - 12y^2$$

And for maxim and minima we have, $\frac{dp}{dy} = 0$

$\Rightarrow 180y^2 - 4y^3 = 0$

$\Rightarrow 4y^2(45 - y) = 0$

$\Rightarrow y = 0, 45$

At y=45,

$\left(\frac{d^2P}{dy^2}\right) = 360 \times 45 - 12 \times (45)^2$

$= 16200 - 24300$

$-8100 < 0$

$\Rightarrow P$ has local maximaat y=45

$\therefore$ By second derrivaties test, x=45 is point of local maxima of P.

Therefore, the function xy^3 is maximum at 45 and $x = 60 - 45 = 15$.

Hence, the required number is 15 and 45.

31. Evaluate: $\int \frac{2x}{x^2 + 3x + 2} dx.$

Sol.

$\int \frac{2x}{x^2+3x+2} dx, \frac{2x}{x^2+3x+2} = \frac{2x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$=2x\text{A}(x+2) + B(x+1)$

$x = -1, \qquad 2(-1) = A(-1+2) + B(0)$

$-2=\text{A}$

$x = 2,$

$-4 = A(0) + B(-2+1)$

$-4 = B(-1)$

$B = 4$

$\int \frac{2x}{x^2+3x+2} dx.$

$= \int \left(\frac{-2}{x+1} + \frac{4}{x+2} \right) dx$

$= -2\log(x+1) 4\log(x+2) + c$

32. Evaluate: $\int e^x \sin x dx$.

Sol.

$$I = \int \left(\underset{(2)}{e^x} \underset{(1)}{\sin x} \right) dx$$

$$= \sin x.e^x - - \int \underset{(2)}{e^x} \underset{(1)}{\cos} xdx$$

$$= \sin x.e^x - \left[\cos xe^x - \int e^x \left(-\sin x \right) dx \right]$$

$$= \sin x.e^x - \left[\cos x.e^x - \int e^x \sin xdx \right]$$

$$I = \sin xe^x - \cos xe^x - 1$$

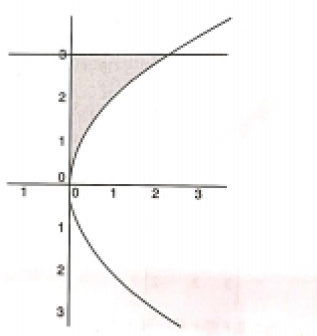
$$2I = e^x \left(\sin x - \cos x \right)$$

$$I = \frac{e^x}{2} \left(\sin x - \cos x \right) + c$$

33. Find area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y=3.

Sol.

Drawing to figure



Area of region:

$$A = \int_0^3 xdx$$

$$= \int_0^3 \frac{y^2}{4} dy$$

Getting:

$$A = \frac{27}{12}$$
$$= \frac{9}{4}$$

34. Form the differential equation of the family of circles having center on y-axis and radius 3 units.

Sol.

$$x^2 + (y-k)^2 = 3$$
$$2x + 2(y-k)\frac{dy}{dx} = 0$$

Substituting and getting the differential equation:

$$x^2 + \left(-\frac{x}{\frac{dy}{dx}}\right)^2 = 9$$

35. Find x, such that the four points A(3,2,1), B(4,x,5), C(4,2,-2) and D(6,5,-1) are coplanar.

Sol.

$$\overrightarrow{OA} = 3\hat{i} + 2\hat{j} + \hat{k}$$
$$\overrightarrow{OB} = 4\hat{i} + x\hat{j} + 5\hat{k}$$
$$\overrightarrow{OC} = 4\hat{i} + 2\hat{j} - 2\hat{k}$$
$$\overrightarrow{OD} = 6\hat{i} + 5\hat{j} - \hat{k}$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + (x-2)\hat{j} + 4\hat{k}$$
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \hat{i} + 0\hat{j} - 3\hat{k}$$
$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = 3\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = 0$$

$$\begin{bmatrix} \overset{+}{1} & x \overset{-}{-} 2 & \overset{+}{4} \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{bmatrix} = 0$$

$$1[0+9]-(x-2)[-2+9]+4[3-0]=0$$

$$9-(x-2)(7)+12=0$$

$$9-7x+14+12=0$$

$$35=7x$$

$$x=5$$

36. Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, evaluate

$\mu = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=1$ $|\vec{b}|=4$ and $|\vec{c}|=2$.

Sol.

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$\vec{a}.(\vec{a}+\vec{b}+\vec{c})=0 \Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$$

$$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-|\vec{a}|^2=-1$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}=-16$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}=-4$$

Adding all these:

$$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-21$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{-21}{2}$$

37. Find the shortest distance between the lines
$\vec{r} = \hat{i} + 2j + k + \lambda\left(\hat{i} - j + k\right)$ and $\vec{r}$=$2\hat{i} - j - k + \mu\left(2\hat{i} + j + 2k\right)$.

Sol.

We have the equation are:

$$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$$

And

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$$

Such that,

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$$
$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

Now,

$$\vec{a}_2 - \vec{a}_1 = \left(2\hat{i} + \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)$$
$$= \hat{i} - 3\hat{j} - 2\hat{k}$$

And,

$$\vec{b}_1 \times \vec{b}_2 = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$
$$= \hat{i}\left(2-1\right) - \hat{j}\left(2-2\right) + \hat{k}\left(1-2\right)$$
$$= 3\hat{i} + 3\hat{k}$$
$$\Rightarrow \left|b_1 \times b_2\right| = \sqrt{\left(-3\right)^2 + \left(3\right)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= z3\sqrt{2}$$

Substituting all the values in equation, we obtain

Shortest distance

$$=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_1-\vec{a}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

$$=\left|\frac{\left(-3\hat{i}+3k\right).\left(\hat{i}-3\hat{j}-2k\right)}{3\sqrt{2}}\right|$$

$$\frac{\left|(-3)\times 1+0\times(-3)+3\times -2\right|}{3\sqrt{2}}$$

$$=\frac{9}{3\sqrt{2}}\times\frac{3\sqrt{2}}{3\sqrt{2}}$$

$$=\frac{27\sqrt{2}}{18}$$

$$=\frac{3\sqrt{2}}{2}\text{units}$$

Therefore, the shortest distance between the two lines is $\frac{3\sqrt{2}}{2}$ units .

38. Given that the two numbers appearing on throwing two dice are different .Find the probability of the event 'the sum of numbers on the dice is 4'.

Sol.

When dice is thrown, number of observation in the sample space $S=6\times6=36$

$i.e., n(S)=36$

Let E : Set of numbers having sum=4

Let F : Set of number in which numbers appearing on the two dice are different

Then,

$$F=\left\{\begin{array}{l}(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,2),(6,3),(6,4),(6,5),(6,6)\end{array}\right\}$$

$$\Rightarrow n(F)=30$$

Here, F contains all points of S expect

$$\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$$\therefore E\cap F=\{(1,3),(3,1)\}$$

$$\therefore P(E)=\frac{\text{Number of favourable outcomes}}{\text{total number of outcomes}}$$

$$=\frac{3}{36}=\frac{1}{2}$$

Similarly, $P(F)=\frac{30}{36}=\frac{5}{6}$ and $P(E\cap F)=\frac{2}{36}=\frac{1}{18}$

Here, the required probability

$$=P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{18}}{\frac{5}{6}}=\frac{1}{15}$$

PART-D

Answer any six questions: **(6 x 5=30)**

39. Let $f : N \rightarrow R$ be a function defined as

$f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$, where S is the range of f, is invertible. Find the inverse of f.

Sol.

Let f $\left(x_1\right) = f\left(x_2\right)$

$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow \left(x_1^2 - x_2^2\right) + 12\left(x_1 - x_2\right) = 0$

$\Rightarrow 4\left(x_1 - x_2\right)\left(x_1 - x_2\right) + 12\left(x_1 - x_2\right) = 0$

$\Rightarrow \left(x_1 - x_2\right)[\left(4x_1 + x_2\right) + 12] = 0$

$\Rightarrow \left(x_1 - x_2\right) = 0\left(\because 4\left(x_1 + x_2\right) + 12 \neq 0\right)$

$\Rightarrow x_1 = x_2 \therefore f$ is one one

For the function

$f : N \rightarrow S,$ co-domain of $f = S =$ Range of f given

$\therefore$ f is onto , f is dijective , f^{-1} exists.

Let y be an arbitrary element of S. Then

$y = 4x^2 + 12x + 15,$ for some x in

N $\Rightarrow$ y$= \left(2x\right)^2 + 2.2x.3 + 3^2 + 6$

$y - 6 = \left(2x + 3\right)^2$

$\sqrt{y-6} = 2x + 3$

$\sqrt{y-6} - 3 = 2x$

$x = \dfrac{\sqrt{y-6} - 3}{2}$

Define a function

$g : S \rightarrow N$ by g$\left(y\right) = \dfrac{\sqrt{y-6} - 3}{2}$

$$gof(x) = g(f(x)) = g(4x^2 + 12x + 15) = g((2x-3)^2 + 6)$$

$$= \frac{\sqrt{(2x-3)^2 + 6 - 6}}{2}$$

$$= \frac{2x + 3 - 3}{2}$$

$$= x$$

Now,

$$fog(y) = f(g(y)) = f\left(\frac{\sqrt{y-6} - 3}{2}\right)$$

$$= 4\left(\frac{\sqrt{y-6} - 3}{2}\right)^2 + 12\left(\frac{\sqrt{y-6} - 3}{2}\right) + 15$$

$$= 4\left(\frac{y - 6 + 9 - 6\sqrt{y-6}}{4}\right)^2 + 12\sqrt{y-6} - 3 + 15$$

$$= y + 3 - 6\sqrt{y-6} + 6\sqrt{y-6} - 18 + 15$$

$$y - 15 + 15$$

$$= y$$

40. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, **Prove that** $A^3 - 6A^2 + 7A + 2I = 0.$

Sol.

Finding:

$$A^2 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$$

Finding:

$$A^3 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 13 \end{pmatrix}$$

Now,

$$A^3 - 6A^2 + 7A + 2I = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 13 \end{pmatrix} - 6\begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{LHS} = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 13 \end{pmatrix} + \begin{pmatrix} -30 & 0 & -48 \\ -12 & -24 & -30 \\ -48 & 0 & -78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\text{LHS} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

=0

=RHS

41. Solve the following system of linear equations by matrix method.

X − y + 2z = 1

2y − 3z = 1 And

3x − 2y + 4z = 2.

Sol.

Let,

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

and $$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Co-factors:

$C_{11} = 2,\ \ C_{12} = -9,\ \ C_{13} = -6$

$C_{21} = 0,\ \ C_{22} = -2,\ \ C_{23} = -1$

$C_{31} = -1,\ \ C_{32} = 3,\ \ C_{33} = 2$

$$\therefore \quad Adj\ A = \begin{pmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{pmatrix}$$

also $|A| = 1(8-6) + 3(3-4) = -1$

then,

$$A^{-1} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$$

now apply X= $A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

So, $x = 0, y = 5, z = 3$

42. If y = (tan^{-1}x)^2. Show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right) y_1 = 2.$

Sol.

Getting: $\frac{dy}{dx}$ or

$$y_1 = 2\tan^{-1} x . \frac{1}{1+x^2}$$

$\Rightarrow (1+x^2)y_1 = 2\tan^{-1} x$ or $(1+x^2)\frac{dy}{dx} = 2\tan^{-1} x$

$(1+x^2)y_2 + y_1(0+2x) = 2 \times \frac{1}{1+x^2}$ or $1+x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{2}{1+x^2}$

$\Rightarrow \left(1+x^2\right)y_2 + 2x\left(1+x^2\right)y_1 = 2$

43. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of

(1)The perimeter and

(2)The area of the rectangle.

Sol. At any instant of time t, let length, breadth, Perimeter, and area of the triangle are x, y, P and A respectively, then

$P = 2(x + y)$ and A=xy

It is given that $\frac{dx}{dt} = -5cm/\min$ and $\frac{dy}{dt} = 4cm/\min$

$(-ve$ sign shows that the length is decreasing$)$

(a) Now, $P = 2(x + y)$. on differentiating w.r.t. t,

We get,

Rate of change of perimeter

$$\frac{dp}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$=2(-5+4)$$

$$= -2cm/\min$$

Hence, perimeter of the rectangle is decreasing (-ve sign) at the rate of 2cm/min.

(b) Here, area of the rectangle $A = xy$. On differentiating w.r.t. t,

Rate of change

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$=8\times 4+6\times(-5)$$

$$=2\text{cm}^2/\min$$

Hence, are a of the rectangle is increasing at the rate of $2cm^2/\min$.

Note: If rate of change is increasing, we take positive sign and if rate of change is decreasing,

Then we take negative sign.

44. Find the integral of $\sqrt{x^2-a^2}$ with respect to x and hence evaluate $\int\sqrt{x^2-8x+7}\,dx$.

Sol. Let,

$$I=\int\sqrt{x^2-a^2}\,dx$$

$$I=\int\sqrt{x^2-a^2}.1.dx$$

$$I=\int\sqrt{x^2-a^2}.x-\int x.\frac{1}{2\sqrt{x^2-a^2}}.2x.dx$$

$$I=x\int\sqrt{x^2-a^2}-\int\frac{x^2}{\sqrt{x^2-a^2}}dx$$

$$I=x\int\sqrt{x^2-a^2}-\int\frac{x^2+a^2-a^2}{\sqrt{x^2-a^2}}dx$$

$$I=x\int\sqrt{x^2-a^2}-\int\frac{x^2-a^2}{\sqrt{x^2-a^2}}dx-a^2\int\frac{dx}{\sqrt{x^2-a^2}}$$

$$I=x\int\sqrt{x^2-a^2}-\int\sqrt{x^2-a^2}\,dx-a^2\int\frac{dx}{\sqrt{x^2-a^2}}$$

$$I=x\int\sqrt{x^2-a^2}-\int\sqrt{x^2-a^2}\,dx-a^2\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$I=x\int\sqrt{x^2-a^2}-I-a^2\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$2I=x\int\sqrt{x^2-a^2}-a^2\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$I=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$\therefore\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$=\int\sqrt{x^2-8x+7}\,dx$$

$$=\int\sqrt{(x-4)^2-3^2}\,dx$$

$$=\frac{x-4}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left(x-4\right)+\sqrt{x^2-8x+7}+C$$

45. Using integration find the area of the triangular region whose sides have the equation y = 2x + 1,   y= 3x + 1 and x = 4.

Sol.

Given equation of sides of the triangle are $y = 2x + 1, y = 3x + 1$ and x=4.

On solving these equations, we obtain the vertices of triangle as A(0,1) B(4,13) and C(4,9).

Therefor,

Required area (shown in shaded region)

$$= \text{Area}(OLBAO) - \text{Area}(OLCAO)$$

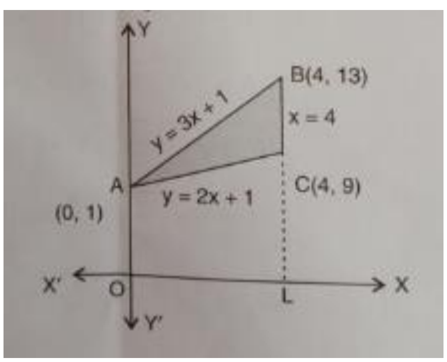
$$= \int_4^4 (3x - 1)\,dx - \int_0^4 (2x + 1)\,dx$$

$$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[x^2 + x\right]_0^4$$

$$= \frac{3 \times 4^2}{2} 0 - [42 + 4 - 0]$$

$$= 24 + 4 - 20$$

$$= 8 \text{ squnit.}$$



46. Solve the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x\left(0\le x<\frac{\pi}{2}\right).$

Sol.

$$\frac{dy}{dx}+\frac{1}{\cos^2 x}y=\frac{\tan x}{\cos^2 x}$$

$$\frac{dy}{dx}+(\sec^2 x)y=\tan x\sec^2 x$$

$$IF=e^{\int \sec^2 x dx}=e^{\tan x}$$

solution is $(IF)=\int Q(IF)\,dx+c$

$$ye^{\tan x}=\int \tan x\sec^2 xe^{\tan x}dx+c$$

put, tanx=$t\sec^2 xdx=dt$

$$ye^{\tan x}=\int e^t.t.dt+c$$

$=\text{te}^t-\int e^t+c$

$=\text{te}^t-e^t+c$

$=\tan xe^{\tan x}-e^{\tan x}+c$

$=e^{\tan x}(\tan x-1)+c$

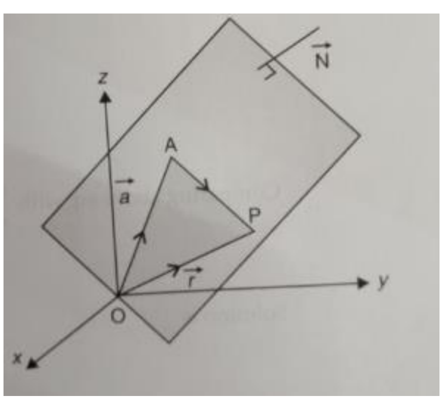
$=\tan x-1+ce^{-\tan x}$

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

Sol.

Let a plane passes through a point A with position vector $\vec{a}$ and perpendicular to the vector $\vec{N}$

Let $\vec{r}$ be the position vector of any point P(x, y, z) in the plane.



Then the point P lies in the plane if and only if $\overrightarrow{AP}$ is perpendicular to $\vec{N}$

i.e. $\overrightarrow{AP}.\vec{N} = 0$

But $\overrightarrow{AP} = \vec{r} - \vec{a}$

So, $(\vec{r} - \vec{a}).\vec{N} = 0$

This is the vector equation of the plane

Cartesian Form

Let the given point A be $\left(x_{1,} y_{1,} z_1\right)$, P be $\left(x_, y_, z\right)$ and direction ratios of $\vec{N}$ are A,B and C. Then $\vec{a} = \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right)$,

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$

now, $(\vec{r} - \vec{a}).\vec{N} = 0$

$\left[(x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k}\right].A\hat{i} + B\hat{j} + C\hat{k} = 0$

i.e., $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

48. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs.

(1) None

(2) Not more than one

(3)More than one

Sol.

$n = 5, p = 0.05$

$P\left(X = x\right) = {}^nC_x q^{n-x}.P^x = {}^5C_x\left(0.95\right)^{5-x}.\left(0.05\right)^x$

$P\left(X = 0\right) = {}^5C_0\left(0.95\right)^{5-0}.\left(0.05\right)^0 = \left(0.95\right)^5$

$P\left(x \le 1\right) = P\left(0\right) + P\left(1\right)$ s

$= \left(0.95\right)^5 + {}^5C_1\left(0.95\right)^4\left(0.05\right)^1$

$= P\left(x > 1\right) = 1 - P\left(x \le 1\right)$

$= 1 - \left(0.95\right)^5 + {}^5C_1\left(0.95\right)^4\left(0.05\right)^1$

PART – E

Answer any one question: **(1 x 10 =10)**

49. (a) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx.$

Sol.

$$\int_0^a f\left(x\right)dx = \int_0^a f\left(x-a\right)dx$$

$Let,\ x = a-t$

$\Rightarrow dx = -dt$ and at $x = 0, t = a,$ at $x = a, t = 0$

Substituting the values

L.H.S.=$\int_a^0 f\left(a-t\right) - dt$

=$\int_a^0 f\left(a-t\right) - dt$

=$\int_0^a f\left(0-t\right)dt$ [we know that $\left(-\int_a^b f\left(x\right)dx = \int_b^a f\left(x\right)dx\right)$]

$t \to x$

$L.H.S. = \int_0^a f\left(a-x\right)dx$ $\left(\because \int f\left(x\right)dx = f\left(t\right)dt\right)$

=R.H.S.

now we take ,

$$I = \int_0^{\frac{\pi}{2}} \left(2\log\sin x - \log\sin 2x\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \log\left[\frac{\left(\sin x\right)^2}{2\sin x\cos x}\right]dx$$

Replacing,

$= \int_0^{\frac{\pi}{2}} \left[\log\left(\tan x\right) - \log 2\right]dx$ …………..(1)

$$I' = \int_0^{\frac{\pi}{2}} [\log(\tan x)]dx$$

$$= \int_0^{\frac{\pi}{2}} \log \tan\left(\frac{\pi}{2} - x\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \log \cot x dx$$

Adding equation (1) and (2),

$$2I' = \int_0^{\frac{\pi}{2}} [\log(\tan x) + \log(\cot x)]dx \qquad \text{...........(2)}$$

$$= \int_0^{\frac{\pi}{2}} [\log(\tan x \cot x)]dx$$

$$= \int_0^{\frac{\pi}{2}} \log(1) = 0$$

from(1),

$$I = \int_0^{\frac{\pi}{2}} (-\log 2)dx$$

$$= [-x \log 2]_0^{\frac{\pi}{2}}$$

$$\Rightarrow I = -\frac{\pi}{2}\log 2$$

(b)Show that:

$$\begin{pmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{pmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$$

Sol:

$$\begin{pmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{pmatrix} = \frac{1}{xyz}\begin{pmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{pmatrix} \text{ using } R_1 \to xR_1 \text{ and } R_2 \to yR_2 \text{ and } R_3 \to zR_3$$

$$= \frac{xyz}{xyz}\begin{pmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} x^2 & x^3 & 1 \\ y^2 - x^2 & y^3 - x^3 & 0 \\ z^2 - x^2 & z^3 - x^3 & 0 \end{pmatrix}$$

Now, expanding along to C_3

$$\begin{pmatrix} & & \\ y^2 - x^2 & y^3 - x^3 & 0 \\ z^2 - x^2 & z^3 - x^3 & 0 \end{pmatrix}$$

$$= \left[\left(y^2 - z^2\right)\left(z^3 - x^3\right) - \left(z^2 - x^2\right)\left(y^3 - x^3\right)\right]$$

$$= (y+x)(y-x)(z-x)\left(z^2 + x^2 + xz\right) - (z+x)(z-x)\left(y^2 + x^2 + xy\right)$$

$$= (y-x)(z-x)\left[(y+x)\left(z^2 + x^2 + xz\right) - (z+x)\left(y^2 + x^2 + xy\right)\right]$$

$$= (y-x)(z-x)\left[yz^2 + yx^2 + xyz + xz^2 + x^3 + x^2z - zy^2 - zx^2 - xyz - xy^2 - x^3 - x^2y\right]$$

$$= (y-x)(z-x)\left[yz^2 + zy^2 - xy^2\right]$$

$$= (y-x)(z-x)\left[yz(z-y) + x(z-y)(z+y)\right]$$

$$= (y-x)(z-x)\left[(z-y)(xy + yz + zx)\right]$$

$$= (y-x)(z-x)(z-y)(xy + yz + zx)$$

Hence, proved.

50. (a) Minimize and Maximize z =600x + 400y

Subject to the constraints:

$x + 2y \le 12$

$2x + y \le 12$

$4x + 5y \ge 20$ and $x \ge 0, y \ge 0$

By graphical method.

Sol:

$x + 2y \le 12$

$y = 0 \Rightarrow x = 12$ P$(12, 0)$

$x = 0 \Rightarrow y = 16$ D$(0, 6)$

$2x + y = 20$

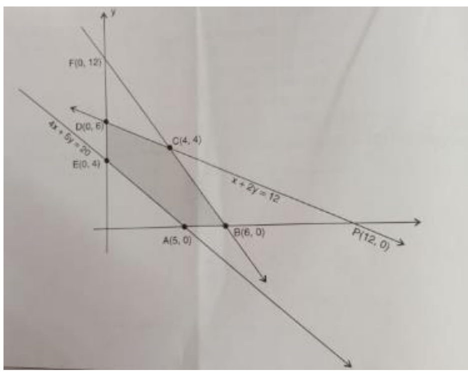
$y = 0 \Rightarrow x = 6$ B$(5, 0)$

$x = 0 \Rightarrow y = 12$ F$(0, 12)$

$4x + 5y = 20$

$y = 0 \Rightarrow x = 5$ B$(5, 0)$

$x = 0 \Rightarrow y = 4$ E$(0, 4)$



Corner Point	Z=600x+400y
(5,0)	**3000**
(6,0)	**3600**
(4,4)	$4000 \leftarrow Maximum$
	2400
	$1600 \leftarrow Maximum$

(b)Find the value of k, if

$$f(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x}, & \text{x} \ne \dfrac{\pi}{2} \\ 3, & \text{x} = \dfrac{\pi}{2} \end{cases}$$

Is continuous at $x = \frac{\pi}{2.}$

Sol:

$$f\left(\frac{\pi}{2}\right) = \lim_{x\to\frac{\pi}{2}}\left(\frac{k\cos x}{\pi - 2x}\right)$$

Taking $\pi - 2x = \theta$ and stating h $\to 0$

Taking $\frac{\pi}{2} - x = h$ and stating h $\to 0$

$$3 = \lim_{\theta\to 0}\frac{k\cos\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}{\theta} = \lim_{\theta\to 0}\frac{k\sin\left(\frac{\theta}{2}\right)}{\theta}$$

$$3 = \lim_{\theta\to 0}\frac{k\cos\left(\frac{\pi}{2}-\theta\right)}{2\theta} = \lim_{\theta\to 0}\frac{k\sin\theta}{2\theta}$$

$$3 = \lim_{\theta\to 0}\frac{k\sin\left(\frac{\pi}{2}+h\right)}{\pi - 2\left(\frac{\pi}{2}+h\right)} = \lim_{\theta\to 0}\frac{k\left(-\sinh\right)}{-2h}$$

$= 6$

We get =6