CBSE Class 12 Mathematics

Sample Paper 01 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \longrightarrow P$ given by (A, B) $\longrightarrow A \cup B$ and $\cap : P \times P \longrightarrow P$ given by (A, B) $\longrightarrow A \cap B$ are binary operations on the set P.

OR

State with reason whether the function has inverse:

f : {1, 2, 3, 4}$\rightarrow${10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

2. Define a function. What do you mean by the domain and range of a function? Give examples.

OR

Determine whether the relation is reflexive, symmetric and transitive:

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y) : x and y live in the same locality}

3. If f : R $\rightarrow$ R is defined by f(x) = x^2, write f^{-1} (25).

4. If $A = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$ find a matrix C such that A + B - C = 0.

5. Consider the matrix $A = \begin{bmatrix} 3 & -2 & 5 \\ 6 & 9 & 1 \end{bmatrix}$.

then write the values of a11, a12, a13, a21, a22 and a23.

OR

If A = $\begin{bmatrix} 5 & 4 & -2 \\ 6 & -1 & 7 \end{bmatrix}$, Find 3A

6. If A = $\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$, X = $\begin{bmatrix} n \\ 1 \end{bmatrix}$, B = $\begin{bmatrix} 8 \\ 11 \end{bmatrix}$ and AX = B, then find n.

7. Write a value of $\int \frac{\left(\tan^{-1} x\right)^3}{1+x^2} dx$

OR

Evaluate: $\int \frac{x}{\left(x^4-x^2+1\right)} dx$

8. Find the value of c for which the area of figure bounded by the curve y = 3, the straight lines x=1 and x=c and the x-axis is equal to $\frac{16}{3}$

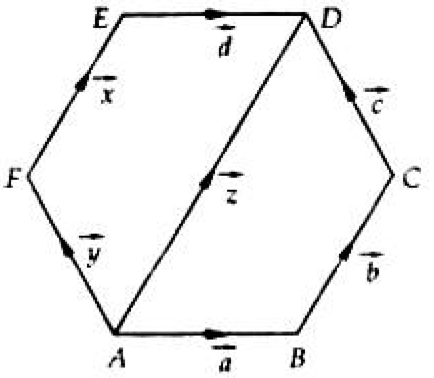
9. Find the general solution for differential equation: $\frac{dy}{dx} = (1+x)\left(1+y^2\right)$

OR

Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$$

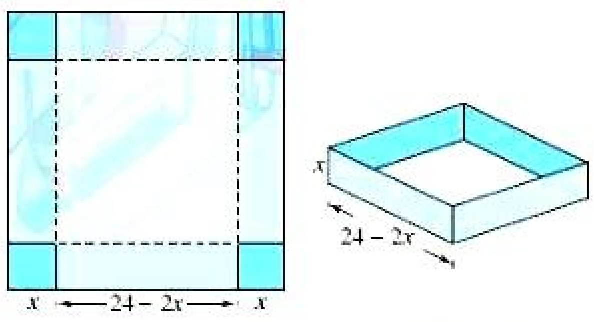
10. Is the measure of 5 seconds vector or scalar?
11. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.
12. In fig. ABCD is a regular hexagon, which vector is Coinitial?



13. Find a normal vector to the plane 2x - y + 2z = 5. Also, find a unit vector normal to the plane.
14. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
15. The probability that a student selected at random from a class will pass in Mathematics is 4/5, and the probability that he/she passes in Mathematics and Computer Science is 1/2. What is the probability that he/she will pass in Computer Science if it is known that he/she has passed in Mathematics?
16. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of getting at least one success.

Section - II

17. A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form a box.



i. Volume of open box formed by folding up the flap:

a. $4(x^3 - 24x^2 + 144x)$

b. $4(x^3 - 34x^2 + 244x)$

c. $x^3 - 24x^2 + 144x$

d. $4x^3 - 24x^2 + 144x$

ii. In the first derivative test, if $\frac{dy}{dx}$ changes its sign from positive to negative as x increases through c_1, then function attains a:

a. Local maxima at $x = c_1$

b. Local minima at $x = c_1$

c. Neither maxima nor minima at $x = c_1$

d. None of these

iii. What should be the side of the square piece to be cut from each corner of the board to be hold the maximum volume?

a. 14 cm

b. 12 cm

c. 4 cm

d. 5 cm

iv. What should be the maximum volume of open box?

a. 1034 cm^3

b. 1024 cm^3

c. 1204 cm^3

d. 4021 cm^3

v. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is:

a. 126

b. 0

c. 135

d. 160

18. A shopkeeper sells three types of flower seeds A_1, A_2, and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%.

A_1 A_2 A_3

Based on the above information answer the following questions:

i. The probability of a randomly chosen seed to germinate:

a. 0.69

b. 0.39

c. 0.49

d. 0.59

ii. The probability that the seed will not germinate given that the seed is of type A_3:

a. $\frac{15}{100}$

b. $\frac{65}{100}$

c. $\frac{75}{100}$

d. $\frac{55}{100}$

iii. The probability that the seed is of the type A_2 given that a randomly chosen seed does not germinate.

a. $\frac{22}{51}$

b. $\frac{55}{51}$

c. $\frac{51}{16}$

d. $\frac{16}{51}$

iv. Calculate the probability that it is of the type A_1 given that a randomly chosen seed does not germinate.

a. $\frac{51}{22}$

b. $\frac{22}{51}$
c. $\frac{16}{51}$
d. $\frac{7}{51}$

v. The probability that it will not germinate given that the seed is of type A_1:

a. $\frac{55}{100}$
b. $\frac{65}{100}$
c. $\frac{35}{100}$
d. $\frac{45}{100}$

Part - B Section - III

19. Prove that: $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\} = \frac{x}{2}, 0 < x < \frac{\pi}{2}$

20. Solve the matrix equation $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$, where X is a 2×2 matrix.

OR

Solve $\begin{bmatrix} 3 & -4 \\ 9 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$ for x and y.

21. Find $\frac{dy}{dx}$ if $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

22. Find the point at which the tangent to the curve y = $\sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.

23. Evaluate: $\int$ x log (1 + x) dx

OR

Evaluate: $\int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x} dx$

24. Find the area of the region common to the parabolas $4y^2 = 9x$ and $3x^2 = 16y$.

25. Find the general solution of the differential equation: $\frac{dy}{dx}$ + (sec x) y = tan x

26. Find a vector of magnitude 49, which is perpendicular to both the vectors $2\hat{i} + 3\hat{j} + 6\hat{k}$ and $3\hat{i} - 6\hat{j} + 2\hat{k}$

27. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ Write its vector form.

28. A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

OR

Two dice were thrown and it is known that the numbers which come up were different.

Find the probability that the sum of the two numbers was 5.

Section - IV

29. Let f : N → N be defined by $f(n)=\begin{cases}\frac{n+1}{2}, \textit{ if n is odd}\\ \frac{n}{2}, \textit{ if n is even}\end{cases}$ for all n ∈ N. State whether the function f is bijective. Justify your answer.

30. Differentiate the function with respect to x: $\sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}, -1 < x < 1$.

31. Find the points of discontinuity, if any, of the function: $f(x)=\begin{cases}\frac{\sin x}{x}, & \text{if } x<0\\ 2x+3, & x\geq 0\end{cases}$

OR

Find $\frac{dy}{dx}$ when y = $(\tan x)^{\cot x} + (\cot x)^{\tan x}$

32. Find the intervals in which the function f(x) = $(x - 1)^3 (x - 2)^2$ is (i) increasing. (ii) decreasing.

33. Evaluate: $\int \frac{(x^2+1)}{(x^4+x^2+1)}dx$

34. Find the area of the region bounded by the parabola $y^2 = 2x + 1$ and the line x - y - 1 = 0.

OR

Find the area enclosed by the curves $3x^2 + 5y = 32$ and $y = |x-2|$.

35. In the differential equation show that it is homogeneous and solve it: $(x - y)\frac{dy}{dx} = x + 3y$.

Section - V

36. If A = $\begin{vmatrix}3 & 2\\ 7 & 5\end{vmatrix}$ and B = $\begin{bmatrix}6 & 7\\ 8 & 9\end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

OR

Show that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1}.

37. Find the equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point (0, 7, -7) and show that the line $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ also lies in the same plane.

OR

Find the vector equation of the plane passing through the intersection of planes

$\overline{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\overrightarrow{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5$ and the point (1,1,1).

38. A company manufactures three kinds of calculators: A, Band C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is Rs 12000 and of factory II is Rs 15000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

OR

An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps. D, E and F whose requirements are 45000 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km.)		
From/To	**A**	**B**
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Rs.1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

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Solution

Part - A Section - I

1. Since, union operation ∪ carries each pair (A, B) in $P \times P$ to a unique element $A \cup B$ in P, ∪ is binary operation on P. Similarly,the intersection operation ∩ carries each pair (A, B) in $P \times P$ to a unique element $A \cap B$ in P, ∩ is a binary operation on P.

OR

f : {1, 2, 3, 4) $\rightarrow$ {10} given by

f{(1, 10), (2, 10), (3, 10), (4, 10)}

clearly f is many-one function

$\Rightarrow$ f is not bijective

$\Rightarrow$ f is not invertible

2. Definition: A relation R from a set A to a set B is called a function if each element of A has a unique image B.

   It is denoted by the symbol f: A$\rightarrow$B which reads 'f' is a function from A to B 'f' maps A to B.

   Let f: A $\rightarrow$ B, then the set A is known as the domain of f & the set B is known as codomain of f.

   The set of images of all the elements of A is known as the range of f.

   Thus, Domain of f = {a | a $\in$ A,(a,f(a)) $\in$ f)

   Range of f = {f(a) | a $\in$ A ,f(a) $\in$ B }

   Example: The domain of y = sin x is all values of x i.e. R, since there are no restrictions on the values for x.

   The range of y is between −1 and 1. We could write this as $-1 \leq y \leq 1$.

OR

Given that R = {(x, y) : x and y live in the same locality}

Clearly, (x,x) $\in$ R as x and x live in the same locality.

$\Rightarrow$ R is reflexive.

Now, if (x, y) ∈ R, then x and y live in the same locality.

⇒ y and x live in the same locality.

⇒ (y,x) ∈ R

⇒ R is symmetric.

Further, let (x, y), (y, z) ∈ R

⇒ x and y live in the same locality and y and z live in the same locality.

⇒ x and z live in the same locality

⇒ (x, z) ∈ R

⇒ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

3. Let $f^{-1}(25) = x$ ...(1)

   Then, we have,

   $f(x) = 25$

   $\Rightarrow x^2 = 25$

   $\Rightarrow x^2 - 25 = 0$

   $\Rightarrow (x - 5)(x + 5) = 0$

   $\Rightarrow x = \pm 5 \Rightarrow f^{-1}(25) = \{-5, 5\}$

4. We have to find C,

   Given A + B – C = 0

$$\begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix} - C = 0$$

$$C = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$$

5. Clearly, the element in the 1st row and 2nd column is -2.

   So, we write $a_{12} = -2$.

   Similarly we can find , $a_{11} = 3$; $a_{12} = -2$; $a_{13} = 5$; $a_{21} = 6$; $a_{22} = 9$ and $a_{23} = 1$

OR

We have to find 3A, i,e;

$$3A = \begin{bmatrix} 3\cdot 5 & 3\cdot 4 & 3\cdot(-2) \\ 3\cdot 6 & 3\cdot(-1) & 3\cdot 7 \end{bmatrix} = \begin{bmatrix} 15 & 12 & -6 \\ 18 & -3 & 21 \end{bmatrix}$$

6. Here,

$$\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} n \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

In the LHS the first matrix is of order 2x2 and the second one is of order 2x1 which will result in the matrix of order 2x1.

$$\Rightarrow \begin{bmatrix} 2n+4 \\ 4n+3 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

$\Rightarrow 2n + 4 = 8$

$\Rightarrow 2n = 4$

$\Rightarrow$ n = 2

7. $I = \int \frac{(\tan^{-1} x)^3}{1+x^2} dx$

Put $\tan^{-1} x = t$

$\Rightarrow \frac{1}{1+x^2} dx = dt$

$\Rightarrow \int \frac{t^3}{1} dt$

$= \frac{t^4}{4} + C$

$= \frac{(\tan^{-1} x)^4}{4} + C$

OR

I = $\int \frac{x}{(x^4 - x^2 + 1)} dx$

Putting $x^2 = t$ and 2x dx = dt, we get

$\frac{1}{2} \cdot \int \frac{dt}{(t^2 - t + 1)}$

$= \frac{1}{2} \cdot \int \frac{dt}{(t - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$.

$= \frac{1}{2} \cdot \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \frac{\left(t - \frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{1}{\sqrt{3}} \cdot \tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right) + C$

$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2-1}{\sqrt{3}}\right) + C$

8. we have, $\int_0^c 3dx = \frac{16}{3}$

$3(x)_0^c = \frac{16}{3}$
$3c = \frac{16}{3}$
$c = \frac{16}{9}$

9. The given differential equation can be rewritten as,

$\frac{1}{1+y^2} dy$ = (1+x)dx

Integrating on both sides

$\int \frac{1}{1+y^2} dy = \int (1+x) dx$

$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$

OR

It is given that equation is $\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$

$\Rightarrow \frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y - \log x = 0$

We can see that the highest order derivative present in the differential is $\frac{d^2y}{dx^2}$.

Thus, its order is two.

The highest power raised to $\frac{d^2y}{dx^2}$ is 1.

Therefore, its degree is one.

10. 5 Seconds is a time period, it has only magnitude i.e; 5 and has no direction, So it is Scalar.

11. We have to find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

Let $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Then, $|\vec{a}| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$

$= \sqrt{4 + 9 + 36} = \sqrt{49}$ = 7 units

Now unit vector in the direction of the given vectors $\vec{a}$ is given as

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{7}(2\hat{i} - 3\hat{j} + 6\hat{k})$

$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

Now, the vector of magnitude equal to 21 units and in the direction of $\vec{a}$ is given by

$21\hat{a} = 21\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 6\hat{i} - 9\hat{j} + 18\hat{k}$

12. Coinitial vectors are vectors originated from same point.

Coinitial vectors are $\overrightarrow{a}\, \overrightarrow{y}$ and $\overrightarrow{z}$

13. We know that the direction ratios of a vector normal to a plane are proportional to the

coefficients of x, y, and z respectively, in the cartesian equation of a plane. Therefore, direction ratios of a vector $\vec{n}$ normal to the given plane are proportional to 2, - 1, 2 and so $\vec{n}=2\hat{i}-\hat{j}+2\hat{k}$. Therefore, a unit vector normal to the plane is given by

$\hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{4+1+4}}=\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$

14. According to the question, equation of line is

$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$

It can be rewritten as

$\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}$

Here, Direction ratios of the line are (-2, 6, -3).

:. Direction cosines of the line are

$\frac{-2}{\sqrt{(-2)^2+6^2+(-3)^2}},\frac{6}{\sqrt{(-2)^2+6^2+(-3)^2}}$

$\frac{-3}{\sqrt{(-2)^2+6^2+(-3)^2}}$ i.e. $\frac{-2}{\sqrt{49}},\frac{6}{\sqrt{49}}$ and $\frac{-3}{\sqrt{49}}$

Thus, Direction cosines of line are $\left(-\frac{2}{7},\frac{6}{7},-\frac{3}{7}\right)$

15. Let M = Mathematics , C = Computer Science

By given data,

P(M) = $\frac{4}{5}$ and P(M ∩ C) = $\frac{1}{2}$

Required probability is given by,

$P\left(\frac{C}{M}\right)=\frac{P(C\cap M)}{P(M)}=\frac{\frac{1}{2}}{\frac{4}{5}}=\frac{5}{4\times 2}=\frac{5}{8}$

16. Let p denote the probability of getting an odd number in a single throw of the die

Then,

$p=\frac{3}{6}=\frac{1}{2}$ and $q=1-p=1-\frac{1}{2}=\frac{1}{2}$

Let X denote the number of successes in 6 trials. Then, X is a binomial variate with parameter $n=6$ and $p=1/2$

We know that, The probability of r successes in 6 trials is given by

$P(X=r)={}^6C_r(1/2)^{6-r}(1/2)^r$, where r = 0,1,2,...6

or, $P(X=r)={}^6C_r(1/2)^6$, where r = 0, 1, 2,...6 ...(i)

Probability of at least one success = $P(X\geq 1)=1-P(X=0)$

$==1-{}^6C_0\left(\frac{1}{2}\right)^6$ [Using (i)]

$=1-\frac{1}{64}=\frac{63}{64}$

Section - II

17. i. (a) $4(x^3 - 24x^2+144x)$

ii. (a) Local maxima at $x = c_1$

iii. (c) 4 cm

iv. (b) 1024 cm^3

v. (b) 0

18. We have, $A_1: A_2: A_3 = 4: 4: 2$

$P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10}$ and $P(A_3) = \frac{2}{10}$

Where A_1, A_2 and A_3 denote the three types of flower seeds.

Let E be the event that seed germinates and $\overline{E}$ be the event that a seed does not germinate.

$\therefore P\left(\frac{E}{A_1}\right) = \frac{45}{100}, P\left(\frac{E}{A_2}\right) = \frac{60}{100}$ and $P\left(\frac{E}{A_3}\right) = \frac{35}{100}$ And

$P\left(\frac{\overline{E}}{A_1}\right) = \frac{55}{100}, P\left(\frac{\overline{E}}{A_2}\right) = \frac{40}{100}$ and $P\left(\frac{\overline{E}}{A_3}\right) = \frac{65}{100}$

i. (c) $\therefore P(E) = P(A_1) \cdot P\left(\frac{E}{A_1}\right) + P(A_2) \cdot P\left(\frac{E}{A_2}\right) + P(A_3) \cdot P\left(\frac{E}{A_3}\right)$

$= \frac{4}{10} \cdot \frac{45}{100} + \frac{4}{10} \cdot \frac{60}{100} + \frac{2}{10} \cdot \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = 0.49$

ii. (b) $P\left(\frac{\overline{E}}{A_3}\right) = 1 - P\left(\frac{E}{A_3}\right) = 1 - \frac{35}{100} = \frac{65}{100}$ [as given above]

iii. (d) $P\left(\frac{A_2}{\overline{E}}\right) = \frac{P(A_2) \cdot P\left(\frac{\overline{E}}{A_2}\right)}{P(A_1) \cdot P\left(\frac{\overline{E}}{A_1}\right) + P(A_2) + P\left(\frac{\overline{E}}{A_2}\right) + P(A_3) \cdot P\left(\frac{\overline{E}}{A_3}\right)}$

$= \frac{\frac{4}{10} \cdot \frac{40}{100}}{\frac{4}{10} \cdot \frac{55}{100} + \frac{4}{10} \cdot \frac{40}{100} + \frac{2}{10} \cdot \frac{65}{100}} = \frac{\frac{160}{1000}}{\frac{220}{1000} + \frac{160}{1000} + \frac{130}{1000}}$

$= \frac{\frac{160}{1000}}{\frac{510}{1000}} = \frac{16}{51}$

iv. (b) $P\left(\frac{A_2}{\overline{E}}\right) = \frac{P(A_1) \cdot P\left(\frac{\overline{E}}{A_1}\right)}{P(A_1) \cdot P\left(\frac{\overline{E}}{A_1}\right) + P(A_2) + P\left(\frac{\overline{E}}{A_2}\right) + P(A_3) \cdot P\left(\frac{\overline{E}}{A_3}\right)}$

$= \frac{\frac{4}{10} \cdot \frac{55}{100}}{\frac{4}{10} \cdot \frac{55}{100} + \frac{4}{10} \cdot \frac{40}{100} + \frac{2}{10} \cdot \frac{65}{100}} = \frac{\frac{220}{1000}}{\frac{220}{1000} + \frac{160}{1000} + \frac{130}{1000}}$

$= \frac{\frac{220}{1000}}{\frac{510}{1000}} = \frac{22}{51}$

v. (a) $P\left(\frac{\overline{E}}{A_1}\right) = 1 - P\left(\frac{E}{A_1}\right) = 1 - \frac{45}{100} = \frac{55}{100}$

Part - B Section - III

19. We know that

$1 \pm \sin x = \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} \pm 2\sin \frac{x}{2}\cos \frac{x}{2} = \left(\cos \frac{x}{2} \pm \sin \frac{x}{2}\right)^2$

$\therefore \quad \cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$

$= \cot^{-1}\left\{\frac{\sqrt{\left(\cos \frac{x}{2}+\sin \frac{x}{2}\right)^2}+\sqrt{\left(\cos \frac{x}{2}-\sin \frac{x}{2}\right)^2}}{\sqrt{\left(\cos \frac{x}{2}+\sin \frac{x}{2}\right)^2}-\sqrt{\left(\cos \frac{x}{2}-\sin \frac{x}{2}\right)^2}}\right\}$

$= \cot^{-1}\left\{\frac{\left|\cos \frac{x}{2}+\sin \frac{x}{2}\right|+\left|\cos \frac{x}{2}-\sin \frac{x}{2}\right|}{\left|\cos \frac{x}{2}+\sin \frac{x}{2}\right|-\left|\cos \frac{x}{2}-\sin \frac{x}{2}\right|}\right\} \left[\because \sqrt{x^2} = |x|\right]$

$= \cot^{-1}\left\{\frac{\left(\cos \frac{x}{2}+\sin \frac{x}{2}\right)+\left(\cos \frac{x}{2}-\sin \frac{x}{2}\right)}{\left(\cos \frac{x}{2}+\sin \frac{x}{2}\right)-\left(\cos \frac{x}{2}-\sin \frac{x}{2}\right)}\right\} \left[\because 0 < \frac{x}{2} < \frac{\pi}{4} \therefore \cos \frac{x}{2} > \sin \frac{x}{2}\right]$

$= \cot^{-1}\left(\cot \frac{x}{2}\right) = \frac{x}{2} \left[\because 0 < \frac{x}{2} < \frac{\pi}{4}\right]$

20. Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

⇒ AX = B

Or, X = $A^{-1}B$

|A| = 1

Now Cofactors of A are

$C_{11} = 1$ $C_{12} = -1$

$C_{21} = -4$ $C_{22} = 5$

⇒ adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T$

= $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

So, X = $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

Hence, X = $\begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$

OR

Here,

$$\begin{bmatrix}3 & -4\\9 & 2\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}10\\2\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}3x-4y\\9x-2y\end{pmatrix}=\begin{pmatrix}10\\2\end{pmatrix}$$

⇒ 3x - 4y = 10 ....(1)

9x + 2y = 2 ....(2)

Solving both the equations, we get

$x=\frac{14}{21}$

$=\frac{2}{3}$

Substituting the value of x in equation (1), we get

$3\times\frac{2}{3}-4y=10$

$\Rightarrow 2-4y=10$

⇒ 2 - 4y = -8

⇒ y = -2

$\therefore x=\frac{2}{3}$ and y = -2.

21. $x=a(\cos\theta+\theta.\sin\theta)$

$\frac{dx}{d\theta}=a[-\sin\theta+\theta.\cos\theta+\sin\theta.1]$

$\frac{dx}{d\theta}=a\theta.\cos\theta$ ...(i)

$y=a(\sin\theta-\theta.\cos\theta)$

$\frac{dy}{d\theta}=a.[\cos\theta-(-\theta\sin\theta+\cos\theta.1)]$

$=a[\cos\theta+\theta.\sin\theta-\cos\theta]$

$=a\theta.\sin\theta$ ...(ii)

$\frac{dy}{dx}=\frac{a\theta.\sin\theta}{a\theta.\cos\theta}=\tan\theta$

22. Slope of tangent to the given curve at (x, y) is

$\frac{dy}{dx}=\frac{1}{2}(4x-3)^{\frac{-1}{2}}\times 4=\frac{2}{\sqrt{4x-3}}$

Given that slope, $\frac{dy}{dx}=\frac{2}{3}$

So, $\frac{2}{\sqrt{4x-3}}=\frac{2}{3}$

or 4x - 3 = 9

or x = 3

Now y = $\sqrt{4x-3}-1$. So when x = 3, y = $\sqrt{4(3)-3}-1=2$

Therefore, the required point is (3, 2).

23. Let I = $\int x\log(1+x)$ dx. Then, we have

$I = \log(x+1)\ \frac{x^2}{2} - \int \frac{1}{x+1} \times \frac{x^2}{2} dx$

$\Rightarrow \quad I = \frac{x^2}{2}\ \log(x+1)\ \frac{1}{2} \int \frac{x^2}{x+1} dx = \frac{x^2}{2}\ \log(x+1) - \frac{1}{2} \int \frac{x^2-1+1}{x+1} dx$

$\Rightarrow \quad I = \frac{x^2}{2}\ \log(x+1) - \frac{1}{2} \int \frac{x^2-1}{x+1} + \frac{1}{x+1} dx$

$\Rightarrow \quad I = \frac{x^2}{2}\ \log(x+1) - \frac{1}{2} \{\int ((x-1) + \frac{1}{x+1})dx\}$

$\Rightarrow \quad I = \frac{x^2}{2}\ \log(x+1) - \frac{1}{2} (\frac{x^2}{2} - x + \log|x+1|\} + C$

OR

$I = \int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x} dx$

Put $\log\tan\frac{x}{2} = t$

$\Rightarrow \frac{1}{\tan\frac{x}{2}} \sec^2\frac{x}{2} \times \frac{1}{2} dx = dt$

$\Rightarrow \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \times \frac{1}{\cos^2\frac{x}{2}} \times \frac{1}{2} dx = dt$

$\Rightarrow \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx = dt$

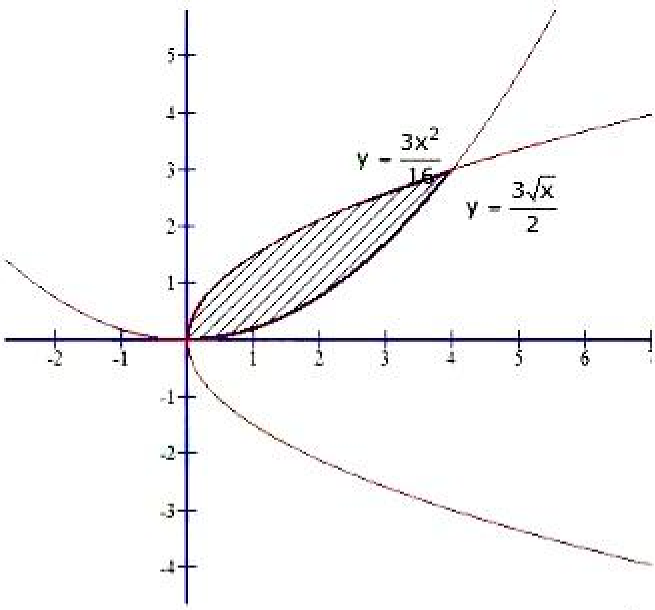
$\Rightarrow \frac{1}{\sin x} dx = dt$

$I = \int t\,dt$

$= \frac{t^2}{2} + c$

$= \frac{(\log\tan\frac{x}{2})^2}{2} + c$

24.



Area of the shaded region is given by $= \int_0^4 \left[\frac{3\sqrt{x}}{2} - \frac{3x^2}{16}\right] dx$

$= \left[x^{3/2} - \frac{x^3}{16}\right]_0^4$

$= \left[(4)^{3/2} - \frac{(4)^3}{16}\right]$

$= \left[8 - \frac{64}{16}\right]$

= [8 - 4] = 4 sq.units

25. The given differential equation is:

$\frac{dy}{dx}$ + py = Q (where p = sec x and Q = tan x)

Now, I.F. $= e^{\int pdt} = e^{\int \sec x dx}$ = $e^{\log(\sec x + \tan x)}$ = sec x + tan x

The general solution of the given differential equation is given by the relation,

y(I.F.) = $\int$(Q $\times$ I.F.) dx + C

$\Rightarrow$ y(sec x + tan x) = $\int$tan x (sec x + tan x)dx + C

$\Rightarrow$ y(sec x + tan x) = $\int$sec x tan xdx + $\int \tan^2$xdx + C

$\Rightarrow$ y (sec x + than x) = sec x + $\int$(sec^2 x - 1)dx + C

$\Rightarrow$ y (sec x + tan x) = sec x + tan x - x + C.

26. Given:

$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$

$= (6+36)\hat{i} - (4-18)\hat{j} + (-12-9)\hat{k}$

$= 42\hat{i} + 14\hat{j} - 21\hat{k}$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{42^2 + 14^2 + (-21^2)}$

$= \sqrt{2401}$

= 49

Required vector $= 49 \times \left\{\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right\}$

$= 49 \times \frac{42\hat{i}+14\hat{j}-21\hat{k}}{49}$

$= 42\hat{i} + 14\hat{j} - 21\hat{k}$

27. Given: The Cartesian equation of the line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} = \lambda$ (say)

$\Rightarrow x - 5 = 3\lambda, y + 4 = 7\lambda, z - 6 = 2\lambda$

$\Rightarrow x = 5 + 3\lambda, y = -4 + 7\lambda, z = 6 + 2\lambda$

General equation for the required line is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Putting the values of x, y, z in this equation,

$\vec{r}=(5+3\lambda)\,\hat{i}+(-4+7\lambda)\,\hat{j}+(6+2\lambda)\,\hat{k}=5\hat{i}+3\lambda\hat{j}-4\hat{j}+7\lambda\hat{j}+6\hat{k}+2\lambda\hat{k}$

$\Rightarrow \vec{r}=\left(5\hat{i}-4\hat{j}+6\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)\left[\sin ce\ \vec{r}=\vec{a}+\lambda\vec{b}\right]$

28. A die has 6 faces and its sample space S = {1, 2, 3, 4, 5, 6}

The total number of outcomes = 6

Let P(A) be the probability of getting an even number

The sample space of A = {2, 4, 6}

$\therefore P(A)=\frac{3}{6}=\frac{1}{2}$

Let P(B) be the probability of getting a number whose value is greater than 2

The sample space of B = {3, 4, 5, 6}

$\therefore$ (A ∩ B) = {4,6}

$\therefore$ P(A ∩ B) = $\frac{2}{6}=\frac{1}{3}$

The probability of getting a number greater than 2 given that the outcome is even is given by: P(B/A)

$=\frac{P(A\cap B)}{P(A)}$

$=\frac{1/3}{1/2}$

$=\frac{2}{3}$

This is the required probability

OR

Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2 = 36

Let P(A) be the probability of getting a sum equal to 5

Let P(B) be the probability of getting 2 different numbers

Probability of getting 2 different numbers

= 1 - probability of getting same numbers

= 1 - $\frac{1}{6}$

= $\frac{5}{6}$

$\therefore$ P(B) = $\frac{5}{6}$

Let P(A ∩ B) be the probability of getting a sum = 5 and two different numbers at the same time

The sample space of (A ∩ B) = {(1,4), (2,3), (3,2), (4,1)}

$\therefore$ P(A ∩ B) = $\frac{4}{36} = \frac{1}{9}$

The probability that the sum = 5 given that two different numbers were thrown: P(A/B)

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/9}{5/6}$

$= \frac{2}{15}$

Section - IV

29. f : N → N be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \ if \ n \ is \ odd \\ \frac{n}{2}, \ if \ n \ is \ even \end{cases}$

 a. f(1) = $\frac{1+1}{2}$ = 1 and f(2) = $\frac{2}{2}$ = 1

 The elements 1, 2, belonging to domain of f have the same image 1 in its co-domain.

 So, f is not one-one, therefore, f is not injective.

 b. Every number of co-domain has pre-image in its domain e.g., 1 has two pre-images 1 and 2.

 So, f is onto, therefore, f is not bijective.

30. Let, y $= \sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}$

Put x = sinθ

$\therefore y = \sin^{-1}\left(\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right)$

$\Rightarrow y = \sin^{-1}\left(\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right)$

$\Rightarrow y = \sin^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right) + \cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$

$\Rightarrow y = \sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right\}$

$\Rightarrow y = \sin^{-1}\left\{\sin\left(\theta + \frac{\pi}{4}\right)\right\}$

Here, -1 < x < 1

$\Rightarrow$ -1 < sin θ < 1

$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \left(-\frac{\pi}{2} + \frac{\pi}{4}\right) < \left(\frac{\pi}{4} + \theta\right) < \frac{3\pi}{4}$

$\Rightarrow -\frac{\pi}{4} < \left(\frac{\pi}{4} + \theta\right) < \frac{3\pi}{4}$

So, from (i)

$y = \theta + \frac{\pi}{4}$ $\left[\text{since}, \sin^{-1}(\sin\alpha) = \alpha, \text{ if } \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$\Rightarrow y = \sin^{-1}x + \frac{\pi}{4}$

Differentiating it with respect to x,

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + 0$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

31. Given; $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ 2x+3, & x \geq 0 \end{cases}$

When x < 0, then

$f(x) = \frac{\sin x}{x}$

We know that sin x, as well as the identity function x, are everywhere continuous.

So, the quotient function $\frac{\sin x}{x}$ is continuous at each x < 0

When x > 0, then

f(x) = 2x + 3, which is a polynomial function

Therefore, f(x) is continuous at each x > 0

Now, Let us consider the point x = 0

We have

(LHL at x = 0) = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} f(-h)$

$= \lim\limits_{h \to 0} f(-h) = \lim\limits_{h \to 0} \left(\frac{\sin(-h)}{-h}\right) = \lim\limits_{h \to 0} \left(\frac{\sin(h)}{h}\right) = 1$

(RHL at x = 0) = $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} f(h) = \lim\limits_{h \to 0} (2h+3) = 3$

$\therefore \lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$

Thus, f(x) is discontinuous at x = 0

Hence, the only point of discontinuity for f(x) is x = 0

OR

We have, y = $(\tan x)^{\cot x} + (\cot x)^{\tan x}$

$y = e^{\log(\tan x)^{\cot x}} + e^{\log(\cot x)^{\tan x}}$

$\Rightarrow$ y = $e^{\cot x \log \tan x} + e^{\tan x \log(\cot x)}$

Differentiating with respect to x using chain rule and product rule,

$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cot x \log \tan x}\right) + \frac{d}{dx}\left(e^{\tan x \log \cot x}\right)$

$= e^{\cot x \log \tan x} \frac{d}{dx}\left(\cot x \log \tan x\right) + e^{\tan x \log \cot x} \frac{d}{dx}\left(\tan x \log \cot x\right)$

$= e^{\log(\tan x)^{\cot x}} \left[\cot x \frac{d}{dx}(\log \tan x) + \log \tan x \frac{d}{dx}(\cot x)\right]$

$+ e^{\log(\cot x)\tan x} \left[\tan x \frac{d}{dx}(\log \cot x) + \log \cot x \frac{d}{dx}(\tan x)\right]$

$= (\tan x)^{\cot x}\left[\cot x \times \left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log\tan x\left(-cosec^2 x\right)\right]$
$+(\cot x)^{\tan x}\left[\tan x \times \left(\frac{1}{\cot x}\right)\frac{d}{dx}(\cot x) + \log\cot x\left(\sec^2 x\right)\right]$
$= (\tan x)^{\cot x}\left[\left(\frac{cosec^2 x}{\sec^2 x}\right)\left(\sec^2 x\right) - cosec^2 x\log\tan x\right]$
$+(\cot x)^{\tan x}\left[\left(\frac{\sec^2 x}{cosec^2 x}\right)\left(-cosec^2 x\right) + \sec^2 x\log\cot x\right]$

$= (\tan x)^{\cot x}[cosec^2 x - cosec^2 x \log \tan x] + (\cot x) \tan x [\sec^2 x \log \cot x - \sec^2 x]$

$= (\tan x) \cot x\, cosec^2 x[1 - \log \tan x] + (\cot x) \tan x \sec^2 x[\log \cot x^{-1}]$

The differentiation of the given function y is as above.

32. $f(x) = (x - 1)^3(x - 2)^2$

Therefore, on differentiating both sides w.r.t. x, we get,

$f'(x) = (x - 1)^3 \frac{d}{dx}(x-2)^2 + (x-2)^2 \cdot \frac{d}{dx}(x-1)^3$

$\Rightarrow f'(x) = (x - 1)^3.2(x - 2) + (x - 2)^2 \cdot 3(x -1)^2$

$= (x - 1)^2 (x - 2)[2(x - 1) + 3(x - 2)]$

$= (x - 1)^2(x - 2)(2x - 2 + 3x - 6)$

$\Rightarrow f'(x) = (x -1)^2(x - 2)(5x - 8)$

Now, put $f'(x) = 0$

$\Rightarrow (x - 1)^2(x - 2)(5x - 8) = 0$

Either $(x - 1)^2 = 0$ or $x - 2 = 0$ or $5x - 8 = 0$

$\therefore x = 1, \frac{8}{5}, 2$

Now, we find intervals and check in which interval f(x) is strictly increasing and strictly decreasing.

Interval	$f'(x) = (x - 1)^2(x - 2)(5x - 8)$	Sign of f'(x)
$x < 1$	(+)(-)(-)	+ve
$1 < x < \frac{8}{5}$	(+)(-)(-)	+ve
$\frac{8}{5} < x < 2$	(+)(-)(+)	-ve
$x > 2$	(+)(+)(+)	+ve

So, the given function f(x) is increasing on the intervals $(-\infty, 1)$ $\left(1, \frac{8}{5}\right)$ and $(2, \infty)$ and decreasing on $\left(\frac{8}{5}, 2\right)$.

$\left[\frac{8}{5}, 2\right]$

33. To find: $\int \frac{(x^2+1)}{(x^4+x^2+1)} dx$

Formula Used: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

On dividing by x^2 in the numerator and denominator of the given equation,

$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx$

$\Rightarrow \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3} dx$

Let $y = x - \frac{1}{x}$

Differentiating write x,

$dy = \left(1 + \frac{1}{x^2}\right) dx$

substituiting in the original equation,

$\Rightarrow \int \frac{dy}{y^2+(\sqrt{3})^2}$

$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{\sqrt{3}} + c$

Substituting for $y = x - \frac{1}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right) + C$

$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + c$

Therefore, we have the value of given integral as.

$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + C$

34. To find area bounded by

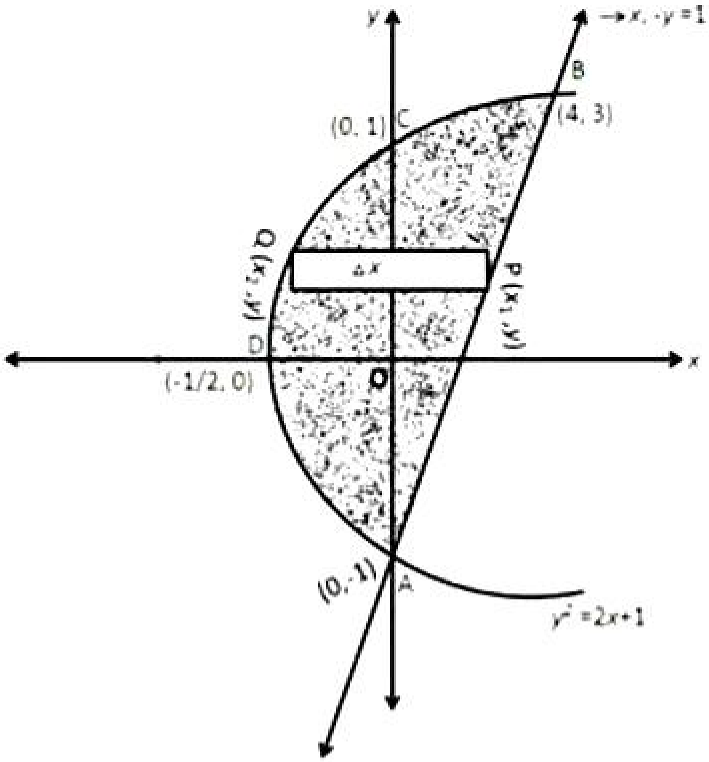
$y^2 = 2x + 1$ ...(i)

and $x - y = 1$ ...(ii)

Equation (i) is a parabola with vertex $\left(-\frac{1}{2}, 0\right)$ and passes through (0, 1) (0, -1).

Equation (ii) is a line passing through (1, 0) and (0, -1), points of intersection of parabola and line are (3, 2) and (0, -1).

A rough sketch of the curve is given as:-



Shaded region represents the required area. It is sliced in rectangles of area $(x_1 - x_2) \triangle y$.

It slides from y = -1 to y = 3, so

Required area of the shaded region =Area of the Region ABCDA

$= \int_{-1}^{3} (x_1 - x_2)\, dy$

$= \int_{-1}^{3} \left(1 + y - \frac{y^2-1}{2}\right) dy$

$= \frac{1}{2} \int_{-1}^{3} (2 + 2y - y^2 + 1)\, dy$

$= \frac{1}{2} \int_{-1}^{3} (3 + 2y - y^2)\, dy$

$= \frac{1}{2} \left[3y + y^2 - \frac{y^3}{3}\right]_{-1}^{3}$

$= \frac{1}{2} \left[(9 + 9 - 9) - \left(-3 + 1 + \frac{1}{3}\right)\right]$

$= \frac{1}{2} \left[9 + \frac{5}{3}\right]$

$= \frac{32}{6}$

Required area $= \frac{16}{3}$ sq. units

OR

To find area enclosed by

$3x^2 + 5y = 32$

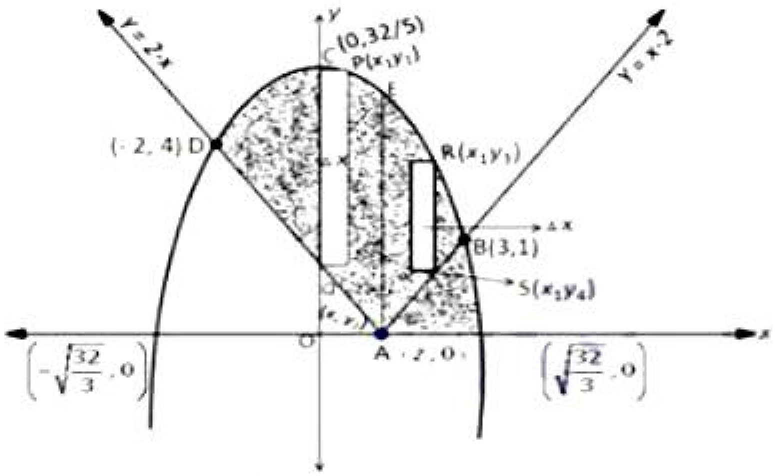
$3x^2 = -5\left(y - \frac{32}{5}\right)$ ...(i)

And

$$y = |x-2|$$
$$\Rightarrow y = \begin{cases} -(x-2), \text{ if } x-2 < 1 \\ (x-2), \text{ if } x-2 \geq 1 \end{cases}$$
$$\Rightarrow y = \begin{cases} 2-x, & \text{if } x < 2 \\ x-2, & \text{if } x \geq 2 \end{cases} \quad \ldots(2)$$

Equation (i) represents a downward parabola with vertex $\left(0, \frac{32}{5}\right)$ and equation (ii) represents lines.

A rough sketch of curves is given as:-



Thus the Required area of the Region = Area of Region ABECDA

A = Region ABEA + Region AECDA

$$= \int_2^3 (y_3 - y_4)\,dx + \int_{-2}^{2} (y_1 - y_2)\,dx$$
$$= \int_2^3 \left(\frac{32-3x^2}{5} - x + 2\right) dx + \int_{-2}^{2} \left(\frac{32-3x^2}{5} - 2 + x\right) dx$$
$$= \int_2^3 \left(\frac{32-3x^2-5x+10}{5}\right) dx + \int_{-2}^{2} \left(\frac{32-3x^2-10+5x}{5}\right) dx$$
$$= \frac{1}{5}\left[\int_2^3 (42 - 3x^2 - 5x)\,dx + \int_{-2}^{2} (22 - 3x^2 + 5x)\,dx\right]$$
$$A = \frac{1}{5}\left[\left(42x - x^3 - \frac{5x^2}{2}\right)_2^3 + \left(22x - x^3 + \frac{5x^2}{2}\right)_{-2}^{2}\right]$$
$$= \frac{1}{5}\left[\left\{\left(126 - 27 - \frac{45}{2}\right) - (84 - 8 - 10)\right\} + \{(44 - 8 + 10) - (-44 + 8 + 10)\}\right.$$
$$= \frac{1}{5}\left[\left\{\frac{153}{2} - 66\right\} + \{46 + 26\}\right]$$
$$= \frac{1}{5}\left[\frac{21}{2} + 72\right]$$
$$A = \frac{33}{2} \text{ sq. units.}$$

35. $(x - y)\frac{dy}{dx} = x + 3y$

$\Rightarrow \frac{dy}{dx} = \frac{x+3y}{x-y}$

$\Rightarrow \frac{dy}{dx} = \frac{1+3\frac{y}{x}}{1-\frac{y}{x}}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put x = vy

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$v + x\frac{dv}{dx} = \frac{1+3\frac{vx}{x}}{1-\frac{vx}{x}}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+3v}{1-v} - v = \frac{1+3v-v+v^2}{1-v} = \frac{1+2v+v^2}{1-v}$

$\Rightarrow \frac{1-v}{1+2v+v^2}dv = \frac{dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int \frac{1-v}{1+2v+v^2}dv = \int \frac{dx}{x} + c$

$\Rightarrow \int \frac{v-1}{1+2v+v^2}dv = -\int \frac{dx}{x} + c$

$\Rightarrow \frac{\ln|1+2v+v^2|}{2} = -\ln|x| + \ln c$

Resubstituting the value of y = vx we get

$\Rightarrow \frac{\ln\left|1+2\frac{y}{x}+\left(\frac{y}{x}\right)^2\right|}{2} = -\ln|x| + \ln c$

$\Rightarrow \log|x+y| + \frac{2x}{(x+y)} = c$

Section - V

36. Clearly, $|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$. So, A is invertible

Let A_{ij} be the cofactor of element a_{ij} in $A = [a_{ij}]$. Then,

$A_{11} = (-1)^{1+1} 5 = 5$, $A_{12} = (-1)^{1+2} 7 = -7$, $A_{21} = (-1)^{2+1} 2 = -2$ and $A_{22} = (-1)^{2+2} 3 = 3$

$$\therefore \quad \text{adj } A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|}$ adj $A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

We have, $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

$\therefore \quad |B| = \begin{vmatrix} 6 & 7 \\ 8 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$

So, B is invertible

Let B_{ij} be the cofactors of b_{ij} in B = [b_{ij}]. Then,

$B_{11} = (-1)^{1+1}\, 9 = 9$, $B_{12} = (-1)^{1+2}\, 8 = -8$, $B_{21} = (-1)^{2+1}\, 7 = -7$ and $B_{22} = (-1)^{2+2}\, 6 = 6$

$$\therefore \quad \text{adj}\, B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^T = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}^T = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Hence, $B^{-1} = \frac{1}{|B|}$ adj B $= -\frac{1}{2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

We know that adj AB = adj B. adj A.

$$\therefore \quad \text{adj}\, AB = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

We also know that $|AB| = |A|\ |B|$

$\therefore |AB| = 1 \times -2 = -2 \neq 0$

So, AB is invertible

Hence, $(AB)^{-1} = \frac{1}{|AB|}$ adj (AB) $= \frac{1}{-2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$ ...(i)

Also, $B^{-1} A^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get

$(AB)^{-1} = B^{-1} A^{-1}$

OR

We have, $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

And $7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$$\therefore A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

=0 Hence proved.

Since, $A^2 - 3A - 7I = 0$

$\Rightarrow A^{-1}[(A^2) - 3A - 7I] = A^{-1}0$

$\Rightarrow A^{-1}A.A - 3A^{-1}A - 7A^{-1}I = 0$ [$\because A^{-1}0 = 0$]

$\Rightarrow IA - 3I - 7A^{-1} = 0$ [$\because A^{-1}A = I$]

$\Rightarrow A - 3I - 7A^{-1} = 0$ [$\because A^{-1}I = A^{-1}$]

$\Rightarrow -7A^{-1} = -A + 3I$

$$= \begin{bmatrix} -5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{-1}{7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

37. We know that the plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ ...(i)

Required plane is passing through (0, 7, -7), so

$a(x - 0) + b(y - 7) + c(z + 7) = 0$

$ax + b(y - 7) + c(z + 7) = 0$ ...(ii)

Plane (ii) also contains line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ so, it passes through point (-1, 3, -2),

$a(-1) + b(3 - 7) + c(-2 + 7) = 0$

$-a - 4b + 5c = 0$ ...(iii)

Also, plane (ii) will be parallel to line, so, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(-3) + (b)(2) + (c)(1) = 0$

$-3a + 2b + c = 0$ .....(iv)

Solving (iii) and (iv) by cross-mulitplication,

$$\frac{a}{(-4)(1)-(5)(2)} = \frac{b}{(-3)(5)-(-1)(1)} = \frac{c}{(-1)(2)-(-4)(-3)}$$

$$\frac{a}{-4-10} = \frac{b}{-15+1} = \frac{c}{-2-12}$$

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{-14} = \lambda \text{ (say)}$$

$\Rightarrow a = -14\lambda, b = -14\lambda, c = -14\lambda$

Put a, b, c in equation (ii);

$ax + b(y - 7) + c(z + 7) = 0$

$(-14\lambda)x + (-14\lambda)(y - 7) + (-14\lambda)(z + 7) = 0$

Dividing by (-14λ), we get,

$x + y - 7 + z + 7 = 0$

$x + y + z = 0$

So, equation of plane containing the given point and line is $x + y + z = 0$

The other line is $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(1) (1) + (1) (-3) + (1) (2) = 0

1 - 3 + 2 = 0

0 = 0

LHS = RHS

So, $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ lie on plane x + y + z = 0

OR

$\vec{n}_1 = \hat{i} + \hat{j} + \hat{k}, \vec{n}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$d_1 = 6$, $d_2 = -5$

Using the relation

$\vec{r} \cdot (\vec{n_1} + \lambda\vec{n_2}) = d_1 + \lambda d_2$

$\vec{r} \cdot \left[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}\right] = 6 - 5\lambda$...... (1)

taking $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$(x\hat{i} + y\hat{j} + z\hat{k}).\left[(1+2\lambda)i + (1+3\lambda)j + (1+4\lambda)\hat{k}\right] = 6 - 5\lambda$

$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z = 6 - 5\lambda$

(x + y + z - 6) + λ(2x + 3y + 4y + 4z + 5) = 0 ..... (2)

plane passes through the point (1, 1, 1)

$\lambda = \frac{3}{14}$

Put λ in eq (1),

$\vec{r} \cdot \left[\left(1+\frac{3}{7}\right)\hat{i} + \left(1+\frac{9}{14}\right)\hat{j} + \left(1+\frac{6}{7}\right)\hat{k}\right] = 6 - \frac{15}{14}$

$\vec{r} \cdot \left(\frac{10}{7}\hat{i} + \frac{23}{14}\hat{j} + \frac{13}{7}\hat{k}\right) = \frac{69}{14}$

$\vec{r} \cdot \left(20\hat{i} + 23\hat{j} + 26\hat{k}\right) = 69$

38. Let factory (I) run x days and factory (II) run y days respectively to produce the three kinds of calculators. The LPP is to minimize the cost of the production of the three kinds of calculators.Hence let the

equation representing the total cost (in Rs) be 12000x + 15000y. Let z be the objective function which represents the total cost.Hence Z = 12000x + 15000y, which is to be minimised

Subject to the constraints

$50x+40y \geq 6400$ or $5x+4y \geq 640$ (by dividing throughout by 10)
$50x+20y \geq 4000$ or $5x+2y \geq 400$ (by dividing throughout by 10)
$30x+40y \geq 4800$ *or* $3x+4y \geq 480$ (by dividing throughout by 10)
$x \geq 0$ and $y \geq 0$ (non negative constraints which will restrict the solution in the first quadrant only.)

Now, considering the inequations as equations, we get

5x + 4y = 640 ...(i)

5x + 2y = 400 ...(ii)

3x + 4y = 480 ...(iii)

Table of values for line 5x + 4y = 640 is given below.

x	128	0
y	0	160

So, the line (i) passes through the points with coordinates (128, 0) and (0, 160)

On replacing the coordinates of the origin O (0, 0) in the inequality $5x+4y \geq 640$, we get

$0+0 \geq 640$ [which is false]

So, the half plane of the inequality of the line (i) is away from the origin, means that the point (0,0) which is the origin is not in the feasible region of the inequality of the line (i).

Table of values for the line (ii) 5x + 2y = 400 is given below.

x	80	0
y	0	200

So, the line (ii) passes through the points with coordinates (80, 0) and (0, 200).

On replacing the coordinates of the origin O (0, 0) in the inequality,$5x+2y \geq 400$ we get

$0+0 \geq 400$ [which is false]

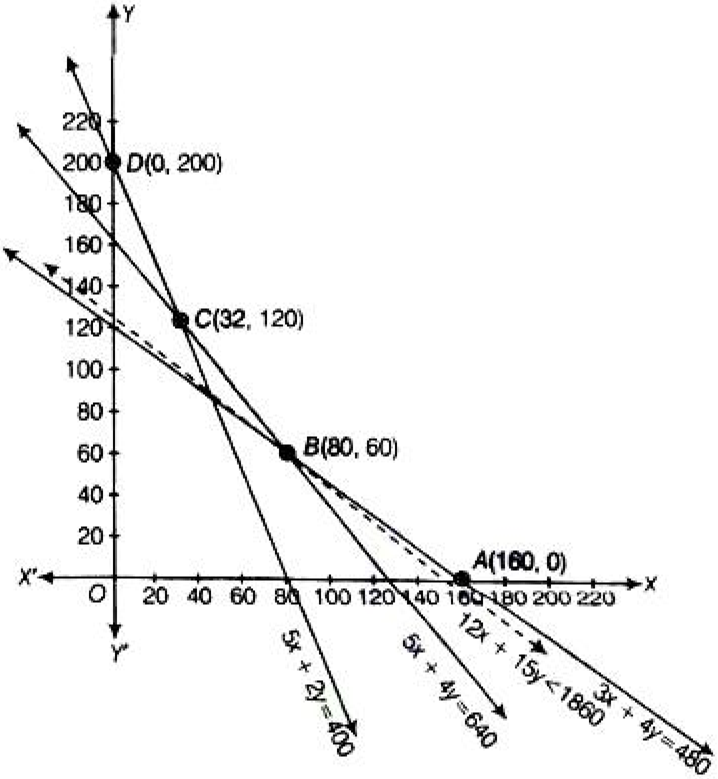
So, the half plane for the inequality of the line (ii) is away from the origin, which means that the point O(0, 0) is not a point in the feasible region of the inequality of line (ii).

Table of values for line (iii) 3x + 4y = 480 is given below.

x	160	0
y	0	120

So, the line (iii) passes through the points with coordinates (160, 0) and (0, 120).

On replacing the coordinates of the origin O (0, 0) in the inequality $3x + 4y \geq 480$,
we get
$0 + 0 \geq 480$ [which is false]
So, the half plane for the inequality of the line (iii) is away from the origin, which means that the origin O(0, 0) is not in the feasible region for the inequality of the line (iii) .
Also, $x \geq 0$ and $y \geq 0$ so the feasible region lies in the first quadrant.
The point of intersection of lines (i) and (iii) is B (80, 60) and lines (i) and (ii) is C (32, 120).
The graphical representations of the system of inequations as given below



Clearly, feasible region is ABCD is an unbounded feasible region, where the coordinates of the corner points are A(160,0),B (80, 60), C (32, 120) and D(0, 200).
The values of Z at corner points are as follows

Comer Points	Z = 12000x + 15000y
A(160,0)	$Z = 12000 \times 160 + 0 = 1920000$
B(80, 60)	$Z = 12000 \times 80 + 15000 \times 60$ = 1860000 (minimum)
C(32,120)	$Z = 12000 \times 32 + 15000 \times 120$ = 2184000
D(0, 200)	$Z = 0 + 15000 \times 200 = 3000000$

In the table above , we find that minimum value of Z is 1860000 occur at the point B(80, 60). But we can't say that it is a minimum value of Z as the region is unbounded.
Therefore, we have to draw the graph of the inequality $12000x + 15000y < 1860000$ or $12x + 15y < 1860$ (when dividing throughout by 1000)
From the figure, we see that the open half plane represented by $12x + 15y < 1860$ has no point in common with the feasible region. Thus, the minimum value of Z is Rs 1860000 attained at the point with coordinates (80, 60). Hence, factory (I) should run for 80 days and factory (II) should run for 60 days to get a minimum cost of Rs. 1860000.

OR

In this problem, we note that the total quantity of oil available

= (7000 + 4000) = 11000 L

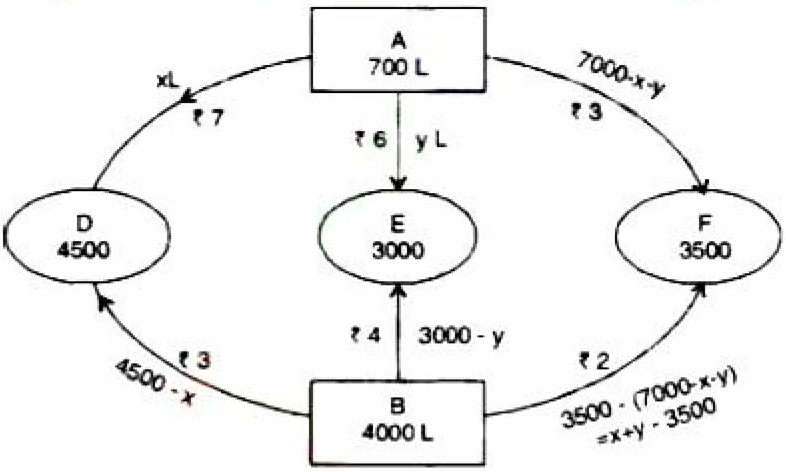
And total requirement of oil

= (4500 + 3000 + 3500) = 11000L

⇒ Total availability = Total requirement

Let depot A supply x-litre of oil to petrol pump D and y litre to E so that supplies to F will be (7000 - x - y)L

All given information can be represented diagrammatically as:



Since petrol pump D requires 4500 L and it has already received x-litres from depot A, it must received (4500 - x)L from depot B, Similarly, E receives (3000 - y)L from depot B and F receives 3500 - (7000 - x - y)L from the depot B.

has already received x-litres from depot A, it must received (4500 - x) L from depot B, Similarly, E receives (3000 - y) L from depot B and F receives 3500 - (7000 -x-y) L from the depot B.

Now, total transportation cost (in Rs)

= 7x + 6y + 3(7000 - x - y) + 3(4500 - x) + 4(3000 - y) + 2 (x + y - 3500)

= 3x + y + 39500

Hence, the given problem can be formulated as an L.P.P. as follows

Minimize Z = 3x + y + 39500

Subject to constraints

$$x + y \leq 7000$$

$$x \leq 4500$$

$$y \leq 3000$$

$$x + y \geq 3500$$

$$x \geq 0, \quad y \geq 0$$

Now, reducing the all inequalities into equations, we have

x + y = 7000 .... (i)

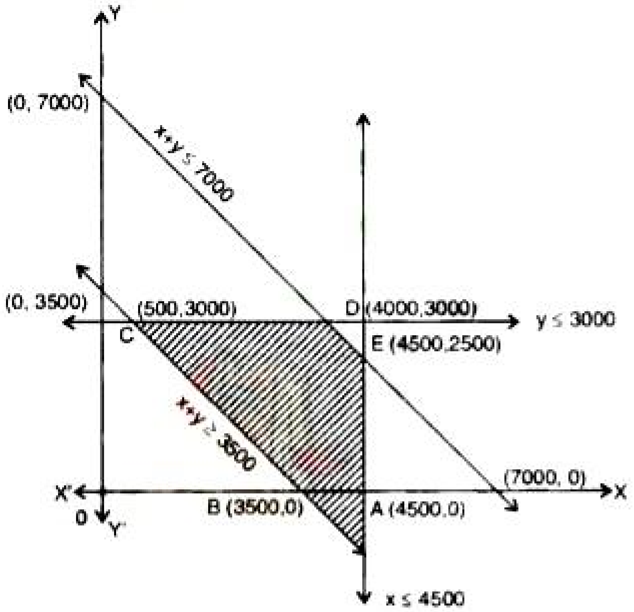
x = 4500 .... (ii)

y = 3000 .... (iii)

x + y = 3500 .... (iv)

x = 0, y = 0 ...... (v)

Now, tracing all the given equation of lines on a graph and shade the region satisfied by all the inequalities.



Here, the feasible region is ABCDEA, which is bounded and comer points are A (4500, 0), B(3500, 0), C(500, 3000), D(4000, 3000) and E (4500, 2500)

Now, evaluating the Z for each comer point i.e.

Corner	**Z = 3x + y + 39500**

A(4500, 0)	Z = 53,000
B(3500, 0)	Z = 50,000
C(500, 3000)	Z = 44,000 → Minimum
D(4000, 3000)	Z = 54500
E(4500, 2500)	Z = 55500

⇒ Transportation cost will be minimum when x = 500 and y = 3000

Hence, 500, 3000, 3500 litres are supplied from depot A and 4000,0,0 litres are supplied from depot B to petrol pump D, E and F respectively with minimum transportation cost.