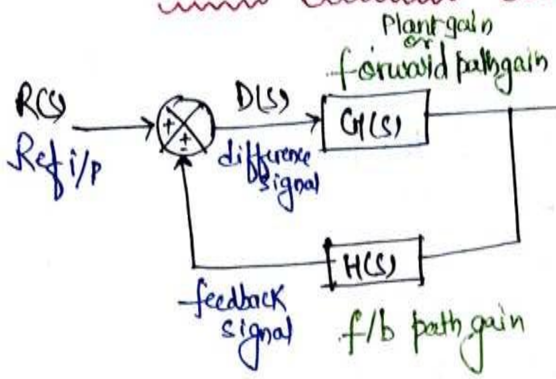
Control System

# BASIC CONCEPT OF CONTROL SYSTEM




| Negative f/b | Positive f/b |
|---|---|
| ↑s stability | ↓s stability |
| Degenerative f/b | Regenerative f/b |
| for designing of Amplifier | for designing of osscilator & multivibrtr |

(i) $OLTF = G(S)H(S)$

- for unity f/b system, $OLTF = G(S)\cdot(1) = G(S)$

(ii) $CLTF = \dfrac{G(S)}{1+G(S)H(S)}$

- for unity f/b system, $CLTF = \dfrac{G(S)}{1+G(S)} = \dfrac{OLTF}{1+OLTF}$

(iii) DC gain of CLTF $= T(S)\big|_{s=0}$

DC gain of OLTF $= G(S)\big|_{s=0}$

$\boxed{\text{DC gain of } \left(G(S)=\dfrac{10}{S(S+2)}\right) = G(S)\Big|_{S=0} = \infty}$ (WRONG)

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$G(s) = \dfrac{C(s)}{R(s)}$ = T.F. of open system

~~$\boxed{OLTF = G(s)}$~~ OLTF is defined only for closed systems.

# Sensitivity

If $A = f(B)$ $\quad S_B^A = \dfrac{\partial A/A}{\partial B/B} = \dfrac{B}{A}\dfrac{\partial A}{\partial B}$

by default Sensitivity $\quad \boxed{S_G^T = \dfrac{G}{T}\dfrac{\partial T}{\partial G}} \qquad \boxed{S_H^T = \dfrac{H}{T}\dfrac{\partial T}{\partial H}}$ where $T(s) = \dfrac{G(s)}{1+G(s)H(s)}$

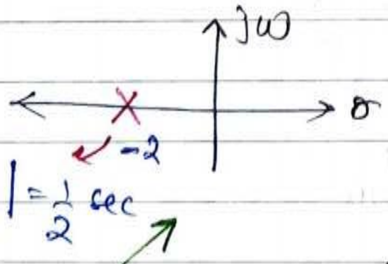
$S_G^T = \dfrac{1}{1+GH} \qquad S_H^T = \dfrac{-GH}{1+GH}$

- Desensitivity, $\boxed{D = 1 + GH}$

# Significance of Poles

- If All poles on LHS of s-plane ⇒ STABLE SYSTEM
- If one of the pole on RHS of s-plane ⇒ UNSTABLE
- If complex conjugate pole on Im axis ⇒ MARGINALLY STABLE
- If repeated pole on Im axis ⇒ UNSTABLE

⇒




Reciprocal of magnitude of -ve real root is equal to time constant of system

$\tau = \left|\dfrac{1}{2}\right| = \dfrac{1}{2}$ sec

$X(s) = \dfrac{10}{s+2} \leftrightarrow 10e^{-2t} = 10e^{-t/(1/2)}$

* Time constant doesn't exist for unstable system or Marginally stable sys.



* Time constant is only valid for absolutely stable system

⊙ $t_{sett} \propto \tau$

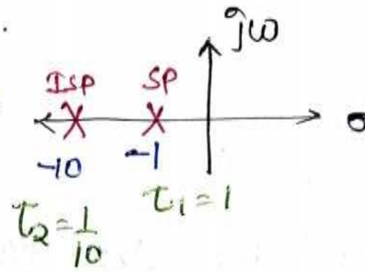
# Initial & final value theorem

IVT: $\boxed{x(0) = \lim_{s\to\infty} sX(s)}$ Valid for both stable & unstable system.

FVT: $\boxed{x(\infty) = \lim_{s\to 0} sX(s)}$ FVT is valid only for absolutely stable system.

⊙ +ve f/b ↓s the stability of system by giving a right shift to poles, But not necessary to +ve real axis. So, it is not necessary that the +ve f/b. sys are unstable.

# Concept of dominant pole




(i) Significant Pole is referred as dominant pole.

(ii) $\tau_{SP} > \tau_{ISP}$

(iii) Time response due to significant pole is slower than that of ISP.

(iv) In dominant pole concept we neglect the ISP for approxⁿ of T(s).

(v) DC gain should be same a/4 approxⁿ

(vi) for existence of dominant pole concept, $\left|\frac{ISP}{SP}\right| \geq 4$ {condition}

(vii) Overall time constant is defined as $\left|\frac{1}{SP}\right|$

(viii) Speed of dominant pole is least.

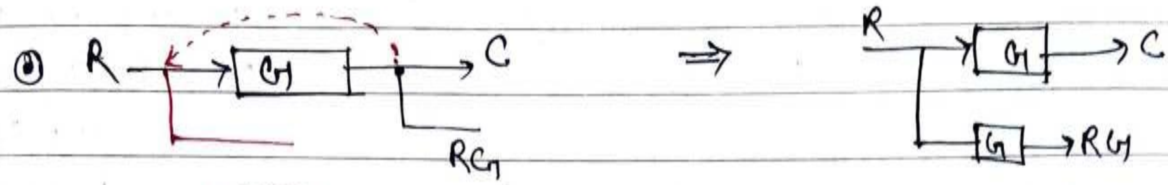
eg: $X(s) = \frac{100}{(s+1)(s+10)}$ ⇒ DC gain $= \frac{100}{1\times 10} = 10$

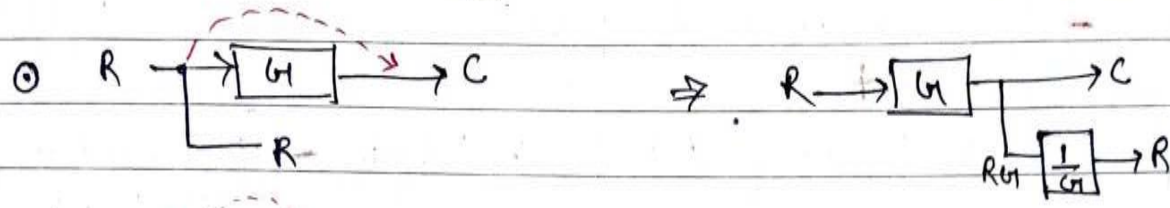
$\boxed{X(s) \approx \frac{100}{(s+1)}}$ ✗ DC gain must be same here DC gain = 100 ≠ 10

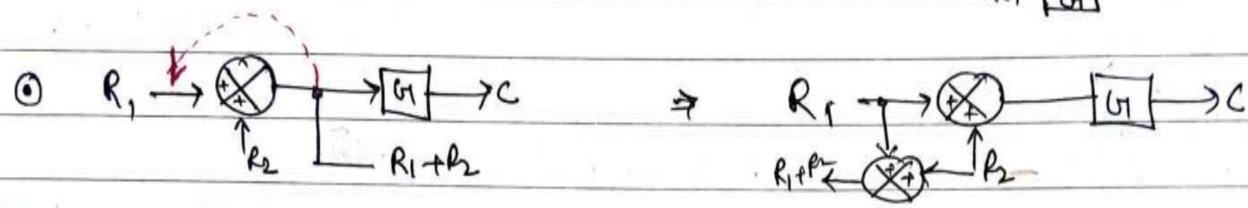
$\boxed{X(s) \approx \frac{10}{s+1}}$ ✓

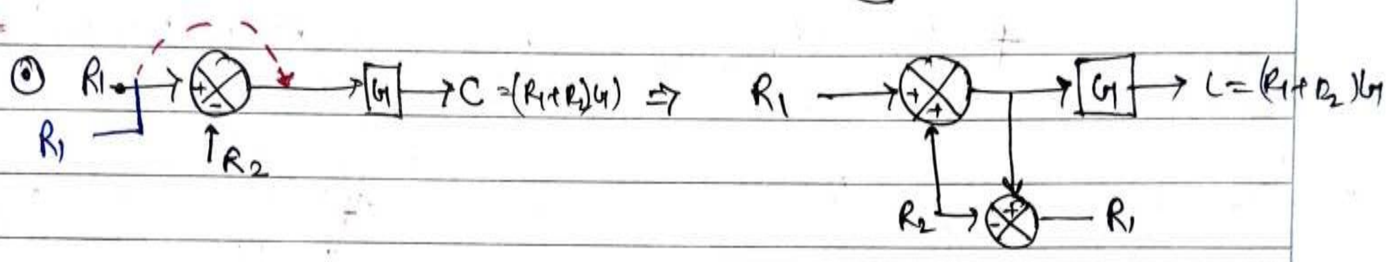
# BLOCK DIAGRAM AND SFG

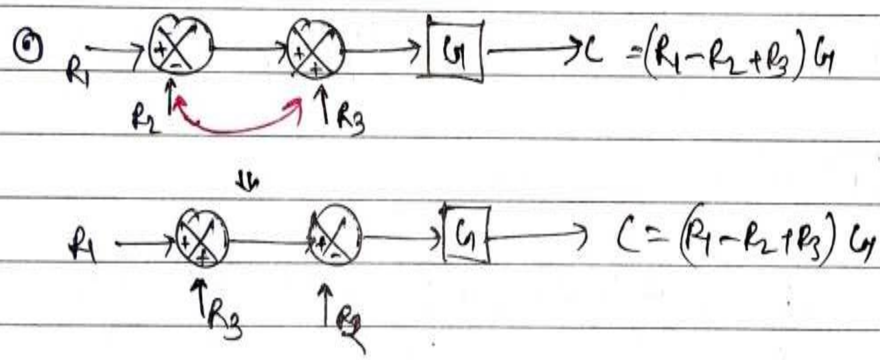


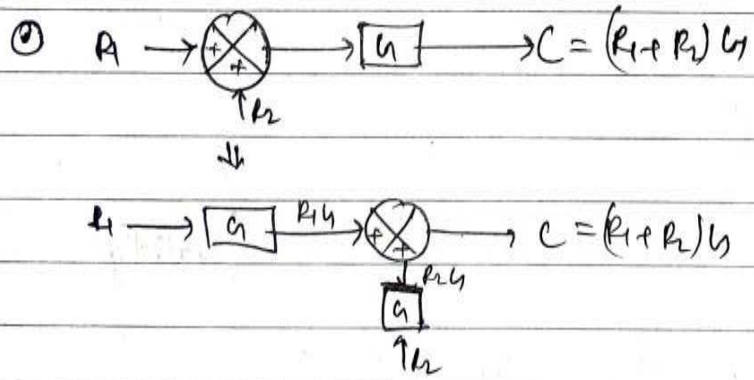


Mason's gain formula : $\boxed{T(s) = \dfrac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}}$

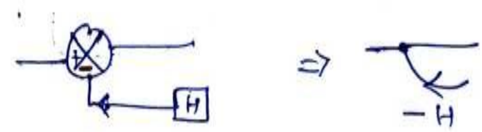
$P_K$ : f/w path gain of $K^{th}$ path
$\Delta$ = Determinant of SFG
$\Delta_K$ = Associated path factor depends on f/w path & isolated loop

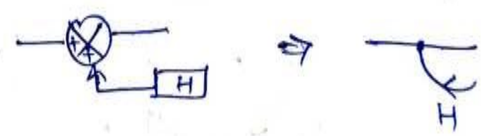
$\Delta = \{$ [1 − sum of all individual loop gain]
+
[sum of product of 2-nontouching loopgain] − [Sum of product of 3 non touching loop gain]
+ ...... $\}$

$\Delta_K = \{$ [1 − sum of all individual isolated loopgain]
+ [sum of product of 2 nontouching isolated loop gain] + .... $\}$

⊙ { − + − + ..... }

⊙

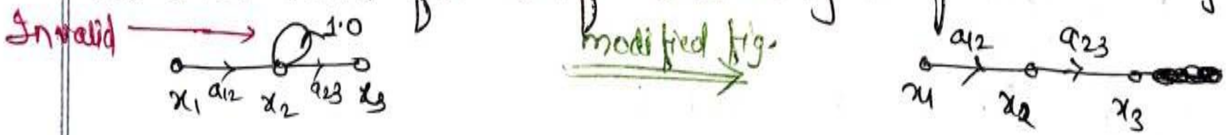


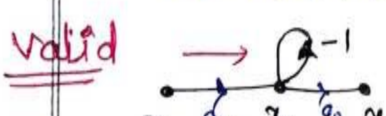

⊙ Repetion of nodes is not allowed.

## Limitation of Mason's gain formula :

(i) Not valid for self sustaining loop with unity gain

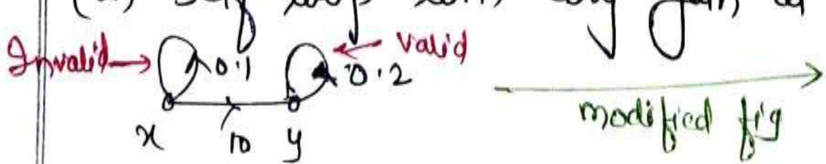


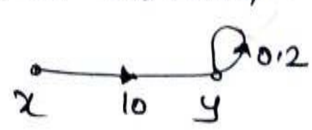

$x_2 = a_{12}x_1 + x_2$ {Invalid eqn}




$x_2 = a_{12}x_1 - x_2 \Rightarrow x_2 = \dfrac{a_{12}x_1}{2}$ {valid}

(ii) Self loop with any gain at input is invalid



Time response $\longrightarrow$

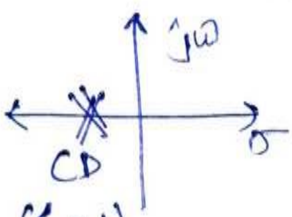
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = -\zeta\omega_n \pm \sqrt{\omega_n^2(\zeta^2-1)}$$

$$D = 4\zeta^2\omega_n^2 - 4\omega_n^2$$
$$= 4\omega_n^2(\zeta^2-1)$$

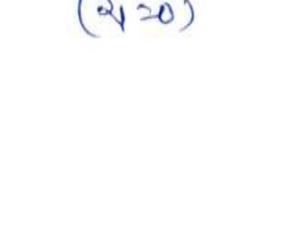
| | | |
|---|---|---|
| $\zeta > 1$ | $D > 0$ | Real roots & unequal |
| $\zeta < 1$ | $D < 0$ | Complex roots |
| $\zeta = 1$ | $D = 0$ | Real & equal roots |
| $\zeta = 0$ | $\alpha = 0$ | Pure Im roots |

OD
($\zeta > 1$)



CD
($\zeta = 1$)

(UD)
($0 < \zeta < 1$)



Undamped
($\zeta = 0$)

# Time Response



* response means $C(s)$ or $c(t)$.
* $1^{st}$ order system $G(s) = \frac{1}{sT}$ CLTF $= \frac{1}{1+sT}$

| $r(t)$ | ess |
|---|---|
| $\delta(t)$ | doesnt exist → as initial & final values can't be calculated |
| $u(t)$ | 0 |
| $t\,u(t)$ | $\tau$ |
| $\frac{t^2}{2}u(t)$ | $\infty$ |

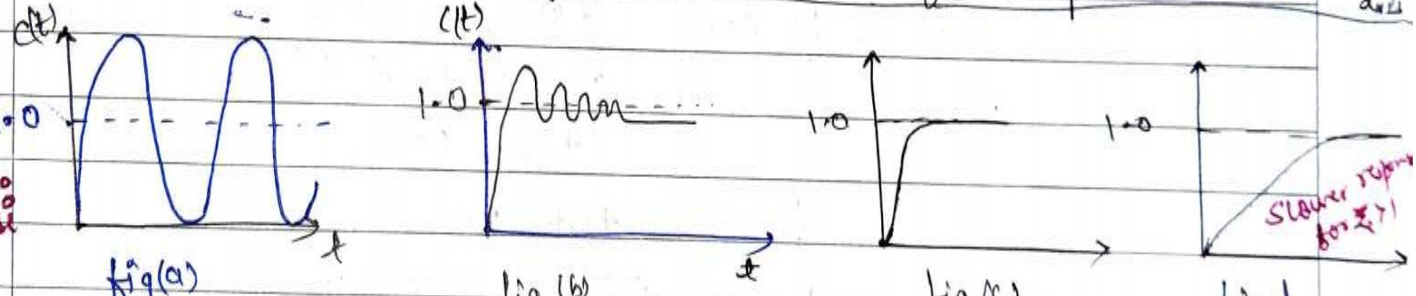
* $2^{nd}$ Order system

$$\boxed{G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}}$$

$$\boxed{CLTF = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

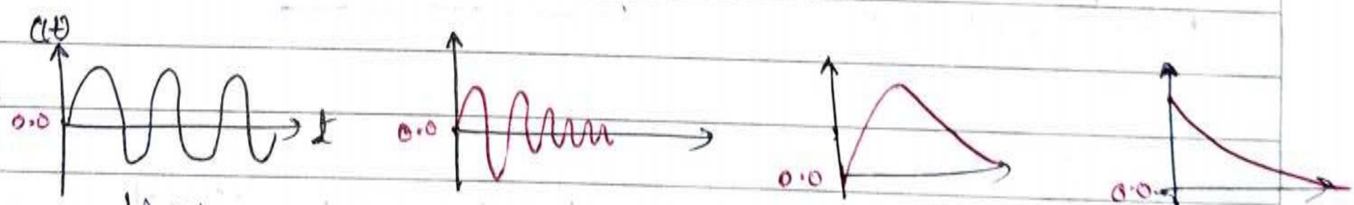
| | | | | |
|---|---|---|---|---|
| M.S. | $\xi = 0$ | Undamped osscilation | fig(a) | Poles on Im axis |
| Stable | $0 < \xi < 1$ | Underdamped osscilation | fig(b) | Poles: $-\alpha \pm j\omega_d$ |
| " | $\xi = 1$ | Critical damped osscilation | fig(c) | Poles:- -ve real axis & equal |
| " | $\xi > 1$ | Over damped osscilation | fig(d) | poles:- unequal on -ve R axis |

Step response:




fig(a) fig (b) fig (c) fig d

Impulse response:

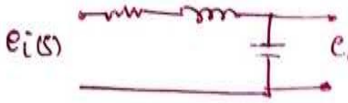


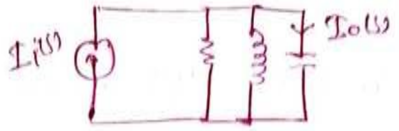

fig(a)

** Over damped gives slower response & critical gives faster for step input although they look almost similar.

Date ................ Teacher's Signature :................

* RLC ckt. for 2nd order system




o/p is taken from 'C' if not mentioned in all questions.

* $\alpha = \xi\omega_n$ & $\omega_d = \omega_n\sqrt{1-\xi^2}$ $\tau = \left|\frac{1}{\alpha}\right|$

* $\theta = \cos^{-1}\xi$

* C/S eqn $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

* Don't take care of N°, always Calculate $\omega_n$ & $\xi$ from D°.

* 1st order:

delay time (to reach 50%) $\boxed{t_d = 0.693\tau}$

rise time 10%–90% $\boxed{t_r = 2.2\tau}$

Settling time 2% to 5% error band

for eg: for 2%. $c(t) = 1 - e^{-t/\tau}$

$0.98 = 1 - e^{-t_{sett}/\tau}$

$t_{sett} = 4\tau$

for 5%. $t_{sett} = 3\tau$

for E%. $\boxed{t_{sett} = \tau \ln \frac{100}{E}}$

Note: $\boxed{t_d < t_r < t_{sett}}$

** 2nd Order:

$\boxed{\frac{dc}{dt} = 0}$, $t_p = \frac{n\pi}{\omega_d}$ where $n = 1, 3, 5 \ldots$ for 1st, 2nd ... maxima

$n = 2, 4, \ldots$ for 1st, 2nd ... minima

$M.P._o = e^{-\pi\cot\theta}$ or $e^{-(2m-1)\pi\cot\theta}$

$MP_u = e^{-2\pi\cot\theta}$ or $e^{-2m\pi\cot\theta}$ // first or Generalised

$$t_r = \frac{\pi - \theta}{\omega_d} \quad \{\text{when } c(t) = 100\%\}$$

$$t_s = \left(\frac{4}{\xi \omega_n} (=4\tau) \text{ for } 2\% \text{ tolerance}\right) = \frac{3}{\xi \omega_n} (=3\tau) \text{ for } 5\% \text{ tolerance}$$

No. of oscillations, $N_o = \frac{2\sqrt{1-\xi^2}}{\pi \xi} = \frac{2}{\pi} \tan\theta$

evergreen formula for error calculatn.

$$\boxed{E_{ss} = \frac{R(s)}{1+G(s)}} \quad ; \quad \boxed{e_{ss} = \lim_{s \to 0} \frac{sE(s)}{} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}}$$

* Only valid for unity f/b system

$K_p = \lim_{s \to 0} G(s)H(s)$ ; $K_v = \lim_{s \to 0} sG(s)H(s)$ ; $K_a = \lim_{s \to 0} s^2 G(s)H(s)$

$$\boxed{e_{ss} = \frac{1}{1+K_p} = \frac{1}{K_v} = \frac{1}{K_a}}$$

* Type of CLTF = Type of OLTF

| Type of system | error coefficients | | | Steady state error | | |
|---|---|---|---|---|---|---|
| | $K_p$ | $K_v$ | $K_a$ | Step i/p | Ramp i/p | Parbolic i/p |
| 0 | K | 0 | 0 | $\frac{1}{1+K}$ | $\infty$ | $\infty$ |
| 1 | $\infty$ | K | 0 | 0 | $1/K$ | $\infty$ |
| 2 | $\infty$ | $\infty$ | K | 0 | 0 | $1/K$ |
| | | | | | | |

for Non unity f/b system : $\left\{ e_{ss} = \lim_{s \to 0} sE(s) \right.$

$$= \lim_{s \to 0} sR(s) \left[\frac{1}{K_H} - T(s)\right]$$

P.T.O

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pg 5.33 $e_{ss} = \lim_{s \to 0} s^N H(s)$

order(finite error)

order 2 (zero error) type order 3 ($\infty$ error)

- If order of i/p is greater than type of system the error will be infinite.
- If type of system > order of i/p ⇒ $e_{ss} = 0$
- If type of system = order of i/p ⇒ $e_{ss}$ = finite
- Error coefficient is valid for stable CLTF

| i/p | order |
|---|---|
| $u(t)$ | 0 |
| $t^1 u(t)$ | 1 |
| $\frac{t^2}{2} u(t)$ | 2 |

In case of non-unity f/b system

(i) when f/b system consist no poles & zeroes at origin

$$K_H = \lim_{s \to 0} H(s)$$

$$e_{ss} = \lim_{s \to 0} sR(s) \left[\frac{1}{K_H} - T(s)\right]$$

(ii) when f/b system consits N number of poles at origin

$$K_H = \lim_{s \to 0} s^N H(s)$$

$$e_{ss} = \lim_{s \to 0} sR(s) \left[\frac{s^N}{K_H} - T(s)\right]$$

(iii) when f/b system consists N no. of zeroes at origin

$$K_H = \lim_{s \to 0} \frac{H(s)}{s^N}$$

$$e_{ss} = \lim_{s \to 0} sR(s) \left[\frac{1}{s^N K_H} - T(s)\right]$$

** Avoid this mistake

Doubt
- Never use $e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$ in case of non-unity f/b system.

why don't we use this for non unity f/b. as H(s) is already included & if H(s) is not to be used in non unity f/b & formula is valid for U.F.S. H(s) = 1 then why H(s) is included in formula?

Ans: In actual $E(s) = R(s) - C(s)$ but in case of non unity f/b $K_H E(s) = R(s) - K_H C(s)$
video lecture by Dhande sir. QR code scan.

# frequency response of 2nd Order system

(1) $T(s) = \dfrac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \dfrac{K}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\dfrac{\omega}{\omega_n}\right)}$

$T(ju)' = \dfrac{K}{1 - u^2 + 2j\xi u}$

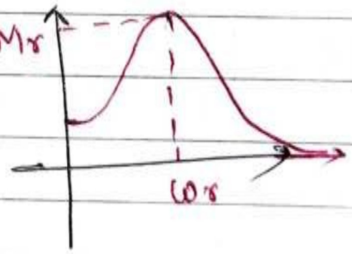
*** $\boxed{|Tju|_{max} = M_r = \text{resonant peak} = \dfrac{K}{2\xi\sqrt{1-\xi^2}}}$ $= K\operatorname{cosec} 2\theta$

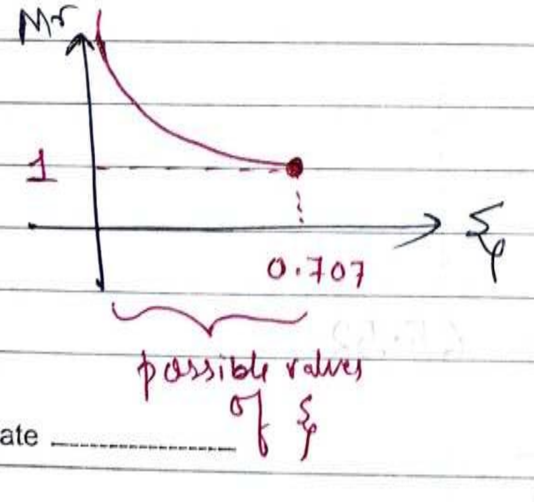
occurs at $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$ ⇐ resonant frequency

or $\boxed{u_r = \sqrt{1-2\xi^2} =}$ Normalised resonant frequency

(2) $\boxed{\xi_{max} = \dfrac{1}{\sqrt{2}} = 0.707}$ as $\xi > 0.707 \Rightarrow \omega_r =$ Imaginary

$\boxed{\omega_{r\,min} = 0}$



(3) ** Let $M_r = 2.5$, & $K=1$

$$2.5 = \frac{1}{\sin 2\theta} \Rightarrow 2\theta = \sin^{-1}(0.4) = (23.57^\circ) \text{ or } (180^\circ - 23.57^\circ)$$

Don't forget to calculate this

$$\Rightarrow \theta = 11.78 \text{ or } 78.215$$

$$\Rightarrow \xi = \cos\theta = \underline{0.978} \text{ or } \underline{0.204}$$

× (> 0.707) ✓

$$\boxed{\xi = 0.204}$$

(4) 3-dB Bandwidth or simply Bandwidth.

$$\Rightarrow |T(j\omega)| = \frac{1}{\sqrt{2}} \text{ for } \omega_b$$

$$\boxed{\omega_b = \omega_n \sqrt{a + \sqrt{a^2+1}}} \quad \text{where } \boxed{a = (1 - 2\xi^2)}$$

** Bandwidth decreases the damping ratio.

(5) Relative stability parameters

** $$\boxed{\omega_{gc} = \omega_n \sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}}$$

** $$\boxed{PM = \tan^{-1}\left(\frac{2\xi}{\sqrt{-2\xi^2 + \sqrt{4\xi^4+1}}}\right)}$$

$\xi_{max} = 0.707 \Rightarrow \max P.M. = 65.52$

** $$\boxed{\xi = 0.5 \sqrt{\sin(PM) \times \tan(PM)}}$$

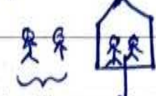
# Routh's stability criteria



* ROZ occurs at only odd powers of s

****** IP = EP for value of Kmax. Marginally stable
IP > EP Stable
IP < EP Unstable

{ Yad rakhne k liye




for safety Internal > Ext. members members

hence for stability IP > EP

* Key pt. Pg 5.58

eg: $a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0 s^0 = 0$

| | | | | |
|---|---|---|---|---|
| $s^6$ | $a_6$ | $a_4$ | $a_2$ | $a_0$ |
| $s^5$ | $a_5$ | $a_3$ | $a_1$ | 0 |
| $s^4$ | $b_1 = \frac{a_5 a_4 - a_3 a_6}{a_5}$ | $b_2$ | $a_0$ | |
| $s^3$ | $c_1$ | | | |
| $s^2$ | $d_1$ | | | |
| $s^1$ | $e_1$ | | | |
| $s^0$ | $f_1$ | | | |

0 0 0 0 (AE) ROZ

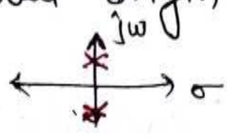
- No. of sign changes = No. of RH poles
- No. of Im axis poles = order(AE) − 2 × No. of sign changes below AE
- No. of LH pole = Total No. of poles − RH pole − Im axis poles

** ~~ROZ~~ afterwards

[ Imaginary axis poles ⇒ ROZ always
But ROZ doesn't always ⇒ Im axis poles ]

Whenever ROZ occurs & if formula for Im axis poles not remembered then check roots of AE. → If roots are Imaginary then Im axis poles.

⊙ ROZ occurs only when the poles are located symmetrical about origin.

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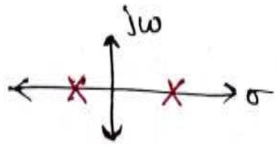


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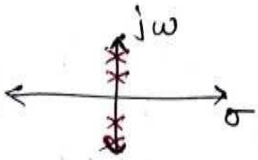


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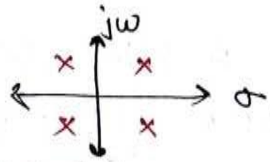


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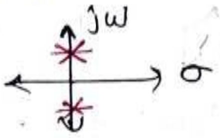


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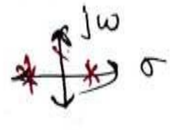


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⊙ ROZ > 1 ⇒ Repeated poles (any where ~~on s-plane~~ Re or Im axis)

No. of repeated poles = No. of ROZ

*** ⊙ $as^3 + bs^2 + cs + d = 0$

EP IP

IP > EP Stable

IP < EP Unstable

IP = EP M.Stable

* This concept is valid only for 3rd order system.

* This concept is valid for hurwitz eqn only

$$\boxed{\begin{array}{l} s^3 - 4s^2 + s + 6 \\ IP = -4 \\ EP = 6 \\ EP > IP \end{array}}$$

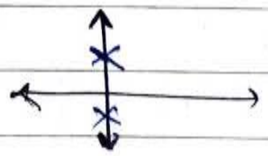
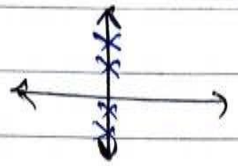
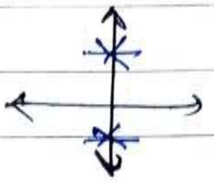
← Not valid ∵ non Hurwitz polynomial

⊙ There is no significance of ROZ in last Row

⊙ If only first element of ROW = 0 & Not a ROZ condn then replace 0 by K & put K=0 at last for checking sign change

(c) Symmetricity about origin is used ↓

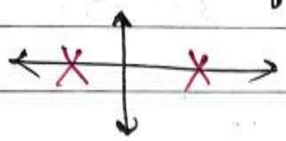
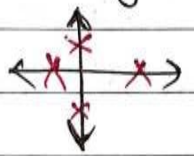
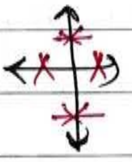
(i) If no. of sign changes below AE = 0 [⇒ No RH pole found below AE]

O[A·E·] = 2  O[A·E·] = 4  O[A·E·] = 6

2 RDZ

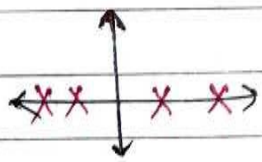
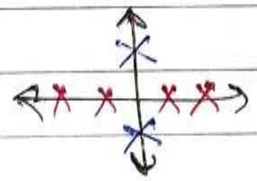
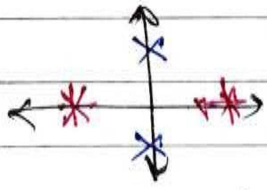
(ii) If no. of sign changes below AE = 1 [1 RH pole found below AE but symmetry ⇒ 1 pair]

O[A·E·] = 2  O[A·E·] = 4  O[A·E·] = 6

2 RDZ

(iii) If no. of sign changes below AE = 2 [2 RH pole found below AE but from symmetry ⇒ 2 pair]

# ROOT LOCUS

Poles of CLTF

- Valid for -ve f/b system
- Path of poles of CLTF for different values of system gain of OLTF Xfer function. $0 < K < \infty$
- In root locus diagram, arrow indicates, movement of poles with ↑ value of K

# Rules for sketching root locus:

(i) Root locus is always symmetrical about real axis.

(ii) Root locus always start from open loop poles & terminate at either $\infty$ open loop zero (Virtual zero) or at finite zero of open loop

If $P > Z$ then RL ends at virtual zero

$$\text{Poles of CLTF} = \text{Poles of OLTF} [K=0]$$
$$= \text{Zeroes of OLTF} [K=\infty]$$

(iii) RL exist on real axis only where where total odd no. of [zeroes + poles open loop] to RHS of that pnt.

or

RL will exist only that section of real axis where total angle contribution by all open loop poles & zeroes towards that observation pt. is odd integral multiples of 180°.

(iv)

(v)

Shortcut $\angle A = 90, 270$ ** diff $= 2\times90$ | $\angle A = 60, 180, 300$ diff: $2\times60$ $2\times60$ ← Just remember the starting ∠s

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(vi) for $P > Z$ $(B = P)$
$Z > P$ $(B = Z)$
$Z = P$ $(B = P = Z)$

(vii) for Break Pnt Calculation : $1 + G H(s) = 0 \Rightarrow K = f(s)$
Roots of $\frac{dK}{ds} = 0$

(viii) No. of Asymptotes : $A = \underset{(P>Z)}{P-Z} = \underset{(Z>P)}{Z-P}$

(ix) Angle of Asymptotes :
$P > Z$ $\angle A = \frac{(2\alpha+1)180^\circ}{P-Z}$ $\alpha = 0, 1, 2, .., P-Z-1$

$P-Z=1 \Rightarrow \angle A = 180^\circ$
$P-Z=2 \Rightarrow \angle A = 90^\circ, 270^\circ$ $= \frac{(2\alpha+1)180^\circ}{Z-P}$ $\alpha = 0, 1, 2, ... Z-P-1$
$P-Z=3 \Rightarrow \angle A = 60, 180^\circ, 300$

$Z = P \Rightarrow$ No Asymptotes

(x) $\sigma = \frac{\sum Re(P) - \sum Re(Z)}{P - Z}$

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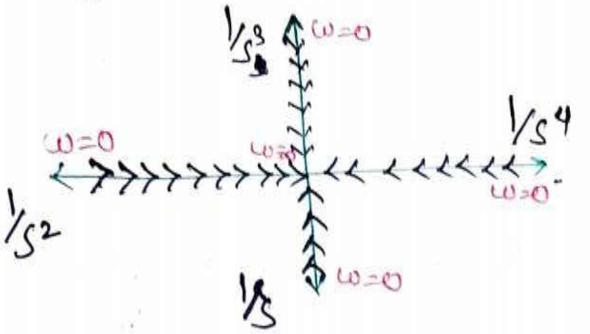
## 6. Polar plot ($\omega = 0$ to $\omega = \infty$)

- Relative stability of CLTF by using frequency response of OLTF



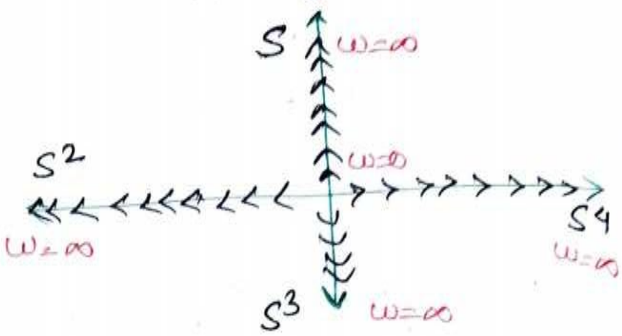
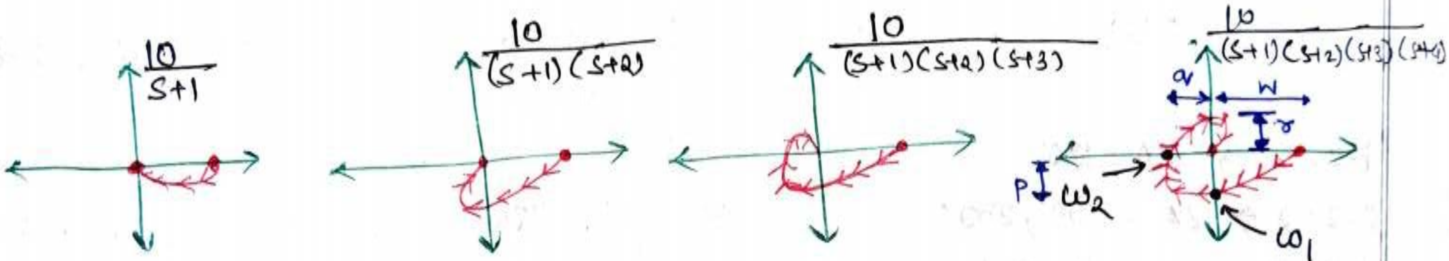




Table:

eg: $\frac{10}{s+1}$

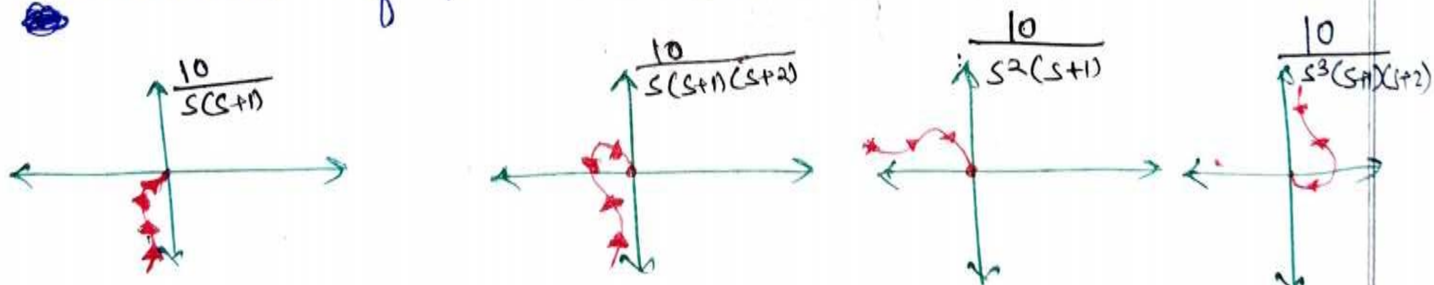
| $\omega$ | $\lvert GH(j\omega)\rvert$ | $\angle GH(j\omega)$ |
|---|---|---|
| 0 | 10.0 | 0° |
| 1.0 | 7.07 | −45° |
| $\infty$ | 0.0 | −90° |

*




- Calculation of $\omega_1$ : $Re[GH(s) = 0]$
- Calculation of $\omega_2$ : $Im[GH(s) = 0]$
- Calculation of $p$ : $|GH(j\omega_1)|$

*

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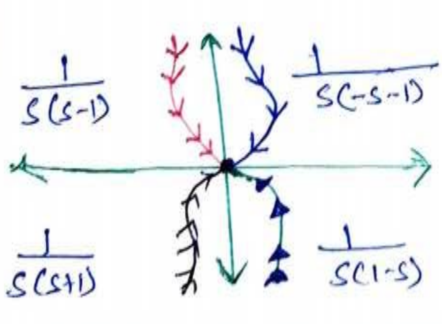



$\swarrow \left(\frac{1}{s^3(s+1)}\right)$

$-\omega^3(j\omega+1)$

$= \omega^3(-j\omega-1)$

- $s \to$ −ve hoga to +s wale ka Im axis ka mirror image
- Pole → −ve hoga to Real ke about

eg: $(-s-1)$ : $(s+1) \xrightarrow{\text{Real}} (s-1) \xrightarrow{\text{Im}} (-s-1)$

## ** # Relative stability Parameter :

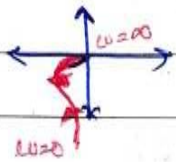
Sure shot Question.

<i> $\angle GH(j\omega_{pc}) = -180°$ or $Im[GH(j\omega_{pc})] = 0$

<ii> $|GH(j\omega_{gc})| = 1$
$= 0dB$

<iii> $GM = \frac{1}{|GH(j\omega_{pc})|}$ (= +ve value) ** {GM dB can be negative}

<iv> $PM = 180° + \angle GH(j\omega_{gc})$




- $\omega_{pc} = \infty$
- B/4 Calculating $Im[GH(j\omega_{pc})] = 0$. first check weather polar plot crosses the 180° line or not, otherwise it will give wrong answer.
- eg: $\frac{5}{s(s+1)} = \frac{5}{-\omega^2 + j\omega}$ $\boxed{\omega_{pc} = 0}$ (WRONG)

So, calculate angle at $\omega = 0$ & $\omega = \infty$ b/4 solving for $\omega_{pc}$.

- If $\omega_{gc}$ = real, if no real values comes then $\omega_{gc}$ doesn't exist.

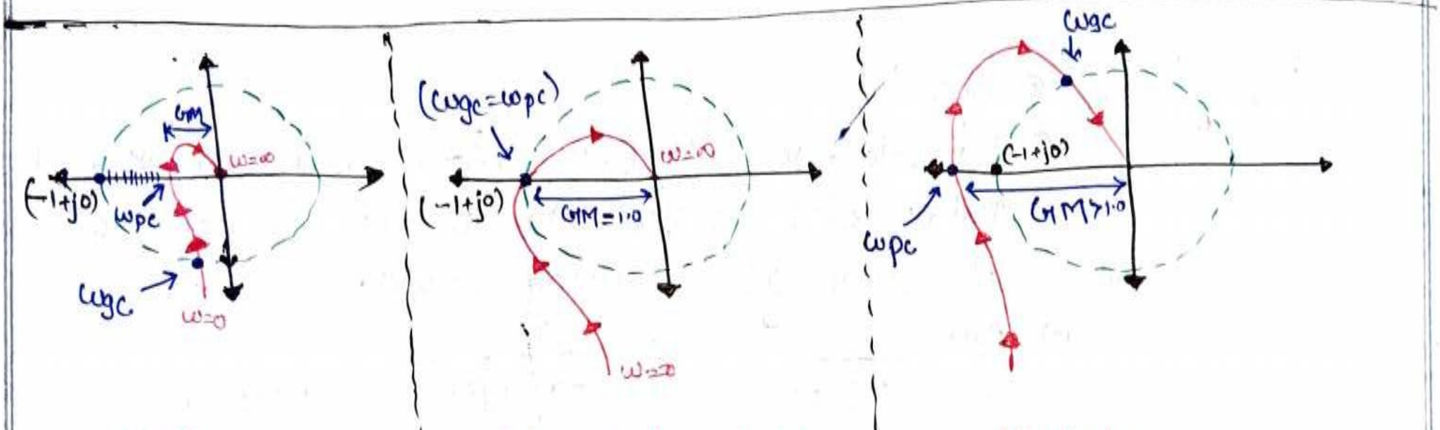
| GH(s) | $\lvert GH(j\omega)\rvert_{\omega=0}$ | $\lvert GH(j\omega=\infty)\rvert$ | $\lvert GH(j\omega)\rvert$ | $\omega_{gc}$ |
|---|---|---|---|---|
| $\frac{10}{(s+1)(s+2)}$ | 5.0 → | 0.0 | $0 < \lvert GH(j\omega)\rvert < 5$ | exist |
| $\frac{1}{(s+1)(s+2)}$ | 0.5 → | 0.0 | $0 < \lvert GH(j\omega)\rvert < 0.5$ | doesn't exist |

* for concept of $\omega_{gc}$ we will calculate magnitude at $\omega = 0$ & $\omega = \infty$ and we will decide the calculation of $\omega_{gc}$.

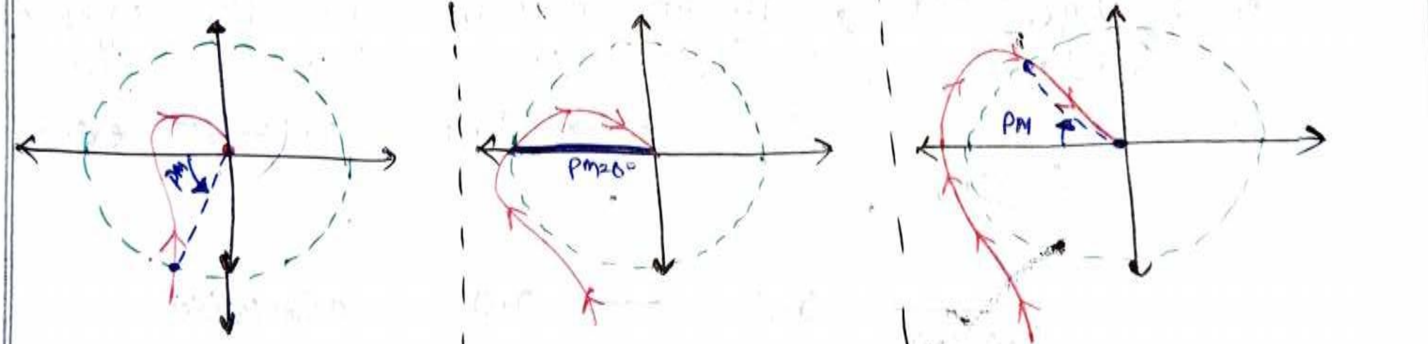
# Stability concept

| | |
|---|---|
| ⟨i⟩ GM > 0 dB & PM > 0° | Stable |
| ⟨ii⟩ GM = 0 dB & PM = 0° | Marginally stable |
| ⟨iii⟩ GM < 0 dB & PM < 0° | Unstable |

- $\omega_{gc} < \omega_{pc}$ → CLTF (or system) will be stable
- $\omega_{gc} > \omega_{pc}$ → System will be unstable
- $\omega_{gc} = \omega_{pc}$ → System will be marginally unstable




Stable | Marginally stable | Unstable

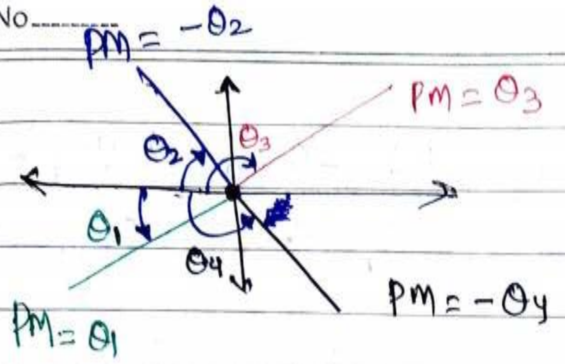



PM > 0 stable | PM = 0 Marginally stable | PM < 0 Unstable

from fig.

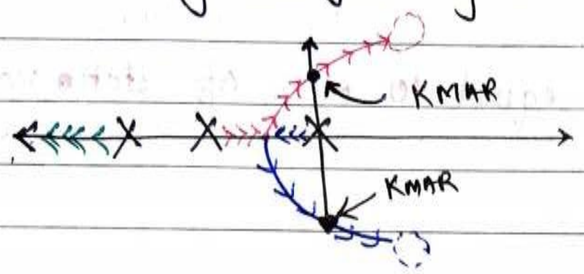
$\angle GH(j\omega) = -180° + PM$

$$\boxed{PM = 180° + \angle GH(j\omega)}$$




- ACW → $\theta$ = +ve   PM = $\theta$
  CW → $\theta$ = −ve

# Gain Margin by using root locus :




- KMAR from root locus
- for order − 3 GH(s)
  IP = EP for Kmar
- Kdesired = K ← $GH(s) = \dfrac{K}{(s+P_1)(s+P_2)}$

$$\boxed{GM = \frac{K_{MAR}}{K_{desired}}}$$

- $|GH(s)| = |GH(s)| \times GM$
  $K_{MAR} = K \times GM$

# State space Analysis

$\dot{X} = AX + BU$ $\qquad$ $[\dot{X}]_{n\times 1} = [A]_{n\times n}[X]_{n\times 1} + [B]_{n\times m}[U]_{m\times 1}$

$Y = CX + DU$ $\qquad$ $[Y]_{p\times 1} = [C]_{p\times n}[X]_{n\times 1} + [D]_{p\times m}[U]_{m\times 1}$

* n: No. of state variables = No. of effective storing elements
= No. of initial values
= order of T.F. = order of D.E.
= No. of time constants.

* No. of $Y_s$ terms in SFG is equal to No. of state variables.

* eg: $\dfrac{Y(s)}{U(s)} = \dfrac{10}{s^3 - 4s^2 + 2s - 6} = T(s)$

No. of state variables = 3

$s^3 Y(s) - 4s^2 Y(s) + 2sY(s) - 6Y(s) = 10U(s)$

** LT: $\dfrac{d^3[y(t)]}{dt^3} - 4\dfrac{d^2[y(t)]}{dt^2} + 2\dfrac{d[y(t)]}{dt} - 6y(t) = 10u(t)$

$\dfrac{d^3[y(t)]}{dt^3} \downarrow \dot{x}_3$, $\quad \dfrac{d^2[y(t)]}{dt^2} \downarrow x_3 = \dot{x}_2$, $\quad \dfrac{d[y(t)]}{dt} \downarrow x_2 = \dot{x}_1$, $\quad y(t) \downarrow x_1$

$$\left.\begin{aligned} \dot{x}_3 &= 4\dot{x}_3 - 2x_2 + 6x_1 + 10u \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ y &= x_1 \end{aligned}\right\} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -2 & 4 \end{bmatrix}}_{[A]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}}_{[B]} [u]$$

** If $N^r$ contains only constant term, then {shortcut}

$\dfrac{Y(s)}{U(s)} = \dfrac{10}{s^3 - 4s^2 + 2s - 6}$ $\longrightarrow$ Last row with sign change & order change (asc./des.)

$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -2 & 4 \end{bmatrix}$

element above diagonal is always 1 except this last row is non zero & rest all terms are zero.

# Recovery of T.F. from state variable representation

** $\dot{X} = AX + BU$ $\qquad Y = CX + DU$

$sX(s) = AX(s) + BU(s)$

$X(s) = (sI - A)^{-1} B U(s) \Rightarrow \boxed{\frac{Y(s)}{U(s)} = [C][(sI-A)^{-1} B] + D}$

for continuous:

for discrete:

** $\boxed{\frac{Y(z)}{U(z)} = T(z) = C\left[(zI - A)^{-1} B\right] + D}$

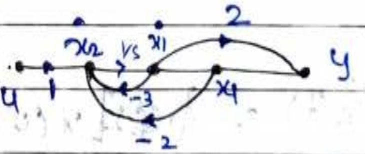
** Shortcut 2: {only valid for proper fraction}

$T(s) = \frac{2s+7}{s^2+3s+2}$ $\qquad Y = 2s+7$, $\quad U = s^2+3s+2$ $\qquad y = 7x_1 + 2x_2$

$u = 2x_1 + 3x_2 + \dot{x}_2$

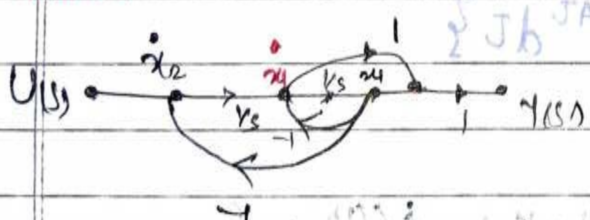
$\dot{x}_1 = x_2$

** $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$




** $[y] = [7 \quad 2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

** Note:




$\dot{x}_1 \neq x_2$

$\dot{x}_1 = x_2 - x_1$

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Note :- $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $e^{At} \neq \begin{bmatrix} 1 & e^{t} \\ e^{t} & 1 \end{bmatrix}$ $e^{At} = L^{-1}\left\{ \begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix}^{-1} \right\}$

# Solution of state space equation

$\dot{X} = AX + BU$ $\quad Y = CX + DU$

Case (i) $U$ (i/p) $= 0$

$[X(s)]_{n\times 1} = [sI - A]^{-1}_{n\times n} [x(0)]_{n\times 1}$

$[\phi(s)]_{n\times n} = [sI - A]^{-1}_{n\times n}$ = Resolvent Matrix

└ State transition matrix

- ZIR $= f[x(0)]$
- $\phi(s) = [sI - A]^{-1} = \frac{1}{s-A} \Rightarrow [\phi(t) = e^{At}] = STM$

** Apply this to eliminate options for $\phi(t) = ?$

- Properties of STM $\{\phi(t)\}$

**(i) $\phi(0) = I$ $\quad$ **(ii) $\phi'(0) = A$ $\quad$ (iii) $\phi^{K}(t) = \phi(Kt)$ $\quad$ (iv) $\phi^{-1}(t) = \phi(-t)$

(v) $\phi(t_1 - t_0)\cdot\phi(t_2 - t_2) = \phi(t_2 - t_2)$

Case (ii) $U \neq 0$ {Non Homogenous equation}

$\dot{X} = AX + BU \Rightarrow sX(s) - x(0) = AX(s) + BU(s)$

$$\boxed{X(s) = \phi(s)\,x(0) + \phi(s)\,B\,U(s)}$$

State eqⁿ $\quad$ ZIR $\quad$ ZSR

$x(t) = \phi(t)[x(0)] + \phi(t) \circledast B\,u(t)$

$x(t) = e^{At}\,x(0) + \int_0^{\infty} B\,U(\tau)\,e^{A(t-\tau)}\,d\tau$

or, $x(t) = e^{At}\left\{x(0) + \int_0^{t} BU(\tau)\,e^{-A\tau}\,d\tau\right\}$

* Calculation of ZIR :

$x(t)_{ZIR} = [\phi(t)]_{2\times 2}\,[x(0)]_{2\times 1}$

$\phi(t) = L^{-1}\left\{(sI - A)^{-1}\right\}$

To check : $\phi'(t) = ?$

$\phi'(0) = ?$

$\phi'(0)$ must be equal to $A$

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** Calculation of ZSR

$$[X(s)]_{2\times1} = [\phi(s)]_{2\times2}[B][U(s)]_{1\times1}$$

$$x(t)\Big|_{ZSR} = L^{-1}[X(s)]$$

** $x(t) = [x(t)]_{ZIR} + [x(t)]_{ZSR}$

Calculate ZSR & put $t=0$

$$x(t)\Big|_{t=0} = ZIR + ZSR = ZIR$$ {satisfy initial value}

# Controllability: {relationship b/w i/p & SV}

SFG: If there is path from i/p to all S.V. the controllable

Kalman's test: $Q_c = [B : AB : A^2B \dots A^{n-1}B]_{n\times n}$

If $|Q_c| \neq 0 \Rightarrow$ controllable

$|Q_c| = 0 \Rightarrow$ uncontrollable

# Observability:

Kalman's test: $Q_o = [C^T : A^TC^T : A^{2T}C^T : \dots A^{(n-1)T}C^T]_{n\times n}$

$|Q_o| \neq 0 \Rightarrow$ system is observable

$|Q_o| = 0 \Rightarrow$ system is unobservable

** If T.F. doesn't involve any pole zero cancellation then system is both controllable & observable

In case of PZ cancellation $\Rightarrow$ may be uncontrollable / unobservable / both.

eg: $T(s) = \dfrac{s+2}{s^3+9s^2+26s+24} = \dfrac{\cancel{(s+2)}}{\cancel{(s+2)}(s^2+7s+12)}$

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# Types of decomposition : (later)

**Note :** $|SI - A| = 0$ is c/s eqⁿ which decides stability of system.

for eg: $A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$ $\quad \begin{vmatrix} s-2 & -3 \\ 0 & s-5 \end{vmatrix} = 0 \quad \cancel{s^2 - 7s + 7 = 0}$

$\Rightarrow (s-2)(s-5) = 0$

unstable

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# Nyquist Plot

- Stability of CLTF from openloop frequency response & open loop poles.
- Zeroes [CE] = Poles [CLTF]

  Poles [CE] = Zeroes [OLTF]

- $N = P - Z$

  P: RH poles of CE = RH poles of OLTF

  Z: RH ~~pol~~ zeroes of CE = RH poles of CLTF

  N: No. of encirclement about $(-1+j0)$ critical pnt. in CCW direction

- By default, Nyquist contour = CW direction, N in CCW dir.

  If Nyquist plot is CCW dir, $N = P - Z$ in CW direction

- <u>Objective Approach:</u>

$G(s) = \frac{10}{s(s+1)}$

(i) $P = 0$ {No. of RH pole of OLTF}

(ii) $1 + G(s) = 0 \Rightarrow s^2 + s + 10 = 0$

$s^2$ 1 10

$s$ 1 0

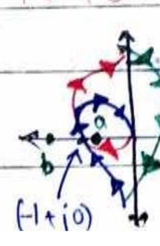
$s^0$ 10, 0

} RH pole of CLTF $Z = 0$

(iii) $N = P - Z = 0$

- Nyquist plot is always symmetrical about real axis.
- $N > 0 \Rightarrow$ CCW encirclement } for CW contour

  $N < 0 \Rightarrow$ CW encirclement

- 


$\omega_{pc} = ?$

$\rightarrow |GH(j\omega)_{pc}| = \frac{2K}{3}$ (say)

Pnt. (a) $N = -2$, $P = 0$, $Z = 2$ } unstable for $\left|\frac{2K}{3}\right| > 1$ or $K > \frac{3}{2}$

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# Bode Plot

- Only for minimum phase system {i.e. All P & Z on LHS}
- Corner frequency represent the frequency at which error will be maximum.

- $T(s) = (1.0)\dfrac{(s+10)}{(s+100)^2}$ $\qquad K \neq 1.0$

$$= \frac{10}{10^4}\frac{(1 + s/10)}{\left(1+\frac{s}{100}\right)^2} \qquad K = 10^{-3}$$

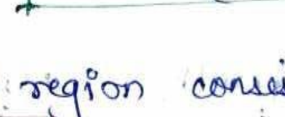
- $K = 1$, zero at $s=10$, 2 poles at $s=100$

$$\boxed{T(s) = \frac{1.0\,(s+10)}{(s+100)^2}} \text{ (WRONG)} \qquad \boxed{T(s) = 1\cdot\frac{(1+s/10)}{(1+s/100)^2}}$$

- If low frequency region or initial region consists of slope then $\boxed{20\log K = |G\,H(j\omega)|_{\omega = 1\ \text{rad/s}}}$

- For initial slope {Low freq. region}

$$\boxed{y_B = 20\log K \pm m\log\omega_1}$$



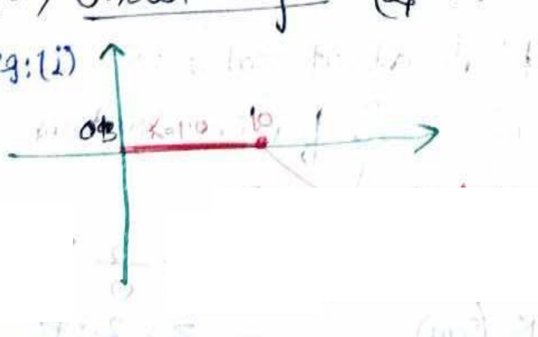


- 'K' is always Calculated from low frequency region.

## # Bode plot for 2nd order System $\quad T(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

(i) Underdamped ($\zeta < 1$)

eg:(i)




$$T(s) = \frac{1}{1 + 2\zeta\left(\frac{s}{\omega_n}\right) + \left(\frac{s}{\omega_n}\right)^2}$$

$$T(s) = \frac{1}{1 + 2\times 0.2\times\frac{s}{10} + \frac{s^2}{100}}$$

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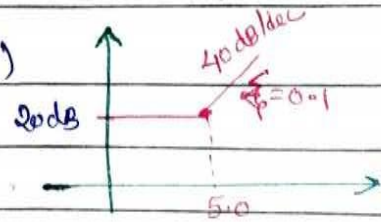
(ii)



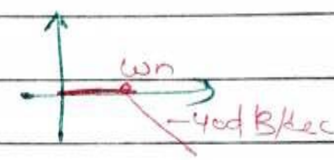

$$G(S) = 10\left[1 + 2\times 0.1 \times \left(\frac{S}{5}\right) + \left(\frac{S}{5}\right)^2\right]$$

- Slope must be multiples of 40 dB/dec in $2^{nd}$ order system. {except Overdamped}

My point → 
- initial slope = 0dB/dec for Underdamped, always.
- $\omega_n = \omega_c$ = corner frequency.

(ii) ($\xi = 1$) C.D.

$$T(s) = \frac{1}{1 + \frac{2s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2} = \frac{1}{\left(1 + \frac{s}{\omega_n}\right)^2}$$




(iii) ($\xi > 1$) O.D.

$$T(s) = \frac{\omega_n^2}{\left(s + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1}\right)\left(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)}$$

$$T(s) = \frac{1}{\left(1 + \frac{s}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}\right)\left(1 + \frac{s}{\xi\omega_n + \omega_n\sqrt{\xi^2-1}}\right)}$$

* In Bode plot when repeated poles ⇒ C.D. $\xi = 1$
" " " " unequal poles ⇒ O.D. $\xi > 1$
we only define $\xi$ for U.D. ⇒ $\xi < 1$

** Bode plot is not defined for undamped ($\xi = 0$) since High frequency region must have constant slope & Undamped system has varying slope always.

Teacher's Signature ________

$$\text{Error (dB)} = \text{Exact}\Big|_{dB,\,u=1} - \text{Approximate}\Big|_{dB,\,u=1}$$

$$= -20\log 2\zeta_p$$

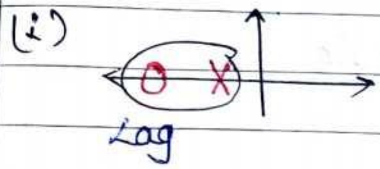
⊙ To remember, lag me pole origin ke pas hota h, so α is small i.e. $(\xi\omega_n)$ is small ⇒ $\xi\downarrow$ $\omega_n\downarrow$ ⇒ $\uparrow t_{sett} = 4/\xi\omega_n$ ⇒ $t_r\uparrow$

$\omega_{gc}, BW\downarrow$ $\quad \uparrow MPO = e^{-\pi\cot\theta}$ ⇒ stability↓ $e_{ss}\downarrow$

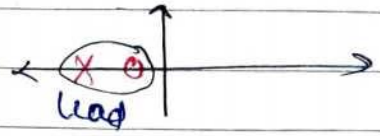


## Compensator

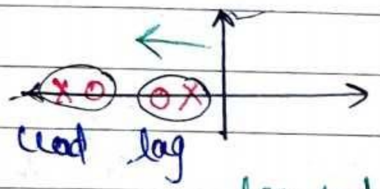
(i)




(ii)

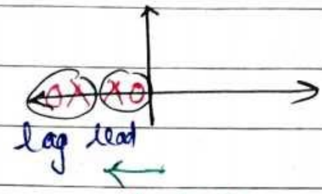



(iii)



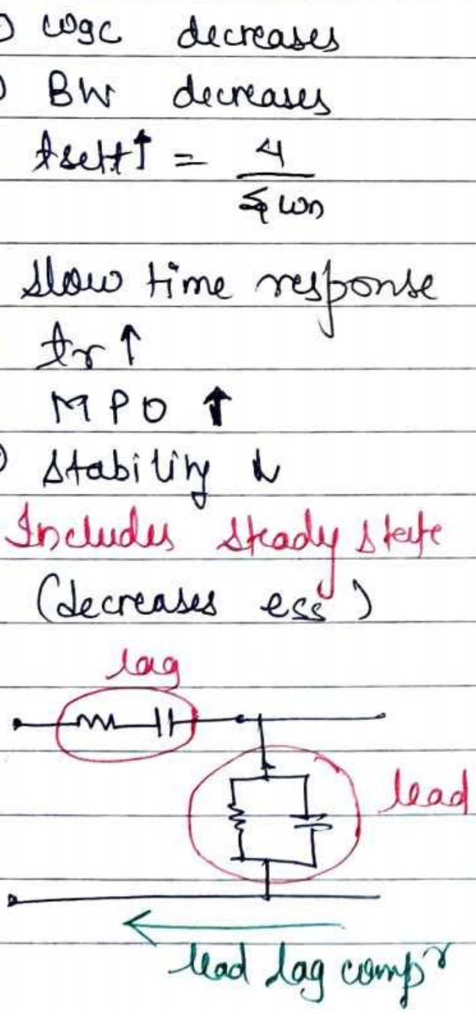

lag lead comp$^r$

(iv)

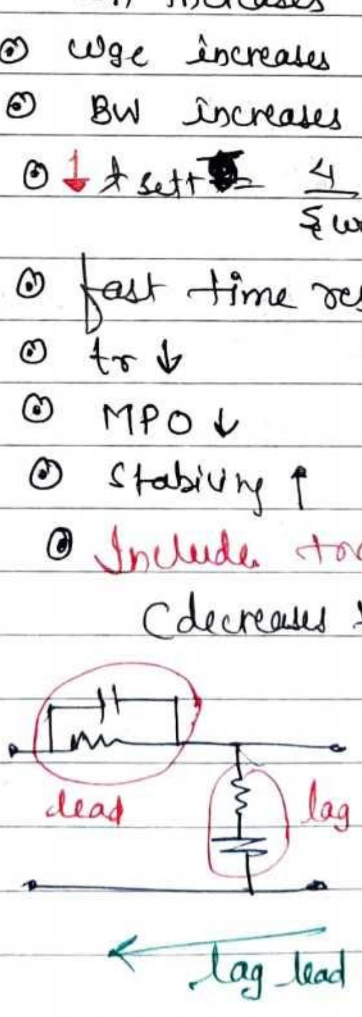



lead lag comp$^r$

| lag compensator | lead Compensator |
|---|---|
| ⊙ Damping ratio ↓ | ⊙ Damping ratio ↑ |
| ⊙ $\omega_n$ decreases | ⊙ $\omega_n$ increases |
| ⊙ $\omega_{gc}$ decreases | ⊙ $\omega_{gc}$ increases |
| ⊙ BW decreases | ⊙ BW increases |
| ⊙ $t_{sett}\uparrow = \frac{4}{\xi\omega_n}$ | ⊙ $\downarrow t_{sett} = \frac{4}{\xi\omega_n}$ |
| ⊙ slow time response | ⊙ fast time response |
| ⊙ $t_r\uparrow$ | ⊙ $t_r\downarrow$ |
| ⊙ MPO ↑ | ⊙ MPO ↓ |
| ⊙ stability ↓ | ⊙ stability ↑ |
| ⊙ Includes steady state performance (decreases $e_{ss}$) | ⊙ Include transient state performance (decreases time domain para) |




lead lag comp$^r$




lag lead comp$^r$