Factors

For a dance competition, 16 dancers have to perform on the stage together. The choreographer drew the different ways they could do it, as shown below.



4 rows of 4 dancers each.

The first way can be shown as 1×16 . The second way as 2×8 . The third way as 4×4 . We see that different numbers can be multiplied to get the same product. The numbers that are multiplied to get a product are called the factors of the product. So, we can say that 1, 2, 4, 8 and 16 are all factors of 16. These numbers divide the number 16 completely. Thus,

"Factors" are the numbers you multiply to get another number. Factors of a number divide the number completely.

Properties of Factors

- 1. 1 is a factor of every number. It is also the smallest factor of a number. **Example:** 1 is a factor of 1, 2, 3, 4, 5, ...
- 2. Every number (other than 1) has at least 2 factors, 1 and the number itself.

Example: 2 has 2 factors, 1 and 2, whereas 6 has 4 factors, 1, 2, 3 and 6.

- 3. The factor of a number is less than or equal to the number.
- A number is its own largest factor.
 Example: 15 is the largest factor of 15.
- 5. A number has limited number of factors. **Example:** 6 has 4 factors, 1, 2, 3 and 6. It has no more factors.

Finding Factors

We can find the factors of a number by any two methods:

1. By Multiplication

Let us find the factors of 24. We have: $1 \times 24 = 24, 2 \times 12 = 24, 3 \times 8 = 24, 4 \times 6 = 24$ Clearly, 1, 2, 3, 4, 6, 8, 12 and 24 are all the factors of 24 in ascending order.

2. By Division

 $24 \div 1 = 24, 24 \div 2 = 12, 24 \div 3 = 8, 24 \div 4 = 6, 24 \div 6 = 4, 24 \div 8 = 3, 24 \div 12 = 2, 24 \div 24 = 1$

The numbers 1, 2, 3, 4, 6, 8, 12 and 24 divide the number 24 exactly. So, 1, 2, 3, 4, 6, 8, 12 and 24 are all factors of 24.

Example 1: Is 3 a factor of 54?

Thus, **3** is a factor of **54**, as it divides 54 without a remainder.

Example 2: Is 8 a factor of 81?



Thus, 8 is not a factor of 81, as it does not divide 81 exactly.

Common Factors

The numbers which are common among the factors of two or more numbers are called the **common factors**.

(a) Find the common factors of 9 and 15. Since $9 = 1 \times 9 = 3 \times 3$. Factors of 9 are 1, 3 and 9. Since $15 = 1 \times 15 = 3 \times 5$. Factors of 15 are 1, 3, 5 and 15. We can see that factors 1 and 3 appear in both the lists of factors. So, 1 and 3 are common factors of 9 and 15.

(b) Find the common factors of 12 and 18.

 Factors of 12 are
 1,
 2,
 3,
 4,
 6,
 12

 Factors of 18 are
 1,
 2,
 3,
 6,
 9,
 18

 Clearly, the common factors of 12 and 18 are
 1,
 2 and
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Multiple

Consider the multiplication, $7 \times 8 = 56$. We call 7 and 8 factors and 56, the product. Another name for product is **multiple**.

Look at the multiplication table of 2 given below:

×	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20

2, 4, 6, 8, 10, 12, 14, 16, 18 are the first 9 multiples of 2. These multiples of 2 have been obtained by multiplying 2 by the counting numbers 1, 2, 3, ..., 9. Thus, a multiple of 2 is a product of 2 and another counting number. A **multiple** of a number is the product of the given number and some other counting numbers.

Properties of Multiples

- 1. Every number is a multiple of 1. **Examples:** $5 \times 1 = 5$, $13 \times 1 = 13$, $108 \times 1 = 108$
- 2. Every number is a multiple of itself.
- 3. Every multiple of a counting number is either **greater than** or **equal** to the number.

Example: The first four multiples of 4 are 4, 8, 12, 16.

- 4. The first and the smallest multiple of a number is the number itself.
- 5. There are infinite multiples of a number. So, the largest multiple of a number cannot be determined.

Tips: To check if bigger number is a multiple of the smaller number, we divide the bigger number by the smaller number. If the remainder is 0, it means the bigger number is a multiple of the smaller number.

Example 3: Is 93 a multiple of 7?

We check this by dividing 93 by 7. Since remainder \neq 0, 7 does not divide 93 exactly.

 $\begin{array}{r}
13 \\
7 93 \\
-7 \\
23 \\
-21 \\
2
\end{array}$

So, 93 is not a multiple 7.

Even and Odd Numbers

A number which is a multiple of 2 is an **even number**.

0, 2, 4, 6, 8, 10 and 12 are some of the even numbers. Even numbers always end in one of the digits 0, 2, 4, 6 or 8. **Tips: Even numbers** are exactly divisible by 2.

A number which is not a multiple of 2 is an **odd number**.

1, 3, 5, 7, 9, 11 and 15 are some of the odd numbers. Odd numbers always end in one of the digits, 1, 3, 5, 7 or 9.

Common Multiples

The numbers which are common among the multiples of two or more numbers are called **common multiples**.

Let us find common multiples of 6 and 8.

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ... Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ... Common multiples of 6 and 8 are 24, 48, ...

Natural Numbers

The counting numbers 1, 2, 3, 4, ... are called **natural numbers**.

Whole Numbers

The natural numbers along with 0, i.e., 0, 1, 2, 3, 4, ... are called **whole numbers**.

Prime and Composite Numbers 1. Prime Numbers

A **prime number** is a natural number greater than 1, which has only two different factors, namely, 1 and the number itself.

Let us find all the factors of 2, 5 and 7. Factors of 2 are 1 and 2. Factors of 5 are 1 and 5. Factors of 7 are 1 and 7. Since each of the numbers 2, 5 and 7 has only two factors, they are prime numbers.

2. Composite Numbers

Numbers that have more than two factors are called **composite numbers**.

Let us find all the factors of 4, 6, 9. Factors of 4 are 1, 2 and 4. Factors of 6 are 1, 3 and 6. Factors of 9 are 1, 3 and 9. Since each of the numbers 4, 6 and 9 has more than two factors, they are composite numbers.

Tips:

- 1 is neither prime nor composite.
- 2 is the smallest prime number.

Sieve of Eratosthenes

About 250 BCE, a Greek mathematician, Eratosthenes, developed a method to find the prime numbers up to 100. In this method, we prepare a table of numbers from 1 to 100 and proceed as follows:

Step 1: Cross out 1 as it is neither prime nor composite.

Step 2: Encircle 2 and cross out all the multiples of 2.

Step 3: Encircle 3 and cross out all the multiples of 3.

Step 4: Encircle 5 and cross out all the multiples of 5.

Step 5: Encircle 7 and cross out all the multiples of 7.

Ж	2	3	Ж	5	X	7	<u>×</u>	×	M
(11)	X	13	14	x	X	17	<u>}§(</u>	19	20
21	22	23	24	25	26	27	28	29	30
31	32	3 3	34	35	36	37	38	39	<u>40</u>
(41)	} 4	43	¥4	<u>}45</u>	¥6	47	<u>}</u>	<u>49</u>	<u>30</u>
Ж	32	53	54	355	36	351	<u>58</u>	59	<u>ðð</u>
61	62	<u>63</u>	64	<u>65</u>	66	67	<u>68</u>	69	70
71	72	73	74	75	76	X	78	79	<u>}</u>
31	82	83	84	85	86	87	88	89	90
M	<u>92</u>	<u>93</u>	94	<u>95</u>	96	97	<u>98</u>	<u>99</u>	100

Now, encircle the numbers which are not crossed.

All of the encircled numbers are prime numbers.

Thus, all prime numbers from 1 to 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

You can see that there are 25 prime numbers from 1 to 100.

Tips: This method is called Sieve of Eratosthenes because factors are used to sift out certain numbers which are multiples as a sieve sifts out chaff from wheat. Finally, only the numbers which are not the multiples of any numbers remain in the list.

Prime Factorisation

There are several ways in which a given number may be factorised (broken into its factors).

For example, the number 36 can be factorised as,

 $36 = 4 \times 9$, $36 = 6 \times 6$, $36 = 1 \times 36$, $36 = 2 \times 18$, $36 = 3 \times 12$.

Here, we see that in all these cases either one factor is prime and the other is composite or both the factors are composite.

Is it possible to break 36 into factors which are all prime numbers? Let us see.

 $36 = 2 \times 18 = 2 \times 2 \times 9 = 2 \times 2 \times 3 \times 3$. These are all prime numbers.

The factors of a number which are prime are called prime factors.

When a number is written as a product of its prime factors only, then it is called **prime factorisation**.

Prime Factorisation of a Number

Prime factorisation can be done by two methods: **Factor tree method** and **division method**.

1. Factor Tree Method

Observe the following factor trees for the number 36.



You can observe that whichever way the factorisation is done, you can see that the prime factors are the same.By convention, **the prime factors are listed from the least to the greatest.**

Thus, the prime factorisation of 36 is $2 \times 2 \times 3 \times 3$.

2. Division Method

Let us find prime factorisation of 24, 36 and 45.

2 24	2 36	3 45
2 12	2 18	3 15
2 6	3 9	5 5
3 3	3 3	1
1	1	
$24 = 2 \times 2 \times 2 \times 3$	$36 = 2 \times 2 \times 3 \times 3$	$45 = 3 \times 3 \times 5$

The steps to be followed in this method can be summarised as under: Step 1: Divide the given number by the smallest prime number that exactly divides it.

Step 2: Divide the quotient further by a prime number that exactly divides it.

Step 3: Continue the repeated division, till the remainder is not further divisible by any number.

Divisibility Rules

One number is divisible by another, if it can be divided with no remainder. Here are some common rules which help us to find at a glance whether a given number is divisible by another number or not, without actually dividing it.

1. Divisibility by 2

A number is divisible by 2 if its last digit is an even number, i.e., the number ends in 0, 2, 4, 6 or 8.

For example: 6, 16, 44, 52, 118, 346 are all divisible by 2.

2. Divisibility by 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

· · · · · · · · · · · · · · · · · · ·							
Numbers	Sum of the Digits	Is the Sum Divisible by 3?	Is the Number Divisible by 3?				
36	3 + 6 = 9	Yes	Yes				
72	7 + 2 = 9	Yes	Yes				
316	3 + 1 + 6 = 10	No	No				

For example:

3. Divisibility by 5

A number is divisible by 5 if the digit at its ones place (last digit) is 0 or 5.

For example: 10, 15, 45, 60, 120, 155 are all divisible by 5.

4. Divisibility by 6

All numbers divisible by 2 and 3 both are divisible by 6.

A number will be divisible by 6 if its ones digit is an even number and the sum of its digits is divisible by 3.

For example:

Numbers	Divisibility by 2	Divisibility by 3	Divisibility by 6
24	Ones digit = 4, Yes	2 + 4 = 6, Yes	Yes
92	Ones digit = 2, Yes	9 + 2 = 11, No	No
345	Ones digit = 5, No	3 + 4 + 5 = 12, Yes	No

5. Divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

Numbers	Sum of the Digits	Is the Sum Divisible by 9?	Is the Number Divisible by 9?
54	5 + 4 = 9	Yes	Yes
477	4 + 7 + 7 = 18	Yes	Yes
118	1 + 1 + 8 = 10	No	No

6. Divisibility by 10

A number is divisible by 10 if the digit at its ones place (last digit) is 0. **For example:** 30, 50, 110, 260, 1860 are all divisible by 10.

Tips: The numbers which are divisible by 10 are also divisible by 5.