Lines and Angles

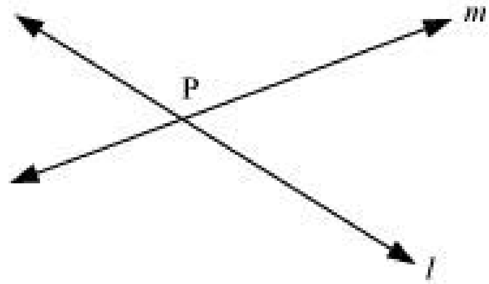
- A **point** determines a location. The tip of a compass, the sharpened end of a pencil, the pointed end of a needle, etc., are the examples of points. Generally, points are denoted by capital letters.
- A **line segment** corresponds to the shortest distance between two points. The line segment joining the points P and Q is denoted as $\overline{PQ}$.

- A **ray** is a portion of a line, which starts at one point and goes endlessly in a direction.

This ray is denoted as $\overrightarrow{PQ}$. Arrow head is towards Q since it is extended along Q.

- When a line segment PQ is extended indefinitely on both sides of points P and Q, it becomes a **line**, $\overleftrightarrow{PQ}$. Line is usually denoted by small letters *l, m, n.*

- Two lines *l* and *m* are said to be **intersecting lines**, if they intersect at a point.



- Two lines are said to be **parallel lines**, if they never intersect each other. We can represent the given lines as *l*||*m*.

- A **plane** is a flat surface having length and width, but no thickness. We can say that a plane is a flat surface, which extends indefinitely in all directions. For example, surface of a wall, floor of a ground, etc.
- **Incidence properties in a plane:**
    1. An unlimited number of lines can be drawn passing through a given point.
    2. There is exactly one line passing through two distinct points in a plane.
    3. Points lying on the same line are known as collinear points and the points which do not lie on the same line are called non-collinear points.
    4. Three or more lines passing through a common point are known as concurrent lines and that point is known as point of concurrence.

- One complete turn of the hand of a clock is one revolution. The angle of one revolution is called a **complete angle.**

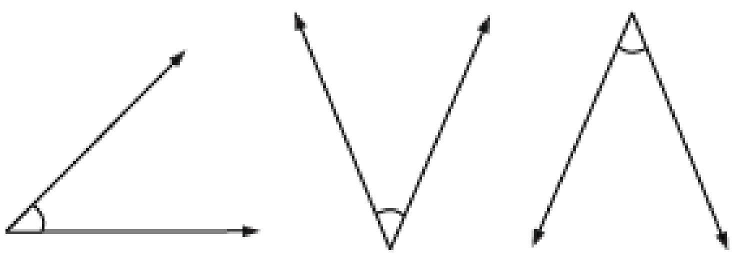
- A right angle is $\left(\frac{1}{4}\right)^{th}$ of a revolution and a straight angle is $\left(\frac{1}{2}\right)^{th}$ of a revolution.

Right angle

Straight angle

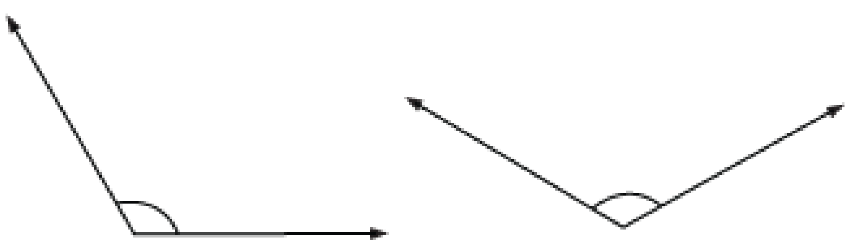
- 1 complete angle = 2 straight angles = 4 right angles
- 1 straight angle = 2 right angles
- If an angle measures less than a right angle then it is known as an **acute angle.**

  The following angles are acute:



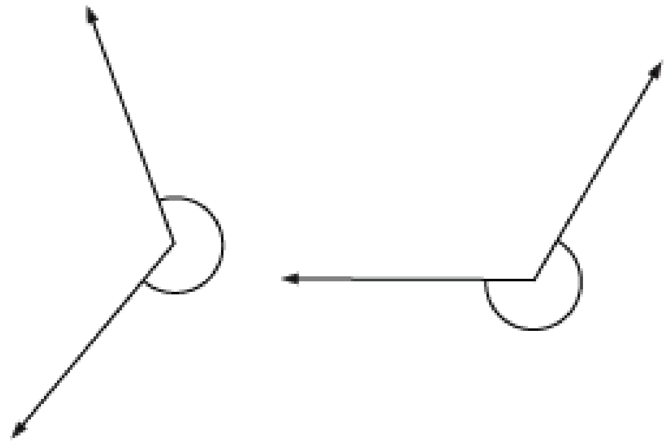
- If an angle measures more than a right angle but less than a straight angle, then it is an **obtuse angle.**

  The following angles are obtuse:



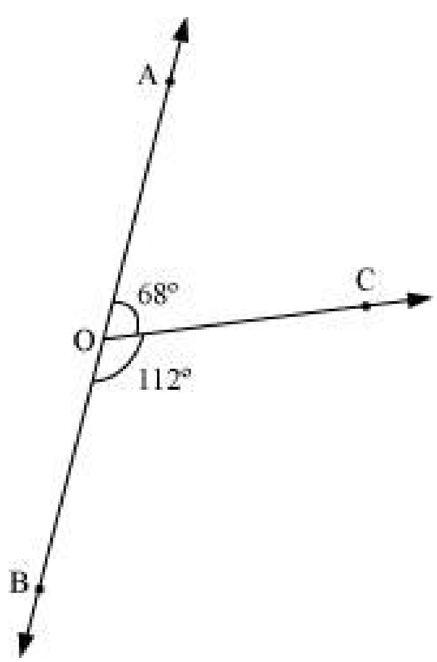
- If an angle measures more than a straight angle, then it is known as a **reflex angle.**

  The following angles are reflex:



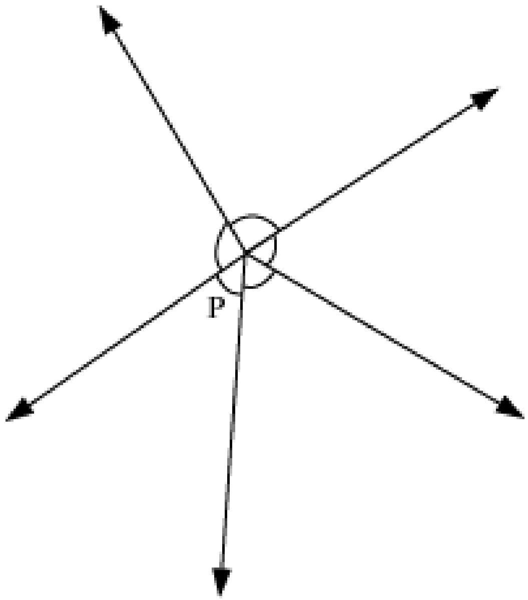
- We use a protractor to measure an angle.
- One complete revolution is divided into 360 equal parts. Each part is called a **degree.** Thus, the unit of angle is degree (°).
- Right angle measures 90°, complete angle measures 360°, and straight angle measures 180°.
- Acute angle is less than 90°, obtuse angle is more than 90° but less than 180°, and reflex angle is more than 180° but less then 360°.

- A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays.
- The sum of the measures of the adjacent angles is 180°.



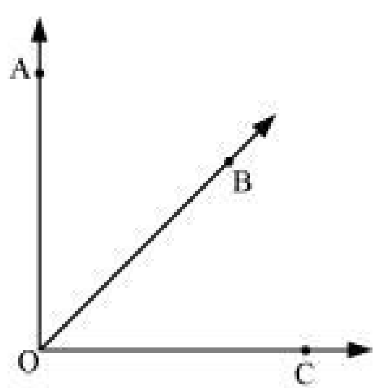
Here, ∠AOC and ∠BOC form a linear pair as ∠AOC + ∠BOC = 180°.

- The sum of angles around a point is equal to 360°.

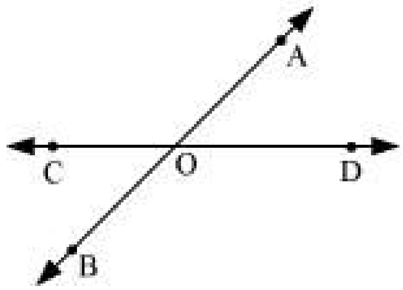


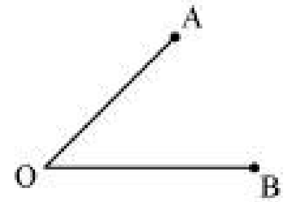
In this figure, five angles have a common vertex, which is point P. In other words, the five angles make a complete turn and therefore the sum of these five angles will be equal to 360°. This is true no matter how many angles make a complete turn.

- A pair of angles are called adjacent angles, if:
    - they have a common vertex
    - they have a common arm
    - the non-common arms are on either side of the common arm
      For example, ∠AOB and ∠BOC are adjacent angles as they have a common vertex O, common arm OB, and non-common arms OA and OC lie on either side of OB.

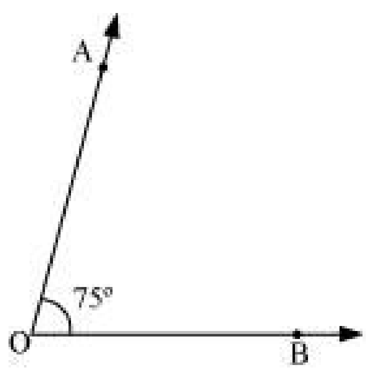


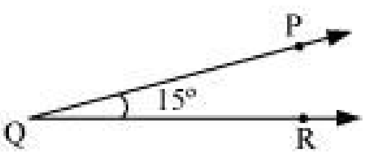
- An angle is made when two lines or line segments meet. For example:





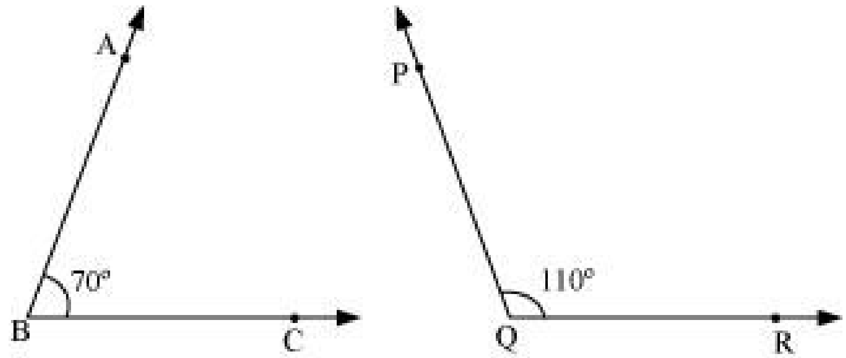
- When the sum of the measures of two angles is 90°, the angles are called **complementary angles**.





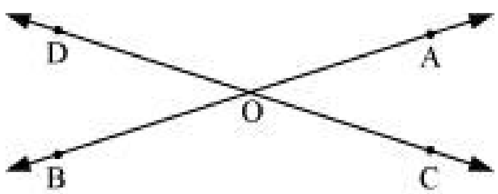
Here, ∠AOB and ∠PQR are complementary as (∠AOB + ∠PQR) = 75° + 15° = 90°.

- When the sum of the measures of two angles is 180°, the angles are called **supplementary angles**.



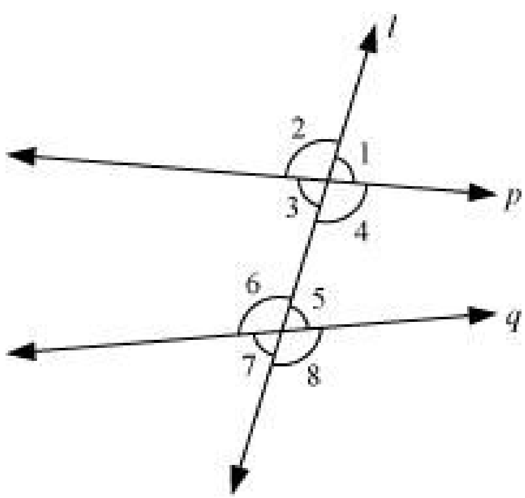
Here, ∠ABC and ∠PQR are supplementary as (∠ABC + ∠PQR) = 110° + 70° = 180°.

- When two lines intersect, the vertically opposite angles so formed are equal.



Here, ∠AOC = ∠BOD and ∠AOD = ∠BOC.

- A line which intersects two or more lines at distinct points is called **transversal** to the lines.



Here, line *l* is a transversal with respect to lines *p* and *q*.

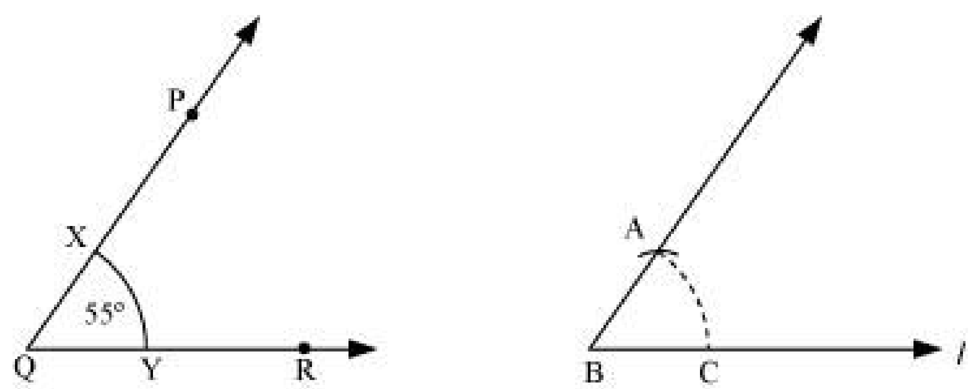
1. ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8 are pairs of corresponding angles.
2. ∠3 and ∠5, ∠4 and ∠6 are pairs of alternate interior angles.
3. ∠1 and ∠7, ∠2 and ∠8 are pairs of alternate exterior angles.

4. ∠3 and ∠6, ∠4 and ∠5 are pairs of interior angles on the same side of the transversal.
5. ∠1 and ∠8, ∠2 and ∠7 are pairs of exterior angles on the same side of the transversal.

- **Steps for the construction of copy of a given angle:**

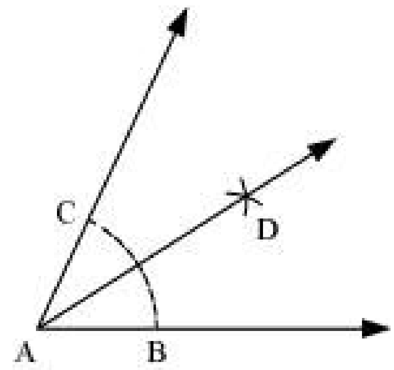
  Given ∠PQR = 55°.

  1. Draw a line *l* and mark a point B on it.
  2. Place the compass at Q and draw an arc to cut the rays QP and QR at points X and Y respectively.
  3. Use the same compass setting to draw an arc with B as the centre, cutting *l* at C.
  4. Set your compass to length XY.
  5. Place the compass pointer at C and draw the arc (with the same setting) that cuts the arc drawn earlier at A.
  6. Join B with A and extend it.

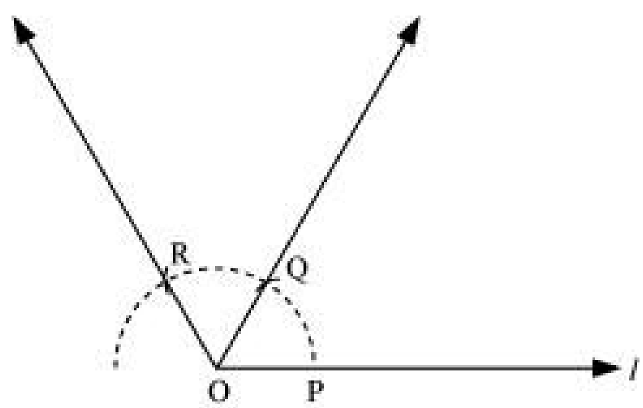


Now, ∠ABC = ∠PQR = 55°

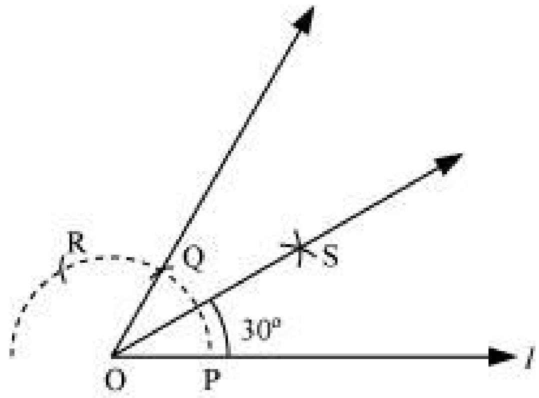
- **Steps of construction for the bisector of a given angle (say 60°):**
  1. Draw ∠A such that ∠A = 60°
  2. With A as the centre, draw an arc that cuts both the rays of ∠A at B and C.
  3. With B and C as centres and radius more than $\frac{1}{2}$ BC, draw two arcs that intersect each other at D.
  4. Join AD. AD is the bisector of ∠A.



- The steps for the construction of angles of measures 60° and 120° are as follows:
    1. Draw a line *l* and mark a point O on it.
    2. Place the pointer of the compass at O and draw an arc of convenient radius that cuts *l* at P.
    3. With the same radius, draw an arc with centre P that cuts the previous arc at Q.
    4. Similarly, with the same radius, draw an arc with centre Q that cuts the arc at R.
    5. Join OQ and OR to get ∠QOP = 60° and ∠ROP = 120°.



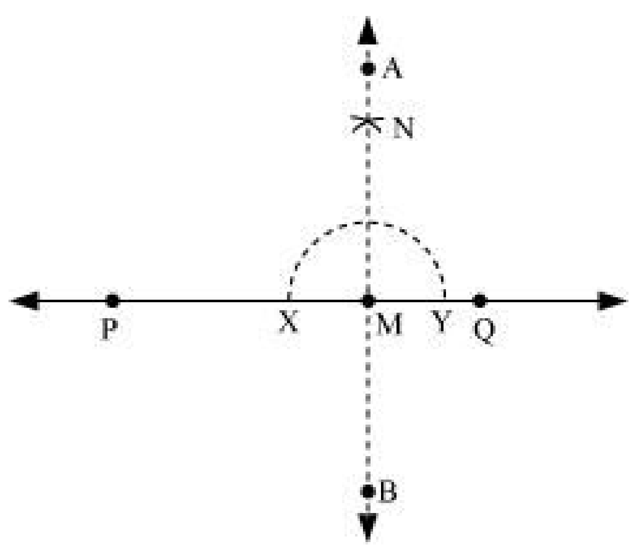
- Now, 30° is nothing but half of angle 60°. Therefore, 30°angle can be obtained by drawing the bisector of ∠QOP.



Here, ∠SOP = 30°.

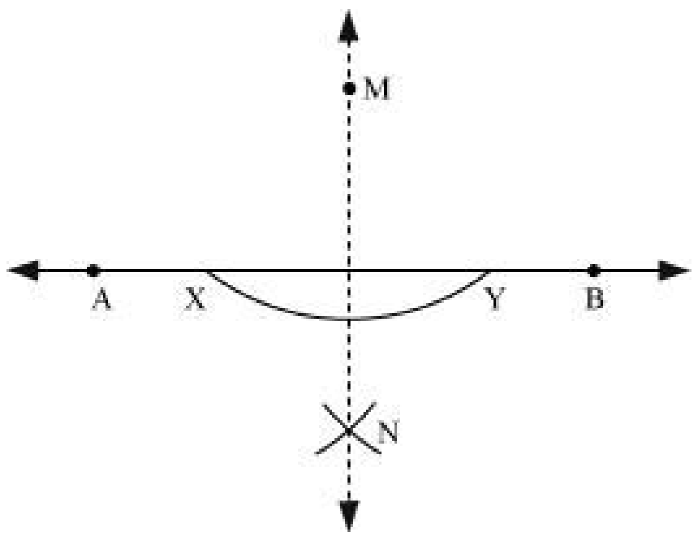
Similarly, we can draw other angles of measures 45°, 90°, 135°, and 150° using the above method.

- **Steps to construct perpendicular to a line PQ through a point M on it:**
  1. Draw a line $\overleftrightarrow{PQ}$ and mark a point M on it.
  2. With M as the centre and a convenient radius, construct an arc intersecting $\overleftrightarrow{PQ}$ at two points i.e., X and Y. With X and Y as centres and radius greater than MX, construct two arcs that cut each other at N.
  3. Draw a line through points M and N and name this line as $\overleftrightarrow{AB}$. Now, $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$.

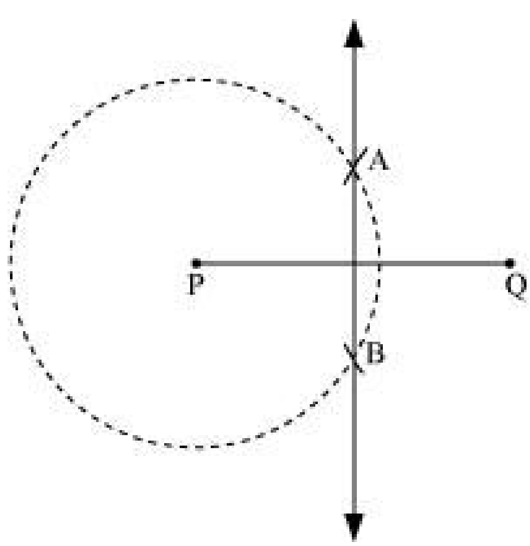


- **Steps to construct perpendicular to a line AB through a point M not on it:**

1. Draw line $\overleftrightarrow{AB}$. Mark a point M outside it.
2. With M as the centre, draw an arc that intersects $\overleftrightarrow{AB}$ at two points i.e., X and Y.
3. Using the same radius and with X and Y as centres, construct two arcs such that they intersect at N on the other side of the line.
4. Join $\overleftrightarrow{MN}$ to get $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$.



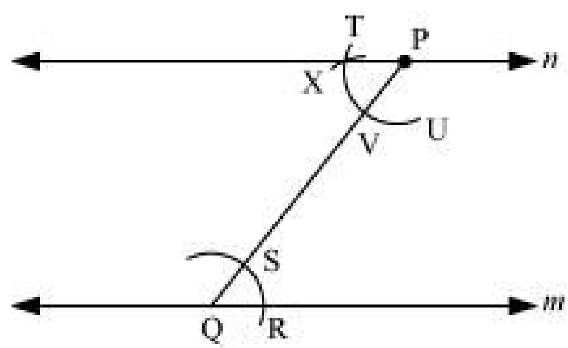
- **Steps of construction for the perpendicular bisector of a line segment $\overline{PQ}$ where $\overline{PQ}$= 9.4 cm:**
  1. Draw a line segment $\overline{PQ}$ whose length is 9.4 cm.
  2. With P as the centre and radius more than half of $\overline{PQ}$, draw a circle using compass.
  3. With the same radius and Q as the centre, draw two arcs that cut the previous circle at points A and B. Join AB to get the perpendicular bisector of $\overline{PQ}$.



- **Construction of line parallel to given line:**
  - **Using ruler and compass:**

Steps of construction to draw a line parallel to a given line *m*, through a point P, outside the line *m*:

1. Take any point Q on *m* and join PQ.
2. With Q as centre and convenient radius, draw an arc cutting *m* at R and PQ at S.
3. With P as centre and the same radius, draw an arc TU cutting PQ at V; then with V as centre and radius equal to RS, draw an arc cutting TU at X.
4. Join PX to draw a line *n*.



Now, the line *n* is parallel to *m*. [Corresponding pairs of angles are equal]

  - **Using ruler and set square:**

Steps of construction of a line parallel to $\overleftrightarrow{AB}$ through point P:

1. Place your set square such that one of its shorter edges i.e., XY lies just along line AB.
2. Place your ruler such that one of its edges lies just along the shorter edge i.e., XZ of the set square. Hold the ruler firmly and slide the set square along the ruler until the edge XY of the set square passes through P.

3. Draw a line along the edge XY of the set square. This is the required line through point P. Note that it is parallel to line AB.