Atoms

basic concepts

1. Geiger-Marsden's α-particle Scattering Experiment

On the suggestion of Rutherford, in 1911, his two associates, H. Geiger and E. Marsden, performed an experiment by bombarding α-particles (Helium nuclei $Z = 2$, $A = 4$) on a gold foil.

Observations:

(*i*) Most of the α-particles pass through the gold foil undeflected.

(*ii*) A very small number of α-particles (1 in 8000) suffered large angle deflection; some of them retraced their path or suffered 180° deflection.

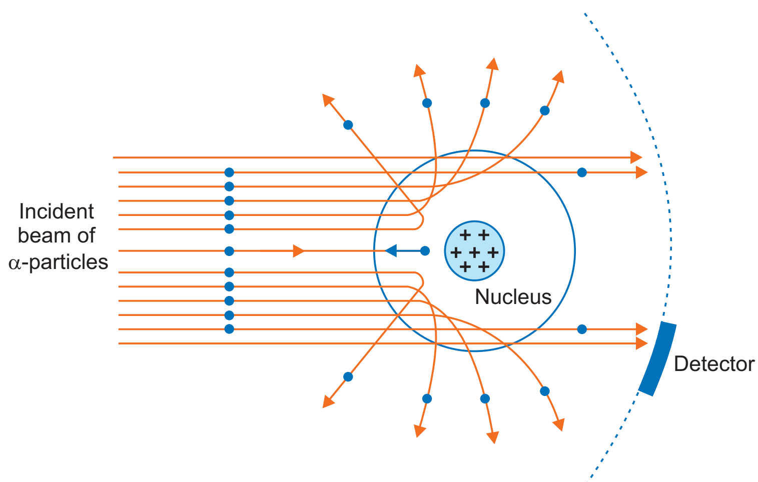
Conclusion:

(*i*) Atom is hollow.

(*ii*) Entire positive charge and nearly whole mass of atom is concentrated in a small centre called nucleus of atom.

(*iii*) Coulomb's law holds good for atomic distances.

(*iv*) Negatively charged electrons are outside the nucleus.



Rutherford Scattering experiment

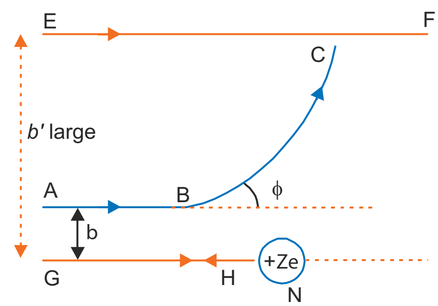
Impact Parameter: The perpendicular distance of initial velocity vector of α-particle from the nucleus, when the particle is far away from the nucleus, is called the impact parameter. It is denoted by b. For head on approach of α-particle, $b = 0$.

Angle of Scattering (ϕ) : The angle by which α-particle is deviated from its original direction is called angle of scattering.

$$b = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{E_K} \cot\frac{\phi}{2}$$

where E_k is the initial kinetic energy for head on approach of alpha particle.

Impact parameter, $b = 0$.



2. Distance of Closest Approach

The smallest distance of approach of α-particle near heavy nucleus is a measure of the size of nucleus.

Distance of nearest approach ≈ size of nucleus $= \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{E_K}$

where E_K is kinetic energy of incident α-particle, Z = atomic number, e = electronic charge.

3. Rutherford's Atom Model

Atom consists of a central heavy nucleus containing positive charge and negatively charged electrons circulating around the nucleus in circular orbits.

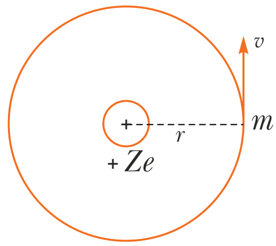
Rutherford model could explain the neutrality of an atom, thermionic emission and photoelectric effect; but it could not explain the stability of an atom and the observed line spectrum of an atom (atomic spectrum).

4. Bohr's Model

Bohr modified Rutherford atom model to explain the line spectrum of hydrogen.

Postulates of Bohr's Theory

(*i*) **Stationary Circular Orbits:** An atom consists of a central positively charged nucleus and negatively charged electrons revolve around the nucleus in certain orbits called **stationary orbits.**



The electrostatic coulomb force between electrons and the nucleus provides the necessary centripetal force.

i.e., $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \qquad \ldots(i)$$

where Z is the atomic number, m is the mass of electrons, r = radius of orbit.

(*ii*) **Quantum Condition:** The stationary orbits are those in which angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$, *i.e.*,

$$mvr = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, \ldots \qquad \ldots(ii)$$

Integer n is called the principal quantum number. This equation is called Bohr's quantum condition.

(*iii*) **Transitions:** The electron does not radiate energy when in a stationary orbit. The quantum of energy (or photon) is emitted or absorbed when an electron jumps from one stationary orbit to the other. The frequency of emitted or absorbed photon is given by

$$h\nu = |E_i - E_f| \qquad \ldots(iii)$$

This is called Bohr's frequency condition.

Radius of Orbit and Energy of Electron in Orbit

Condition of motion of electron in circular orbit is

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \qquad \ldots(i)$$

Bohr's quantum condition is

$$mvr = n\frac{h}{2\pi} \qquad \ldots(ii)$$

$$\Rightarrow \qquad v = \frac{nh}{2\pi mr}$$

Substituting this value of v in (*i*), we get

$$\frac{m}{r}\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

This gives $r = \dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$

Denoting radius of nth orbit by r_n, we have

$$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} \qquad \ldots(iii)$$

For hydrogen atom $Z = 1$,

$$\therefore \qquad (r_n)_H = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$$

The radius of first orbit of hydrogen atom is called Bohr's radius. It is denoted by a_0

$$\Rightarrow \qquad a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.529 \times 10^{-10}\,\text{m} = 0.529\,\text{Å}$$

Energy of Orbiting Electron

From equation (i), $\qquad mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r}$

Kinetic energy, $\qquad K = \dfrac{1}{2}mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{2r}$

Potential energy, $\qquad U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(-e)}{r} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r}$

Total energy $\qquad E = K + U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{2r} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r}$

$$\Rightarrow \qquad E = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

For nth orbit, writing E_n for E, we have

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n} \qquad \ldots(iv)$$

Substituting the value of r_n from (iii) in (iv), we get

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2\left(\dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right)} = -\frac{mZ^2 e^4}{8\varepsilon_0^2 h^2 n^2} \qquad \ldots(v)$$

For convenience introducing Rydberg constant, $R = \dfrac{me^4}{8\varepsilon_0^2 c h^3}$ $\qquad \ldots(vi)$

The value of Rydberg constant is $1.097 \times 10^7\ \text{m}^{-1}$.

We have

$$E_n = -\frac{Z^2 Rhc}{n^2} \qquad \ldots(vii)$$

For hydrogen atom $Z = 1$,

Energy of orbiting electron in H-atom

$$E_n = -\frac{Rhc}{n^2}$$

$$\Rightarrow \qquad E_n = -\frac{13.6}{n^2}\,\text{eV} \qquad \ldots(viii)$$

Equations (iii) and (vii) indicate that radii and energies of hydrogen like atoms ($i.e.$, atoms containing one electron only) are quantised.

5. Energy Levels of Hydrogen Atom

The energy of electron in hydrogen atom ($Z = 1$) is given (or series of hydrogen spectrum) by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2}\text{eV};$$

when $n = 1$, $E_1 = -13.6$ eV

when $n = 2$, $E_2 = -\frac{13.6}{4}\text{eV} = -3.4\text{ eV}$

when $n = 3$, $E_3 = -\frac{13.6}{9}\text{eV} = -1.51\text{ eV}$

when $n = 4$, $E_4 = -\frac{13.6}{16}\text{eV} = -0.85\text{ eV}$

when $n = 5$, $E_5 = -\frac{13.6}{25}\text{eV} = -0.54\text{ eV}$

when $n = 6$, $E_6 = -\frac{13.6}{36}\text{eV} = -0.38\text{ eV}$

when $n = 7$, $E_7 = -\frac{13.6}{49}\text{eV} = -0.28\text{ eV}$

....

....

when $n = \infty$, $E_\infty = -\frac{13.6}{(\infty)^2}\text{eV} = 0\text{ eV}$

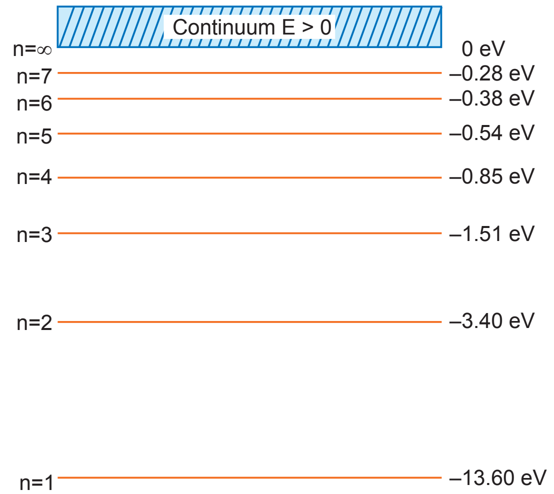


Fig. (a) Energy Level Diagram

If these energies are expressed by vertical lines on proper scale, the diagram obtained is called the energy level diagram. The energy level diagram of hydrogen atom is shown in fig. (*a*). Clearly the separation between lines goes on decreasing rapidly with increase of n (*i.e.*, order of orbit). The series of lines of H-spectrum are shown in fig. (*b*).

If the total energy of electron is above zero, the electron is free and can have any energy. Thus there is a continuum of energy states above $E = 0$ eV.

6. Hydrogen Spectrum

Hydrogen emission spectrum consists of 5 series.

(*i*) **Lyman series:** This lies in ultraviolet region.

(*ii*) **Balmer series:** This lies in the visible region.

(*iii*) **Paschen series:** This lies in near infrared region.

(*iv*) **Brackett series:** This lies in mid infrared region.

(*v*) **Pfund series:** This lies in far infrared region.

Hydrogen absorption spectrum consists of only Lyman series.

Explanation of Hydrogen Spectrum: n_i and n_f are the quantum numbers of initial and final states and E_i and E_f are energies of electron in H-atom ($Z = 1$) in initial and final states then we have

$$E_i = -\frac{Rhc}{n_i^2} \text{ and } E_f = -\frac{Rhc}{n_f^2}$$

Energy of absorbed photon

$$\Delta E = E_f - E_i = Rhc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

If ν is the frequency of emitted radiation, we have from Bohr's fourth postulate

$$\nu = \frac{E_i - E_f}{h} = -\frac{Rc}{n_i^2} - \left(-\frac{Rc}{n_f^2}\right) = Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \ldots(ix)$$

The wave number (*i.e.*, reciprocal of wavelength) of the emitted radiation is given by

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The relation explains successfully the origin of various lines in the spectrum of hydrogen atom. The series of lines are obtained due to the transition of electron from various other orbits to a fixed inner orbit.

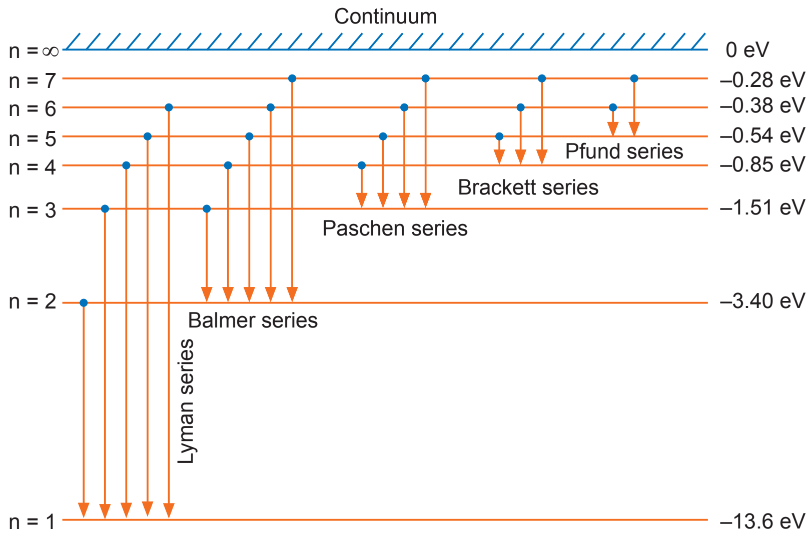


Fig. (b) Series of H-spectrum

(*i*) **Lyman series:** This series is produced when electron jumps from higher orbits to the first stationary orbit (*i.e.*, $n_f = 1$). Thus for this series

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right) \qquad \text{where } n_i = 2, 3, 4, 5, ...$$

For longest wavelength of Lyman series $n_i = 2$

$$\therefore \qquad \frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\therefore \qquad \lambda_{max} = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} \text{m}$$

$$= 1.215 \times 10^{-7} \text{ m} = 1215 \text{ Å}$$

For shortest wavelength of Lyman series $n_i = \infty$

$$\therefore \qquad \frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R$$

$$\lambda_{min} = \frac{1}{R} = \frac{1}{1.097 \times 10^7} \text{m} = 0.9116 \times 10^{-7} \text{m} = 911.6 \text{ Å}$$

This is called series limit of Lyman series $\lambda_{limit} = 911.6$ Å

Obviously the lines of Lyman series are found in ultraviolet region.

(*ii*) **Balmer series:** The series is produced when an electron jumps from higher orbits to the second stationary orbit ($n_f = 2$). Thus for this series,

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right) \qquad \text{where } n_i = 3, 4, 5, 6, ...$$

For Longest wavelength of Balmer series ($n_i = 3$)

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\lambda_{max} = \frac{36}{5R} = \frac{36}{5\times 1.097\times 10^7}\text{m} = 6.563\times 10^{-7}\text{m} = 6563\text{ Å}$$

For Shortest wavelength (or series limit) of Balmer series $n_i \to \infty$

$$\therefore \qquad \frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4}$$

$$\lambda_{min} = \frac{4}{R} = \frac{4}{1.097\times 10^{-7}}\text{m} = 3.646\times 10^{-7}\text{ m} = 3646\text{ Å}$$

Obviously the lines of Balmer series are found in the visible region and first, second, third … lines are called H_α, H_β, H_γ..., lines respectively.

(iii) **Paschen series:** This series is produced when an electron jumps from higher orbits to the third stationary orbit ($n_f = 3$).

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right) \qquad \text{where } n_i = 4, 5, 6, 7,...$$

For Longest wavelength of Paschen series ($n_i = 4$)

$$\therefore \qquad \frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$$

$$\therefore \qquad \lambda_{max} = \frac{144}{7R} = \frac{144}{7\times 1.097\times 10^7}\text{m} = 18.752\times 10^{-7}\text{ m} = 18752\text{ Å}$$

For Series limit of Paschen series ($n_i = \infty$)

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right) = \frac{R}{9}$$

$$\lambda_{min} = \frac{9}{R} = \frac{9}{1.097\times 10^7} = 8.204\times 10^{-7}\text{ m} = 8204\text{ Å}$$

Obviously lines of Paschen series are found in infrared region.

(iv) **Brackett series:** This series is produced when an electron jumps from higher orbits to the fourth stationary orbit ($n_f = 4$)

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right) \qquad \text{where } n_i = 5, 6, 7, 8, ...$$

(v) **Pfund series:** This series is produced when an electron jumps from higher orbits to the fifth stationary orbit ($n_f = 5$)

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right) \qquad \text{where } n_i = 6, 7, 8, ...$$

The last three series are found in infrared region.

The series spectrum of hydrogen atom is represented in figure.

Selected NCERT Textbook Questions

Q. 1. Suppose you are given a chance to repeat the alpha particle scattering experiment using a thin sheet of solid hydrogen in place of gold foil (hydrogen is a solid at temperature below 14 K). What results do you expect?

Ans. Size of hydrogen nucleus = 1.2×10^{-15} m.

$\therefore$ Electrostatic potential energy of α-particle at nuclear surface

$$U_e = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(e)}{r} = 9\times 10^9 \times \frac{2\times(1.6\times 10^{-19})^2}{1.2\times 10^{-15}}\text{ J}$$

$$= \frac{9\times 10^9\times 2\times 1.6\times 10^{-19}}{1.2\times 10^{-15}}\text{ eV}$$

$$= 2.4\times 10^6\text{ eV} = \mathbf{2.4\ MeV}$$

This is much less than incident energy 5.5 MeV of α-particle; therefore α-particle will penetrate the nucleus and no scattering will be observed.

Aliter: The de Broglie wavelength of α-particle is much less than inter-proton distance in solid hydrogen, so α-particle will move directly penetrating the nucleus.

Q. 2. A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes transition from the upper level to the lower level?

Ans. According to Bohr's postulate

$$E_1 - E_2 = h\nu$$

∴ Frequency of emitted radiation

$$\nu = \frac{E_1 - E_2}{h} = \frac{2.3\,\text{eV}}{h}$$

$$= \frac{2.3 \times 1.6 \times 10^{-19}\,\text{J}}{6.63 \times 10^{-34}\,\text{J - s}} = \mathbf{5.55 \times 10^{14}\ Hz}$$

Q. 3. The ground state energy of hydrogen atom is –13.6 eV. What is the kinetic and potential energies of the electron in the ground and second excited state?

[*CBSE (AI) 2010, 2011, Bhubaneshwar 2015*]

Ans. Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2r}$ [for H-atom, $Z = 1$] ...(*i*)

Potential energy, $U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ ...(*ii*)

Total energy $E = K + U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}$ ...(*iii*)

Comparing equations (*i*), (*ii*), (*iii*), we have

$K = -E$ and $U = 2E$

Given $E = -13.6$ eV (For ground state $n = 1$)

∴ Kinetic energy, $K =$ **13.6 eV**

Potential energy $U = 2\times(-13.6\text{ eV}) =$ **–27.2 eV**

For second excited state, $n=3$

$$\therefore \quad K = -E = \frac{+13.6}{9}\,\text{eV} = \mathbf{1.51\,eV}$$

$$\text{and} \quad U = 2E = \frac{2\times(-13.6\,\text{eV})}{9} = \mathbf{-3.02\ eV}$$

Q. 4. A hydrogen atom initially in the ground state absorbs a photon, which excites it to the $n=4$ level. Determine the wavelength and frequency of photon.

Ans. The energy levels of H-atom are given by

$$E_n = -\frac{Rhc}{n^2}$$

For given transition $n_1=1$, $n_2=4$

$$\therefore \quad E_1 = -\frac{Rhc}{1^2}, \quad E_2 = -\frac{Rhc}{4^2}$$

∴ Energy of absorbed photon

$$\Delta E = E_2 - E_1 = Rhc\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

or $$\Delta E = \frac{15}{16}Rhc \quad \text{...(}i\text{)}$$

$\therefore$ Wavelength of absorbed photon λ is given by

$$\Delta E = \frac{hc}{\lambda}$$

$$\therefore \quad \frac{hc}{\lambda} = \frac{15}{16}Rhc \quad \Rightarrow \quad \lambda = \frac{16}{15R}$$

or $$\lambda = \frac{16}{15 \times 1.097 \times 10^7} \text{m} = \mathbf{9.72 \times 10^{-8}\ m}$$

Frequency, $$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{9.72\times10^{-8}} = \mathbf{3.09 \times 10^{15}\ Hz}$$

Q. 5. (*a*) Using the Bohr's model, calculate the speed of electron in the hydrogen atom in n=1, 2 and 3 levels.

(*b*) Calculate the orbital period in each of these levels.

Ans. (*a*) The speed of electron in stable orbit of H-atom is

$$v = \frac{e^2}{2\varepsilon_0 h}.\frac{1}{n} = \frac{(1.6\times10^{-19})^2}{2\times8.85\times10^{-12}\times6.63\times10^{-34}}\left(\frac{1}{n}\right)$$

$$= \frac{2.18\times10^6}{n}\text{m/s}$$

For n=1, $v_1 = \mathbf{2.18 \times 10^6\ m/s.}$

For n=2, $v_2 = \frac{2.18\times10^6}{2} = \mathbf{1.09\times10^6\ m/s}$

For n=3, $v_3 = \frac{2.18\times10^6}{3} = \mathbf{7.27\times10^5\ m/s}$

Obviously the speed of electron goes on decreasing with increasing n.

(*b*) Time period, $$T = \frac{2\pi r}{v} = \frac{2\pi(\varepsilon_0 h^2 n^2/\pi m e^2)}{(e^2/2\varepsilon_0 h n)}$$

$$= \frac{4\varepsilon_0^2 h^3 n^3}{me^4} = \frac{4\times(8.85\times10^{-12})^2\times(6.63\times10^{-34})^3\times n^3}{9.1\times10^{-31}\times(1.6\times10^{-19})^4}$$

$$= 1.53\times10^{-16}\, n^3 \text{ seconds}$$

For n=1, $T_1 = \mathbf{1.53\times10^{-16}\ s}$

For n=2, $T_2 = 1.53\times10^{-16}\times(2)^3 = \mathbf{12.24\times10^{-16}\ s}$

For n=3, $T_3 = 1.53\times10^{-16}\times(3)^3 = \mathbf{41.31\times10^{-16}\ s}$

Q. 6. The radius of innermost orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of n=2 and n=3 orbits?

Ans. The radii of Bohr's orbits are given by

$$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2} \quad \Rightarrow \quad r_n \propto n^2$$

For ground state n = 1, $r_1 = 5.3 \times 10^{-11}$ m (given)

$$\frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2$$

$$\Rightarrow \quad r_2 = \left(\frac{2}{1}\right)^2 r_1 = 4r_1 = 4\times5.3\times10^{-11} = \mathbf{2.12\times10^{-10}\ m}$$

For n =3, $r_3 = (3)^2\, r_1 = 9 \times 5.3 \times 10^{-11}$

$$= \mathbf{4.77 \times 10^{-10}\ m}$$

Q. 7. In accordance with Bohr's model, find the quantum number, that characterises the earth's revolution around the sun in an orbit of radius 1.5 × 10^{11} m with orbital speed 3 × 10^4 m/s. (Mass of earth = 6.0 × 10^{24} kg)

Ans. According to Bohr's model, angular momentum

$$mvr = n\frac{h}{2\pi} \qquad \Rightarrow \qquad n = \frac{2\pi mvr}{h}$$

Given $m = 6.0 \times 10^{24}$ kg, $v = 3 \times 10^4$ m/s, $r = 1.5 \times 10^{11}$ m

$$\therefore \qquad n = \frac{2\times 3.14\times 6\times 10^{24}\times 3\times 10^4\times 1.5\times 10^{11}}{6.6\times 10^{-34}} = \mathbf{2.57\times 10^{74}}$$

Q. 8. Obtain the first Bohr's radius and the ground state energy of a 'muonic' hydrogen atom [*i.e.*, an atom in which a negatively charged muon (μ^-) of mass about 207 m_e orbits around a proton].

Ans. If m_μ is the mass of muon, then from Bohr's theory

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m_\mu v^2}{r} \quad \text{and} \quad m_\mu vr = \frac{nh}{2\pi} \qquad \text{[for H-atom, } Z = 1]$$

Eliminating v from these equations, we get

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m_\mu e^2}$$

As $m_\mu = 207 m_e$, where m_e is mass of electron

$$\therefore \qquad r = \frac{\varepsilon_0 h^2 n^2}{207\pi m_e e^2}$$

For ground state for muon, we have

$$r_\mu = \frac{\varepsilon_0 h^2}{207\pi m_e . e^2}$$

But $$\frac{\varepsilon_0 h^2}{\pi m_e e^2} = \text{ground state radius of H-atom} = 0.53\times 10^{-10} \text{ m}$$

$$\therefore \qquad r_\mu = \frac{0.53\times 10^{-10}}{207} = \mathbf{2.56\times 10^{-13}\,m}$$

Also energy $$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\cdot\frac{1}{n^2}$$

Obviously, $E_n \propto m$ $$\frac{E_\mu}{E_e} = \frac{m_\mu}{m_e} \Rightarrow E_\mu = \frac{m_\mu}{m_e}\times E_e$$

Ground state energy of an electron in H-atom, $E_e = -13.6$ eV

$$\therefore \qquad E_\mu = \frac{207 m_e}{m_e}\times(-13.6 \text{ eV}) = -2.8\times 10^3 \text{ eV} = -\mathbf{2.8\ keV}$$

Multiple Choice Questions [1 mark]

Choose and write the correct option(s) in the following questions.

1. The size of the atom is proportional to

(*a*) A (*b*) $A^{1/3}$ (*c*) $A^{2/3}$ (*d*) $A^{-1/3}$

2. To explain his theory, Bohr used

(*a*) conservation of linear momentum
(*b*) quantisation of angular momentum
(*c*) conservation of quantum
(*d*) none of these

3. **Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about** **[*NCERT Exemplar*]**
(*a*) 53 pm (*b*) 27 pm (*c*) 18 pm (*d*) 13 pm

4. **The ratio of energies of the hydrogen atom in its first to second excited state is**
(*a*) 1 : 4 (*b*) 4 : 1 (*c*) – 4 : – 9 (*d*) $-\frac{1}{4} : -\frac{1}{9}$

5. **The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is $B = -\frac{me^4}{8n^2 \varepsilon_0^2 h^2}$ (m = electron mass).**
If one decides to work in a frame of reference where the electron is at rest, the proton would be moving arround it. By similar arguments, the binding energy would be
$$B = -\frac{me^4}{8n^2 \varepsilon_0^2 h^2} \quad (\text{M = proton mass})$$ **[*NCERT Exemplar*]**
This last expression is not correct because
(*a*) n would not be integral
(*b*) Bohr-quantisation applies only to electron
(*c*) the frame in which the electron is at rest is not inertial
(*d*) the motion of the proton would not be in circular orbits, even approximately

6. **The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because** **[*NCERT Exemplar*]**
(*a*) of the electrons not being subject to a central force
(*b*) of the electrons colliding with each other
(*c*) of screening effects
(*d*) the force between the nucleus and an electron will no longer be given by Coulomb's law

7. **The ratio of the speed of the electrons in the ground state of hydrogen to the speed of light in vacuum is**
(*a*) 1/2 (*b*) 2/237 (*c*) 1/137 (*d*) 1/237

8. **For the ground state, the electron in the H-atom has an angular momentum = h, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,** **[*NCERT Exemplar*]**
(*a*) because Bohr model gives incorrect values of angular momentum.
(*b*) because only one of these would have a minimum energy.
(*c*) angular momentum must be in the direction of spin of electron.
(*d*) because electrons go around only in horizontal orbits.

9. **O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms** **[*NCERT Exemplar*]**
(*a*) is not important because nuclear forces are short-ranged.
(*b*) is as important as electrostatic force for binding the two atoms.
(*c*) cancels the repulsive electrostatic force between the nuclei.
(*d*) is not important because oxygen nucleus have equal number of neutrons and protons.

10. **In the following transitions of the hydrogen atom, the one which gives an absorption line of highest frequency is**
(*a*) $n = 1$ to $n = 2$ (*b*) $n = 3$ to $n = 8$ (*c*) $n = 2$ to $n = 1$ (*d*) $n = 8$ to $n = 3$

11. **The wavelength of the first line of Lyman series in hydrogen is 1216 Å. The wavelength of the second line of the same series will be**
(*a*) 912 Å (*b*) 1026 Å (*c*) 3648 Å (*d*) 6566 Å

12. **Two H atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is** *[NCERT Exemplar]*

(*a*) 10.20 eV (*b*) 20.40 eV (*c*) 13.6 eV (*d*) 27.2 eV

13. **When an electron in an atom goes from a lower to a higher orbit, its**

(*a*) kinetic energy (*KE*) increases, potential energy (*PE*) decreases

(*b*) *KE* increases, *PE* increases

(*c*) *KE* decreases, *PE* increases

(*d*) *KE* decreases, *PE* decreases

14. **According to Bohr's theory, the energy of radiation in the transition from the third excited state to the first excited state for a hydrogen atom is**

(*a*) 0.85 eV (*b*) 13.6 eV (*c*) 2.55 eV (*d*) 3.4 eV

15. **Given the value of Rydberg constant is $10^7\ m^{-1}$, the wave number of the last line of the Balmer series in hydrogen spectrum will be**

(*a*) $0.25 \times 10^7\ m^{-1}$ (*b*) $2.5 \times 10^7\ m^{-1}$ (*c*) $0.025 \times 10^4\ m^{-1}$ (*d*) $0.5 \times 10^7\ m^{-1}$

16. **If an electron in a hydrogen atom jumps from the 3rd orbit to the 2nd orbit, it emits a photon of wavelength λ. When it jumps from the 4th orbit to the 3rd orbit, the corresponding wavelength of the photon will be**

(*a*) $\frac{16}{25}\lambda$ (*b*) $\frac{9}{16}\lambda$ (*c*) $\frac{20}{7}\lambda$ (*d*) $\frac{20}{13}\lambda$

17. **Hydrogen H, deuterium D, singly-ionised helium He^+ and doubly-ionised lithium Li^{++} all have one electron around the nucleus. Consider $n = 2$ to $n = 1$ transition. The wavelengths of the emitted radiations are $\lambda_1, \lambda_2, \lambda_3$, and λ_4 respectively. Then approximately**

(*a*) $\lambda_1 = 2\lambda_2 = 2\sqrt{2}\lambda_3 = 3\sqrt{2}\lambda_4$ (*b*) $\lambda_1 = \lambda_2 = 2\lambda_3 = 3\lambda_4$

(*c*) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (*d*) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

18. **The Bohr model for the spectra of a H-atom** *[NCERT Exemplar]*

(*a*) will not be applicable to hydrogen in the molecular from.

(*b*) will not be applicable as it is for a He-atom.

(*c*) is valid only at room temperature.

(*d*) predicts continuous as well as discrete spectral lines.

19. **Let $E_n = \frac{1}{8\varepsilon_0^2}\frac{me^4}{n^2h^2}$ be the energy of the *n*th level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $(E_2 - E_1)/h$ falls on it,** *[NCERT Exemplar]*

(*a*) it will not be absorbed at all.

(*b*) some of atoms will move to the first excited state.

(*c*) all atoms will be excited to the $n = 2$ state.

(*d*) no atoms will make a transition to the $n = 3$ state.

20. **A set of atoms in an excited state decays.**

(*a*) in general to any of the states with lower energy.

(*b*) into a lower state only when excited by an external electric field.

(*c*) all together simultaneously into a lower state.

(*d*) to emit photons only when they collide.

Answers

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*a*)	**7.** (*c*)
8. (*a*)	**9.** (*a*)	**10.** (*a*)	**11.** (*b*)	**12.** (*a*)	**13.** (*c*)	**14.** (*c*)
15. (*a*)	**16.** (*c*)	**17.** (*c*)	**18.** (*a*), (*b*)	**19.** (*b*), (*d*)	**20.** (*a*)	

Fill in the Blanks [1 mark]

1. The angle of scattering θ for zero value of impact parameter b is ________________.
2. The frequency spectrum of radiation emitted as per Rutherford's model of atom is ____________.
3. The force responsible for scattering of alpha particle with target nucleus is ______________.
4. According to de Broglie a stationary orbit is that which contains an ______________ number of de Broglie waves associated with the revolting electron.
5. ______________ is a physical quantity whose dimensions are the same as that of Plank's constant.
6. ______________ series of hydrogen spectrum lies in the visible region electromagnetic spectrum.
7. ______________ is the ionisation potential of hydrogen atom.
8. Total energy of electron in a stationary orbit is ________________, which means the electron is bound to the nucleus and is not free to leave it.
9. The value of Rydberg constant is ________________.
10. When an electron jumps from 2nd stationary orbit of hydrogen atom to 1st stationary orbit, the energy emitted is ________________.

Answers

1. 180° 2. continuous 3. electrostatic force 4. integral
5. Angular momentum 6. Balmer 7. 13.6 eV
8. negative 9. 1.09×10^7 m^{-1} 10. 10.2 eV

Very Short Answer Questions [1 mark]

Q. 1. Write the expression for Bohr's radius in hydrogen atom. [*CBSE Delhi 2010*]

Ans. Bohr's radius, $r_1 = \dfrac{\varepsilon_0 h^2}{\pi m e^2} = \mathbf{0.529 \times 10^{-10}\,m}$

Q. 2. In the Rutherford scattering experiment the distance of closest approach for an α-particle is d_0. If α-particle is replaced by a proton, how much kinetic energy in comparison to α-particle will it require to have the same distance of closest approach d_0? [*CBSE (F) 2009*]

Ans. $E_k = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(2e)}{d_0}$ (for α-particle, $q = 2e$)

$E_k' = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(e)}{d_0}$ (for proton, $q = e$)

$\dfrac{E_k'}{E_k} = \dfrac{1}{2} \quad \Rightarrow \quad \boldsymbol{E_k' = \dfrac{E_k}{2}}$

That is KE of proton must be half on comparison with KE of α-particle.

Q. 3. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atom? [*CBSE Delhi 2010*]

Ans. $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m e^2} \propto n^2$

For 1st excited state, $n = 2$

For ground state, $n = 1$

$\therefore \quad \boldsymbol{\dfrac{r_2}{r_1} = \dfrac{4}{1}}$

Q. 4. Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its:

(*i*) second permitted energy level to the first level, and

(*ii*) the highest permitted energy level to the first permitted level. *[CBSE (AI) 2010]*

Ans. $E_I = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}Rhc$

$$E_{II} = Rhc\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = Rhc$$

Ratio $\frac{E_I}{E_{II}} = \mathbf{\frac{3}{4}}$

Q. 5. State Bohr's quantisation condition for defining stationary orbits. *[CBSE (F) 2010]*

Ans. **Quantum Condition:** The stationary orbits are those in which angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$ *i.e.*,

$$mvr = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, \ldots$$

Integer n is called the **principal quantum number**. This equation is called Bohr's quantum condition.

Q. 6. The radius of innermost electron orbit of a hydrogen atom is 5.1 × 10^{-11} m. What is the radius of orbit in the second excited state? *[CBSE Delhi 2010]*

Ans. In ground state, $n = 1$

In second excited state, $n = 3$

As $r_n \propto n^2$

$$\therefore \quad \frac{r_3}{r_1} = \left(\frac{3}{1}\right)^2 = 9$$

$$r_3 = 9r_1 = 9 \times 5.1 \times 10^{-11}\text{ m} = \mathbf{4.59 \times 10^{-10}\text{ m}}$$

Q. 7. The mass of H-atom is less than the sum of the masses of a proton and electron. Why is this so? *[NCERT Exemplar] [HOTS]*

Ans. Einstein's mass-energy equivalence gives $E = mc^2$. Thus the mass of an H-atom is $m_p + m_e - \frac{B}{C^2}$ where $B \approx 13.6$ eV is the binding energy. It is less than the sum of masses of a proton and an electron.

Q. 8. When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of electromagnetic radiation. Why cannot it be emitted as other forms of energy? *[NCERT Exemplar] [HOTS]*

Ans. This is because electrons interact only electromagnetically.

Q. 9. Would the Bohr formula for the H-atom remain unchanged if proton had a charge (+4/3)*e* and electron had a charge (–3/4)*e*, where *e* = 1.6 × 10^{-19} C? Give reasons for your answer. *[NCERT Exemplar] [HOTS]*

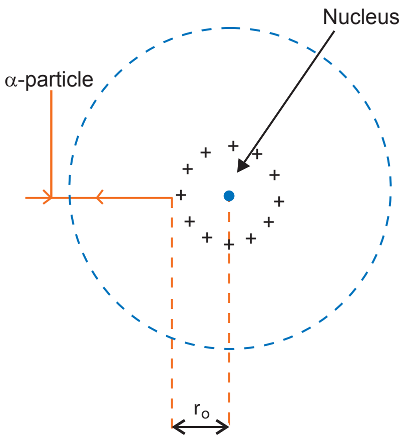
Ans. Yes, since the Bohr formula involves only the product of the charges.

Q. 10. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model? *[NCERT Exemplar] [HOTS]*

Ans. No, because according to Bohr model, $E_n = -\frac{13.6}{n^2}$, and electrons having different energies belong to different levels having different values of n. So, their angular momenta will be different, as $mvr = \frac{nh}{2\pi}$.

Short Answer Questions–I [2 marks]

Q. 1. Define the distance of closest approach. An α-particle of kinetic energy 'K' is bombarded on a thin gold foil. The distance of the closest approach is 'r'. What will be the distance of closest approach for an α-particle of double the kinetic energy? [*CBSE Delhi 2017*]



Ans. Distance of closest approach is the distance of charged particle from the centre of the nucleus, at which the entire initial kinetic energy of the charged particles gets converted into the electric potential energy of the system.

Distance of closest approach (r_o) is given by

$$r_o = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K}$$

If 'K' is doubled, r_o becomes $\frac{r_o}{2}$.

Q. 2. Write two important limitations of Rutherford nuclear model of the atom. [*CBSE Delhi 2017*]

Ans. Two important limitations of Rutherford Model are:

(*i*) According to Rutherford model, electron orbiting around the nucleus, continuously radiates energy due to the acceleration; hence the atom will not remain stable.

(*ii*) As electron spirals inwards; its angular velocity and frequency change continuously, therefore it should emit a continuous spectrum.

But an atom like hydrogen always emits a discrete line spectrum.

Q. 3. Define ionization energy. How would the ionization energy change when electron in hydrogen atom is replaced by a particle of mass 200 times than that of the electron but having the same charge? [*CBSE Central 2016*]

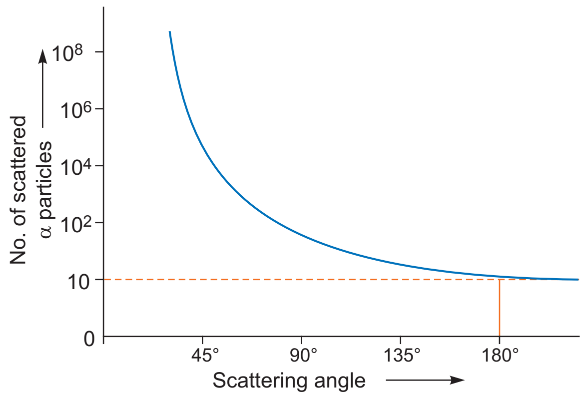
Ans. The minimum energy required to free the electron from the ground state of the hydrogen atom is known as ionization energy.

$$E_0 = \frac{me^4}{8\varepsilon^2 h^2},\quad i.e., E_0 \propto m$$

Therefore, ionization energy will become 200 times.

Q. 4. In an experiment on α-particle scattering by a thin foil of gold, draw a plot showing the number of particles scattered versus the scattering angle θ.

Why is it that a very small fraction of the particles are scattered at θ > 90°? [*CBSE (F) 2013*]



Ans. A small fraction of the alpha particles scattered at angle θ > 90° is due to the reason that if impact parameter 'b' reduces to zero, coulomb force increases, hence alpha particles are scattered at angle θ > 90°, and only one alpha particle is scattered at angle 180°.

Q. 5. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. [*CBSE Delhi 2017*]

Ans. Radius of ground state of hydrogen atom, $r = 0.53\ \text{Å} = 0.53 \times 10^{-10}\ \text{m}$

According to de Broglie relation, $2\pi r = n\lambda$

For ground state, $n = 1$

$$2 \times 3.14 \times 0.53 \times 10^{-10} = 1 \times \lambda$$

$$\therefore \quad \lambda = 3.32 \times 10^{-10}\ \text{m}$$

$$= \mathbf{3.32\ \text{Å}}$$

Q. 6. When is H_α line in the emission spectrum of hydrogen atom obtained? Calculate the frequency of the photon emitted during this transition. [*CBSE North 2016*]

Ans. The line with the longest wavelength of the Balmer series is called H_α.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where λ = wavelength

$R = 1.097 \times 10^7\ \text{m}^{-1}$ (Rydberg constant)

When the electron jumps from the orbit with $n = 3$ to $n = 2$,

we have

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \frac{1}{\lambda} = \frac{5}{36}R$$

The frequency of photon emitted is given by

$$\nu = \frac{c}{\lambda} = c \times \frac{5}{36}R$$

$$= 3 \times 10^8 \times \frac{5}{36} \times 1.097 \times 10^7\ \text{Hz}$$

$$= \mathbf{4.57 \times 10^{14}\ Hz}$$

Q. 7. Calculate the de-Broglie wavelength of the electron orbiting in the $n = 2$ state of hydrogen atom. [*CBSE Central 2016*]

OR

The kinetic energy of the electron orbiting in the first excited state of hydrogen atom is 3.4 eV. Determine the de Broglie wavelength associated with it. [*CBSE (F) 2015*]

Ans. Kinetic Energy for the second state

$$E_k = \frac{13.6\ \text{eV}}{n^2} = \frac{13.6\ \text{eV}}{2^2} = \frac{13.6\ \text{eV}}{4} = 3.4 \times 1.6 \times 10^{-19}\ \text{J}$$

de Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mE_k}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} = \mathbf{0.67\ nm}$$

Q. 8. Calculate the orbital period of the electron in the first excited state of hydrogen atom. [*CBSE 2019 (55/1/1)*]

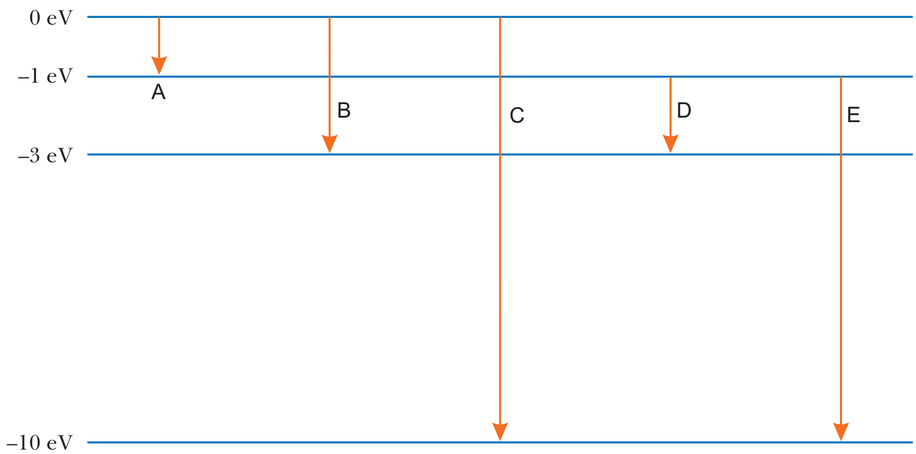
Ans. For ground state, $n = 1$

For first excited state, $n = 2$

Now, $T_n \propto n^3$

$$\frac{T_2}{T_1} = \frac{2^3}{1^3} \Rightarrow T_2 = 8T_1$$ = 8 times of orbital period of the electron in the ground state.

Q. 9. The energy levels of an atom are given below in the diagram.



Which of the transitions belong to Lyman and Balmer series? Calculate the ratio of the shortest wavelengths of the Lyman and the Balmer series of the spectra.

[*CBSE Chennai 2015, CBSE 2019 (55/2/3)*]

Ans. Transition *C* and *E* belong to Lyman series.

Reason: In Lyman series, the electron jumps to lowest energy level from any higher energy levels.

Transition *B* and *D* belong to Balmer series.

Reason: The electron jumps from any higher energy level to the level just above the ground energy level.

The wavelength associated with the transition is given by

$$\lambda = \frac{hc}{\Delta E}$$

Ratio of the shortest wavelength

$$\lambda_L : \lambda_B = \frac{hc}{\Delta E_L} : \frac{hc}{\Delta E_B}$$

$$= \frac{1}{0-(-10)} : \frac{1}{0-(-3)} = \mathbf{3:10}$$

Q. 10. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. [*CBSE Delhi 2015*]

Ans. Hydrogen atom

Let r be the radius of the orbit of a hydrogen atom. Forces acting on electron are centrifugal force (F_c) and electrostatic attraction (F_e)

At equilibrium, $F_c = F_e$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \qquad \text{[for H-atom, } Z = 1]$$

According to Bohr's postulate

$$mvr = \frac{nh}{2\pi} \quad \Rightarrow \quad v = \frac{nh}{2\pi mr}$$

$$m\left(\frac{nh}{2\pi mr}\right)^2 \cdot \frac{1}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \Rightarrow \quad \frac{mn^2h^2}{4\pi^2 m^2 r^2 . r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2} \quad \Rightarrow \quad \therefore \quad r \propto n^2$$

Q. 11. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, show how the de Broglie wavelength associated with it would be affected.

[*CBSE Ajmer 2015*]

Ans. We know,

de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv}$

$\Rightarrow \quad \lambda \propto \frac{1}{v},$

Also $\quad v \propto \frac{1}{n}$

$\therefore \quad \lambda \propto n$

$\therefore$ de Broglie wavelength will increase.

Q. 12. When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de Broglie wavelength associated with the electron change? Justify your answer.

[*CBSE Allahabad 2015*]

Ans. de Broglie wavelength associated with a moving charge particle having a *KE* '*K*' can be given as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \qquad \left[K = \frac{1}{2}mv^2 = \frac{p^2}{2m}\right] \qquad \ldots(i)$$

The kinetic energy of the electron in any orbit of hydrogen atom can be given as

$$K = -E = -\left(\frac{13.6}{n^2}\text{eV}\right) = -\frac{13.6}{n^2}\text{eV} \qquad \ldots(ii)$$

Let K_1 and K_4 be the *KE* of the electron in ground state and third excited state, where $n_1 = 1$ shows ground state and $n_2 = 4$ shows third excited state.

Using the concept of equation (*i*) & (*ii*), we have

$$\frac{\lambda_1}{\lambda_4} = \sqrt{\frac{K_4}{K_1}} = \sqrt{\frac{n_1^2}{n_2^2}}$$

$$\frac{\lambda_1}{\lambda_4} = \sqrt{\frac{1^2}{4^2}} = \frac{1}{4}$$

$$\Rightarrow \quad \lambda_1 = \frac{\lambda_4}{4}$$

i.e., the wavelength in the ground state will decrease.

Q. 13. A photon emitted during the de-excitation of electron from a state *n* to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2 eV, in a photo cell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state *n*.

[*CBSE 2019 (55/2/1)*]

Ans. From photoelectric equation,

$$h\nu = \phi_0 + eV_s$$

$$= 2 + 0.55 = 2.55 \text{ eV}$$

Given, $\quad E_n = -\frac{13.6}{n^2}$

The energy difference, $\Delta E = -3.4 - (-2.55) \text{ eV} = -0.85 \text{ eV}$

$$-\frac{13.6}{n^2} = -0.85$$

$\therefore \quad n = \mathbf{4}$

Q. 14. A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state. [*CBSE 2019 (55/2/1)*]

Ans. Energy in ground state, $E_1 = -13.6$ eV

Energy supplied = 12.5 eV

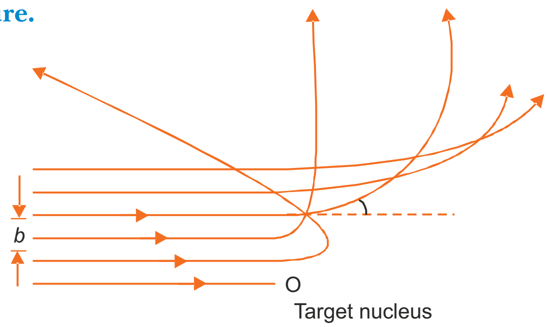
Energy in excited state, $-13.6 + 12.5 = -1.1$ eV

But, $E_n = -\dfrac{13.6}{n^2} = -1.1$

$n \simeq 3$

Maximum number of lines = 3.

Q. 15. The trajectories, traced by different α-particles, in Geiger-Marsden experiment were observed as shown in the figure.



(*a*) What names are given to the symbols '*b*' and 'θ' shown here?

(*b*) What can we say about the values of *b* for (*i*) θ = 0° (*ii*) θ = π radians? **[*HOTS*]**

Ans. (*a*) The symbol '*b*' represents **impact parameter** and 'θ' represents the **scattering angle**.

(*b*) (*i*) When θ = 0°, the **impact parameter** will be maximum and represent the **atomic size.**

(*ii*) When θ = π radians, the impact parameter '*b*' will be minimum and represent the nuclear size.

Q. 16. Which is easier to remove: orbital electron from an atom or a nucleon from a nucleus? [*HOTS*]

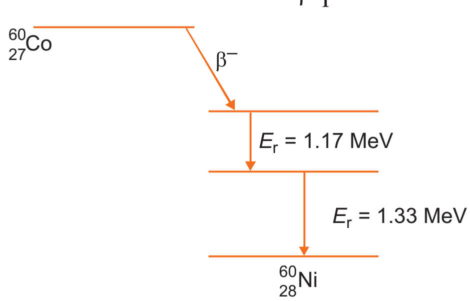
Ans. It is easier to remove an orbital electron from an atom. The reason is the binding energy of orbital electron is a few electron-volts while that of nucleon in a nucleus is quite large (nearly 8 MeV). This means that the removal of an orbital electron requires few electron volt energy while the removal of a nucleon from a nucleus requires nearly 8 MeV energy.

Q. 17. (*a*) Draw the energy level diagram showing the emission of β-particles followed by γ-rays by a $^{60}_{27}Co$ nucleus.

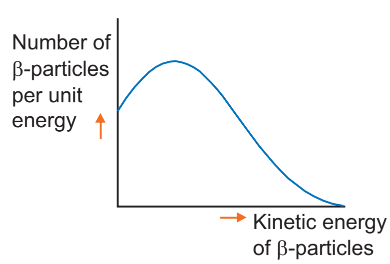
(*b*) Plot the distribution of kinetic energy of β-particles and state why the energy spectrum is continuous. **[*HOTS*]**

Ans. (*a*) The energy level diagram is shown in Fig. (*a*).

(*b*) Plot of distribution of *KE* of β-particles is shown in Fig. (*b*).



(*a*) Energy level diagram



(*b*) Energy distribution of β-particles

The energy spectrum of β-particles is continuous because an **antineutrino** is simultaneously emitted in β-decay; the total energy released in β-decay is shared by β-particle and the antineutrino so that momentum of system may remain conserved.

Short Answer Questions–II [3 marks]

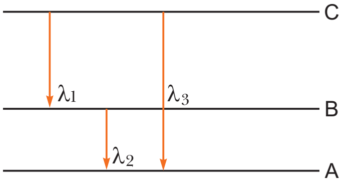
Q. 1. **(*a*) Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the n^{th} orbital state in hydrogen atom is *n* times the de Broglie wavelength associated with it.** [*CBSE (F) 2017*]

(*b*) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

OR

(*a*) State Bohr's quantization condition for defining stationary orbits. How does de Broglie hypothesis explain the stationary orbits?

(*b*) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown below. [*CBSE Delhi 2016*]



Ans. (*a*) Only those orbits are stable for which the angular momentum of revolving electron is an integral multiple of $\left(\frac{h}{2\pi}\right)$ where *h* is the planck's constant.

According to Bohr's second postulate

$$mvr_n = n\frac{h}{2\pi} \quad \Rightarrow \quad 2\pi r_n = \frac{nh}{mv}$$

But $\frac{h}{mv} = \frac{h}{p} = \lambda$ (By de Broglie hypothesis)

$\therefore \quad 2\pi r_n = n\lambda$

(*b*) For third excited state, $n = 4$

For ground state, $n = 1$

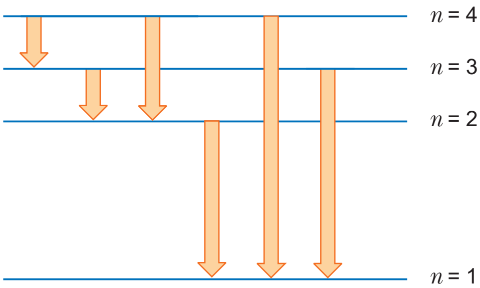
Hence possible transitions are

$n_i = 4$ to $n_f = 3, 2, 1$

$n_i = 3$ to $n_f = 2, 1$

$n_i = 2$ to $n_f = 1$

Total number of transitions = 6



$$E_C - E_B = \frac{hc}{\lambda_1} \quad ...(i)$$

$$E_B - E_A = \frac{hc}{\lambda_2} \quad ...(ii)$$

$$E_C - E_A = \frac{hc}{\lambda_3} \quad ...(iii)$$

Adding (*i*) and (*ii*), we have

$$E_C - E_A = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \quad ...(iv)$$

From (*iii*) and (*iv*), we have

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \quad \Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Q. 2. (*i*) **State Bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition.**

(*ii*) **An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond?** [*CBSE (F) 2016*]

Ans. (*i*) **Bohr's third postulate:** It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is given by

$$h\nu = E_i - E_f$$

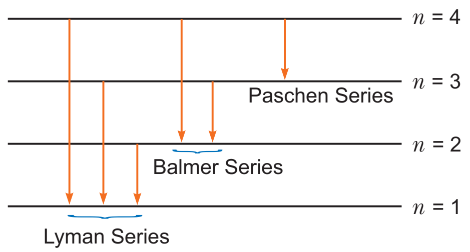
where E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

(*ii*) Electron jumps from fourth to first orbit in an atom

$\therefore$ Maximum number of spectral lines can be

$$^4C_2 = \frac{4!}{2!2!} = \frac{4\times 3}{2} = 6$$

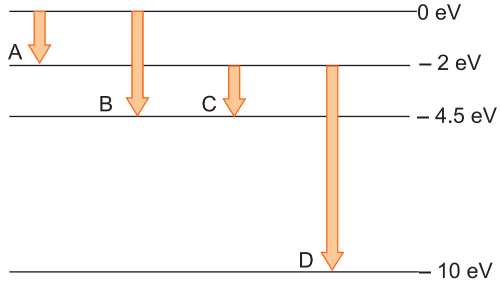
In diagram, possible way in which electron can jump (above).



The line responds to Lyman series (e^- jumps to 1st orbit), Balmer series (e^- jumps to 2nd orbit), Paschen series (e^- jumps to 3rd orbit).

Q. 3. The energy levels of a hypothetical atom are shown alongside. Which of the shown transitions will result in the emission of a photon of wavelength 275 nm?

Which of these transitions correspond to emission of radiation of (*i*) maximum and (*ii*) minimum wavelength? [*CBSE Delhi 2011*]



Ans. Energy of photon wavelength 275 nm

$$E = \frac{hc}{\lambda} = \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{275\times 10^{-9}\times 1.6\times 10^{-19}}\text{ eV} = 4.5\text{ eV}.$$

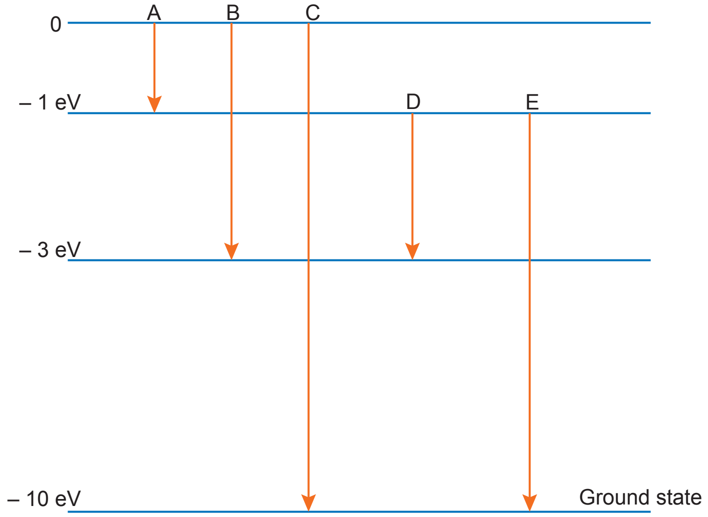
This corresponds to transition '*B*'.

(*i*) $\Delta E = \dfrac{hc}{\lambda} \Rightarrow \lambda = \dfrac{hc}{\Delta E}$

For maximum wavelength ΔE should be minimum. This corresponds to transition *A*.

(*ii*) For minimum wavelength ΔE should be maximum. This corresponds to transition *D*.

Q. 4. The energy levels of an atom of element X are shown in the diagram. Which one of the level transitions will result in the emission of photons of wavelength 620 nm? Support your answer with mathematical calculations. [*CBSE Sample Question Paper 2018*]



Ans.

$$E = \frac{hc}{\lambda}$$

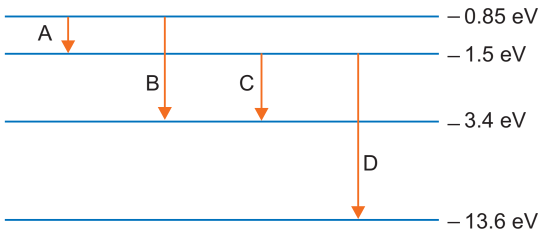
$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{620\times10^{-9}}$$

$$= 3.2 \times 10^{-19}\,\text{J}$$

$$= \frac{3.2\times10^{-19}}{1.6\times10^{-19}} = \mathbf{2\ eV}$$

This corresponds to the transition 'D'. Hence level transition D will result in emission of wavelength 620 nm.

Q. 5. The energy level diagram of an element is given below. Identify, by doing necessary calculations, which transition corresponds to the emission of a spectral line of wavelength 102.7 nm. [*CBSE Delhi 2008*]



Ans.

$$\Delta E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{102.7\times10^{-9}}\,\text{J}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{102.7\times10^{-9}\times1.6\times10^{-19}}\,\text{eV}$$

$$= \frac{66\times3000}{1027\times16} = 12.04\ \text{eV}$$

Now, $\Delta E = |-13.6-(-1.50)|$

$$= \mathbf{12.1\ eV}$$

Hence, transition shown by arrow D corresponds to emission of $\lambda = 102.7$ nm.

Q. 6. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de Broglie's hypothesis explain the stability of these orbits?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. [*CBSE 2018*]

Ans. (*a*) Bohr's postulate, for stable orbits, states

"The electron, in an atom, revolves around the nucleus only in those orbits for which its angular momentum is an integral multiple of $\frac{h}{2\pi}$ (h = Planck's constant)."

As per de Broglie's hypothesis

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

For a stable orbit, we must have circumference of the orbit= $n\lambda$ (n=1,2,3,.......)

$$\therefore \quad 2\pi r = \frac{nh}{mv}$$

$$\text{or} \quad mvr = \frac{nh}{2\pi}$$

Thus de-Broglie showed that formation of stationary pattern for integral 'n' gives rise to stability of the atom.

This is nothing but the Bohr's postulate.

(*b*) Energy in the $n = 4$ level $= \frac{-E_0}{4^2} = -\frac{E_0}{16}$

$\therefore$ Energy required to take the electron from the ground state, to the

$$n = 4 \text{ level} = \left(-\frac{E_0}{16}\right) - (-E_0)$$

$$= \left(\frac{-1 + 16}{16}\right)E_0 = \frac{15}{16}E_0$$

$$= \frac{15}{16} \times 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

Let the frequency of the photon be ν, we have

$$h\nu = \frac{15}{16} \times 13.6 \times 1.6 \times 10^{-19}$$

$$\therefore \quad \nu = \frac{15 \times 13.6 \times 1.6 \times 10^{-19}}{16 \times 6.63 \times 10^{-34}} \text{Hz}$$

$$= \mathbf{3.07 \times 10^{15}\ Hz}$$

Q. 7. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of $Z = 80$, stops and reverses its direction. [*CBSE Ajmer 2015*]

Ans. Let r be the centre to centre distance between the alpha particle and the nucleus ($Z = 80$). When the alpha particle is at the stopping point, then

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r}$$

$$\text{or} \quad r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K}$$

$$= \frac{9 \times 10^9 \times 2 \times 80\, e^2}{4.5 \text{ MeV}} = \frac{9 \times 10^9 \times 2 \times 80 \times (1.6 \times 10^{-19})^2}{4.5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= \frac{9 \times 160 \times 1.6}{4.5} \times 10^{-16} = 512 \times 10^{-16} \text{m}$$

$$= \mathbf{5.12 \times 10^{-14}\ m}$$

Q. 8. A 12.3 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited?

Calculate the wavelengths of the second member of Lyman series and second member of Balmer series. [*CBSE Delhi 2014*]

Ans. The energy of electron in the nth orbit of hydrogen atom is

$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

when the incident beam of energy 12.3 eV is absorbed by hydrogen atom. Let the electron jump from $n = 1$ to $n = n$ level.

$$E = E_n - E_1$$

$$12.3 = -\frac{13.6}{n^2} - \left(-\frac{13.6}{1^2}\right)$$

$$\Rightarrow \quad 12.3 = 13.6\left[1 - \frac{1}{n^2}\right] \quad \Rightarrow \quad \frac{12.3}{13.6} = 1 - \frac{1}{n^2}$$

$$\Rightarrow \quad 0.9 = 1 - \frac{1}{n^2} \quad \Rightarrow \quad n^2 = 10 \quad \Rightarrow \quad n = \mathbf{3}$$

That is the hydrogen atom would be excited upto second excited state.

For Lyman Series

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{1}{1} - \frac{1}{9}\right] \quad \Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{8}{9}$$

$$\Rightarrow \quad \lambda = \frac{9}{8 \times 1.097 \times 10^7} = 1.025 \times 10^{-7} = \mathbf{102.5\ nm}$$

For Balmer Series

$$\frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{1}{4} - \frac{1}{16}\right] \quad \Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{16}$$

$$\Rightarrow \quad \lambda = 4.86 \times 10^{-7}\text{m} \quad \Rightarrow \quad \lambda = \mathbf{486\ nm}$$

Q. 9. The ground state energy of hydrogen atom is – 13.6 eV. If an electron makes a transition from an energy level – 1.51 eV to – 3.4 eV, calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs. [*CBSE (AI) 2017*]

Ans. Energy difference = Energy of emitted photon

$$= E_1 - E_2$$

$$= -1.51 - (-3.4) = 1.89\text{ eV} = 1.89 \times 1.6 \times 10^{-19}\text{ J}$$

$$\lambda = \frac{hc}{E_1 - E_2}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.89 \times 1.6 \times 10^{-19}} = \frac{19.8}{3.024} \times 10^{-7}$$

$$= 6.548 \times 10^{-7}\text{ m} = \mathbf{6548\ Å}$$

This wavelength belongs to Balmer series of hydrogen spectrum.

Q. 10. A hydrogen atom initially in its ground state absorbs a photon and is in the excited state with energy 12.5 eV. Calculate the longest wavelength of the radiation emitted and identify the series to which it belongs.

[Take Rydberg constant $R = 1.1 \times 10^7\ \text{m}^{-1}$] [*CBSE East 2016*]

Ans. Let n_i and n_f are the quantum numbers of initial and final states, then we have

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The energy of the incident photon = 12.5 eV.

Energy of ground state = –13.6 eV

∴ Energy after absorption of photon can be –1.1 eV.

This means that electron can go to the excited state n_i =3. It emits photon of maximum wavelength on going to n_f =2, therefore,

$$\frac{1}{\lambda_{max}} = \left\{\frac{1}{2^2} - \frac{1}{3^2}\right\}R$$

$$\lambda_{max} = \frac{36}{5R} = \frac{36}{5 \times 1.1 \times 10^7} = 6.545 \times 10^{-7} \text{ m} = \mathbf{6545\ Å}$$

It belongs to Balmer Series.

Q. 11. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for Balmer series of the hydrogen spectrum. [*CBSE (AI) 2017*]

Ans. $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For short wavelength of Lyman series, $n_1 = 1, n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R$$

$$\lambda_L = \frac{1}{R} = \mathbf{913.4\ Å}$$

For short wavelength of Balmer series, $n_1 = 2, n_2 = \infty$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4}$$

$$\therefore \quad \lambda_B = \frac{4}{R} = 4 \times 913.4\ Å = \mathbf{3653.6\ Å}$$

Q. 12. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted.[*CBSE (AI) 2017*]

Ans. It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV.

Also, the energy of the gaseous hydrogen in its ground state at room temperature is –13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes –13.6 + 12.5 eV = –1.1 eV.

Orbital energy related to orbit level (n) is

$$E = \frac{-13.6}{(n)^2} \text{eV}$$

For $n = 3$,

$$E = \frac{-13.6}{(3)^2} \text{eV} = \frac{-13.6}{9} \text{eV} = -1.5 \text{ eV}$$

This energy is approximately equal to the energy of gaseous hydrogen.

This implies that the electron has jumped from $n = 1$ to $n = 3$ level.

During its de-excitation, electrons can jump from $n = 3$ to $n = 1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

Relation for wave number for the Lyman series is

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

For first member $n = 3$

$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{(3)^2}\right] = R\left[\frac{1}{1} - \frac{1}{9}\right]$

$\therefore \quad \frac{1}{\lambda_1} = 1.097 \times 10^7 \left[\frac{9-1}{9}\right]$ (where Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$)

$\therefore \quad \frac{1}{\lambda_1} = 1.097 \times 10^7 \times \frac{8}{9} \quad \Rightarrow \quad \lambda_1 = \mathbf{1.025 \times 10^{-7} \text{ m}}$

For $n = 2$,

$\therefore \quad \frac{1}{\lambda_2} = R\left[\frac{1}{1^2} - \frac{1}{(2)^2}\right] = R\left[\frac{1}{1} - \frac{1}{4}\right]$

$\therefore \quad \frac{1}{\lambda_2} = 1.097 \times 10^7 \left[\frac{4-1}{4}\right]$

$\therefore \quad \frac{1}{\lambda_2} = 1.097 \times 10^7 \times \frac{3}{4} \quad \Rightarrow \quad \lambda_2 = \mathbf{1.215 \times 10^{-7} \text{ m}}$

Relation for wave number for the Balmer series is

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For first member, $n = 3$

$\therefore \quad \frac{1}{\lambda_3} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 1.097 \times 10^7 \times \left[\frac{1}{4} - \frac{1}{9}\right]$

$\Rightarrow \quad \lambda_3 = \mathbf{6.56 \times 10^{-7} \text{ m}}$

Q. 13. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom, *i.e.*, an atom where the electron is replaced by a negatively charged muon (μ^-) of mass about 207 m_e that orbits around a proton.

(Given for hydrogen atom, radius of first orbit and ground state energy are 0.53×10^{-10} m and – 13.6 eV respectively) **[*CBSE 2019 (55/5/1)*]**

Ans. In Bohr's Model of hydrogen atom the radius of nth orbit is given by

$$r_n = \frac{n^2 h^2}{4\pi^2 e^2 m_e} \qquad \text{[for H-atom, } Z = 1\text{]}$$

$$r_1 \propto \frac{1}{m_e} \qquad (\because n = 1)$$

Similarly,

$$r_\mu \propto \frac{1}{m_\mu}$$

$$\frac{r_\mu}{r_e} = \frac{m_e}{m_\mu} = \frac{1}{207}$$

$\therefore \quad r_\mu = \frac{1}{207} r_e = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13} \text{ m}$

Energy of electron in nth orbit

$$E_n = -\frac{Z^2 m e^4}{8E_0 h^2 n^2}$$

$$E_n \propto m \qquad (\because n = 1)$$

$\therefore \quad \frac{E_\mu}{E_e} = \frac{m_\mu}{m_e} = 207$

$\therefore \qquad E_\mu = 207\, E_e$

$= -207 \times 13.6$ eV

$= \mathbf{-2.8\ keV}$

Long Answer Questions [5 marks]

Q. 1. Draw a schematic arrangement of Geiger-Marsden experiment for studying α-particle scattering by a thin foil of gold. Describe briefly, by drawing trajectories of the scattered α-particles. How this study can be used to estimate the size of the nucleus? [*CBSE Delhi 2010*]

OR

Describe Geiger-Marsden experiment. What are its observations and conclusions?

Ans. At the suggestion of Rutherford, in 1911, H. Geiger, and E. Marsden performed an important experiment called Geiger-Marsden experiment (or Rutherford's scattering experiment). It consists of

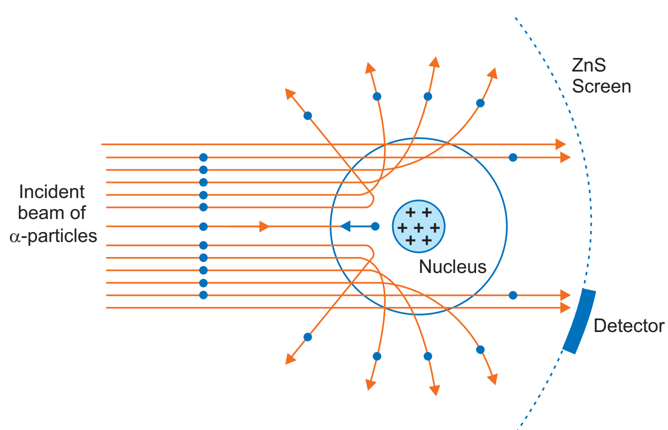
1. **Source of α-particles:** The radioactive source polonium emits high energetic alpha (α) particles. Therefore, polonium is used as a source of α-particles. This source is placed in an enclosure containing a hole and a few slits A_1, A_2, ..., etc., placed in front of the hole. This arrangement provides a fine beam of α-particles.
2. **Thin gold foil:** It is a gold foil of thickness nearly 10^{-6} m, α-particles are scattered by this foil. The foil taken is thin to avoid multiple scattering of α-particles, *i.e.*, to ensure that α-particle be deflected by a single collision with a gold atom.
3. **Scintillation counter:** By this the number of α-particles scattered in a given direction may be counted. The entire apparatus is placed in a vacuum chamber to prevent any energy loss of α-particles due to their collisions with air molecules.

Method: When α-particle beam falls on gold foil, the α-particles are scattered due to collision with gold atoms. This scattering takes place in all possible directions. The number of α-particles scattered in any direction is counted by scintillation counter.

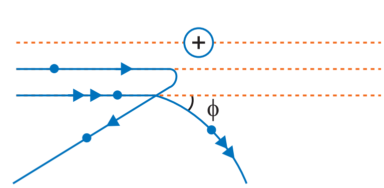
Observations and Conclusions

(*i*) Most of α-particles pass through the gold foil undeflected. This implies that "most part of the atom is hollow."

(*ii*) α-particles are scattered through all angles. Some α-particles (nearly 1 in 2000), suffer scattering through angles more than 90°, while a still smaller number (nearly 1 in 8000) retrace their path. This implies that when fast moving positively charged α-particles come near gold-atom, then a few of them experience such a strong repulsive force that they turn back. On this basis Rutherford concluded that whole of positive charge of atom is concentrated in a small central core, called the nucleus.



The distance of closest approach of α-particle gives the estimate of nuclear size. If Ze is charge of nucleus, E_k–kinetic energy of α particle, $2e$–charge on α-particle, the size of nucleus r_0 is given by



$$E_k = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_0} \qquad \Rightarrow \qquad r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{E_k}$$

Calculations show that the size of nucleus is of the order of 10^{-14} m, while size of atom is of the order of 10^{-10}m; therefore the size of nucleus is about $\frac{10^{-14}}{10^{-10}} = \frac{1}{10,000}$ times the size of atom.

(*iii*) The negative charges (electrons) do not influence the scattering process. This implies that nearly whole mass of atom is concentrated in nucleus.

Q. 2. Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when atom make a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number n_f ($n_f < n_i$). **[*CBSE (AI) 2013, (F) 2012, 2011*]**

OR

Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels.
[*CBSE Delhi 2013, Guwahati 2015*]

OR

Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron? **[*CBSE (AI) 2014*]**

Ans. Suppose m be the mass of an electron and v be its speed in nth orbit of radius r. The centripetal force for revolution is produced by electrostatic attraction between electron and nucleus.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \qquad \text{[from Rutherford model]} \qquad \ldots(i)$$

or, $$mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

So, Kinetic energy $[K] = \frac{1}{2}mv^2$

$$K = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

Potential energy $= \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$

Total energy, $E = KE + PE$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

For nth orbit, E can be written as E_n

so, $$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n} \qquad \ldots(ii)$$

Negative sign indicates that the electron remains bound with the nucleus (or electron-nucleus form an attractive system)

From Bohr's postulate for quantization of angular momentum

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

Substituting this value of v in equation (*i*), we get

$$\frac{m}{r}\left[\frac{nh}{2\pi mr}\right]^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \quad \text{or} \quad r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

or, $$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} \qquad \ldots(iii)$$

For Bohr's radius, $n = 1$, *i.e.*, for K shell $r_B = \dfrac{\varepsilon_0 h^2}{\pi Z m e^2}$

Substituting value of r_n in equation (*ii*), we get

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2\left(\dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right)} = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2}$$

or, $$E_n = -\frac{Z^2 Rhc}{n^2}, \text{ where } R = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

R is called Rydberg constant.

For hydrogen atom $Z=1$, $E_n = \dfrac{-Rhc}{n^2}$

If n_i and n_f are the quantum numbers of initial and final states and E_i & E_f are energies of electron in H-atom in initial and final state, we have

$$E_i = \frac{-Rhc}{n_i^2} \text{ and } E_f = \frac{-Rch}{n_f^2}$$

If ν is the frequency of emitted radiation, we get

$$\nu = \frac{E_i - E_f}{h}$$

$$\nu = \frac{-Rc}{n_i^2} - \left(\frac{-Rc}{n_f^2}\right) \Rightarrow \nu = Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

For Balmer series $n_f = 2$, while $n_i = 3, 4, 5, \ldots\infty$.

Q. 3. Derive the expression for the magnetic field at the site of a point nucleus in a hydrogen atom due to the circular motion of the electron. Assume that the atom is in its ground state and give the answer in terms of fundamental constants. **[*CBSE Sample Paper 2016*]**

Ans. To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic force of attraction. Therefore,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \text{(For H atom, } Z = 1) \qquad \ldots(i)$$

From Bohr's quantisation condition

$$mvr = \frac{nh}{2\pi} \quad \Rightarrow \quad v = \frac{nh}{2\pi mr}$$

For K shell, $n=1$

$$v = \frac{h}{2\pi mr} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$\frac{m}{r}\left(\frac{h}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\frac{m}{r}\frac{h^2}{4\pi^2 m^2 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \Rightarrow \quad \pi r m e^2 = \varepsilon_0 h^2$$

$$r = \frac{\varepsilon_0 h^2}{\pi m e^2} \qquad \ldots(iii)$$

From (*ii*) and (*iii*), we have

$$v = \frac{h \times \pi m e^2}{2\pi m \varepsilon_0 h^2} = \frac{e^2}{2\varepsilon_0 h}$$

Magnetic field at the centre of a circular loop $B = \frac{\mu_0 I}{2r}$

$$I = \frac{\text{Charge}}{\text{Time}} \text{ and Time} = \frac{2\pi r}{v}$$

$\therefore$ $$I = \frac{ev}{2\pi r}$$

So, $$B = \frac{\mu_0 ev}{2r \times 2\pi r} = \frac{\mu_0 ev}{4\pi r^2} \quad ...(iv)$$

From (*ii*), (*iii*) (*iv*), we have

$$B = \frac{\mu_0 e . e^2 \pi^2 m^2 e^4}{2\varepsilon_0 h \times 4\pi \times \varepsilon_0^2 h^4} \quad \Rightarrow \quad B = \frac{\mu_0 e^7 \pi m^2}{8\varepsilon_0^3 h^5}$$

Self-Assessment Test

Time allowed: 1 hour **Max. marks: 30**

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly-ionised Li atom (Z = 3) is

(*a*) 1.51 (*b*) 13.6

(*c*) 40.8 (*d*) 122.4

(*ii*) The ratio of kinetic energy to the total energy of an electron in a Bohr orbit of the hydrogen atom, is

(*a*) 1 : 1 (*b*) 1 : –1

(*c*) 2 : –1 (*d*) 1 : –2

(*iii*) The ratio of wavelengths of the last line of Balmer series and the last line of Lyman series is

(*a*) 1 (*b*) 4

(*c*) 0.5 (*d*) 2

2. Fill in the blanks. **(2 × 1 = 2)**

(*i*) The scattering angle will decreases with the ________________ in impact parameter.

(*ii*) When an electron jumps from an outer stationary orbit of energy E_2 to an inner stationary orbit of energy E_1, the frequency of radiation emitted = ________________.

3. When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$, identify the spectral series to which the emission lines belong. **1**

4. The energy of electron in *n*th orbit of H-atom is $E_n = -\frac{13.6}{n^2}$ eV. What is the energy required for transition from ground state to first excited state? **1**

5. Define ionisation energy. What is its value for a hydrogen atom? **1**

6. The ground state energy of hydrogen atom is –13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to – 1.51 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong? **2**

7. Calculate the shortest wavelength of the spectral lines emitted in Balmer series. **2**
[Given Rydberg constant, $R = 10^7 \text{ m}^{-1}$]

8. The ground state energy of hydrogen atom is – 13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to –3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong? **2**

9. Determine the value of the de Broglie wavelength associated with the electron orbiting in the ground state of hydrogen atom (Given $E_n = -(13.6/n^2)$ eV and Bohr radius $r_0 = 0.53$ Å). How will the de Broglie wavelength change when it is in the first excited state? **2**

10. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. **3**

11. The spectrum of a star in the visible and the ultraviolet region was observed and the wavelength of some of the lines that could be identified were found to be:

824 Å, 970 Å, 1120 Å, 2504 Å, 5173 Å, 6100 Å

Which of these lines cannot belong to hydrogen atom spectrum? (Given Rydberg constant $R = 1.03\times10^7 \text{ m}^{-1}$ and $\frac{1}{R} = 970$ Å). Support your answer with suitable calculations. **3**

12. Given the ground state energy $E_0 = -13.6$ eV and Bohr radius $a_0 = 0.53$ Å. Find out how the de Broglie wavelength associated with the electron orbiting in the ground state would change when it jumps into the first excited state. **3**

13. (*a*) Using Bohr's postulates, derive the expression for the total energy of the electron in the stationary states of the hydrogen atom.
(*b*) Using Rydberg formula, calculate the wavelengths of the spectral lines of the first member of the Lyman series and of the Balmer series. **5**

Answers

1. (*i*) (*d*) (*ii*) (*b*) (*iii*) (*b*)
2. (*i*) increase (*ii*) $\nu = \frac{(E_2 - E_1)}{h}$
4. 10.2 eV
7. 3.646×10^{-7} m
8. $\lambda = 4853$ Å
10. 6.54×10^{-7} m