PARTIAL DIFFERENTIATION

- **DEFINITION :** If $U = f(x,y)$ then the derivative 'u' of with respect to x, when x varies and y remains constant is called the partial derivative of u with respect to x and is denoted by

$$\frac{\partial u}{\partial x} \, or \, U_x \, or \, f_x$$

$$\therefore \frac{\partial u}{\partial x} = \underset{\delta x \to 0}{Lt} \frac{f(x+\delta x, y) - f(x,y)}{\delta x}$$

Similarly $\frac{\partial u}{\partial y} = \underset{\delta y \to 0}{Lt} \frac{f(x, y+\delta y) - f(x,y)}{\delta y}$

- Partial derivative of $u = f(x,y)$ wrt x is the ordinary derivative of 'u' w.r.t 'x' treating 'y' as constant

  similarly partial derivative of $u = f(x,y)$ w.r.t y is the ordinary derivative of 'u' w.r.t 'y' treating x as constant

- $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial x^2} \, or \, U_{xx} \, or \, f_{xx};$

  $\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial y^2} \, or \, U_{yy} \, or \, f_{yy}$

- $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial x \partial y} \, or \, U_{xy} \, or \, f_{xy};$

  $\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial y \partial x} \, or \, U_{yx} \, or \, f_{yx}$

- $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$, when $u = f(x,y)$ is continuous function.

- If $z = f(u)$ and $u = g(x,y)$ then

  $\frac{\partial z}{\partial x} = \frac{dz}{du} \cdot \frac{\partial u}{\partial x}$ and $\frac{\partial z}{\partial y} = \frac{dz}{du} \cdot \frac{\partial u}{\partial y}$

- <u>Relation between ordinary derivative and partial derivative</u>

  $If \; f(x,y) = c \; then \; \frac{dy}{dx} = \frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$

- **<u>HOMEGENEOUS FUNCTION :</u>**

  $u = f(x,y)$; *If* $f(tx, ty) = t^n f(x,y)$ then $f(x,y)$ is called homogeneous function of degree 'n'

  If $u = f(x,y)$ is a homogeneous function of degree 'n' then

  $$f(x,y) = x^n \phi\left(\frac{y}{x}\right) (or) \, f(x,y) = y^n \psi\left(\frac{x}{y}\right)$$

- **<u>Euler's Theorem :</u>** If $u = f(x,y)$ is a homogeneous fuction of degree n in x and y then

  g $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

  g $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$

  g $x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y}$

  g $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

- If $f(u)$ is a homogenous fucntion in x and y of degree 'n' then

  g $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{nf(u)}{f'(u)}$

  g $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \phi(u)\left[\phi'(u) - 1\right]$

  where, $\phi(u) = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

- If $f(x,y,z)$ is a homogeneous function in x,y and z of degree 'n' then

  g $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf$

  g $x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + z^2\frac{\partial^2 f}{\partial z^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + 2yz\frac{\partial^2 f}{\partial y \partial z}$

  $+ \, 2zx\frac{\partial^2 f}{\partial z \partial x} = n(n-1)f$

- If $u = f(x,y)$ then $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$
- If $u = f(x,y)$ and $x = g(t); y = h(t)$ then
$$\frac{du}{dt} = \frac{\partial u}{\partial x}.\frac{dx}{dt} + \frac{\partial u}{\partial y}.\frac{dy}{dt}$$
- If α is a function of u and v where u, v are functions of x and y, then
$$\frac{\partial \alpha}{\partial x} = \frac{\partial \alpha}{\partial u}.\frac{\partial u}{\partial x} + \frac{\partial \alpha}{\partial v}.\frac{\partial v}{\partial x}$$
$$\frac{\partial \alpha}{\partial y} = \frac{\partial \alpha}{\partial u}.\frac{\partial u}{\partial y} + \frac{\partial \alpha}{\partial v}.\frac{\partial v}{\partial y}$$
- If $u = f(r)$ where $r^2 = x^2 + y^2$ then
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{f'(r)}{r}$$
- If $u = f(r)$ where $r^2 = x^2 + y^2 + z^2$ then
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r}f'(r)$$
- If $u = f(x-y, y-z, z-x)$ then
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$
- If $u = r^m$ and $r^2 = x^2 + y^2 + z^2$ then
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$$

CONCEPTUAL QUESTIONS

1. $\lim\limits_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h}$ if exists=

   1) $\frac{df}{dx}$ 2) $\frac{\partial f}{\partial x}$ 3) $\frac{df}{dy}$ 4) $\frac{\partial f}{\partial y}$

2. If f(x,y) is a homogeneous function of degreen 'n' then $\frac{\partial f}{\partial y}$ is

   1) not a homogeneous function
   2)homogeneous function of n^{th} degree
   3) homogeneous function of $(n-1)^{th}$ degree
   4) homogeneous function of $(n-2)^{th}$ degree

3. If $f(u) = g(x,y)$ and $g(x,y)$ is a homogeneous function of degreen n, then $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} =$

   1) $nf(u)$ 2) $n.\frac{f'(u)}{f(u)}$
   3) $n.\frac{f(u)}{f'(u)}$ 4. n

4. If u = log v and v is a homogeneous function of degree 'n' in x and y then $xu_x + yu_y =$

   1) 0 2) n
   3) ne^u 4) $n\log v$

5. If $f = x(y-z) + y(z-x) + z(x-y)$ then $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} =$

   1) 0 2) 1 3) 2 4) -1

6. If $f(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$ then $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} =$

   1) 0 2) 2f 3) 4f 4) f

7. If $u = \frac{x^2(x^2-y^2)^3}{(x^2+y^2)^2}$ then 'u' is a homogenous function of degree

   1) 1 2) 8 3) 6 4) 4

8. If $u = \sin^{-1}\left\{\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$ then
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$$

   1) $\frac{\tan u}{12}(13+\tan^2 u)$ 2) $\frac{\tan u}{144}(13+\tan^2 u)$
   3) $\frac{\sin u}{144}(13-\tan^2 u)$ 4) $\frac{13}{144}u$

9. if $z = f(ax+by) + g(ax-by)$ then $b^2 z_{xx} - a^2 z_{yy} =$

   1) z 2) -z
   3) $a^2 - b^2$ 4) 0

10. If f is homogenous function of degree 'n' in x,y then $(n-1)\begin{vmatrix} f_{xx} & f_{xy} & f_x \\ f_{xy} & f_{yy} & f_y \\ f_x & f_y & 0 \end{vmatrix}$

1) $n\left(f_{xy}{}^2 - f_{xx}f_{yy}\right)f$

2) $n\left(f_{xy}{}^2 + f_{xx}f_{yy}\right)f$

3) $\left(f_{xy}{}^2 - f_{xx}f_{yy}\right)f$

4) $n\left(f_{xx} + f_{yy}\right)$

KEY

1)2	2)3	3)3	4)2	5)1
6)1	7)4	8)2	9)4	10)1

LEVEL - I

1. If $u=\sqrt{x^2+y^2+z^2}$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=$

1)1/u 2)u 3)0 4)2/u

2. If $u=x^3+y^3-3xy$ then $\frac{\partial^2 u}{\partial x \partial y}=$

1)0 2)1 3)-3 4)4

3. If $u=e^x.\cos y, V=e^x \sin y$ then $\frac{\partial u}{\partial x}=$

1)$\frac{\partial v}{\partial y}$ 2)$-\frac{\partial v}{\partial y}$ 3)$-\frac{1}{u}\frac{\partial u}{\partial y}$ 4)$\frac{1}{v}\frac{\partial u}{\partial y}$

4. If $u=Tan^{-1}\left(\frac{y}{x}\right)$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1)u 2)2u 3)3u 4)0

5. If $u=\frac{1}{\sqrt{x^2+y^2}} : r^2=x^2+y^2$ then

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1) $\frac{1}{r}$ 2) $\frac{1}{r^2}$ 3) $\frac{1}{r^3}$ 4) $\frac{2}{r^2}$

6. If $f=\cosh(y+\cos x)$ then $f_{xy}=$

1) $-\sin x\cosh(y+\cos x)$

2) $\sin x\cosh(y+\cos x)$

3) $\cos x\sinh(y+\cos x)$

4) $-\cos x\sinh(y+\cos x)$

7. If $f(x,y)=\sin\left(e^{ax}+e^{by}\right)$ then $f_{xy}=$

1) $ab\,e^{ax+by}\sin\left(e^{ax}+e^{by}\right)$

2) $-ab\,e^{ax+by}\sin\left(e^{ax}+e^{by}\right)$

3) $ab\,e^{ax+by}\cos\left(e^{ax}+e^{by}\right)$

4) $-ab\,e^{ax+by}\cos\left(e^{ax}+e^{by}\right)$

8. If $f(x,y)=x^y$ then $f_{yy}=$

1) $(\log x)^2$ 2) $y^x(\log x)^2$

3) $x^y\log x$ 4) $x^y(\log x)^2$

9. If $u=\sin(ax+by+cz)$ then

$\sec(ax+by+cz).\left[\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right]=$

1) ax+by+cz 2) abc
3) a+b+c 4) x+y+z

10. If $u=\log(ax+by+cz)$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) $\frac{a+b+c}{ax+by+cz}$ 2) $\frac{ax+by+cz}{a+b+c}$

3) $(a+b+c).(ax+by+cz)$

4) a+b+c

11. If $u=\frac{1}{\sqrt{x^2+y^2+z^2}}$

then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=$

1)u 2)-u 3)2u 4)-2u

12. If $u=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=$

1)u 2)-u 3)2u 4)0

13. If $u=\dfrac{x-y}{\sqrt{x^2+y^2}}$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1)u 2)-u 3)3u 4)0

14. If $u=\sin^{-1}\dfrac{x}{y}+\tan^{-1}\dfrac{y}{x}$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1)u 2)2u 3)3u 4)0

15. If $u=xy^2f\left(y/x\right)$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1)u 2)2u 3)3u 4)-3u

16. If $u=x^3y^3\tan\left(x/y\right)$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1)u 2)2u 3)6u 4)7u

17. If $u=\left(x^3+y^3+z^3\right)\left(x+y+z\right)$

then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=$

1)u 2)-3u 3)12u 4)4u

18. If $u=\log\left(\dfrac{x^4+y^4}{x^2+y^2}\right)$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1)2 2)2u 3)3u 4)u

19. If $f\left(x,y\right)=x^4y^2\sin^{-1}\left(\dfrac{y}{x}\right)$ then $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=$

1)f 2)0 3)4f 4)6f

20. If $\sin u=\dfrac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}$ then $\Sigma x\dfrac{\partial u}{\partial x}=$

1)3sinu 2)-3 cosecu
3)3tanu 4)-3 tanu

21. If $u=\dfrac{\sqrt[3]{y}-\sqrt[3]{x}}{x+y}$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1)(2/3)u 2)(-2/3)u
3)(3/2)u 4)-(3/2)u

22. If $z=\dfrac{x^4y^4\left(x-y\right)^3}{\left(x^2+y^2\right)^{5/2}}$ then $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=$

1) 2z 2) 4z 3) 6z 4) 8z

23. If $u=\sin^{-1}\left(\dfrac{x}{y}\right)+\tan^{-1}\left(\dfrac{y}{x}\right)$ then

$x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=$

1) 0 2) u 3) 2u 4) 4u

24. If $f\left(x,y,z\right)=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}-1$ then $\Sigma x\dfrac{\partial f}{\partial x}=$

1) 2f 2) f+2
3) 2f+2 4) 2f-2

25. If $Z=x\tan^{-1}\left(\dfrac{y}{x}\right)+xe^{x/y}$ then

$x^2\dfrac{\partial^2 z}{\partial x^2}+2xy\dfrac{\partial^2 z}{\partial x\partial y}+y^2\dfrac{\partial^2 z}{\partial y^2}=$

1) 2z 2) z 3) 3z 4) 0

26. If $u=x\phi\left(\dfrac{y}{x}\right)-y\psi\left(\dfrac{y}{x}\right)$ then

$x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=$

1) 0 2) 4 3) -4 4) 1

27. If $u=x^3-3x^2y+y^3$ then $x\dfrac{\partial^2 u}{\partial x^2}+y\dfrac{\partial^2 u}{\partial x\partial y}=$

1) $6\left(x^2-2xy\right)$ 2) $6x\left(x-y\right)$

3) $6\left(x-2y\right)$ 4) $6\left(2y-x\right)$

28. If $u=ax^2+2hxy+by^2$ then $x\dfrac{\partial^2 u}{\partial x\partial y}+y\dfrac{\partial^2 u}{\partial y^2}=$

1) $2\left(hx+by\right)$ 2) $2\left(hx-by\right)$

3) $2\left(bx+hy\right)$ 4) $2\left(bx-hy\right)$

29. If $f\left(x,y\right)=x\cos\left(\dfrac{y}{x}\right)+y\tan\left(\dfrac{y}{x}\right)$ then

$x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=$

1)0 2)1

3) $f\left(x,y\right)$ 4) $2f\left(x,y\right)$

30. If $u=x^3-3x^2y+y^3$ then $x\dfrac{\partial^2 u}{\partial x\partial y}+y\dfrac{\partial^2 u}{\partial y^2}=$

1) $6\left(y^2-x^2\right)$ 2) $6\left(x^2-y^2\right)$

3) $6\left(x^2+y^2\right)$ 4) $6\left(x-2y\right)$

31. If $z=e^{xy}$ then dz=

1)z(dx+dy) 2)z(xdx+ydy)
3)z(ydx+xdy) 4)xdx+ydy

32. If $u=\frac{x^4+y^4}{x-y}$ then $x\frac{\partial^2 u}{\partial x\partial y}+y\frac{\partial^2 u}{\partial y^2}=$

1) $\frac{\partial u}{\partial y}$ 2) $2\frac{\partial u}{\partial y}$ 3) $\frac{\partial u}{\partial x}$ 4) $4\frac{\partial u}{\partial x}$

33. If $z=f\left(x^k y\right)$ satisfies $x\frac{\partial z}{\partial x}=2y\frac{\partial z}{\partial y}$ then k=

1)1 2)2 3)3 4)0

34. $\frac{\partial f}{\partial x}=e^x\cos y$ and $\frac{\partial f}{\partial y}=-e^x\sin y$ then f(x,y)=

1) $e^x\sin y$ 2) $e^x\cos y$

3) $e^x\left(\sin y+\cos y\right)$ 4) $e^x+\sin y$

35. If $z=xe^y, x=t, y=1+3t$ then $\frac{dz}{dt}=$

1) $e^y.3t$ 2)xz

3) $e^y\left(1+3t\right)$ 4)xyz

36. $x=e^{-t}\cos\theta; y=e^{-t}\sin\theta$ then $\frac{\partial t}{\partial x}=$

1) $\frac{x}{x^2+y^2}$ 2) $\frac{y}{x^2+y^2}$

3) $-\frac{x}{x^2+y^2}$ 4) $-\frac{y}{x^2+y^2}$

37. If $x=e^{-t}\cos\theta; y=e^{-t}\sin\theta$ then $\frac{\partial\theta}{\partial x}=$

1) $\frac{x}{x^2+y^2}$ 2) $\frac{y}{x^2+y^2}$

3) $-\frac{x}{x^2+y^2}$ 4) $-\frac{y}{x^2+y^2}$

38. $u=\sin^{-1}\left(x-y\right); x=3t; y=4t^3$ then $\frac{du}{dt}=$

1) $\frac{1}{\sqrt{1-t^2}}$ 2) $\frac{2}{\sqrt{1-t^2}}$

3) $\frac{3}{\sqrt{1-t^2}}$ 4) $\frac{4}{\sqrt{1-t^2}}$

39. If $u=\cos^{-1}\left(x-y\right); x=4t^3; y=3t$ then $\frac{du}{dt}=$

1) $-\frac{1}{\sqrt{1-t^2}}$ 2) $-\frac{2}{\sqrt{1-t^2}}$

3) $-\frac{3}{\sqrt{1-t^2}}$ 4) $-\frac{4}{\sqrt{1-t^2}}$

40. If $u=\tan^{-1}\left[\frac{x-y}{1-z}\right]$ and $x=3t; y=t^3; z=3t^2$ then $\frac{du}{dt}=$

1) $\frac{1}{1+t^2}$ 2) $\frac{2}{1+t^2}$

3) $\frac{3}{1+t^2}$ 4) $\frac{4}{1+t^2}$

41. If $U=\log\left(e^x+e^y\right)$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=$

1)1 2) e^x+e^y

3) $e^{-x}+e^{-y}$ 4) $e^{-x}-e^{-y}$

42. If $U=x^n+y^n+z^n$ then $\frac{\partial^n U}{\partial x^n}+\frac{\partial^n U}{\partial y^n}+\frac{\partial^n U}{\partial z^n}=$

1)0 2) $3(n!)$ 3) $n!$ 4)3

43. If $z=f\left(x^2+y^2\right)$ then $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=$

1) $\left(x-y\right)z$ 2) $\left(x+y\right)z$

3) x^2+y^2 4) 0

44. If $z=ax^2+2hxy+by^2$ then

$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=$

1)0 2)z 3)2z 4)4z

45. If $f\left(x,y\right)=\left[\frac{x-y}{x+y}\right]^n$ then $xf_x+yf_y=$

1)0 2)f 3) nf 4) f/n

46. $u=y^x$ then $\frac{\partial^2 u}{\partial x\partial y}=$

1) $y^{x-1}\left(y\log x+1\right)$ 2) $y^{x-1}\left(x\log y+1\right)$

3) $y^{x-1}\left(x\log y-1\right)$ 4) $y^x\left(x\log y+1\right)$

47. If $z=\tan^{-1}\sqrt{x^y}$ then $\frac{\partial z}{\partial x}=$

1) $\frac{y\left(x^y\right)}{2\left(1-x^y\right)}$ 2) $\frac{\left(x^y\right)}{2\left(1+x^y\right)}$

3) $\frac{\sqrt{y^2-1}}{\sqrt{x^2-1}}$ 4) $\frac{y\sqrt{\left(x^y\right)}}{2x\left(1+x^y\right)}$

48. $u=\frac{\sin(x+y)}{\sin(x-y)}$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=$

1) $\frac{2\cos(x+y)}{\cos(x-y)}$ 2) $\frac{\cos(x+y)}{\sin(x-y)}$

3) $\frac{2\cos(x+y)}{\sin(x-y)}$ 4. $2\cot(x-y)$

49. If $u=f(x+\tan y)+g(x-\tan y)$ then $u_{yy}=$

1) $(\sec^4 y)u_{xx}-(2\tan y)u_y$

2) $(\sec^4 y)u_{xx}+(2\tan y)u_y$

3) $(\sec^4 y)u_{xx}+(\tan y)u_y$

4) $(2\sec^4 y)u_{xx}+(2\tan y)u_y$

50. If $a^2x^2+b^2y^2=c^2z^2$ then $\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2}+\frac{1}{b^2}\frac{\partial^2 z}{\partial y^2}=$

1) $\frac{1}{z^2}$ 2) $\frac{1}{z}$ 3) $\frac{1}{c^2 z}$ 4) 1

51. If $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n+\left(\frac{z}{c}\right)^n=1$, then $\frac{\partial z}{\partial x}=$

1) $-\left(\frac{c}{a}\right)^n\left(\frac{x}{z}\right)^{n-1}$ 2) $-\left(\frac{a}{c}\right)^n\left(\frac{z}{x}\right)^{n-1}$

3) $-\left(\frac{a}{b}\right)^n\left(\frac{x}{y}\right)^{n-1}$ 4) $-\left(\frac{b}{c}\right)^n\left(\frac{y}{x}\right)^{n-1}$

52. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ then $\frac{dy}{dx}=$

1) $-\left(\frac{ax+hy+g}{hx+by+f}\right)$ 2) $-\left(\frac{ax+hy+g}{bx+hy+f}\right)$

3) $-\left(\frac{hx+by+f}{ax+hy+g}\right)$ 4) $-\left(\frac{hx+by+f}{hx+ay+g}\right)$

53. If $x^3y^3+3x\sin y=e^y$ then $\frac{dy}{dx}=$

1) $\frac{3x^2y^3+3\sin y}{e^y-3x^3y^2-3x\cos y}$

2) $\frac{3x^2y^3+3\sin y}{e^y+3x^3y^2+3x\cos y}$

3) $\frac{3x^2y^3-3\sin y}{e^y+3x^3y^2-3x\cos y}$

4) $\frac{3xy^3+3\cos y}{e^y-3x^3y^2-3x\cos y}$

54. If $x^4+y^4-a^2xy=0$ defines y implicitly as function of $'x'$, $\frac{dy}{dx}=$

1) $\frac{4x^3-a^2y}{4y^3-a^2x}$ 2) $-\left(\frac{4x^3-a^2y}{4y^3-a^2x}\right)$

3) $\frac{4x^3}{4y^3-a^2x}$ 4) $\frac{-4x^3}{4y^3-a^2x}$

KEY

1)4	2)3	3)1	4)4	5)3
6)1	7)2	8)4	9)3	10)1
11)2	12)4	13)4	14)4	15)3
16)3	17)4	18)1	19)4	20)4
21)2	22)3	23)1	24)3	25)4
26)1	27)1	28)1	29)1	30)1
31)3	32)2	33)2	34)2	35)3
36)3	37)4	38)3	39)3	40)3
41)1	42)2	43)4	44)3	45)1
46)2	47)4	48)3	49)2	50)3
51)1	52)1	53)1	54)2	

HINTS

16. $n=3+3+0=6$ and apply Eluer's theorem

27. Apply $xu_{xx}+yu_{xy}=(n-1)u_x$ where n=3

30. Apply $xu_{xy}+yu_{yy}=(n-1)u_y$ where n=3

31. $h\text{int}:\frac{dz}{dx}=e^{xy}\left[x\frac{dy}{dx}+y\right].$

$dz=e^{xy}[xdy+ydx]$

33. $h\text{int}:x.f'k.x^{k-1}y=2yf'.x^k \Rightarrow k=2$

34. Hint: Verify

35. $h\text{int}:\frac{dz}{dt}=\frac{\partial z}{\partial x}.\frac{dx}{dt}+\frac{\partial z}{\partial y}.\frac{dy}{dt}$

42. $h\text{int}:\frac{\partial^n u}{\partial x^n}=n!$

44. Hint: homogenous function of degree 2.

45. Hint: homogenous function of degree 0.

47. $\frac{\partial z}{\partial x}=\frac{y.x^y.x^{-1}}{2\sqrt{x^y}\left(1+x^y\right)}$

51. $n\left(\frac{x}{a}\right)^{n-1}.\frac{1}{a}+n\left(\frac{z}{c}\right)^{n-1}.\frac{1}{c}.\frac{\partial z}{\partial x}=0$

LEVEL - II

1. If $u=\log(\tan x+\tan y)$ then

$\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}=$

1) 1 2) 2 3) 1/2 4) -1

2. If $u=\tan(y+ax)+(y-ax)^{3/2}$ then $\frac{\partial^2 u}{\partial x^2}=$

1) $\frac{\partial^2 u}{\partial y^2}$ 2) $\frac{1}{a^2}\frac{\partial^2 u}{\partial y^2}$

3) $a^2\frac{\partial^2 u}{\partial y^2}$ 4) $a\frac{\partial^2 u}{\partial y^2}$

3. If $Z=f(x+ay)+\phi(x-ay)$ then $\frac{\partial^2 z}{\partial y^2}=$

1) $a^2\frac{\partial^2 z}{\partial x^2}$ 2) $-a^2\frac{\partial^2 z}{\partial x^2}$

3) $\frac{1}{-a^2}\frac{\partial^2 z}{\partial x^2}$ 4) $-a\frac{\partial^2 u}{\partial x^2}$

4. $x^x y^y z^z=C$ if $x=y=z$ then $\frac{\partial^2 z}{\partial x\partial y}=$

1) $\frac{1}{x(1+\log x)}$ 2) $\frac{-1}{x(1+\log x)}$

2) $\frac{1}{y(1+\log y)}$ 4) $\frac{1}{1+\log x}$

5. If $u=r^5$, where $r^2=x^2+y^2+z^2$ then

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=$

1) $30r^3$ 2) $30r^2$ 3) $20r^3$ 4) $20r^2$

6. If $u=\log$ r where $r=\sqrt{x^2+y^2+z^2}$ then

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=$

1) 1/r 2) $1/r^2$ 3) $1/r^3$ 4) $1/r^4$

7. If $x=r\cos\theta, y=r\sin\theta$ then

$\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}=$

1) 1/r 2) $1/r^2$ 3) $1/r^3$ 4) $1/r^4$

8. If $V=\left(x^2+y^2+z^2\right)^{-1/2}$ then

$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=$

1) v 2) 2v 3) 0 4) 3v

9. If $Z=\sin(x+y)+\log(x+y)$ then $\frac{\partial^2 z}{\partial x^2}=$

1) $\frac{\partial z}{\partial y}$ 2) $\frac{-\partial^2 z}{\partial y^2}$ 3) $\frac{1}{x}\frac{\partial^2 z}{\partial y^2}$ 4) $\frac{\partial^2 z}{\partial y^2}$

10. If $u=\frac{x}{x^2+y^2}$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1) u 2) $u''+\frac{u'}{u}$ 3) 0 4) -u

11. If $u=e^x\cos y$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1) 0 2) u 3) -u 4) 2u

12. If $z=\sin\left(x^2y^2\right)$ then $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=$

1) $4x^2y^2\left(x^2+y^2\right)\cos^2\left(x^2y^2\right)$

2) $4\left(x^2+y^2\right)\cos^2 x^2y^2$

3) $4x^2y^2\cos^2\left(x^2+y^2\right)$

4) $2x^2y^2\left(x^2+y^2\right)\cos^2\left(x^2y^2\right)$

13. If $u=\frac{1}{\sqrt{1-2xy+y^2}}$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=$

1) $(x-y)u^3$ 2) xu^3

3) yu^3 4) $(x+y)u^3$

14. If $u=x^y+y^x$ then $\frac{\partial u}{\partial x}=$

1) $yx^{y-1}+y^x.\log y$ 2) $xy\log x+xy^{x-1}$

3) $yx^{y-1}+yx$ 4) $yx^{y-1}+x^y\log x$

15. If $z\,e^{x^2+y^2}=1$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) $2z\log z$ 2) $\frac{2}{z}$

3) $\frac{2\log z}{z}$ 4) $2\log z$

16. If $u=\cos\left(x^2+y^2\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) $-2\cos^{-1}u\sqrt{1-u^2}$ 2) $\frac{-2\cos^{-1}u}{\sqrt{1-4u}}$

3) $2u$ 4) $2\cos^{-1}u$

17. If $x=r\cos\theta, y=r\sin\theta$ then

$\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2=$

1) 0 2) 1 3) u^2 4) u^3

18. If $F=x^2yz-2xz^3+xz^2$ then $\frac{\partial^2 F}{\partial x\partial y}$ at $(1,0,-2)$ is

1) 0 2) 4 3) -4 4) 1

19. If $f(x,y)=x\cos y+ye^x$ then $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$ at (1,0) is

1) 0 2) 1 3) -1 4) e

20. If $f(x,y)=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ then $f_x+f_y=$

1) x 2) y 3) x+y 4) x-y

21. If $u=\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^3 & y^3 & z^3\end{vmatrix}$ then $\frac{\partial u}{\partial z}$ at (1,2,3) =

1) 20 2) 40 3) 60 4) 80

22. If $f=x+y+z; g=x^2+y^2+z^2; h=x^3+y^3+z^3$

then $\begin{vmatrix}f_x & f_y & f_z\\ g_x & g_y & g_z\\ h_x & h_y & h_z\end{vmatrix}=$

1) -1 2) 0

3) 1 4) $6(x-y)(y-z)(z-x)$

23. If $z=e^{x^y}$ then $\frac{\partial z}{\partial x}=$

1) $x^{y-1}.(yz)$ 2) $xy.z\log x$

3) $(yzx)y^{-1}$ 4) $(xyz)^y$

24. If $u=x\sin x.\cosh y-y.\cos x.\sinh y$,
$v=y.\sin x.\cosh y+x.\cos x.\sinh y$ then

1) $u_x=v_y$ 2) $u_y=v_x$

3) $u_x+v_y=0$ 4) $u_x-v_y=0$

25. If $u=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$ then $\sum\frac{\partial u}{\partial x}=$

1) $\sum\frac{1}{1+x^2}$ 2) $\sum\frac{3}{1+y^2}$

3) $\frac{1}{1-x^2}$ 4) $\frac{1}{1-y^2}$

26. If $r=\sqrt{x^2+y^2+z^2}$ and

$x=2\sin 3t, y=2\cos 3t, z=8t$ then $\frac{dr}{dt}=$

1) $\frac{32t}{\sqrt{1+16t^2}}$ 2) $\frac{16t}{\sqrt{1+16t^2}}$

3) $\frac{t}{\sqrt{1+16t^2}}$ 3) $\frac{4t}{\sqrt{1+16t^2}}$

27. If $F=e^{\lambda x}\left(c_1\sin\lambda y+c_2\cos\lambda y\right)$ then

$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}=$

1) F 2) 2F 3) 1 4) 0

28. If $z=\log\left(x+\sqrt{x^2+y^2}\right)$ then $e^z\frac{\partial z}{\partial y}=$

1) z_x 2) $y.z_x$ 3) $z.z_x$ 4) $x.z_x$

29. If $u=\frac{x^2\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 4u 2) 0 3) 1 4) u

30. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) $\sin u$ 2) $\cos u$ 3) $-\sin u$ 4) $\tan u$

31. If $u=\tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) $\sin 2u$ 2) $(1/2)\sin 2u$

3) $(1/3)\sin 2u$ 4) $2\sin 2u$

32. If $u=\log\left(\frac{x^3+y^3}{x-y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 1 2) 0 3) 2 4) 4

33. If $u=\log_a\left(x^3+y^3+z^3\right)$ then

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=$

1) $3\log_a e$ 2) $2\log_a e$ 3) $\log_a e$ 4) $4\log_a e$

34. If $u=\sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) cotu 2) tan u 3) 2cot u 4) 2 tan u

35. If $u=\frac{e^{x/y}\sin\left(y/x\right)}{x^3}$ then

$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=$

1) 0 2) 2u 3) 12u 4) 4u

36. If $u=\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) u 2) (1/4)u 3) (1/5)u 4) (1/20)u

37. If $u=\tan^{-1}\left(\frac{x^6+y^6}{x^2+y^2}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 4 tan u 2) 4 sin u 3) 2 sin 2u 4) 2tan 2u

38. If $u=\sin^{-1}\left(\frac{2x-y}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) u 3) sin u 4) tan u

39. If $u=\tan^{-1}\left(\frac{3x^2y+2xy^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) tan u 3) sin u 4) sin 2u

40. If $e^u=\frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) e^u 3) 1 4) $2e^u$

1) $2\lambda f$ 2) λf 3) 0 4) f

41. If $u=\left(x+y\right)f\left(\frac{y}{x}\right)$ then

$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=$

1) f 2) 2f 3) 3f 4) 0

42. If $u=\begin{vmatrix}1&1&1\\x&y&z\\x^2&y^2&z^2\end{vmatrix}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=$

1) u 2) 2u 3) 3u 4) 4u

43. If $z=3x^3\sin\frac{y}{x}+\left(x^3+y^3\right)e^{y/x}$ then

$x^2\frac{\partial^2 z}{\partial x^2}+y^2\frac{\partial^2 z}{\partial y^2}+2xy\frac{\partial^2 z}{\partial x\partial y}=$

1) z 2) 2z 3) 4z 4) 6z

44. If $f\left(x,y\right)=\frac{1}{x^2}+\frac{1}{xy}+\frac{\log x-\log y}{x^2+y^2}$ then

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=$

1) 0 2) f 3) 2f 4) -2f

45. If $Z=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$ then $\frac{\partial^2 z}{\partial x\partial y}$

1) $\frac{x^2+y^2}{x^2-y^2}$ 2) $\frac{x^2-y^2}{x^2+y^2}$

3) $\frac{2x}{x^2+y^2}$ 4) $\frac{-2x}{x^2+y^2}$

46. If $u=\frac{x^2y^2}{x+y}$ then $\frac{x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\partial y}}{\frac{\partial u}{\partial x}}=$

1) 0 2) u 3) 2 4) 3

47. If $u^2\left(x^2+y^2+z^2\right)=1$ then $u+\sum x\frac{\partial u}{\partial x}=$

1) $u^{-3/2}$ 2) $\frac{5}{2}u^{-5/2}$ 3) u 4) 0

48. If $u=\log r; r=\sqrt{x^2+y^2}$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1) 0 2) 4 3) $\frac{1}{r}$ 4) $\frac{2}{r}$

49. If x,y,z are the measurements of a rectangular box and 'S' is its total surface area then $S_{xy}=$

1) 0 2) 1 3) 2 4) $x+y+z$

50. If V is the volume of a variable cone with base radius 'r' and vertical height 'h' then $V_{rh} =$

1) $\frac{2\pi r}{3}$ 2) $\frac{2\pi h}{3}$ 3) $\frac{2\pi}{3}$ 4) $\frac{\pi}{3}$

51. If the edges of a rectangular parallelopiped are x, y, z and V is its volume then

1) $V_{xy} = V_{yz} = V_{zx}$ 2) $V_{xx} = V_{yy} = V_{zz}$

3) $V_x + V_y + V_z = V$ 4) $V_{xyz} = 0$

52. If $V = \sqrt{xyz}$ then $V_{xyz} =$

1) $\frac{1}{2v}$ 2) $\frac{8}{v}$ 3) $\frac{1}{8v}$ 4) $8v$

53. $V = \frac{1}{xyz}$ then $V_{xyz} =$

1) $\frac{1}{2v^2}$ 2) $\frac{1}{v}$ 3) $\frac{1}{8v}$ 4) $-v^2$

54. If $u = e^{ax}\sin by$ then $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} =$

1) $(a^2 - b^2)u$ 2) $(a^2 + b^2)u$

3) abu 4) abu^2

55. If $u = (ax+by)^2 - (x^2+y^2)$ and $a^2 + b^2 = 2$ then $u_{xx} + u_{yy} =$

1) 0 2) 1 3) ab 4) 2ab

KEY

1)2	2)3	3)1	4)2	5)1
6)2	7)1	8)3	9)4	10)3
11)1	12)1	13)2	14)1	15)1
16)1	17)2	18)3	19)1	20)3
21)1	22)4	23)1	24)1	25)1
26)1	27)4	28)2	29)2	30)4
31)2	32)3	33)1	34)3	35)3
36)4	37)3	38)1	39)4	40)1
41)4	42)3	43)4	44)4	45)2
461)3	47)4	48)1	49)3	50)1
51)2	52)3	53)4	54)2	55)1

HINTS

4. Take 'log' on both sides

$x\log x + y\log y + z\log z = \log c$

$(1+\log y) + (1+\log z)\frac{\partial z}{\partial y} = 0$

$\frac{\partial z}{\partial y} = -\left(\frac{1+\log y}{1+\log z}\right)$

$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{x(1+\log x)} (\because x = y = z)$

17. By elimination 'θ' then the equation is $x^2 + y^2 = r^2$

$\Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial r}{\partial y} = \frac{y}{r}$ and replace

in $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2$

23. take 'log for $z = e^{x^y}$

24. Hint : verify

25. Hint : $u = \tan^{-1} x + \tan^{-1} y + \tan^{-1} z$

$\sum u_x = \sum \frac{1}{1+x^2}$

26. Hint : $r = \sqrt{4\cos^2 3t + 4\sin^2 3t + 64t^2}$

$\Rightarrow r = \sqrt{4 + 64t^2}$

27. $h\text{int} : \frac{\partial^2 F}{\partial x^2} = \lambda^2 F, \frac{\partial^2 F}{\partial y^2} = -\lambda^2 F$

28. $z = \log\left(x + \sqrt{x^2+y^2}\right) \Rightarrow e^z = x + \sqrt{x^2+y^2}$

30. Sinu is a homogenous of degree '1' and apply Euler's theorem

33. $a^u = x^3 + y^3 + z^3$ is a homogenous function of degree '3'

apply $\sum x\frac{\partial u}{\partial x} = n\frac{f(u)}{f^{|}(u)} here f(u) = a^u$

35. Apply $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = n(n-1)u$ where $n = 0 + 0 - 3 = -3$

36. $n = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$

47. $u=\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^2 & y^2 & z^2\end{vmatrix}=(x-y)(y-z)(z-x)$

54. $S=2(xy+yz+zx)$

55. $V=\frac{1}{3}\pi r^2h$

LEVEL - III

1. If $u=(y-z)(z-x)(x-y)$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) u 2) 2u 3) 0 4) 3u

2. If $u=(x-y)^4+(y-z)^4+(z-x)^4$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) 3u 2) u 3) 0 4) 2u

3. If $u=f(x+3y)+\varphi(x-3y)$ then $\dfrac{\frac{\partial^2 u}{\partial x^2}}{\frac{\partial^2 u}{\partial y^2}}=$

1) 9 2) -9 3) -1/9 4) 1/9

4. If $z=\log t, t^2=x^2+y^2$ then $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=$

1) z 2) 2z 3) 3z 4) 0

5. If $u=e^{xyz}$ then $\frac{\partial^3 u}{\partial x\partial y\partial z}=$

1) $u(xy+yz+zx)$ 2) $u(1+3xyz)$

3) $u(1+3xyz+x^2y^2z^2)$

4) $u(x+y+z)$

6. If $f(x,y)=xy+\frac{1}{x}+\frac{1}{y}$; then $f_{xx}.f_{yy}-f_{xy}^2$ at (1,1) is

1) 1 2) 2 3) 3 4) 4

7. If $f(x,y)=2(x-y)^2-x^4-y^4$; then the value of $f_{xx}.f_{yy}-f_{xy}^2$ at (0,0) is

1) 0 2) 1 3) 16 4) 32

8. If $V=\log r; r^2=(x-a)^2+(y-b)^2$ then $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=$

1) 0 2) v 3) 2v 4) 3v

9. If $r^2=(x-a)^2+(y-b)^2+(z-c)^2$ then $r\left(\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}+\frac{\partial^2 r}{\partial z^2}\right)=$

1) 1 2) 2 3) 3 4) 4

10. If $r^2=x^2+y^2$ then $\dfrac{\left(\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}\right)}{\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2}=$

1) r 2) r^2 3) $\frac{1}{r}$ 4) $\frac{1}{r^2}$

11. If $u^3(1+a^3)=8(x+ay+b)^3$ then $u_x^3+u_y^3=$

1) 0 2) 8 3) 3 4) 1

12. If $z=\tan^{-1}\left[\frac{\sqrt{x^2-1}+\sqrt{y^2-1}}{1-\sqrt{x^2-1}\sqrt{y^2-1}}\right]$ then $\frac{\partial z}{\partial x}=$

1) $\frac{1}{|y|\sqrt{y^2-1}}$ 2) $\frac{\sqrt{x^2-1}}{\sqrt{y^2-1}}$

3) $\frac{\sqrt{y^2-1}}{\sqrt{x^2-1}}$ 4) $\frac{1}{|x|\sqrt{x^2-1}}$

13. If $u=\tan(\tan^{-1}x+\tan^{-1}y)$ then $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}=$

1) $\frac{x^2+y^2}{(1-xy)^2}$ 2) $\frac{x^2-y^2}{(1-xy)^2}$

3) u 4) $\frac{y^2-x^2}{(1-xy)^2}$

14. If $f(x,y)=\cos^{-1}\left\{\frac{1-xy}{\sqrt{1+x^2}\sqrt{1+y^2}}\right\}$ then $f_x=$

1) $\frac{1}{1+y^2}$ 2) $\frac{1}{1+x^2}$

3) $\frac{1+y^2}{1+x^2}$ 4) $\frac{-1}{1+x^2}$

15. If $u=\cos^{-1}\left(\frac{x-y}{\sqrt{1+x^2}\sqrt{1+y^2}}\right)$ then

$\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=$

1) $\frac{x+y}{(1+y^2)(1+x^2)}$ 2) $\frac{2(y+x)}{(1+x^2)^2(1+y^2)^2}$

3) $\frac{2y(1+x^2)^2+2x(1+y^2)^2}{[(1+x^2)(1+y^2)]^2}$ 4) 0

16. If $u=\log\left(\frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}}\right)$ then $\frac{\partial u}{\partial x}=$

1) $\frac{2}{\sqrt{x^2-y^2}}$ 2) $\frac{2}{y\sqrt{x^2-y^2}}$

3) $\frac{2x}{y}$ 4) $\frac{2y}{x}$

17. If $z^3-xz-y=0$ then

1) $z_x=z_y$ 2) $z.z_x=z_y$

3) $z_x=z.z_y$ 4) $z_x=-z.z_y$

18. If $f(x,y)=x^3y\tan^{-1}\left(\frac{x^2}{y^2}\right)$ then

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=$

1) 6u 2) 4u 3) 2u 4) u

19. If $u=\sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ then $\frac{\partial u}{\partial x}=$

1) $\frac{y}{x}\frac{\partial u}{\partial y}$ 2) $-\frac{y}{x}\frac{\partial u}{\partial y}$ 3) $\frac{x}{y}\frac{\partial u}{\partial y}$ 4) $\frac{-x}{y}\frac{\partial u}{\partial y}$

20. If $z=\sin^{-1}\left(\frac{x^3+y^3}{2(x^2+y^2)}\right)$ then at $(x,y)=(1,1)$ the value of $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ is

1) 1 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

21. If $u=\tan^{-1}(xyz)$ then $\Sigma\, x\frac{\partial u}{\partial x}=$

1) $\frac{xyz}{1+x^2y^2z^2}$ 2) $\frac{2xyz}{1+x^2y^2z^2}$

3) $\frac{3xyz}{1+x^2y^2z^2}$ 4) $\frac{4xyz}{1+x^2y^2z^2}$

22. If $u=\tan^{-1}\left[\left(\frac{x-y}{x+y}\right)^{3/2}\right]$ then $\Sigma x\frac{\partial u}{\partial x}=$

1) 0 2) sinu 3) tanu 4) 1

23. If $z=\tan(xy+x^2+y^2)$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) $\frac{1}{1+z^2}$ 2) $2\tan^{-1}z.(1+z^2)$

3) $\frac{2z}{1+z^2}$ 4) $\frac{2\tan^{-1}z}{1+z^2}$

24. If $u=\sin\sqrt{\frac{x-y}{x+y}}$ then $\Sigma x\frac{\partial u}{\partial x}=$

1) 0 2) u 3) 2u 4) 3u

25. If $u=\log\left(\frac{x^n+y^n}{x^e+y^e}\right)$ then $\Sigma x\frac{\partial u}{\partial x}=$

1) n 2) e 3) n-e 4) $\frac{n}{e}$

26. If $u=\sin^{-1}\left[(x^2+y^2)^{\frac{1}{5}}\right]$ then $\Sigma x\frac{\partial u}{\partial x}=$

1) $\frac{2}{5}\cot u$ 2) $\frac{5}{2}\cot u$

3) $\frac{2}{5}\tan u$ 4) $\frac{5}{2}\tan u$

27. If $u=\sin\left\{\frac{\Pi(x^2+y^2+zx)^{\frac{1}{2}}}{2(x^2+xy+2yz+z^2)^{\frac{1}{3}}}\right\}$ then the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$ for x = 0; y = 1; z = 2 is

1) $\frac{\Pi}{12\sqrt{2}}$ 2) $\frac{1}{12\sqrt{2}}$ 3) $\frac{\Pi}{12}$ 4) $\frac{\Pi}{\sqrt{2}}$

28. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then

$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=$

1) $(2\cos 2u-1)\sin 2u$ 2) $(2\cos 2u+1)\sin 2u$

3) $(\sin 2u-1)\cos 2u$ 4) $(\sin 2u+1)\cos 2u$

29. If $u=x^2+y^2+z^2, x=e^t, y=e^t\sin t, z=e^t\cos t$

then $\frac{\partial u}{\partial t}=$

1) e^{2t} 2) $4e^{2t}$ 3) u 4) $3e^{2t}$

30. If $z=x^2+y^2, x=a\cos\theta, y=a\sin\theta$ then $\frac{\partial z}{\partial\theta}=$

1) 0 2) 1 3) z 4) -z

31. If $z=f(x,y); x=u^2-v^2; y=v^2-u^2$ then

$u\frac{\partial z}{\partial v}+v\frac{\partial z}{\partial u}=$

1) 0 2) 1 3) 2 4) -3

32. If $u=\begin{vmatrix}1&1&1\\x&y&z\\x^2&y^2&z^2\end{vmatrix}$ then $\Pi\left(\frac{\partial^2 u}{\partial x^2}\right)=$

1) u 2) -2u 3) 8u 4) -8u

33. If $u=\begin{vmatrix}1&1&1\\x&y&z\\yz&zx&xy\end{vmatrix}$ then $\Sigma\frac{\partial z}{\partial x}=$

1) 0 2) 1 3) x+y+z 4) xyz

34. If $u=\begin{vmatrix}1&1&1\\x&y&z\\x^2&y^2&z^2\end{vmatrix}$ then $U_{xyz}=$

1) 0 2) -1 3) 1 4) x+y+z

35. If $U=\begin{vmatrix}y+z&z&y\\z&z+x&x\\y&x&x+y\end{vmatrix}$ then $U_{xyz}=$

1) 0 2) 1 3) x+y+z 4) 4

36. If $U=\begin{vmatrix}x&y&z\\y&z&x\\z&x&y\end{vmatrix}$ then $\Sigma xu_x=$

1) 1 2) u 3) 2u 4) 3u

37. If $u(x,y)=\begin{vmatrix}x^2&y^2&z^2\\x&y&z\\1&1&1\end{vmatrix}$ then $\Sigma x\frac{\partial u}{\partial x}=$

1) 0 2) 1 3) 3u 4) 2u

38. If $u=\begin{vmatrix}1&1&1\\x&y&z\\x^2&y^2&z^2\end{vmatrix}$ then $\Sigma\frac{\partial u}{\partial x}=$

1) 0 2) 1
3) 2u 4) 3u

39. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) $\frac{z^2-c^2}{z}$ 2) $\frac{z^2+c^2}{z}$ 3) $\frac{c^2}{z}$ 4) $\frac{z^2}{2}$

40. If $u=\log(x^3+y^3+z^3-3xyz)$ and

$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{k}{(x+y+z)^2}$ then k =

1) -6 2) -3
3) -9 4) -5

41. If $u=xf(x+y)+yg(x+y)$ then

$\frac{\partial^2 u}{\partial x^2}-2\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=$

1) -1 2) 0
3) 1 4) 2

42. If $z=\sin(x-2y)-\log(x+2y)$ then

1) $z_{xx}=z_{yy}$ 2) $z_{xx}=4z_{yy}$

3) $4z_{xx}=z_{yy}$ 4) $z_{xx}+z_{yy}=0$

43. The focal length of mirror is given by $\frac{2}{f}=\frac{1}{v}-\frac{1}{u}$ if du,dv,df are the differentials in u,v,f and du=dv then $\frac{df}{f}=$

1) $\left(\frac{1}{u}-\frac{1}{v}\right)du$ 2) $\left(\frac{1}{u}+\frac{1}{v}\right)du$

3) 1 4) $\left(\frac{1}{u^2}+\frac{1}{v^2}\right)$

44. If u is a homogeneous function of degree n in x,y such that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and $V=\left(x^2+y^2\right)u$ then $V_{xx}+V_{yy}=$

1) $(x+2)u$ 2) $4(n+1)v$

3) $4(n+1)nV$ 4) $4(n+1)u$

45. If A,B,C are angles of a triangle such that $\sin A+\sin B+\sin(A+B)=$ constant then $\frac{dA}{dB}=$

1) $\frac{\cos C-\cos B}{\cos A-\cos C}$ 2) $\frac{\cos B-\cos C}{\cos A-\cos C}$

3) 0 4) 1

KEY

1)3	2)3	3)4	4)4	5)3
6)3	7)1	8)1	9)2	10)3
11)2	12)4	13)4	14)2	15)3
16)1	17)3	18)2	19)2	20)3
21)3	22)1	23)2	24)1	25)3
26)3	27)1	28)1	29)2	30)1
31)1	32)4	33)1	34)1	35)4
36)4	37)3	38)1	39)1	40)3
41)2	42)3	43)2	44)4	45)1

HINTS

12. Hint: Put $x = \sec\theta$, $y = \sec\phi$

13. Hint : $u=\frac{x+y}{1-xy}$

14. Put $x=\tan\alpha$ *and* $y=\tan\beta$

19. Sinu is a homogenous funtion of degree '0' and apply Euler's theorem

29. Substitute x,y,z, in 'u' and then find $\frac{\partial u}{\partial t}$

33. Hint: $u=(x-y)(y-z)(z-x)$

$$\frac{\partial u}{\partial x}=(y-z)(z-x)-(x-y)(y-z)$$

34. $f_x=\begin{vmatrix}0&1&1\\1&y&z\\2x&y^2&z^2\end{vmatrix}, f_{xyz}=\begin{vmatrix}0&0&0\\1&1&1\\2x&2y&2z\end{vmatrix}=0$

35. $f=4xyz \therefore f_{xyz}=4$

36. f is homogeneous of or degree 3

43. $\frac{-2}{f^2}df=\left(\frac{-1}{v^2}+\frac{1}{u^2}\right)du$

$$\frac{-2}{f^2}df=-\left(\frac{1}{v}-\frac{1}{u}\right)\left(\frac{1}{v}+\frac{1}{u}\right)du$$

44. $V_{xx}=\left(x^2+y^2\right)u_{xx}+4xu_x+2u.$ Similarly calculate v_{yy}

45. $\sin A+\sin B+\sin(A+B)=K$

$$\Rightarrow \cos A\frac{\partial A}{\partial B}+\cos B+\cos(A+B)\left(\frac{\partial A}{\partial B}+1\right)=0$$

NEW PATTERN QUESTIONS

1. Observe the following statements :

I: If $Z=\log\left(\frac{x^2+y^2}{xy}\right)$ then $\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y\partial x}=0$

II: If $F=x^2yz-2xz^3+xz^2$ then $\frac{\partial^2 F}{\partial x\partial y}$ at $(1,0,-3)$ is 6

Which of the above statement is correct?

1) only I 2) only II

3) both I and II 4) Neither I nor II

2. Observe the following statements :

I: If $Z=xy.\tan\left(\frac{y}{x}\right)$ then $x.\frac{\partial z}{\partial x}+y.\frac{\partial z}{\partial y}=z$

II: The degree of the homogeneous fucntion $f(x,y)=\frac{1}{x^2y}+\frac{1}{xy^2}-\frac{\log x-\log y}{x^3+y^3}$ can not be defined.

Which of the above statements are correct?

1) only I 2) only II

3) both I and II 4) Neither I nor II

3. Observe the following statements :

I: If $\upsilon = f(r), r^2 = x^2 + y^2$ then

$$\frac{\partial^2\upsilon}{\partial x^2}+\frac{\partial^2\upsilon}{\partial y^2}=f''(r)+\frac{2}{r}f'(r)$$

II: If $\upsilon = f(r), r^2 = x^2 + y^2 + z^2$ then

$$\frac{\partial^2\upsilon}{\partial x^2}+\frac{\partial^2\upsilon}{\partial y^2}+\frac{\partial^2\upsilon}{\partial z^2}=f''(r)+\frac{1}{r}f'(r)$$

Which of the above statements is correct

1) only I 2) only II

3) both I and II 4) Neither I nor II

4. Observe the following statements :

I: If $x = r\cos\theta, y = r\sin\theta$, then $\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=0$

II: If $x = r\cos\theta, y = r\sin\theta$, then $\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2=0$

Which of the above statement is correct?

1) only I 2) only II

3) both I and II 4) Neither I nor II

5. I. If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$$

II. If $u = \left(x^2 + y^2 + z^2\right)^{3/2}$ then

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=9u^{3/2}$$

1) only I is true 2) only II is true

3) both I and II are true

4) neither I nor II is true

6. If $v = f(x, y)$ is a homogenous function of degree 'n'

I. $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=nv$

II. $x^2\frac{\partial^2 v}{\partial x^2}+2xy\frac{\partial^2 v}{\partial x\partial y}+y^2\frac{\partial^2 v}{\partial y^2}=n(n-2)v$

III. $x\frac{\partial^2 v}{\partial x^2}+y\frac{\partial^2 v}{\partial x\partial y}=(n-1)\frac{\partial v}{\partial x}$

1) only I is true 2) only II, III are true

3) only I and III are true 4) I,II,III are true

7. I : If $f(x, y) = \tan\left[\tan^{-1}x + \tan^{-1}y\right]$ then

$$\frac{f_x}{f_y}=\frac{1+y^2}{1+x^2}$$

II: If $u=\frac{\left(x^2+y^2\right)^2}{\sqrt{x^8+y^8}}$ then $xu_x + yu_y = -4u$

which of the above statement is true

1) only I 2) only II

3) both I and II 4) neither I nor II

8. Match the following :

List-I	List-II
I. If $f(x,y,z)=\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ then $\sum xf_x$	a) 5μ
II. If $\mu=\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ then $\sum x\frac{\partial\mu}{\partial x}$	b) 3f
III. If $\mu = x + y, V = x^2 - y^2$ then $\begin{vmatrix} \mu_x & \mu_y \\ V_x & V_y \end{vmatrix}$	c) 3μ
IV. If $f(x,y)=\tan^{-1}\left(\frac{y}{x}\right)$ then $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$	d) -2μ
	e) 0

The correct match from list-1 and list-2 is :

a) b,a,c,e 2) a,b,c,d

3) b,c,d,e 4) a,b,d,e

9. Observe the following statements :

List-I	List-II
A. If $V=\left(x^2+y^2+z^2\right)^{-1/2}$ then $\sum x.\frac{\partial v}{\partial x}=$	1) V
B. If $V=\frac{xy}{x+y}$ then $x.\frac{\partial v}{\partial x}+y.\frac{\partial v}{\partial y}=$	2) 3V
C. If $V=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$ then $\sum x.\frac{\partial v}{\partial x}=$	3) -V
D. If $V = xy\tan\left(\frac{y}{x}\right)$ then $\sum x.\frac{\partial v}{\partial x}=$	4) 0
	5) 2V

The correct match from list-1 and list-2 is :

1) A-3,B-1,C-2,D-5

2) A-3,B-1,C-4,D-5

3) A-3,B-2,C-5,D-5

4) A-3,B-1,C-5,D-2

10. Arrange the following in ascending order of degree of the homogenous fucntion.

A. $f(x,y)=x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{x}{y}\right)$

B. $f(x,y)=\frac{xy}{\sqrt{x^6+y^6}}$

C. $f(x,y)=\sin^{-1}\left(\frac{y}{x}\right)+\tan^{-1}\left(\frac{x}{y}\right)$

D. $f(x,y)=\frac{1}{x^2}+\frac{1}{y^2}+\frac{\log(x/y)}{x^2+y^2}$

1) A,B,D,A 2) D,B,C,A
3) A,B,,D,C 4) B,C,A,D

11. 1. $u=\sqrt{\sin^2 x+\sin^2 y+\sin^2 z}$ then

$\frac{\partial u}{\partial z} at\left(0,0,\frac{\pi}{4}\right)=A$

2. $u=(x-y)(y-z)(z-x)$ then

$u_x+u_y+y_z=B$

3. $u=\log\left(x^3+y^3+z^3-3xyz\right)$ then

$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} at(1,-1,3)=C$

The ascending order of A,B,C is

1) A,B,C 2) B,C,A 3) C,A,B 4) B,A,C

12. Arrange the following in the decreasing order of their values:

A. If $f(x,y)=x^3+y^3-2x^2y^2$ then $(f_{xx})_{(1,1)}$

B. If $f(x,y,z)=\left(x^2+y^2+z^2\right)^{-1/2}$ then $\Sigma\frac{\partial^2 f}{\partial x^2}$

C. If $\mu=\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}$ then $\frac{1}{\mu}\left[x\frac{\partial\mu}{\partial x}+y\frac{\partial\mu}{\partial y}\right]$

1) A,C,B 2) B,C,A 3) A,B,C 4) C,A,B

13. Write the decending order of k values of the following statements

A: If $z=x^3\sin\left(\frac{x}{y}\right)$ and $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=kz$ then k=

B: If $z=\frac{x^3+y^3}{x-y}$ then $\frac{x\frac{\partial^2 z}{\partial x^2}+y\frac{\partial^2 z}{\partial x\partial y}}{\frac{\partial z}{\partial x}}=k$

C: $z=\log\left(\frac{x^3+y^3+3xy^2}{x+y}\right)$ and

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=k$ then k=

D: If $z=\sin^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{x-y}\right)$ and

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=k\tan z$ then k=

1) A,C,D,B 2) B,A,C,D
3) D,C,B,A 4) A,C,B,D

14. Assertion: If $f(x,y,z)=\sqrt{yz}+\sqrt{zx}+\sqrt{xy}$ then $xf_x+yf_y+zf_z=f(x,y,z)$

Reason: If $F(u)=f(x,y,z)$ is a homogeneous function of degree 'n' in x : y,z then

$xu_x+yu_y+z.u_z=\frac{n.F(u)}{F'(u)}$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true but R is false
4) A is false but R is true

15. Assertion: If $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)=z$ then

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{1}{2}\tan z$

Reason: If $f(x,y)=x^y$ then $f_{yy}=x^{y-1}(\ln x)^2$

1) Both A and R are tru and R is the correct explanation of A
2) Both A and R are true and R is not the currect explation of A
3) A is true but R is false
4) A is false but R is true

16. Assertion : If $u=\sqrt{x^2+y^2+z^2}$ then

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{u}$$

Reason: If $u=f(r)$ when $r^2=x^2+y^2+z^2$

then $\sum\frac{\partial^2 u}{\partial x^2}=f''(r)+\frac{2}{r}f'(r)$

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true and R is not the curfect explation of A

3)A is true but R is false

4) A is false but R is true

KEY

1)1	2)4	3)4	4)1	5)1
6)3	7)1	8)3	9)2	10)2
11)4	12)1	13)4	14)1	15)3
16)4				

PREVIOUS EAMCET QUESTIONS

2005

1. If $z=\cos^{-1}(x+y)+\sec^{-1}(y+2x)\Rightarrow$

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial y^2}=$$

1) $3\frac{\partial^2 z}{\partial x\partial y}$ 2) $-\frac{\partial^2 z}{\partial x\partial y}$

3) $-3\frac{\partial^2 z}{\partial x\partial y}$ 4) $\frac{\partial^2 z}{\partial x\partial y}$

2004

2. $f(x,y)=2(x-y)^2-x^4-y^4$

$\Rightarrow\left(f_{xx}f_{yy}-f_{xy}{}^2\right)_{(0,0)}=$

1) 32 2) 16 3) 0 4) -1

2003

3. If $\mu(x,y)=y\log x+x\log y$ then

$\mu_x\mu_y-\mu_x\log x-\mu_y\log y+\log x\log y=$

1) 0 2) -1 3) 1 4) 2

2002

4. If $z=\frac{y}{x}\left[\sin\frac{x}{y}+\cos\left(1+\frac{y}{x}\right)\right]$ then $x\frac{\partial z}{\partial x}=$

1) $y\frac{\partial z}{\partial y}$ 2) $-y\frac{\partial z}{\partial y}$ 3) $2y\frac{\partial z}{\partial y}$ 4) $2y\frac{\partial z}{\partial x}$

5. If $z=\sec(y-ax)+\tan(y+ax)$ then

$$\frac{\partial^2 z}{\partial x^2}-a^2\frac{\partial^2 z}{\partial y^2}=$$

1) z 2) $2z$ 3) 0 4) $-z$

2001

6. If $u=e^{-x^2-y^2}$ then

1) $xu_x=yu_y$ 2) $yu_x=xu_y$

3) $yu_x+xu_y=0$ 4) $x^2u_y+y^2u_x=0$

7. If $u=xy^2\tan^{-1}\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$

1) 2u 2) u 3) 3u 4) $\frac{1}{3}u$

2000

8. If $u=\log_e\left(x^2+y^2\right)+\tan^{-1}\left(\frac{y}{x}\right)$ then

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}=$$

1) 0 2) 2u 3) 1/u 4) u

9. If $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) $(1/2)\cot u$ 2) $2\cot u$

3) $(-1/2)\cot u$ 4) $3\cot u$

1999

10. If $z^2=\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) z/6 2) z/3 3) z/2 4) z/12

11. If $u=x+y\ and\ v=x^2-y^2$ then $\begin{vmatrix}u_x & u_y\\ v_x & v_y\end{vmatrix}=$

1) u 2) v 3) -2u 4) u+v

1998

12. If $x^x.y^y.z^z = a$ constant then $\frac{\partial z}{\partial x} =$

1) $-\left(\frac{1+\log_e x}{1+\log_e z}\right)$ 2) $-\left(\frac{1+\log_e z}{1+\log_e x}\right)$

3) $x^y.y^y$ 4) $x^y.y^y.z$

13. If $z = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then

$2\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right) =$

1) $\cot z$ 2) $(1/2)\tan z$

3) $(1/2)\cot z$ 4) $\tan z$

1996

14. If $u = \left(x^2+y^2+z^2\right)^{3/2}$ then

$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 =$

1) $9u$ 2) $9u^{4/3}$ 3) $9u^2$ 4) $u^{4/3}$

15. If $u = \left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 + \left(\frac{x}{y}\right)$ then $\sum x\frac{\partial u}{\partial x} =$

1) 0 2) 1 3) u 4) -u

16. If $u = \log\left(x^3+y^3+z^3-3xyz\right)$ then

$(x+y+z)\left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right] =$

1) 0 2) 1 3) u 4) 3

17. If $u = \tan^{-1}(x+y)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) $\sin 2u$ 2) $(1/2)\sin 2u$

3) $2\tan u$ 4) $\sec^2 u$

18. If $z = \log\left(\frac{x^2+y^2}{xy}\right)$ then $\frac{\partial^2 z}{\partial x.\partial y} - \frac{\partial^2 z}{\partial y.\partial x} =$

1) 1 2)0 3) -1 4) -2

19. If $z = e^{ax+by} f(ax-by)$ then $b.\frac{\partial z}{\partial x} + a.\frac{\partial z}{\partial y} =$

1) 0 2) ab 3) 2abz 4) z

1995

20. If $u = \log(\sec x + \sec y + \sec z)$ then

$\sum \cot x \frac{\partial u}{\partial x} =$

1) u 2) 2 3) 3 4) 1

21. If $z = \frac{x^{1/25} - y^{1/25}}{x^{1/20} + y^{1/20}}$ then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is

1) -z / 100 2) z 3) z/2 4) z/4

1994

22. If $u = \log\left(x^2+y^2\right)$ then $u_{xx} + u_{yy} =$

1) $-\frac{1}{x^2+y^2}$ 2) 0

3) $-\frac{x^2-y^2}{\left(x^2+y^2\right)^2}$ 4) $-\frac{y^2-x^2}{\left(x^2+y^2\right)^2}$

23. If the edges of a rectangular parallelopiped are x,y,z and υ is its volume then

1) $\upsilon_{xy} = \upsilon_{yz} = \upsilon_{zx}$ 2) $\upsilon_{xx} = \upsilon_{yy} = \upsilon_{zz}$

3) $\upsilon_x + \upsilon_y + \upsilon_z = \upsilon$ 4) $\upsilon_{xyz} = 0$

24. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) $\sin 2u$ 2) $\cos 2u$

3) $\tan 2u$ 4) $\sec 2u$

25. If $u = \frac{x+y}{x-y}$ then $u_x + u_y =$

1) $\frac{1}{x-y}$ 2) $\frac{x^2+y^2}{x-y}$

3) $\frac{2}{x+y}$ 4) $\frac{2}{x-y}$

1990

26. If $x = \sin^{-1}\left(z+y^2\right)$ then $\frac{\partial z}{\partial y} =$

1) $\cos x$ 2) 2y 3) -2y 4) 1-2y

1988

27. 1) If $u = \frac{x^2\left(x^2-y^2\right)^3}{\left(x^2+y^2\right)^2}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) 4u 2) 0 3) 1 4) u

1987

28. If $z = \cos\left(\frac{x}{y}\right) + \sin\left(\frac{x}{y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) z 2) 1 3) 0 4) 2z

KEY

1) 1	2)3	3)3	4)2	5)3
6)2	7)3	8)1	9)3	10)4
11)3	12)1	13)4	14)2	15)1
16)4	17)2	18)2	19)3	20)4
21)1	22)2	23)2	24)1	25)4
26)3	27)1	28)3		

LEVEL-V

I. If $u = f(x, y)$ is a Homogenous Function of n^{th} degree then

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

1. $Z = \frac{\sqrt[3]{x^4 + y^4}}{\sqrt[4]{x^3 + y^3}}$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$ __

1) $\frac{1}{12}z$ 2) $\frac{1}{4}z$

3) $\frac{1}{3}z$ 4) $\frac{7}{12}z$

2. If $u = x\sin^{-1}\left(\frac{y}{x}\right)$ then

$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} =$ ___

1) 0 2) u

3) 2u 4) 1/2 u

KEY

1) 4 2) 1

** ** **