Chapter 10

OPERATIONS RESEARCH

Learning Objectives

After studying this chapter, the students will be able to understand

- Formulation of Linear Programming Problem
- Solution of LPP by graphical method.
- Construction of network of a project
- Project completion time by Critical Path Method(CPM)

Introduction

L.V. Kantorovich

During the world war II, the military management in England recruited a team of scientists, engineers and mathematicians to study the strategic and tactical problems of air and land defence. Their objective was to determine the best utilization of limited military resources like ammunition, food and other things needed for war. This group of scientists formed the first operations research team. The name operations research was apparently coined because the team was dealing with the research on (military) operations. Operations research team helped in developing strategies for mining operations, inventing new flight pattern and planning of sea mines. Following the end of war the success of military team attracted the attention of industrial managers who were seeking solutions to their complex type of problems.

It is not possible to give uniformly acceptable definition of operations research. The following is the definition of operations research published on behalf of UK operational research society. "operations research is the application of scientific methods to complex problems arising from operations involving large system of men, machines, materials and money in industry, business, government and defence."

An operations research model is defined as any abstract or idealized representation of real life system or situation. The objective of the model is to identify significant factors and inter-relationship. Here we study only two models namely linear programming problem and network analysis.

10.1 Linear Programming Problem

The Russian Mathematician L.V. Kantorovich applied mathematical model to solve linear programming problems. He pointed out in 1939 that many classes of problems which arise in production can be defined mathematically and therefore can be solved numerically. This decision making technique was further developed by George B. Dantziz. He formulated the general linear programming problem and developed simplex method (1947) to solve complex real time applications. Linear programming is one of the best optimization technique from theory, application and computation point of view.

Definition:

Linear Programming Problem(LPP) is a mathematical technique which is used to optimize (maximize or minimize) the objective function with the limited resources.

Mathematically, the general linear programming problem (LPP) may be stated as follows.

Maximize or Minimize

$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to the conditions (constraints)

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq (or = or \geq) b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq (or = or \geq) b_2$$
$$\ldots \quad \ldots. \quad \ldots. \quad \ldots. \quad \ldots$$
$$\ldots \quad \ldots. \quad \ldots. \quad \ldots. \quad \ldots$$
$$a_{m1}x1 + a_{m2}x_2 + \ldots + a_{mn}x_n (\leq or = or \geq) b_m$$
$$x_1, x_2 \ldots\ldots x_n \geq 0$$

Short form of LPP

Maximize or Minimize $Z = \sum_{j=1}^{n} c_jx_j$

Subject to $\sum_{j=1}^{n} a_{ij}x_j \leq (or = or \geq) b_i$,

i = 1,2,3,…,m ... (1)

and $x_j \geq 0$ ... (2)

Some useful definitions:

Objective function:

A function $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ which is to be optimized (maximized or minimized) is called objective function.

Decision variable:

The decision variables are the variables, which has to be determined $x_j, j = 1,2,3,\ldots,n$, to optimize the objective function.

Constraints:

There are certain limitations on the use of limited resources called constraints.

$\sum_{j=1}^{n} a_{ij}x_j \leq (or = or \geq) b_i$, i = 1,2,3,…, m are the constraints.

Solution:

A set of values of decision variables x_j, j=1,2,3,…, n satisfying all the constraints of the problem is called a solution to that problem.

Feasible solution:

A set of values of the decision variables that satisfies all the constraints of the problem and non-negativity restrictions is called a feasible solution of the problem.

Optimal solution:

Any feasible solution which maximizes or minimizes the objective function is called an optimal solution.

Feasible region:

The common region determined by all the constraints including non-negative constraints $x_j \geq 0$ of a linear programming problem is called the feasible region (or solution region) for the problem.

DO YOU KNOW?

Linear Programming Problem helps a farmer to produce the best crop by minimising risk and maximizing profit.

10.1.1 Mathematical formulation of a linear programming problem:

The procedure for mathematical formulation of a linear programming problem consists of the following steps.

(i) Identify the decision variables.

(ii) Identify the objective function to be maximized or minimized and express it as a linear function of decision variables.

(iii) Identify the set of constraint conditions and express them as linear inequalities or equations in terms of the decision variables.

DO YOU KNOW?

Linear Programming Problem is used in determining shortest routes for travelling salesman.

Example 10.1

A furniture dealer deals only two items viz., tables and chairs. He has to invest ₹10,000/- and a space to store atmost 60 pieces. A table cost him ₹ 500/– and a chair ₹ 200/–. He can sell all the items that he buys. He is getting a profit of ₹ 50 per table and ₹ 15 per chair. Formulate this problem as an LPP, so as to maximize the profit.

Solution:

(i) Variables:

Let x_1 and x_2 denote the number of tables and chairs respectively.

(ii) Objective function:

Profit on x_1 tables = $50\ x_1$

Profit on x_2 chairs = $15\ x_2$

Total profit = $50\ x_1 + 15x_2$

Let $Z = 50\ x_1 + 15\ x_2$, which is the objective function.

Since the total profit is to be maximized, we have to maximize $Z=50x_1+15x_2$

(iii) Constraints:

The dealer has a space to store atmost 60 pieces

$x_1+x_2 \leq 60$

The cost of x_1 tables = ₹ 500 x_1

The cost of x_2 tables = ₹ 200 x_2

Total cost = $500\ x_1 + 200\ x_2$,which cannot be more than 10000

$500\ x_1 + 200\ x_2 \leq 10000$

$5\ x_1 + 2x_2 \leq 100$

(iv) Non-negative restrictions:

Since the number of tables and chairs cannot be negative, we have $x_1 \geq 0$, $x_2 \geq 0$

Thus, the mathematical formulation of the LPP is

Maximize $Z=50\ x_1+15x_2$

Subject to the constrains

$x_1+x_2 \leq 60$

$5x_1+2x_2 \leq 100$

$x_1, x_2 \geq 0$

Example 10.2

A company is producing three products P_1, P_2 and P_3, with profit contribution of ₹ 20, ₹ 25 and ₹ 15 per unit respectively. The resource requirements per unit of each of the products and total availability are given below.

Product	P_1	P_2	P_3	Total availability
Man hours/unit	6	3	12	200
Machine hours/unit	2	5	4	350
Material/unit	1kg	2kg	1kg	100kg

Formulate the above as a linear programming model.

Solution:

(i) **Variables:** Let x_1 , x_2 and x_3 be the number of units of products P_1, P_2 and P_3 to be produced.

(ii) **Objective function:** Profit on x_1 units of the product $P_1 = 20\ x_1$

Profit on x_2 units of the product $P_2 = 25\ x_2$

Profit on x_3 units of the product $P_3 = 15\ x_3$

Total profit = $20\ x_1 + 25\ x_2 + 15\ x_3$

Since the total profit is to be maximized, we have to maximize $Z = 20\ x_1 + 25\ x_2 + 15\ x_3$

Constraints: $6x_1 + 3\ x_2 + 12\ x_3 \leq 200$

$2\ x_1 + 5\ x_2 + 4\ x_3 \leq 350$

$x_1 + 2\ x_2 + x_3 \leq 100$

Non-negative restrictions: Since the number of units of the products A,B and C cannot be negative, we have $x_1, x_2, x_3 \geq 0$

Thus, we have the following linear programming model.

Maximize $Z = 20\ x_1 + 25\ x_2 + 15\ x_3$

Subject to $6\ x_1 + 3\ x_2 + 12\ x_3 \leq 200$

$2x_1 + 5x_2 + 4\ x_3 \leq 350$

$x_1 + 2\ x_2 + x_3 \leq 100$

$x_1, x_2, x_3 \geq 0$

Example 10.3

A dietician wishes to mix two types of food F_1 and F_2 in such a way that the vitamin contents of the mixture contains atleast 6 units of vitamin A and 9 units of vitamin B. Food F_1 costs ₹ 50 per kg and F_2 costs ₹ 70 per kg. Food F_1 contains 4 units per kg of vitamin A and 6 units per kg of vitamin B while food F_2 contains 5 units per kg of vitamin A and 3 units per kg of vitamin B. Formulate the above problem as a linear programming problem to minimize the cost of mixture.

Solution:

(i) **Variables:**

Let the mixture contains x_1 kg of food F_1 and x_2 kg of food F_2

(ii) **Objective function:**

Cost of x_1 kg of food $F_1 = 50\ x_1$

Cost of x_2 kg of food $F_2 = 70x_2$

The cost is to be minimized

Therefore minimize $Z = 50\ x_1 + 70x_2$

(iii) **Constraints:**

We make the following table from the given data

Resources	Food (in kg)		Requirement
	$F_1\ (x_1)$	$F_2\ (x_2)$	
Vitamin A (units/kg)	4	5	6
Vitamin B (units/kg)	6	3	9
Cost (₹/kg)	50	70	

Table 10.1

$4\ x_1 + 5x_2 \geq 6$ (since the mixture contains ‘atleast 6’ units of vitamin A, we have the inequality of the type ≥)

$6\ x_1 + 3x_2 \geq 9$(since the mixture contains ‘atleast 9’ units of vitamin B ,we have the inequality of the type ≥)

(iv) **Non-negative restrictions:**

Since the number of kgs of vitamin A and vitamin B are non-negative, we have $x_1, x_2 \geq 0$

Thus, we have the following linear programming model

Minimize $Z = 50\ x_1 + 70x_2$ subject to

$4\ x_1 + 5x_2 \geq 6$

$6\ x_1 + 3x_2 \geq 9$ and $x_1, x_2 \geq 0$

Example 10.4

A soft drink company has two bottling plants C_1 and C_2. Each plant produces three different soft drinks S_1,S_2 and S_3. The production of the two plants in number of bottles per day are:

Product	Plant	
	C_1	C_2
S_1	3000	1000
S_2	1000	1000
S_3	2000	6000

A market survey indicates that during the month of April there will be a demand for 24000 bottles of S_1, 16000 bottles of S_2 and 48000 bottles of S_3. The operating costs, per day, of running plants C_1 and C_2 are respectively ₹ 600 and ₹ 400. How many days should the firm run each plant in April so that the production cost is minimized while still meeting the market demand? Formulate the above as a linear programming model.

Solution:

(i) Variables:

Let x_1 be the number of days required to run plant C_1 and x_2 be the number of days required to run plant C_2.

Objective function: Minimize

$Z = 600\ x_1 + 400\ x_2$

(ii) Constraints: $3000\ x_1 + 1000\ x_2 \geq 24000$ (since there is a demand of 24000 bottles of drink A, production should not be less than 24000)

$1000\ x_1 + 1000\ x_2 \geq 16000$

$2000\ x_1 + 6000\ x_2 \geq 48000$

(iii) Non-negative restrictions: Since be the number of days required of a firm are non-negative, we have $x_1, x_2 \geq 0$.

Thus we have the following LP model.

Minimize $Z = 600\ x_1 + 400\ x_2$ subject to $3000\ x_1 + 1000\ x_2 \geq 24000$

$1000\ x_1 + 1000\ x_2 \geq 16000$

$2000\ x_1 + 6000\ x_2 \geq 48000$ and $x_1, x_2 \geq 0$

10.1.2 Solution of LPP by graphical method

After formulating the linear programming problem, our aim is to determine the values of decision variables to find the optimum (maximum or minimum) value of the objective function. Linear programming problems which involve only two variables can be solved by graphical method. If the problem has three or more variables, the graphical method is impractical.

The major steps involved in this method are as follows:

(i) State the problem mathematically

(ii) Write all the constraints in the form of equations and draw the graph

(iii) Find the feasible region

(iv) Find the coordinates of each vertex (corner points) of the feasible region. The coordinates of the vertex can be obtained either by inspection or by solving the two equations of the lines intersecting at the point

(v) By substituting these corner points in the objective function we can get the values of the objective function

(vi) If the problem is maximization then the maximum of the above values is the optimum value. If the problem is minimization then the minimum of the above values is the optimum value.

Example 10.5

Solve the following LPP

Maximize $Z = 2x_1 + 5x_2$ subject to the conditions $x_1 + 4x_2 \leq 24$ $3x_1 + x_2 \leq 21$ $x_1 + x_2 \leq 9$ and $x_1, x_2 \geq 0$

Solution:

First we have to find the feasible region using the given conditions.

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant.

Write all the inequalities of the constraints in the form of equations.

Therefore we have the lines $x_1 + 4x_2 = 24$; $3x_1 + x_2 = 21$; $x_1 + x_2 = 9$

$x_1 + 4x_2 = 24$ is a line passing through the points (0 , 6) and (24 , 0).

[(0,6) is obtained by taking $x_1 = 0$ in $x_1 + 4x_2 = 24$, (24 , 0) is obtained by taking $x_2 = 0$ in $x_1 + 4x_2 = 24$].

Any point lying on or below the line $x_1 + 4x_2 = 24$ satisfies the constraint $x_1 + 4x_2 \leq 24$.

$3x_1 + x_2 = 21$ is a line passing through the points (0, 21) and (7, 0). Any point lying on or below the line $3x_1 + x_2 = 21$ satisfies the constraint $3x_1 + x_2 \leq 21$.

$x_1 + x_2 = 9$ is a line passing through the points (0 , 9) and (9 , 0) .Any point lying on or below the line $x_1 + x_2 = 9$ satisfies the constraint $x_1 + x_2 \leq 9$.

Now we draw the graph.

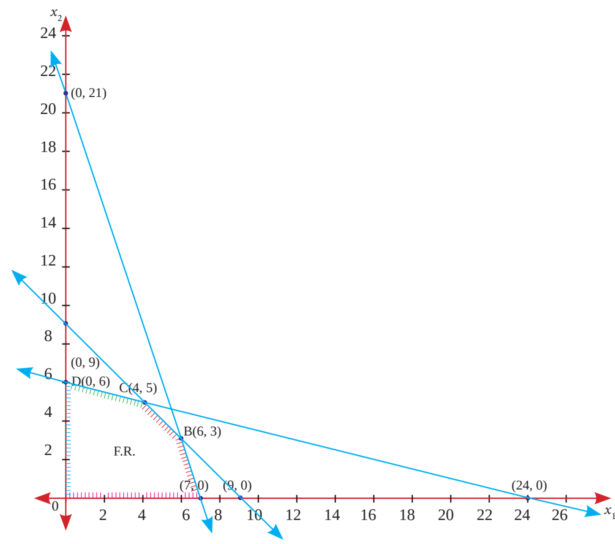


Fig 10.1

The feasible region satisfying all the conditions is OABCD. The co-ordinates of the points are O(0,0) A(7,0);B(6,3) [the point B is the intersection of two lines $x_1 + x_2 = 9$ and $3x_1 + x_2 = 21$];C(4,5) [the point C is the intersection of two lines $x_1 + x_2 = 9$ and $x_1 + 4x_2 = 24$] and D(0,6).

Corner points	$Z = 2x_1 + 5x_2$
O(0,0)	0
A(7,0)	14
B(6,3)	27
C(4,5)	33
D(0,6)	30

Table 10.2

Maximum value of Z occurs at C. Therefore the solution is $x_1 = 4$, $x_2 = 5$, $Z_{max} = 33$.

Example 10.6

Solve the following LPP by graphical method

Minimize $z = 5x_1 + 4x_2$ Subject to constraints $4x_1 + x_2 \geq 40$; $2x_1 + 3x_2 \geq 90$ and $x_1, x_2 \geq 0$

Solution:

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

Consider the equations $4x_1+x_2 = 40$ and $2\,x_1+3\,x_2 = 90$.

$4x_1+x_2 = 40$ is a line passing through the points (0,40) and (10,0). Any point lying on or above the line $4x_1+x_2= 40$ satisfies the constraint $4x_1+ x_2 \geq 40$.

$2x_1+3x_2 = 90$ is a line passing through the points (0,30) and (45,0). Any point lying on or above the line $2\,x_1+3x_2= 90$ satisfies the constraint $2x_1+3x_2 \geq 90$.

Draw the graph using the given constraints.

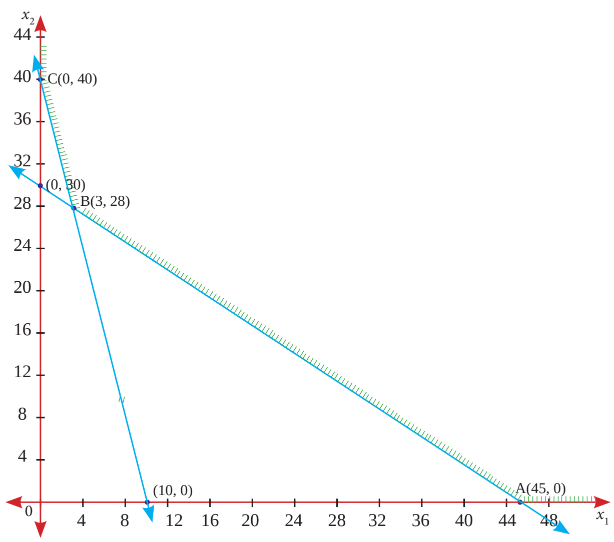


Fig 10.2

The feasible region is ABC (since the problem is of minimization type we are moving towards the origin.

Corner points	$z = 5x_1+4x_2$
A(45,0)	225
B(3,28)	127
C(0,40)	160

Table 10.3

The minimum value of Z occurs at B(3,28).

Hence the optimal solution is $x_1 = 3$, $x_2 = 28$ and $Z_{min}=127$.

Example 10.7

Solve the following LPP.

Maximize $Z= 2\,x_1 +3x_2$ subject to constraints $x_1 + x_2 \leq 30$; $x_2 \leq 12$; $x_1 \leq 20$ and $x_1, x_2 \geq 0$.

Solution:

We find the feasible region using the given conditions.

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

Write all the inequalities of the constraints in the form of equations.

Therefore we have the lines

$x_1+x_2=30$; $x_2 =12$; $x_1= 20$

$x_1+x_2=30$ is a line passing through the points (0,30) and (30,0)

$x_2 = 12$ is a line parallel to x_1–axis

$x_1 = 20$ is a line parallel to x_2–axis.

The feasible region satisfying all the conditions $x_1+ x_2\leq 30$; $x_2\leq 12$; $x_1\leq 20$ and $x_1, x_2 \geq 0$ is shown in the following graph.

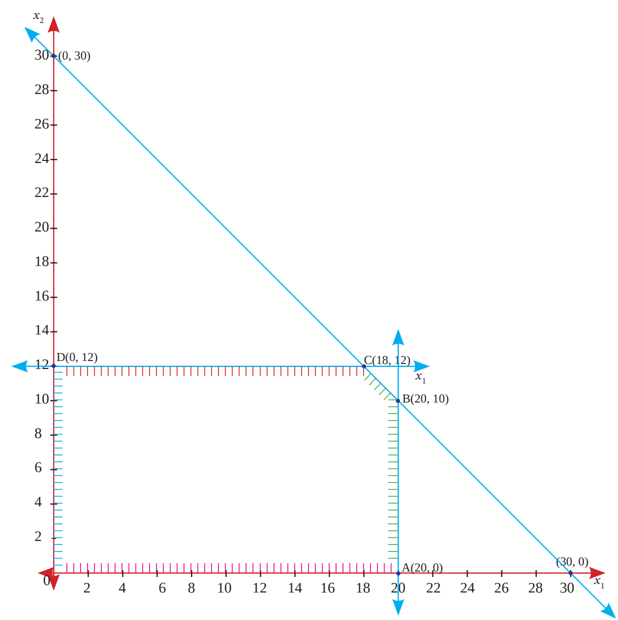


Fig 10.3

The feasible region satisfying all the conditions is OABCD.

The co-ordinates of the points are O(0,0); A(20,0); B(20,10); C(18,12) and D(0,12).

Corner points	$Z = 2x_1 + 3x_2$
O(0,0)	0
A(20,0)	40
B(20,10)	70
C(18,12)	72
D(0,12)	36

Table 10.4

Maximum value of Z occurs at C. Therefore the solution is $x_1 = 18$, $x_2 = 12$, $Z_{max} = 72$.

Example 10.8

Maximize $Z = 3x_1 + 4x_2$ subject to $x_1 - x_2 \leq -1$; $-x_1 + x_2 \leq 0$ and $x_1, x_2 \geq 0$.

Solution:

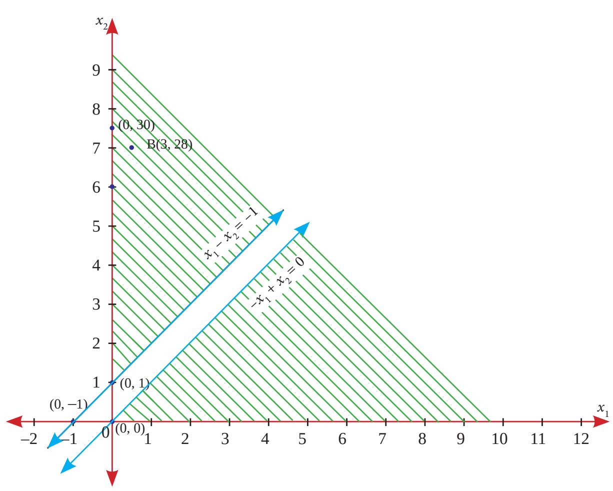


Fig 10.4

Since both the decision variables x_1, x_2 are non-negative ,the solution lies in the first quadrant of the plane.

Consider the equations $x_1 - x_2 = -1$ and $-x_1 + x_2 = 0$.

$x_1 - x_2 = -1$ is a line passing through the points (0,1) and (–1,0).

$-x_1 + x_2 = 0$ is a line passing through the point (0,0).

Now we draw the graph satisfying the conditions $x_1 - x_2 \leq -1$; $-x_1 + x_2 \leq 0$ and $x_1, x_2 \geq 0$.

There is no common region(feasible region) satisfying all the given conditions. Hence the given LPP has no solution.

Exercise 10.1

1. A company produces two types of pens A and B. Pen A is of superior quality and pen B is of lower quality . Profits on pens A and B are ₹ 5 and ₹ 3 per pen respectively. Raw materials required for each pen A is twice as that of pen B. The supply of raw material is sufficient only for 1000 pens per day . Pen A requires a special clip and only 400 such clips are available per day. For pen B, only 700 clips are available per day . Formulate this problem as a linear programming problem.

2. A company produces two types of products say type A and B. Profits on the two types of product are ₹ 30/- and ₹ 40/- per kg respectively. The data on resources required and availability of resources are given below.

	Requirements		Capacity available per month
	Product A	Product B	
Raw material (kgs)	60	120	12000
Machining hours / piece	8	5	600
Assembling (man hours)	3	4	500

Formulate this problem as a linear programming problem to maximize the profit.

3. A company manufactures two models of voltage stabilizers viz., ordinary and auto-

cut. All components of the stabilizers are purchased from outside sources , assembly and testing is carried out at company's own works. The assembly and testing time required for the two models are 0.8 hour each for ordinary and 1.20 hours each for auto-cut. Manufacturing capacity 720 hours at present is available per week. The market for the two models has been surveyed which suggests maximum weekly sale of 600 units of ordinary and 400 units of auto-cut. Profit per unit for ordinary and auto-cut models has been estimated at ₹ 100 and ₹ 150 respectively. Formulate the linear programming problem.

4. Solve the following linear programming problems by graphical method.
   (i) Maximize Z = $6x_1 + 8x_2$ subject to constraints $30x_1+20x_2 \leq 300$; $5x_1+10x_2 \leq 110$; and $x_1, x_2 \geq 0$.
   (ii) Maximize $Z = 22x_1 + 18x_2$ subject to constraints $960x_1 + 640x_2 \leq 15360$; $x_1 + x_2 \leq 20$ and $x_1, x_2 \geq 0$.
   (iii) Minimize $Z = 3x_1 + 2x_2$ subject to the constraints $5x_1 + x_2 \geq 10$; $x_1 + x_2 \geq 6$; $x_1 + 4x_2 \geq 12$ and $x_1, x_2 \geq 0$.
   (iv) Maximize $Z = 40x_1 + 50x_2$ subject to constraints $3x_1 + x_2 \leq 9$; $x_1 + 2x_2 \leq 8$ and $x_1, x_2 \geq 0$
   (v) Maximize $Z = 20x_1 + 30x_2$ subject to constraints $3x_1 + 3x_2 \leq 36$; $5x_1 + 2x_2 \leq 50$; $2x_1 + 6x_2 \leq 60$ and $x_1, x_2 \geq 0$
   (vi) Minimize $Z = 20x_1 + 40x_2$ subject to the constraints $36x_1 + 6x_2 \geq 108$, $3x_1 + 12x_2 \geq 36$, $20x_1 + 10x_2 \geq 100$ and $x_1, x_2 \geq 0$

10.2 Network Analysis

Networks are diagrams easily visualized in transportation system like roads, railway lines, pipelines, blood vessels, etc.

A project will consist of a number of jobs and particular jobs can be started only after finishing some other jobs. There may be jobs which may not depend on some other jobs. Network scheduling is a technique which helps to determine the various sequences of jobs concerning a project and the project completion time. There are two basic planning and control techniques that use a network to complete a pre-determined schedule. They are Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM). The critical path method (CPM) was developed in 1957 by JE Kelly of Ramington R and M.R. Walker of Dupon to help schedule maintenance of chemical plants. CPM technique is generally applied to well known projects where the time schedule to perform the activities can exactly be determined.

Some important definitions in network

Activity:

An activity is a task or item of work to be done, that consumes time, effort, money or other resources. It lies between two events, called the starting event and ending event. An activity is represented by an arrow indicating the direction in which the events are to occur.

Event:

The beginning and end points of an activity are called events (or nodes). Event is a point in time and does not consume any resources. The beginning and completion of an activity are known as tail event and head event respectively. Event is generally represented by a numbered circle. The head event, called the j^{th} event, has always a number higher than the tail event, called the i^{th} event, ie., $j > i$.

Predecessor Activity:

Activities which must be completed before a particular activity starts are called predecessor activities. If an activity *A* is predecessor of an activity *B*, it is denoted by $A<B$. (i.e.,) activity *B* can start only if activity *A* is completed.

Successor Activity:

An activity that cannot be started until one or more of the other activities are completed, but immediately succeed them is referred to as successor activity.

Network:

Network is a diagrammatic representation of various activities concerning a project arranged in a logical manner.

Path:

A path is defined as a set of nodes connected by lines which begin at the initial node and end at the terminal node of the network.

10.2.1 Construction of network:

Rules for constructing network

For the construction of a network, generally, the following rules are followed:

(i) Each activity is represented by one and only one arrow. (i.e) only one activity can connect any two nodes.

(ii) No two activities can be identified by the same head and tail events.

(iii) Nodes are numbered to identify an activity uniquely. Tail node (starting point) should be lower than the head node (end point) of an activity.

(iv) Arrows should not cross each other.

(v) Arrows should be kept straight and not curved or bent.

(vi) Every node must have atleast one activity preceding it and atleast one activity following it except for the node at the beginning and at the end of the network.

Numbering the Events

After the network is drawn in a logical sequence, every event is assigned a number. The number sequence must be such as to reflect the flow of the network. In event numbering, the following rules should be observed:

(i) Event numbers should be unique.

(ii) Event numbering should be carried out on a sequential basis from left to right.

(iii) The initial event is numbered 0 or 1.

(iv) The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow.

(v) Gap should be left in the sequence of event numbering to accommodate subsequent inclusion of activities, if necessary.

Remark: The above procedure of assigning numbers to various events of a network is known as Fulkerson's Rule.

Example 10.9

Draw the logic network for the following:

Activities C and D both follow A, activity E follows C, activity F follows D, activity E and F precedes B.

Solution:

The required network for the above information.

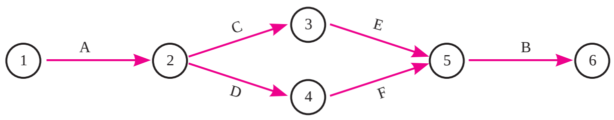


Fig 10.5

Example 10.10

Develop a network based on the following information:

Activity :	A	B	C	D	E	F	G	H
Immediate predecessor:	-	-	A	B	C,D	C,D	E	F

Solution:

Using the immediate precedence relationships and following the rules of network construction, the required network is shown in following figure.

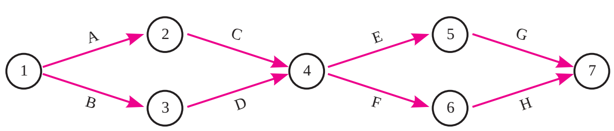


Fig 10.6

Dummy activity:

An activity which does not consume any resource or time, but merely depict the technological dependence is called a dummy activity. It is represented by dotted lines.

Example 10.11

Draw a network diagram for the project whose activities and their predecessor relationships are given below:

Activity :	A	B	C	D	E	F	G	H	I	J	K
Predecessor activity:	-	-	-	A	B	B	C	D	F	H,I	F,G

Solution:

Using the precedence relationships and following the rules of network construction, the required network diagram is shown in following figure

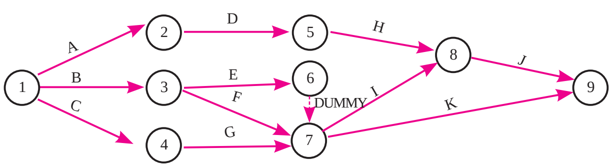


Fig 10.7

Example 10.12

Construct a network diagram for the following situation:

$A < D, E;$ $B, D < F;$ $C < G$ and $B < H.$

Solution:

Using the precedence relationships and following the rules of network construction, the required network is shown in following figure.

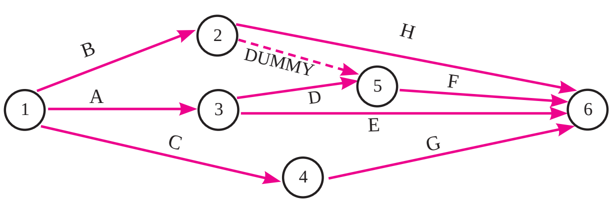


Fig 10.8

10.2.2 Critical path analysis

For each activity an estimate must be made of time that will be spent in the actual accomplishment of that activity. Estimates may be expressed in hours, days, weeks or any other convenient unit of time. The time estimate is usually written in the network immediately above the arrow. For the purpose of calculating various times of events and activities, the following terms shall be used in critical path calculations:

E_i = Earliest start time of event i

L_j = Latest start time of event j

t_{ij} = Duration of activity (i,j)

The next step after making the time estimates is the calculations which are done in the following ways:

(i) Forward Pass Calculations

(ii) Backward Pass Calculations.

Forward pass calculations:

We start from the initial node 1 with starting time of the project as zero. Proceed through the network visiting nodes in an increasing order of node number and end at the final node of the network. At each node, we calculate earliest start times for each activity by considering E_i as the earliest occurrence of node i.

The method may be summarized as below:

Step 1: Set $E_1 = 0$; $i = 1$ (initial node).

Step 2: Set the earliest start time(EST) for each activity that begins at node i as $ES_{ij} = E_i$; for all activities (i, j) that start at node i.

Step 3: Compute the earliest finish time (EFT) of each activity that begins at node i by adding the earliest start time of the activity to the duration of the activity. Thus $EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$.

Step 4: Move on to next node, say node $j(j > i)$ and compute the earliest start time at node j, using $E_j = \max i\{EF_{ij}\} = \max_i \{E_i + t_{ij}\}$ for all immediate predecessor activities.

Step 5: If $j = n$ (final node), then the earliest finish time for the project is given by $E_n = \max\{EF_{ij}\} = \max\{E_{n-1} + t_{ij}\}$.

Backward pass calculations:

We start from the final (last) node n of the network, proceed through the network visiting nodes in the decreasing order of node numbers and end at the initial node 1. At each node, we calculate the latest finish time and latest start time for each activity by considering L_j as the latest occurrence of node j. The method may be summarized below:

Step 1: $L_n = E_n$; for $j = n$

Step 2: Set the latest finish time (LFT)of each activity that ends at node j as $LF_{ij} = L_j$

Step 3: Compute the latest start time (LST) of all activities ending at node j, subtracting the duration of each activity from the latest finish time of the activity. Thus, $LS_{ij} = LF_{ij} - t_{ij} = L_j - t_{ij}$

Step 4: Proceed backward to the next node i($i<j$) in the sequence and compute the latest occurrence time at node i using $L_i = \min_j \{LS_{ij}\} = \min_j \{L_j - t_{ij}\}$.

Step 5: If $j = 1$ (initial node), then $L_1 = \min\{LS_{ij}\} = \min\{L_2 - t_{ij}\}$

Critical path:

The longest path connected by the activities in the network is called the critical path. A path along which it takes the longest duration.

For the activity (i,j), to lie on the critical path, following conditions must be satisfied:

(i) $E_i = L_i$ and $E_j = L_j$

(ii) $E_j - E_i = L_j - L_i = t_{ij}$

Example 10.13

Compute the earliest start time, earliest finish time ,latest start time and latest finish time of each activity of the project given below:

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days)	8	4	10	2	5	3

Solution:

Earliest start time (EST) and latest finish time (LFT) of each activity are given ithe following network.

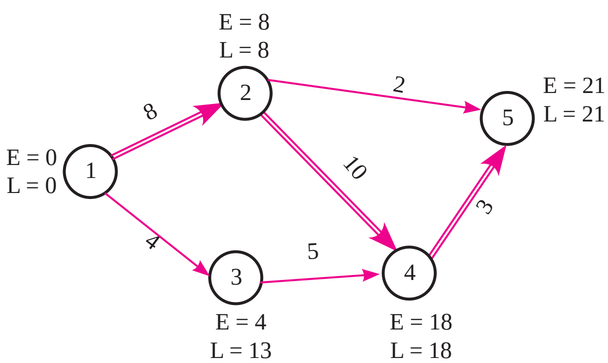


Fig 10.9

$E_1 = 0$

$E_2 = E_1 + t_{12} = 0 + 8 = 8$

$E_3 = E_1 + t_{13} = 0 + 4 = 4$

$E_4 = E_2 + t_{24}$ or $E_3 + t_{34}$

$= 8 + 10 = 18$

(take $E_2 + t_{24}$ or $E_3 + t_{34}$ whichever is maximum)

$E_5 = (E_2 + t_{25}$ or $E_4 + t_{45})$

$= 18 + 3 = 21$

(take $E_2 + t_{25}$ or $E_4 + t_{45}$ whichever is maximum)

$L_5 = 21$

$L_4 = L_5 - t_{45} = 21 - 3 = 18$

$L_3 = L_4 - t_{34} = 18 - 5 = 13$

$L_2 = L_5 - t_{25}$ or $L_4 - t_{24}$

$= 18 - 10 = 8$

(take $L_5 - t_{25}$ or $L_4 - t_{24}$ whichever is minimum)

$L_1 = L_2 - t_{12}$ or $L_3 - t_{13}$

$= 8 - 8 = 0$

(take $L_2 - t_{12}$ or $L_3 - t_{13}$ whichever is minimum)

Here the critical path is 1-2-4-5, which is denoted by double lines.

Activity	Duration (t_{ij})	EST	EFT=EST+t_{ij}	LST=LFT-t_{ij}	LFT
1–2	8	0	8	0	8
1–3	4	0	4	9	13
2–4	10	8	18	8	18
2–5	2	8	10	19	21
3–4	5	4	9	13	18
4–5	3	18	21	18	21

Table 10.5

The longest duration to complete this project is 21 days.

The path connected by the critical activities is the critical path (the longest path).

Critical path is 1-2-4-5 and project completion time is 21 days.

Example 10.14

Calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity of the project given below and determine the Critical path of the project and duration to complete the project.

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration (in week)	8	7	12	4	10	3	5	10	7	4

Solution:

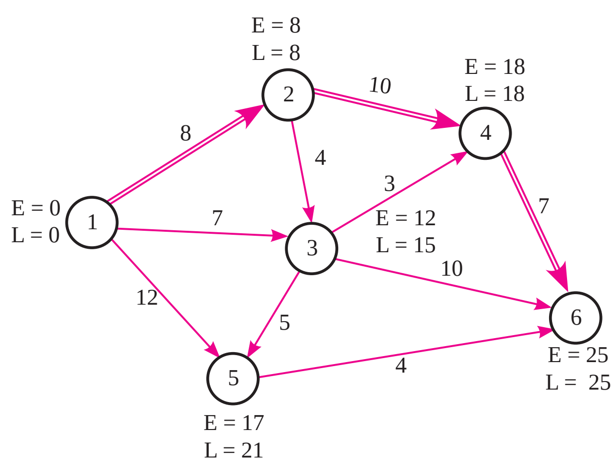


Fig 10.10

Activity	Duration (in week)	EST	EFT	LST	LFT
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Table 10.6

Here the critical path is 1–2–4–6

The project completion time is 25 weeks

Exercise 10.2

1. Draw the network for the project whose activities with their relationships are given below:

   Activities A,D,E can start simultaneously; B,C>A; G,F>D,C; H>E,F.

2. Draw the event oriented network for the following data:

Events	1	2	3	4	5	6	7
Immediate Predecessors	–	1	1	2,3	3	4,5	5,6

3. Construct the network for the projects consisting of various activities and their precedence relationships are as given below:

   A,B,C can start simultaneously A<F, E; B<D, C; E, D<G

4. Construct the network for each the projects consisting of various activities and their precedence relationships are as given below:

Activity	A	B	C	D	E	F	G	H	I	J	K
Immediate Predecessors	–	–	–	A	B	B	C	D	E	H,I	F,G

5. Construct the network for the project whose activities are given below.

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (in week)	3	8	12	6	3	3	8	5	3	8

   Calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity. Determine the critical path and the project completion time.

6. A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time	4	1	1	1	6	5	4	8	1	2	5	7

   Construct the network and calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

7. Draw the network and calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

Jobs	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration	6	5	10	3	4	6	2	9

8. The following table gives the activities of a project and their duration in days

Activity	1-2	1-3	2-3	2-4	3-4	3-5	4-5
Duration	5	8	6	7	5	4	8

   Construct the network and calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

9. A Project has the following time schedule

Activity	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
Duration (in days)	7	6	14	5	11	7	11	4	18

Construct the network and calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

10. The following table use the activities in a construction projects and relevant information

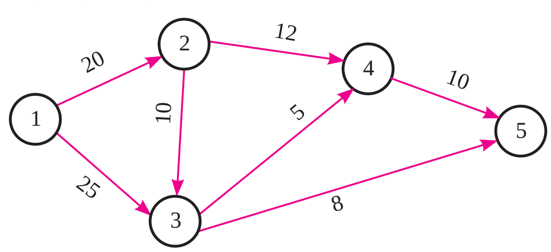
Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration (in days)	22	27	12	14	6	12

Draw the network for the project ,calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and find the critical path. Compute the project duration.

Exercise 10.3

Choose the correct answer

1. The critical path of the following network is



(a) 1 – 2 – 4 – 5 (b) 1– 3– 5
(c) 1 – 2 – 3 – 5 (d) 1 – 2 – 3 – 4 – 5

2. Maximize: $z = 3x_1 + 4x_2$ subject to $2x_1 + x_2 \leq 40$, $2x_1 + 5x_2 \leq 180$, $x_1, x_2 \geq 0$. In the LPP, which one of the following is feasible corner point?

(a) $x_1 = 18, x_2 = 24$
(b) $x_1 = 15, x_2 = 30$
(c) $x_1 = 2 \cdot 5, x_2 = 35$
(d) $x_1 = 20 \cdot 5, x_2 = 19$

3. One of the conditions for the activity (*i, j*) to lie on the critical path is

(a) $E_j - E_i = L_j - L_i = t_{ij}$
(b) $E_i - E_j = L_j - L_i = t_{ij}$
(c) $E_j - E_i = L_i - L_j = t_{ij}$
(d) $E_j - E_i = L_j - L_i \neq t_{ij}$

4. In constructing the network which one of the following statement is false?

(a) Each activity is represented by one and only one arrow. (i.e.) only one activity can connect any two nodes.
(b) Two activities can be identified by the same head and tail events.
(c) Nodes are numbered to identify an activity uniquely. Tail node (starting point) should be lower than the head node (end point) of an activity.
(d) Arrows should not cross each other.

5. In a network while numbering the events which one of the following statement is false?

(a) Event numbers should be unique.
(b) Event numbering should be carried out on a sequential basis from left to right.
(c) The initial event is numbered 0 or 1.
(d) The head of an arrow should always bear a number lesser than the one assigned at the tail of the arrow.

6. A solution which maximizes or minimizes the given LPP is called

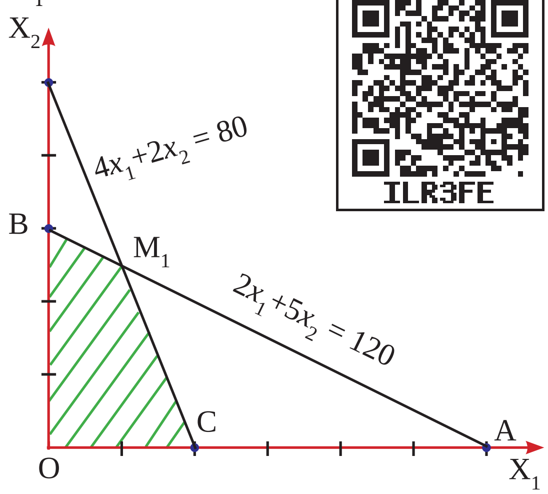
(a) a solution

(b) a feasible solution

(c) an optimal solution

(d) none of these

7. In the given graph the coordinates of M_1 are



(a) $x_1 = 5, x_2 = 30$ (b) $x_1 = 20, x_2 = 16$

(c) $x_1 = 10, x_2 = 20$ (d) $x_1 = 20, x_2 = 30$

8. The maximum value of the objective function $Z = 3x + 5y$ subject to the constraints $x \geq 0, y \geq 0$ and $2x + 5y \leq 10$ is

(a) 6 (b) 15 (c) 25 (d) 31

9. The minimum value of the objective function $Z = x + 3y$ subject to the constraints $2x + y \leq 20$, $x + 2y \leq 20, x > 0$ and $y > 0$ is

(a) 10 (b) 20 (c) 0 (d) 5

10. Which of the following is not correct?

(a) Objective that we aim to maximize or minimize

(b) Constraints that we need to specify

(c) Decision variables that we need to determine

(d) Decision variables are to be unrestricted.

11. In the context of network, which of the following is not correct

(a) A network is a graphical representation .

(b) A project network cannot have multiple initial and final nodes

(c) An arrow diagram is essentially a closed network

(d) An arrow representing an activity may not have a length and shape

12. The objective of network analysis is to

(a) Minimize total project cost

(b) Minimize total project duration

(c) Minimize production delays, interruption and conflicts

(d) All the above

13. Network problems have advantage in terms of project

(a) Scheduling (b) Planning

(c) Controlling (d) All the above

14. In critical path analysis, the word CPM mean

(a) Critical path method

(b) Crash project management

(c) Critical project management

(d) Critical path management

15. Given an L.P.P. maximize $Z = 2x_1 + 3x_2$ subject the constrains $x_1 + x_2 \leq 1$, $5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$ using graphical method, we observe

(a) No feasible solution

(b) unique optimum solution

(c) multiple optimum solution

(d) none of these

Miscellaneous Problems

1. A firm manufactures two products *A* and *B* on which the profits earned per unit are ₹ 3 and ₹ 4 respectively. Each product is processed on two machines M_1 and M_2. Product *A* requires one minute of processing time on M_1 and two minutes on M_2, While *B* requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 7 hrs 30 minutes while M_2 is available for 10 hrs during any working day. Formulate this problem as a linear programming problem to maximize the profit.

2. A firm manufactures pills in two sizes *A* and *B*. Size *A* contains 2 mgs of aspirin, 5 mgs of bicarbonate and 1 mg of codeine. Size *B* contains 1 mg. of aspirin, 8 mgs. of bicarbonate and 6 mgs. of codeine. It is found by users that it requires atleast 12 mgs. of aspirin, 74 mgs.of bicarbonate and 24 mgs. of codeine for providing immediate relief. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LLP.

3. Solve the following linear programming problem graphically.

   Maximise $Z = 4x_1 + x_2$ subject to the constraints $x_1 + x_2 \leq 50$; $3x_1 + x_2 \leq 90$ and $x_1 \geq 0,\ x_2 \geq 0$.

4. Solve the following linear programming problem graphically.

   Minimize $Z = 200x_1 + 500x_2$ subject to the constraints: $x_1 + 2x_2 \geq 10$; $3x_1 + 4x_2 \leq 24$ and $x_1 \geq 0, x_2 \geq 0$.

5. Solve the following linear programming problem graphically.

   Maximize $Z = 3x_1 + 5x_2$ subject to the constraints: $x_1 + x_2 \leq 6$, $x_1 \leq 4$; $x_2 \leq 5$, and $x_1, x_2 \geq 0$.

6. Solve the following linear programming problem graphically. Maximize $Z = 60x_1 + 15x_2$ subject to the constraints: $x_1 + x_2 \leq 50$; $3x_1 + x_2 \leq 90$ and $x_1, x_2 \geq 0$.

7. Draw a network diagram for the following activities.

Activity code	A	B	C	D	E	F	G	H	I	J	K
Predecessor activity	–	A	A	A	B	C	C	C,D	E,F	G,H	I,J

8. Draw the network diagram for the following activities

Activity code	A	B	C	D	E	F	G
Predecessor activity	–	–	A	A	B	C	D,E

9. A Project has the following time schedule

Activity	1-2	2-3	2-4	3-5	4-6	5-6
Duration (in days)	6	8	4	9	2	7

   Draw the network for the project, calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and find the critical path. Compute the project duration.

10. The following table gives the characteristics of project

Activity	1-2	1-3	2-3	3-4	3-5	4-6	5-6	6-7
Duration (in days)	5	10	3	4	6	6	5	5

   Draw the network for the project ,calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and find the critical path. Compute the project duration.

Summary

- Linear programming problem(LPP) is a mathematical modeling technique which is used to allocate limited available resources in order to optimize (maximize or minimize) the objective function.

- **Short form of LPP**

  Maximize or Minimize $Z = \sum_{j=1}^{n} c_j x_j$

  Subject to $\sum_{j=1}^{n} a_{ij} x_j \leq (\text{or} = \text{or} \geq) b_i, \; i = 1, 2, 3, \ldots, m$

  and $x_j \geq 0$

- **Objective function:** A function $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ which is to be optimized (maximized or minimized) is called objective function.

- **Decision variable:** The decision variables are the variables we seek to determine to optimize the objective function. x_j, $j = 1,2,3,\ldots,n$, are the decision variables.

- **Solution:** A set of values of decision variables $x_j, j = 1,2,3,\ldots,n$ satisfying all the constraints of the problem is called a solution to that problem.

- **Feasible solution:** A set of values of the decision variables that satisfies all the constraints of the problem and non-negativity restrictions is called a feasible solution of the problem.

- **Optimal solution:** Any feasible solution which maximizes or minimizes the objective function is called an optimal solution.

- **Feasible region:** The common region determined by all the constraints including non-negative constraints $x_j \geq 0$ of a linear programming problem is called the feasible region (or solution region) for the problem.

- Linear programming problems which involve only two variables can be solved by graphical method.

- It should be noted that the optimal value of LPP occurs at the corner points of the feasible region

- Network is a diagrammatic representation of various activities concerning a project arranged in a logical manner.

- An activity is a task or item of work to be done, that consumes time, effort, money or other resources
- The beginning and end points of an activity are called events (or nodes).
- **Critical path:** The longest path connected by the activities in the network is called the critical path. A path along which it takes the longest duration.

GLOSSARY (கலைச்சொற்கள்)	
Abstract	பண்பு தொகை
Activity	செயல்பாடு
Backward pass calculations	பின் நோக்கி செல்லும் கணக்கீடு
Critical path analysis	தீர்வுக்கு உகந்த பகுப்பாய்வு
Critical path method	தீர்வுக்கு உகந்த முறை
Decision variables	தீர்மான மாறிகள்
Dummy activities	ஒப்புக்கான செயல்
Earliest start time	முன்கூட்டியே தொடங்கும் நேரம்
Event	நிகழ்வு
Feasible solution	ஏற்புடைய தீர்வு
Forward pass calculations	முன் நோக்கி செல்லும் கணக்கீடு
Head event	ஆரம்ப நிகழ்வு
Latest start time	சமீபத்திய தொடங்கும் நேரம்
Linear programming problem	நேரியல் திட்டமிடல் கணக்கு
Logical sequence	தர்க்க தொடர் வரிசை
Network analysis	வலையமைப்பு பகுப்பாய்வு
Predecessor activity	முந்தைய செயல்
Successor activity	பிந்தைய செயல்
Tail event	இறுதி நிகழ்வு

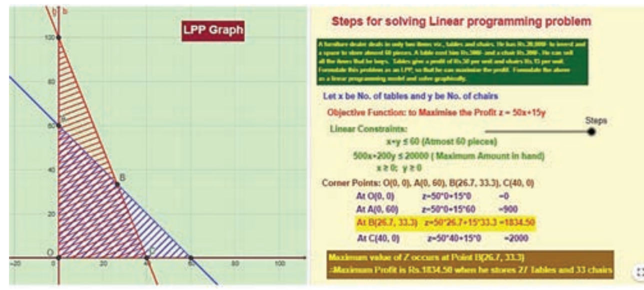
ICT Corner

Operational Research

Step – 1

Open the Browser type the URL Link given below (or) Scan the QR Code.

GeoGebra Workbook called "11th **BUSINESS MATHEMATICS and STATISTICS**" will appear. In that there are several worksheets related to your Text Book.

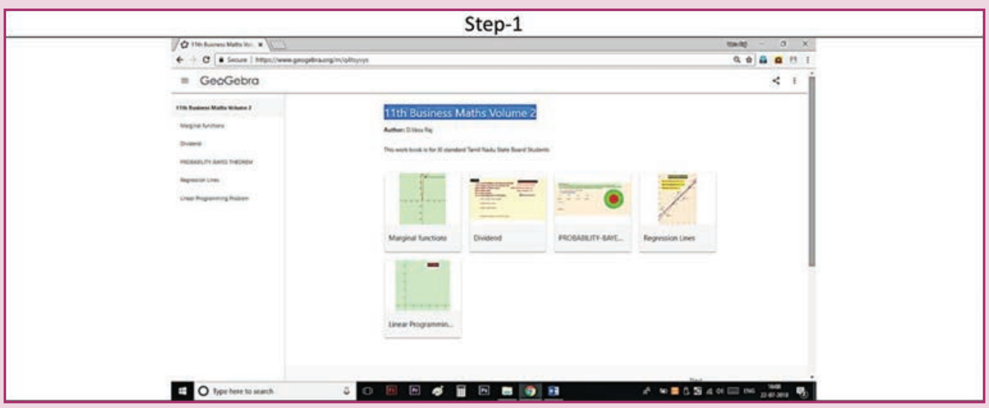


Expected Outcome ⇒

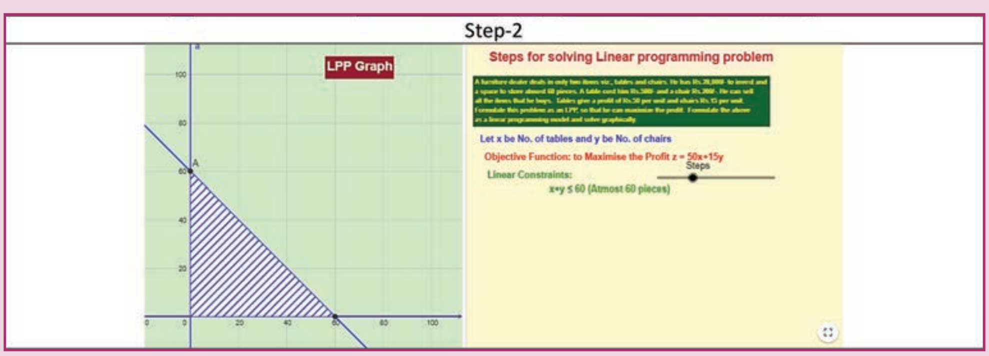
Step-2

Select the work sheet "Linear Programming Problem" Move the slider on Right side to see the steps for working Linear Programming Problem. Work out the problem as given. Graphical representation is given on left side. Also refer the worksheet "Inequality video" in the work book.

Step 1



Step 2



Browse in the link

11th Business Mathematics and Statistics
https://ggbm.at/qKj9gSTG (or) scan the QR Code

B184_11_BM_EM

ANSWERS

1. MATRICES AND DETERMINANTS

Exercise 1.1

1.(i) $M_{11} = -1 \quad M_{12} = 0 \quad M_{21} = 20 \quad M_{22} = 5$

$A_{11} = -1 \quad A_{12} = 0 \quad A_{21} = -20 \quad A_{22} = 5$

(ii) $M_{11} = -12 \quad M_{12} = 2 \quad M_{13} = 23 \quad M_{21} = -16 \quad M_{22} = -4 \quad M_{23} = 14$

$M_{31} = -4 \quad M_{32} = -6 \quad M_{33} = 11$

$A_{11} = -12 \quad A_{12} = -2 \quad A_{13} = 23 \quad A_{21} = 16 \quad A_{22} = -4 \quad A_{23} = -14$

$A_{31} = -4 \quad A_{32} = 6 \quad A_{33} = 11$

2. 20 3. $\frac{1}{2}$ 4. –30 5. $\frac{13}{2}, 2$ 6. 0

Exercise 1.2

1. $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$ 2. $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ 3.(i) $\frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ (ii) $\frac{1}{10}\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$ (iv) $\frac{1}{151}\begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$

7. $\frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$ 10. $\lambda = \frac{7}{4}$ 11. $p = 2, q = -3$

Exercise 1.3

1. $x = 1, y = 1$ 2.(i) $x = 3, y = -2, z = 1$

(ii) $x = -1, y = 2, z = 3$ (iii) $x = 1, y = -1, z = 2$ 3. 17.95, 43.08, 103.85

4. 3000, 1000, 2000 5. 13,2,5 6. 700, 600, 300

Exercise 1.4

1. It is viable
2. It is not viable
3. It is viable
4. $A = 27.82$ tonnes $B = 98.91$ tonnes
5. 181.62, 84.32
6. 34.16, 17.31,
7. 42 and 78

Exercise 1.5

1	2	3	4	5	6	7	8	9	10	11	12	13
(b)	(d)	(b)	(b)	(c)	(c)	(c)	(c)	(c)	(d)	(c)	(b)	(a)
14	15	16	17	18	19	20	21	22	23	24	25	
(c)	(c)	(c)	(b)	(a)	(d)	(b)	(b)	(d)	(d)	(b)	(a)	

Miscellaneous Problems

1. $x = 3.\ x = -1$
2. 0
6. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$
7. $\frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$
8. $x = 1, y = 2, z = 3$
9. $x = 20, y = 30, z = 50$
10. ₹1200 Crores, ₹1600 Crores

2. ALGEBRA

Exercise 2.1

1. $\frac{13}{x-2} - \frac{10}{x-1}$
2. $\frac{3}{x-2} + \frac{1}{x+1}$
3. $\frac{1}{9(x-1)} - \frac{1}{9(x+2)} - \frac{1}{3(x+2)^2}$
4. $\frac{1}{2(x-1)} - \frac{1}{2(x+1)}$
5. $\frac{4}{9(x-1)} - \frac{4}{9(x+2)} - \frac{1}{3(x-1)^2}$
6. $\frac{2}{x-2} + \frac{3}{(x-2)^2} - \frac{9}{(x-2)^3}$
7. $\frac{-7}{2x} + \frac{1}{x^2} + \frac{9}{2(x+2)}$
8. $\frac{1}{5(x+2)} + \frac{4(x-2)}{5(x^2+1)}$
9. $\frac{3}{16(x-1)} - \frac{3}{16(x+3)} + \frac{1}{4(x+3)^2}$
10. $\frac{1-x}{5(x^2+4)} + \frac{1}{5(x+1)}$

Exercise 2.2

1. 64
2. 10
3. 3
4. 336
5. 85

Exercise 2.3

1. 6
2. 14400
3. 1680
4. $\frac{13!}{4!\,3!\,2!\,2!}$
5. (a) $\frac{7!}{2}$ (b) 7!
6. Rank of the word CHAT = 9.

Exercise 2.4

1. 4
2. $8C_4 + 8C_3 = 9C_4 = 126$
3. 210 cards
4. 20
5. 25200
6. 671
7. $\frac{13!}{7!\,6!} \times 9! \times 9!$
8. 11

9.(i) 1365 (ii) 1001 (iii) 364

10.(i) 186 (ii) 186

Exercise 2.6

1.(i) $16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$

(ii) $x^7 + \frac{7x^6}{y} + \frac{21x^5}{y^2} + \frac{35x^4}{y^3} + \frac{35x^3}{y^4} + \frac{21x^2}{y^5} + \frac{7x}{y^6} + \frac{1}{y^7}$

(iii) $x^6 + 6x^3 + 15 + \frac{20}{x^3} + \frac{15}{x^6} + \frac{6}{x^9} + \frac{1}{x^{12}}$

2.(i) 10, 40,60,401 (ii)995009990004999

3. $11440\ x^9\ y^4$

4.(i) $11C_5 x$, $\frac{11C_5}{x}$ (ii) $\frac{8C_4(81)}{16}x^{12}$ (iii) $\frac{-10C_5(6)^5}{x^5}$

5.(i) $9C_6\frac{2^6}{3^6}$ (ii) $-32(15C_5)$ (iii) 7920.

Exercise 2.7

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(c)	(d)	(c)	(a)	(c)	(d)	(b)	(b)	(c)	(b)	(d)	(b)
14	15	16	17	18	19	20	21	22	23	24	25	
(a)	(c)	(a)	(c)	(a)	(c)	(a)	(b)	(c)	(d)	(b)	(a)	

Miscellaneous Problems

1. $\frac{3}{x-1} + \frac{2}{x+3}$
2. $\frac{3}{x-1} - \frac{2}{x-2}$
3. $\frac{1}{x+2} + \frac{5}{x-3} - \frac{3}{(x-3)^2}$
4. $\frac{3}{x-2} + \frac{2x-1}{x^2-x+1}$

5.(i) $\frac{7}{2}$ (ii) $\frac{7}{24}$ (iii) 12

6.(a) 720 (b) 7776 (c) 120 (d) 6

7. 74 ways
8. 85 ways
9. 210
10. 544

3. ANALYTICAL GEOMETRY

Exercise 3.1

1. $x^2 - 2x - 6y + 10 = 0$
2. $x^2 + y^2 - 6x + 4y - 3 = 0$
3. $3x^2 + 3y^2 - 4x - 14y + 15 = 0$
4. $\left(\frac{15}{2}, 0\right)$
5. $2x - 3y + 21 = 0$

Exercise 3.2

1. Angle between the line is 45°
2. 4 units
4. $a = 5$
5. $y = 1500x + 100000$, cost of 95 TV sets ₹ 2,42,500

Exercise 3.3

1. $a = 6,\ c = 6$
2. $2x - y + 2 = 0$ and $6x - 2y + 1 = 0$
3. $2x + 3y - 1 = 0$ and $2x + 3y - 2 = 0$
4. $\theta = \tan^{-1}(7)$

Exercise 3.4

1.(i) $x^2 + y^2 - 6x - 10y + 9 = 0$ (ii) $x^2 + y^2 - 4 = 0$

2.(i) $C(0, 0)$, $r = 4$ (ii) $C(11, 2)$, $r = 10$

(iii) $C\left(\frac{-2}{5}, \frac{4}{5}\right)$, $r = 2$ (iv) $C\left(\frac{3}{2}, \frac{3}{2}\right)$, $r = \frac{5}{\sqrt{2}}$

3. $x^2 + y^2 + 6x + 4y - 51 = 0$
4. $x^2 + y^2 - 4x - 6y + 11 = 0$
5. $x^2 + y^2 - 5x - 2y + 1 = 0$
6. $x^2 + y^2 - x - y = 0$
7. $x^2 + y^2 - 16x + 4y - 32 = 0$
8. $x^2 + y^2 - 2x - 12y + 27 = 0$
9. $x^2 + y^2 = 9$

Exercise 3.5

1. $x + 2 = 0$
2. R lies inside, P lies outside and Q lies on the circle.
3. $\sqrt{20}$ units
4. $p = \pm 20$.

Exercise 3.6

1. $9x^2 + 16y^2 + 24xy + 34x + 112y + 121 = 0$
2. Length of latus rectum = 1, Focus $F\left(\frac{1}{4}, 0\right)$

3.

Axis	Vertex	Focus	Equation of directrix	Length of latus rectum
$y = 4$	$V(1, 4)$	$F(3, 4)$	$x + 1 = 0$	8 units

4.

Problem	Axis	Vertex	Focus	Equation of directrix	Length of latusrectum
(a) $y^2 = 20x$	$y = 0$	$V(0, 0)$	$F(5, 0)$	$x = -5$	20 units
(b) $x^2 = 8y$	$x = 0$	$V(0, 0)$	$F(0, 2)$	$y = -2$	8 units
(c) $x^2 = -16y$	$x = 0$	$V(0, 0)$	$F(0, -4)$	$y = 4$	16 units

5. $(x-15)^2 = 5(y-55)$. The output and the average cost at the vertex are 15 kg. and ₹ 55.

6. when $x = 5$ months

Exercise 3.7

1	2	3	4	5	6	7	8	9	10	11	12	13
(b)	(c)	(c)	(c)	(b)	(c)	(a)	(c)	(a)	(b)	(c)	(a)	(d)
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
(a)	(a)	(d)	(a)	(c)	(b)	(b)	(a)	(d)	(b)	(b)	(b)	

Miscellaneous problems

1. $3x + y - 8 = 0$ 2. $y = 6x + 3000$ 4. $p = 1, p = 2$ 6. $a = 9, b = 8$

7. $(-1, -2)$ is on the line, $(1, 0)$ is above the line and $(-3, -4)$ is below the line

8. $(-2, -7)$ 9. $y^2 = -\frac{9}{2}x$

10. Axis : $y = 2$, Vertex : $(1, 2)$, Focus : $(2, 2)$,
Equation of directrix : $x = 0$ and Length of latus rectum : 4 units

4. TRIGONOMETRY

Exercise 4.1

1.(i) $\frac{\pi}{3}$ (ii) $\frac{5\pi}{6}$ (iii) $\frac{4\pi}{3}$ (iv) $\frac{-16\pi}{9}$

2.(i) $22^\circ 30'$ (ii) 324^0 (iii) $-171^\circ 48'$ (iv) 110°

3.(i) 1^{st} quadrant (ii) 3^{rd} quadrant (iii) 2^{nd} quadrant.

4.(i) $\frac{-\sqrt{3}}{2}$ (ii) $\frac{-\sqrt{3}}{2}$ (iii) $\frac{2}{\sqrt{3}}$ (iv) 1

(v) $\sqrt{2}$ 10. $\frac{-7}{2}$

Exercise 4.2

1.(i) $\frac{2\sqrt{2}}{\sqrt{3}-1}$ (ii) $-\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$ (iii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

2.(i) $\sin 92^\circ$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\cos 80^\circ$ (iv) $\frac{\sqrt{3}}{2}$

3.(i) $\frac{-33}{65}$ (ii) $\frac{-16}{65}$ (iii) $\frac{16}{33}$

6. $\frac{2}{11}$, $\alpha+\beta$ lies in 1^{st} quadrant. 7. $\pm\sqrt{2}-1$

9.(i) $\frac{9}{13}$ (ii) $-\frac{828}{2197}$ 10. $-\frac{44}{125}, -\frac{117}{44}$ 14. $\sqrt{2}-1$

Exercise 4.3

1.(i) $\frac{1}{2}\left(\cos\frac{A}{4}-\cos\frac{A}{2}\right)$ (ii) $\frac{1}{2}\left(-\sin 2A+\frac{\sqrt{3}}{2}\right)$

(iii) $\frac{1}{2}\left(\sin 4A-\sin\frac{2A}{3}\right)$ (iv) $\frac{1}{2}(\sin 10\theta - \sin 4\theta)$

2.(i) $2\sin\frac{3A}{2}\cos\frac{A}{2}$ (ii) $2\cos 3A\cos A$

(iii) $2\sin 2\theta\cos 4\theta$ (iv) $-2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}$

Exercise 4.4

1.(i) $\frac{-\pi}{6}$ (ii) $\frac{-\pi}{4}$ (iii) $\frac{\pi}{6}$ (iv) $\frac{3\pi}{4}$

4. $x=\frac{1}{6}$ 5. $x=\frac{1}{2}$

6.(i) $\frac{4}{5}$ (ii) $\frac{1}{\sqrt{10}}$ 7. $\frac{-33}{65}$ 10. $\frac{\pi}{4}+\frac{x}{2}$

Exercise 4.5

1	2	3	4	5	6	7	8	9	10	11	12	13
(b)	(a)	(c)	(b)	(b)	(d)	(a)	(d)	(a)	(c)	(c)	(d)	(b)
14	15	16	17	18	19	20	21	22	23	24	25	
(c)	(b)	(a)	(c)	(b)	(c)	(c)	(b)	(c)	(a)	(b)	(d)	

11_11th_BM-STAT_Answer-EM.indd 261 21-04-2020 12:08:57 PM

Miscellaneous Problems

4. $\dfrac{3}{\sqrt{10}}$ and $\dfrac{-1}{\sqrt{10}}$ 6.(i) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (ii) $2-\sqrt{3}$

7. $\dfrac{\sqrt{15}+2\sqrt{2}}{12}$ 9. $\dfrac{56}{33}$ 10. $\dfrac{\pi}{4}-x$

5. DIFFERENTIAL CALCULUS

Exercise 5.1

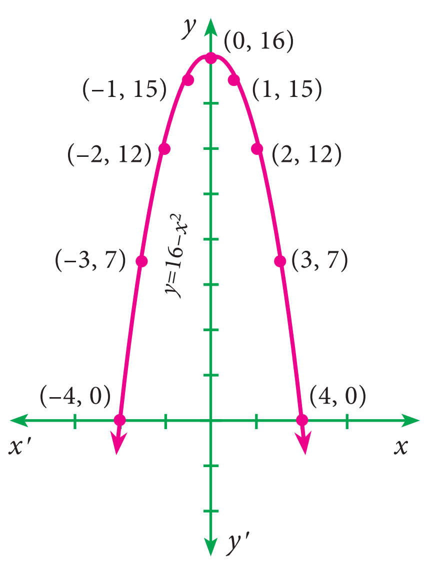
1.(i) Odd function   (ii) Even function

(iii) Neither even nor odd function   (iv) Even function

(v) Neither even nor odd function   2. $k = 0$ 5. $\dfrac{1-x}{3+x}$, $\dfrac{3x+1}{x-1}$

6.(i) e (ii) 0   (iii) $3e$ (iv) 0

7.(i)



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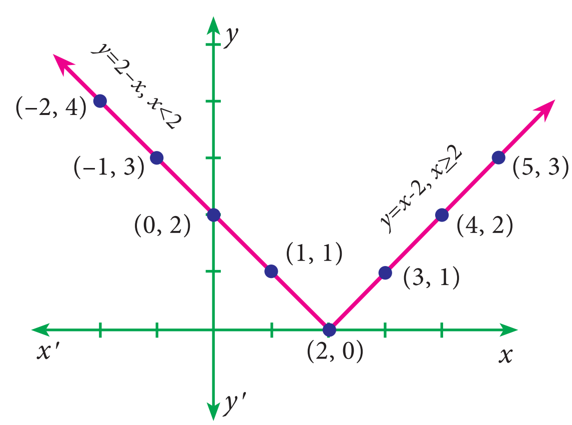

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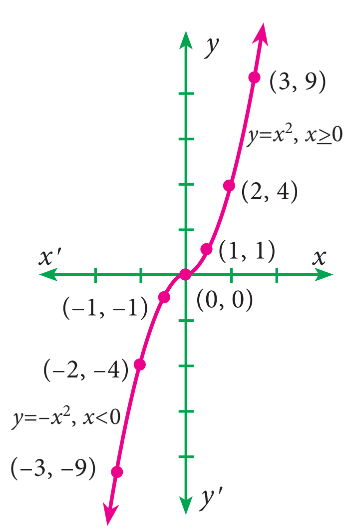

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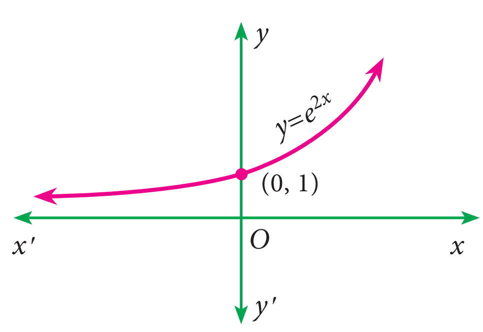

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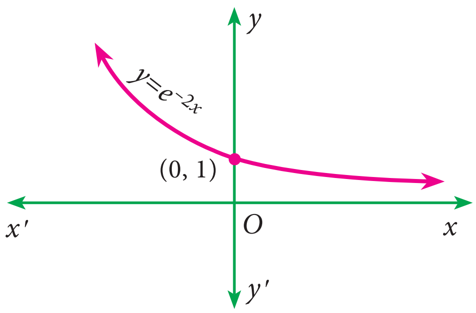

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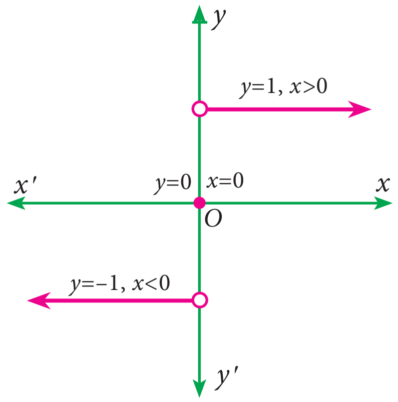


Exercise 5.2

1.(i) $\frac{10}{3}$ (ii) 0 (iii) $\frac{1}{2}$ (iv) 1

(v) $\frac{15}{16}a^{-\frac{1}{24}}$ (vi) 9

2. $a = \pm 1$ 3. $n = 7$ 4. $\frac{28}{5}$ 5. $f(-2) = 0$

Exercise 5.3

1.(a) Not a continuous function at $x = 2$ (b) continuous function at $x = 3$

Exercise 5.4

(i) $2x$ (ii) $-e^{-x}$ (iii) $\frac{1}{x+1}$

Exercise 5.5

1.(i) $12x^3 - 6x^2 + 1$ (ii) $\frac{-20}{x^5} + \frac{6}{x^4} - \frac{5}{x^2}$ (iii) $\frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^4}} + e^x$

(iv) $-\frac{3+x^2}{x^2}$ (v) $x^2 e^x (x+3)$ (vi) $3x^2 - 4x - 1$

(vii) $4x^3 - 3\cos x - \sin x$ (viii) $1 - \frac{1}{x^2}$

2.(i) $\frac{xe^x}{(1+x)^2}$ (ii) $\frac{2(1-x^2)}{(x^2-x+1)^2}$ (iii) $\frac{e^x}{(1+e^x)^2}$

3.(i) $x\cos x + \sin x$ (ii) $e^x(\sin x + \cos x)$

(iii) $e^x\left(1 + \frac{1}{x} + x + \log x\right)$ (iv) $\cos 2x$ (v) $e^x x^2 (x+3)$

4.(i) $\sin 2x$ (ii) $-\sin 2x$ (iii) $\frac{-3}{2}\cos x \sin 2x$ (iv) $\frac{x}{\sqrt{1+x^2}}$

(v) $n(ax^2+bx+c)^{n-1}(2ax+b)$ (vi) $2x(\cos(x^2))$ (vii) $\frac{-x}{(1+x^2)\sqrt{1+x^2}}$

Exercise 5.6

1.(i) $-\frac{y}{x}$ (ii) $\frac{y-2x}{2y-x}$ (iii) $-\frac{(x^2+ay)}{(y^2+ax)}$ 3. $\frac{4}{3}\left(\frac{1-4x+3y}{1+4x-3y}\right)$

Exercise 5.7

1.(i) $x^{\sin x}\left[\frac{\sin x}{x}+\cos x\log x\right]$ (ii) $(\sin x)^x\left[x\cot x+\log(\sin x)\right]$

(iii) $(\sin x)^{\tan x}\left[1+\sec^2 x\log(\sin x)\right]$

(iv) $\frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x^2+x+1)}}\left\{\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{2x+1}{x^2+x+1}\right\}$

Exercise 5.8

1.(i) $-\frac{1}{t^2}$ (ii) $t\cos t$ (iii) $-\tan\theta$ (iv) $\cot\frac{\theta}{2}$

2. $-\tan x$ 3. $\frac{\sin 2x}{2x}$

Exercise 5.9

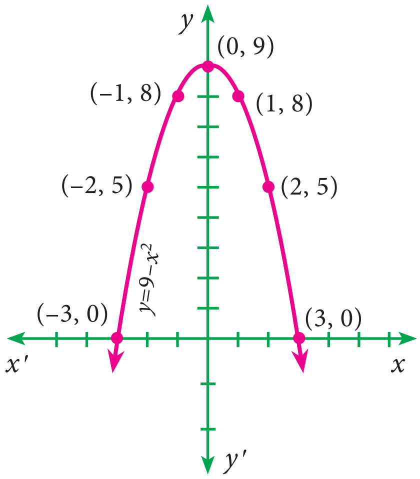
1.(i) $9y$ (ii) $-\frac{1}{x^2}+a^x(\log a)^2$ (iii) $-\frac{1}{a}\operatorname{cosec}^3\theta$

Exercise: 5.10

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(c)	(a)	(b)	(a)	(a)	(c)	(a)	(d)	(a)	(a)	(d)	(b)
14	15	16	17	18	19	20	21	22	23	24	25	
(a)	(a)	(a)	(c)	(b)	(d)	(a)	(a)	(c)	(a)	(b)	(d)	

Miscellaneous Problems

2.



4. $-\frac{1}{10}$

6. (i) continuous at $x = 1$; differentiable at $x = 1$

(ii) not continuous at $x = 2$; not differentiable at $x = 2$

6. APPLICATIONS OF DIFFERENTIATION

Exercise 6.1

1. $AC = \frac{1}{10}x^2 - 4x - 20 + \frac{7}{x}$, $AVC = \frac{1}{10}x^2 - 4x - 20$, $AFC = \frac{7}{x}$,

   $MC = \frac{3}{10}x^2 - 8x - 20$, $MAC = \frac{1}{5}x - 4 - \frac{7}{x^2}$

2. $C = ₹\,\frac{121}{6}$, $AC = ₹\,\frac{29}{12}$, $MC = ₹\,\frac{2}{3}$

3. $AC = x^2 - 2$, $MC = 3x^2 - 2$, $AR = 14 - x$, $MR = 14 - 2x$

4. $\eta_d = \frac{2}{x}$

5(i) $\eta_d = \frac{a - bx}{2bx}$, $x = \frac{a}{3b}$ (ii) $\eta_d = \frac{a - bx^2}{2bx^2}$, $x = \sqrt{\frac{a}{3b}}$

6. $\eta_s = \frac{4p^2}{2p^2 + 5}, \frac{36}{23}$

7. $MR = \frac{50 - 2x}{5}$, 10, 0

8. $\eta_s = \frac{p}{2(p - b)}$, 1

11. $\eta_d = 4$

12. $P = -\frac{x^2}{100} + 160x - 120$, $AP = \frac{-x}{100} + 160 - \frac{120}{x}$, ₹ 147.90

    $MP = \frac{-x}{50} + 160$, ₹159.80, $MAP = -\frac{1}{100} + \frac{120}{x^2}$, ₹ 1.19

13. $x = -8, 2$

15. $\eta_d = \frac{p}{10 - p}, |\eta_d| = \frac{3}{2} > 1 \Rightarrow$ elastic.

16. P_E=₹ 30, $x_E = 40$ units

17. $x = 2100$ units, p = ₹ 130

18. $x = 6$ units

Exercise 6.2

1. AC is increasing when $x > 5$
3. P is maximum when $x \approx 46$, maximum profit ₹ 1107.68
4. Revenue is maximum when $x = 220$
5. Minimum value is –71, maximum value is 54

Exercise 6.3

1.

Items	EOQ in units	Minimum Inventory Cost	EOQ in ₹	EOQ in years of supply	Number of order per year
A	2000	₹4	40	2.5	0.4
B	200	₹20	200	0.5	2
C	2627	₹52.54	525.40	0.19	5.26

2 (i) ≈ 913 units per order (ii) ≈ ₹20, 066 per week.

Exercise 6.4

1. $\dfrac{\partial z}{\partial x} = a\left(cy+d\right), \dfrac{\partial z}{\partial y} = c\left(ax+b\right)$

Exercise 6.5

1. 23, 25 3. −2, 8 4. 0.8832, 2.2081 5. $-\dfrac{4}{3}, -8$ 6. $\dfrac{10}{79}, -\dfrac{3}{79}$

Exercise 6.6

1	2	3	4	5	6	7	8	9	10
(d)	(a)	(b)	(a)	(b)	(a)	(b)	(c)	(b)	(a)
11	12	13	14	15	16	17	18	19	20
(a)	(c)	(b)	(b)	(a)	(b)	(b)	(b)	(b)	(c)

Miscellaneous Problems

1. $AC = \dfrac{10}{x} - 4x^2 + 3x^3, MC = -12x^2 + 12x^3, MAC = \dfrac{-10}{x^2} - 8x + 9x^2$

2.(i) $\eta_s = \dfrac{1}{x+1}$ (ii) $\eta_d = \dfrac{3}{x}$

3. $\eta_s = \dfrac{4p^2}{2p^2+5}, \dfrac{4}{7}$

6. TC is increasing in [0.20] ∪ [30, ∞) and decreasing in [20, 30]

7. x = 3, TC is minimum

7. FINANCIAL MATHEMATICS

Exercise 7.1

1. ₹ 68,429 2. ₹ 1,20,800 3. ₹ 18,930 4. ₹ 500

5. ₹ 23,79,660 6. ₹ 14,736 7. ₹ 8,433 8. ₹ 1,17,612

9. ₹ 14,339 10. ₹ 1,000

11_11th_BM-STAT_Answer-EM.indd 266 21-04-2020 12:09:08 PM

Exercise 7.2

1. ₹ 8,184 2. ₹ 2,250 3. 900 shares

4.(i) 242 (ii) ₹ 3630 (iii) $12\frac{1}{2}\%$

5.(i) ₹ 5,000 (ii) ₹ 480 (iii) 9.6% 6. ₹ 897.50

7. ₹ 7000, ₹ 6500 8. 99 shares

9.(i) ₹ 945 (ii) ₹ 960 2nd investment is better.

10.(i) 1400 (ii) 1400. For the same investment both stocks fetch the same income. Therefore they are equivalent shares.

Exercise 7.3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(c)	(c)	(d)	(c)	(a)	(b)	(a)	(b)	(b)	(b)	(a)	(a)	(b)

Miscellaneous Problems

1. ₹ 9,282 2. ₹ 15,638 3. ₹ 4,327; ₹ 1,25,780; ₹ 29,334; ₹ 1,39,560.

4. Required months ≈ 24 5. ₹ 12,500

6. ₹ 13,250; ₹ 36,443.75; Machine B may be purchased

7. 270, 216, 300, 450

8. ₹ 700, ₹ 900, ₹ 200, 2.5% 9. 1,500 shares, ₹ 625 10. 20%

8. DESCRIPTIVE STATISTICS AND PROBABILITY

Exercise 8.1

1. $Q_1 = 6$, $Q_3 = 18$ 2. $Q_1 = 5$, $Q_3 = 6.5$, $D_8 = 6.5$ and $P_{67} = 6$

3. $Q_1 = 47.14$, $Q_3 = 63.44$, $D_5 = 55.58$, $D_7 = 61.56$ and $P_{60} = 58.37$

4. $GM = 142.5$ lbs 5. $GM = 26.1\%$

6. 192 km/hr 7. 38.92 km/hr

8. $AM=36$ $GM=25.46$ $HM=17.33$

9. $AM=21.96$ $GM=18.31$ $HM=14.32$

10. AM=33, GM=29.51, HM=24.10

11. $Q_1 = 30$, $Q_3 = 70$, $Q_D = 20$, Coefficient of $QD = 0.4$

12. QD= 11.02 , Coefficient of QD = 0.3384

13. Median = 61, MD =1.71

14. Mean = 13, MD = 4.33

15. Median = 45.14 , MD = 14.83

Exercise 8.2

1. $\frac{1}{3}$ 2. $\frac{2}{5}$

3. A and B are independent events 4.(i) $\frac{2}{3}$ (ii) $\frac{1}{2}$

5. 3/10 6.(i) 42/625 (ii) 207/625

7. 33/68 8.(i) 7/29 (ii) 5/29 (iii) 17/29

9. 4/11 10. P(A)=4/7 P(B) = 2/7 P(C) = 1/7

11.(i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ 12. $\frac{1}{2}$ 13. 0.2

14. 0.012 15.(i) 1/221 (ii) 1/17 16. P(C/D)= 0.5208

Exercise 8.3

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(c)	(d)	(a)	(c)	(a)	(d)	(c)	(b)	(a)	(d)	(c)	(a)
14	15	16	17	18	19	20	21	22	23	24	25	
(b)	(d)	(a)	(a)	(b)	(c)	(d)	(d)	(b)	(a)	(b)	(a)	

Miscellaneous Problems

1. 16.02 tons 2. 16 3. Median=28, MD = 10.16

4. Mean=7.5, MD =2.3 5. QD=10, Coefficient of QD=0.5

6. 0.45 7.(i) 3/10 (ii) 3/5 (iii) 1/10

8. 0.948 9. 0.727 10. 0.393

9. CORRELATION AND REGRESSION ANALYSIS

Exercise 9.1

1. 0.575
2. 0.947
3. 0.996
4. 0.891
5. 0.225
6. −0.0735
7. 0.9
8. 0.224
9. 0.905
10. −0.37

Exercise 9.2

1.(a) Y=−0.66X+59.12; X=−0.234Y+40.892

(b) r=-0.394 (c) Y= 39.32

2. Y=0.6102X+66.12; X=0.556Y+74.62 Height of son is 166.19

3. Y=2.3X−35.67; weight of the student is 125.79 lb

4. Y=0.24X+1.04; X=1.33Y+1.34

5. Y=1.6X; estimated yield = 46.4 unit per acre

6. Y=0.942X+6.08; Estimated sales =34.34 (in crores of rupees)

7. Y=0.48X+67.72; X=0.91Y−41.35;Y=72.52

8. b_{yx}=1.33; Y=1.33X+3.35

9. Y=0.1565X+19.94; estimated expenditure on food and entertainment (Y) is 51.24

10. X=0.8Y−1 and estimated value of X is 5.4 when Y=8

Y=0.8X+2.6 and estimated value of Y is 12.2 when X=12

11. $\bar{X}$ = 13; $\bar{Y}$ =17 and r = 0.6

12. $b_{xy} = -\frac{3}{2}; b_{yx} = -\frac{1}{2}; r = -0.866$

Exercise 9.3

1	2	3	4	5	6	7	8	9	10	11	12	13
(a)	(b)	(a)	(b)	(c)	(a)	(a)	(b)	(a)	(b)	(a)	(a)	(c)
14	15	16	17	18	19	20	21	22	23	24	25	
(a)	(b)	(a)	(a)	(a)	(a)	(a)	(b)	(b)	(b)	(a)	(d)	

Miscellaneous Problems

1. 0.906 2. 0.382 3. 0.95 4. 0.667

5. 0.905 6. Y=0.653X+21.71; Marks in B = 55.67

7. Y=0.576X+2.332; Y =5.788 8. Y=0.867X+7.913

9. Y=–0.652X+63.945, Y=10.481≈10 10. b_{yx}=1.422, Y = 141.67

10. OPERATIONS RESEARCH

Exercise: 10.1

1. Maximize $Z = 5x_1 + 3x_2$ subject to constraints $2x_1 + x_2 \leq 1000$; $x_1 \leq 400; x_2 \leq 700$ and $x_1, x_2 \geq 0$.

2. Maximize $Z = 30x_1 + 40x_2$ subject to constraints $60x_1 + 120x_2 \leq 12000$ $8x_1 + 5x_2 \leq 600; 3x_1 + 4x_2 \leq 500$ and $x_1, x_2 \geq 0$.

3. Maximize Z=100x_1+150x_2 subject to constraints 0.8$x_1 + 1.2x_2 \leq 720; x_1 \leq 600;$ $x_2 \leq 400$ and $x_1, x_2 \geq 0$.

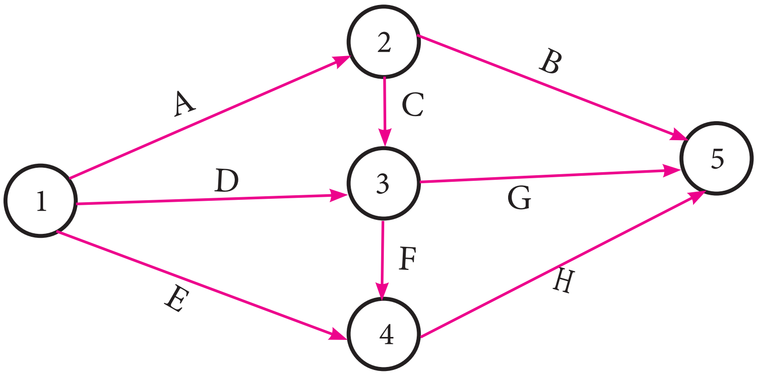
4.(i) $x_1 = 4$; $x_2 = 9$ and $Z_{max} = 96$ (ii) $x_1 = 8$; $x_2 = 12$ and $Z_{max} = 392$

(iii) $x_1 = 1$; $x_2 = 5$ and $Z_{min} = 13$ (iv) $x_1 = 2$; $x_2 = 3$ and $Z_{max} = 230$

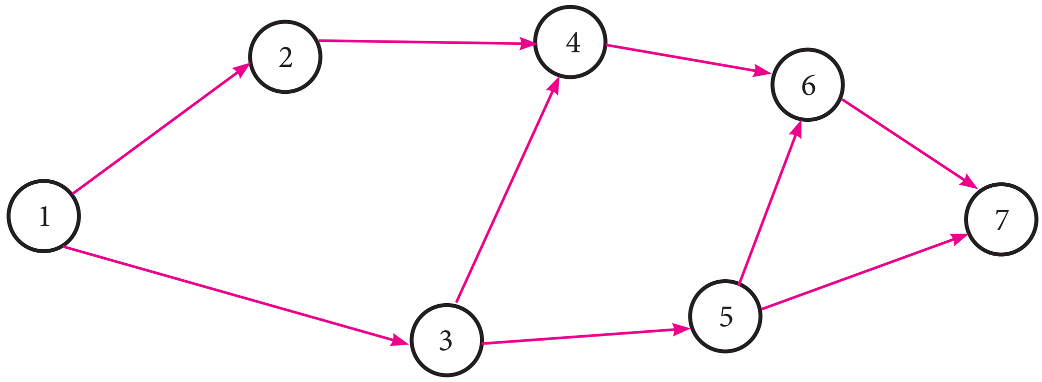
(v) $x_1 = 3$; $x_2 = 9$ and $Z_{max} = 330$ (vi) $x_1 = 4$; $x_2 = 2$ and $Z_{min} = 160$

Exercise: 10.2

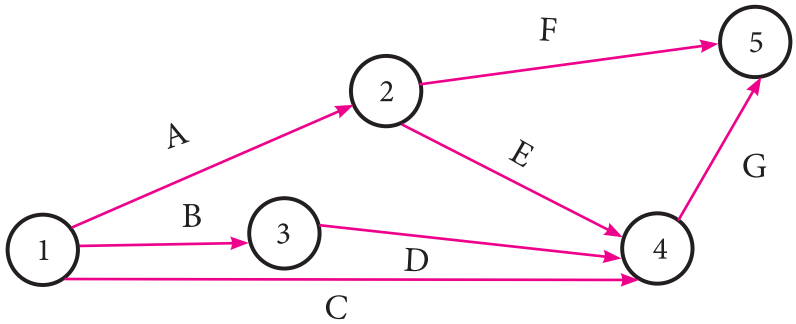
1.



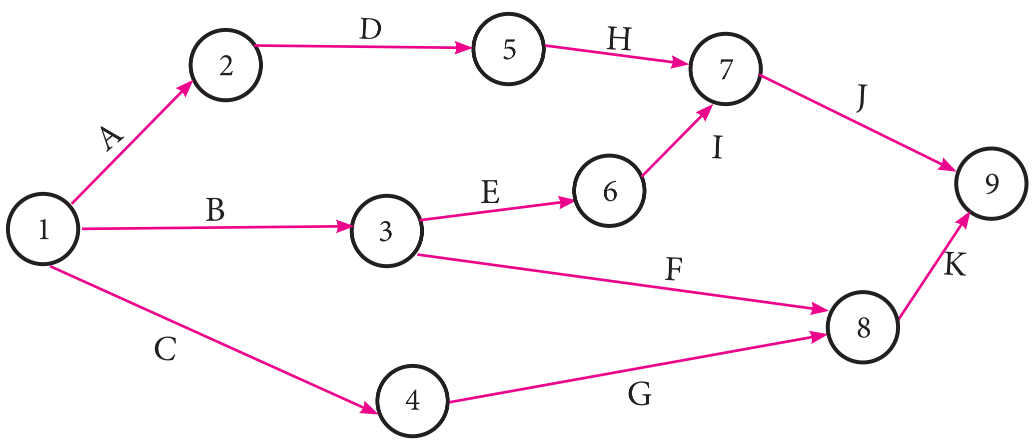
2.



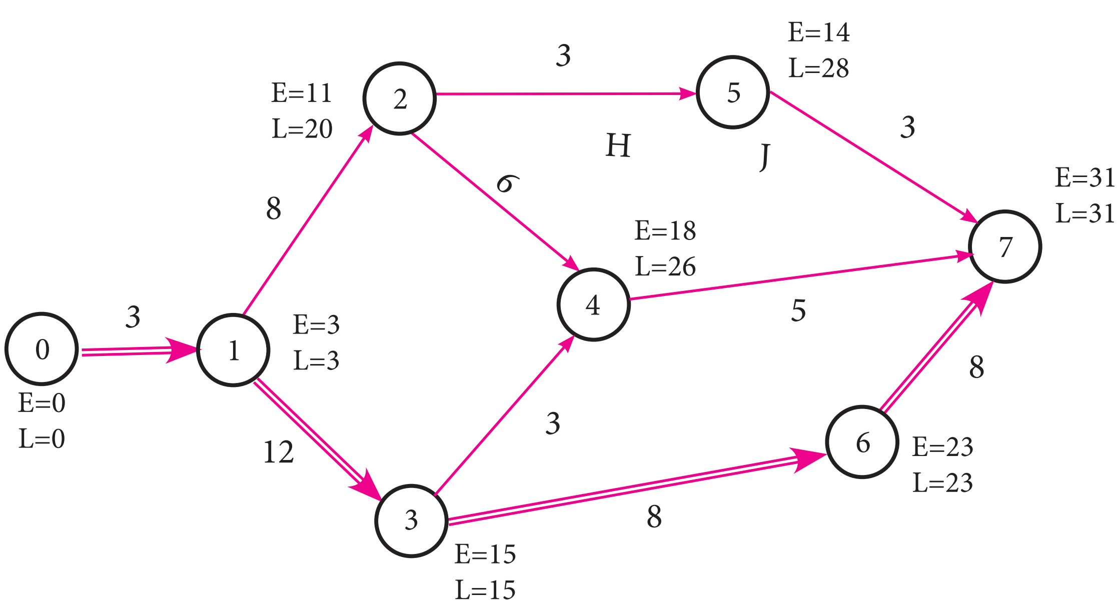
3.



4.

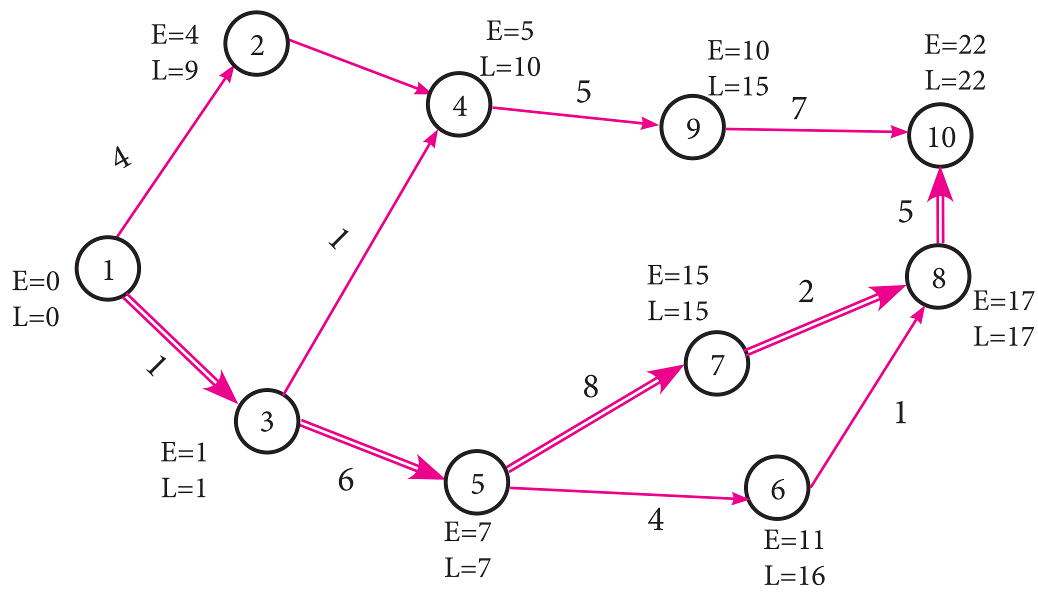


5.



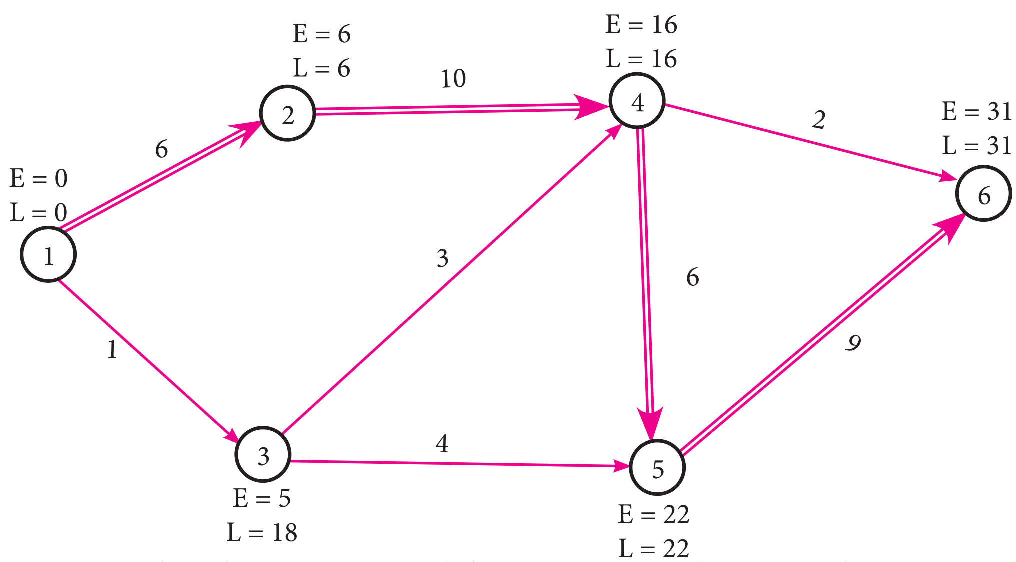
Critical path 0-1-3-6-7 and the duration is 31 weeks

6.



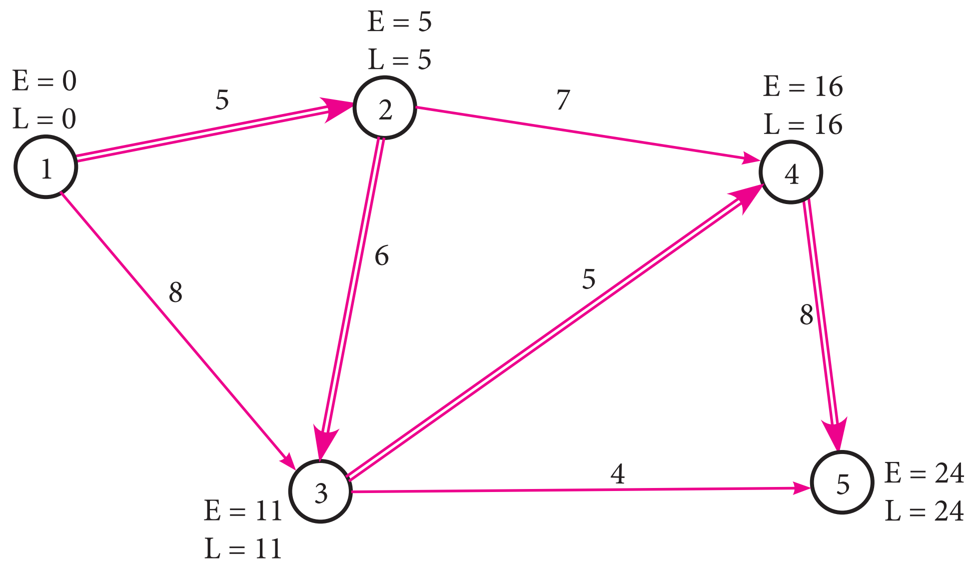
Critical path 1-3-5-7-8-10 and duration 22 time units

7.



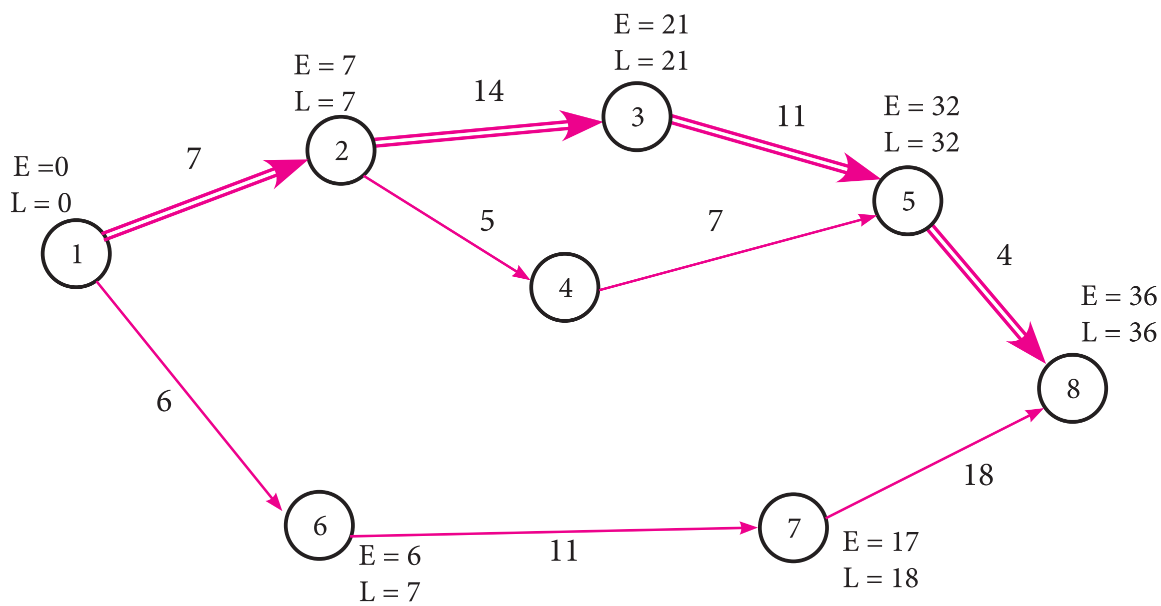
Critical path 1-2-4-5-6 and duration time taken is 31 days

8



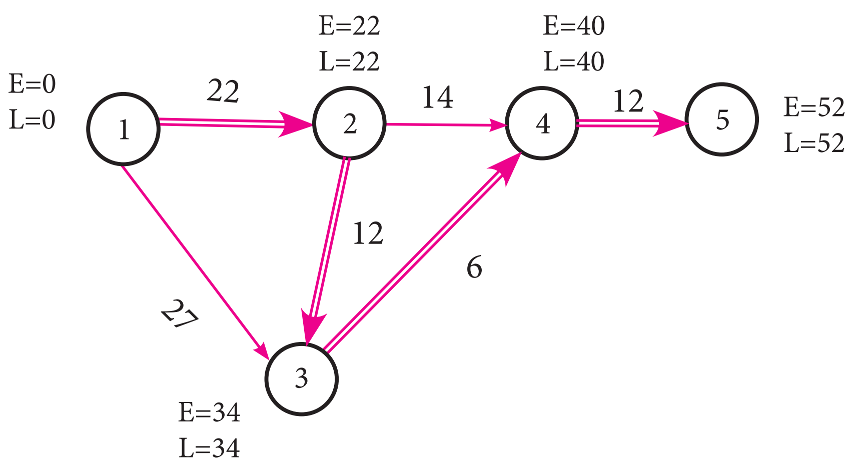
Critical path 1-2-3-4-5 and duration time taken is 24 days

9.



Critical path 1-2-3-5-8 and duration time taken is 36 days

10.



Critical path 1-2-3-4-5 and duration time taken is 52 days

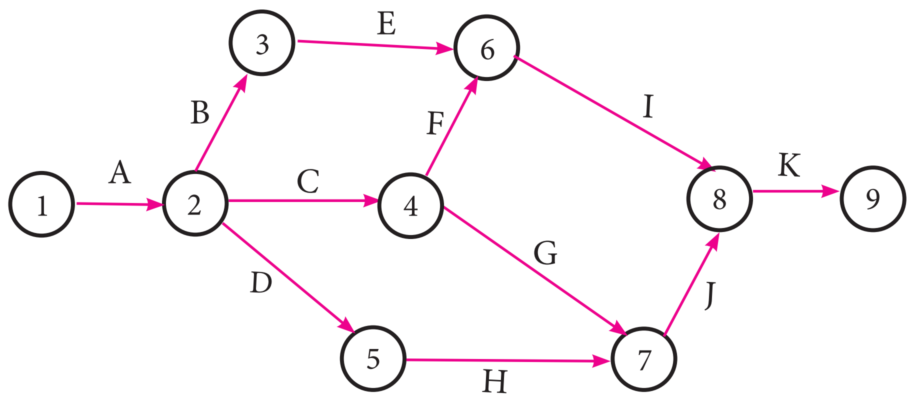
Exercise-10.3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(c)	(a)	(b)	(d)	(c)	(c)	(b)	(c)	(d)	(d)	(b)	(d)	(a)	(a)

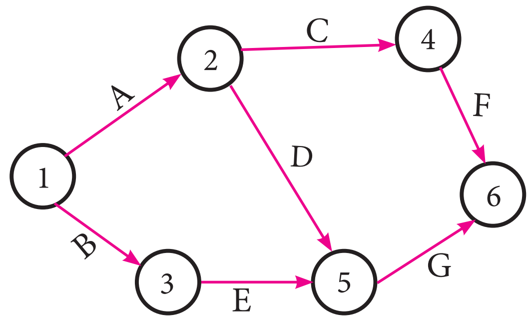
Miscellaneous problems

1. Maximize $Z = 3x_1 + 4x_2$ subject to constraints $x_1 + x_2 \le 450$; $2x_1 + x_2 \le 600$ and $x_1, x_2 \ge 0$
2. Minimize $Z = x_1 + x_2$ subject to constraints $2x_1 + x_2 \ge 12$; $5x_1 + 8x_2 \ge 74$; $x_1 + 6x_2 \ge 24$ and $x_1, x_2 \ge 0$
3. x_1=30; x_2 = 0 and Z_{max} = 120
4. x_1 = 4; x_2 = 3 and Z_{min} = 2300
5. x_1 = 1 ; x_2 = 5 and Z_max= 28
6. x_1 = 20; x_2=30 and Z_{max} = 1650

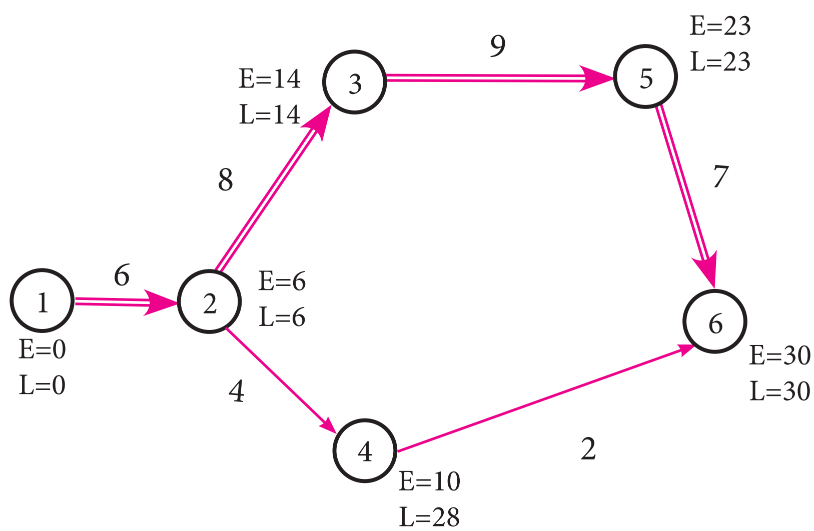
7.



8.

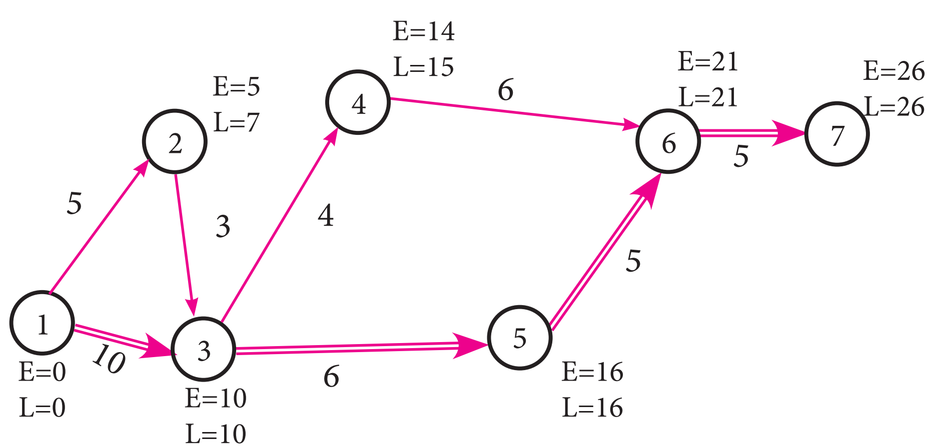


9.



Critical path 1-2-3-5-6 and duration time taken is 30 days.

10.



Critical path 1-3-5-6-7 and duration time taken is 26 days.

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.0374	4	8	12	17	21	25	29	33	37
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0719	0.0755	4	8	11	15	19	23	26	30	34
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1072	0.1106	3	7	10	14	17	21	24	28	31
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1399	0.1430	3	6	10	13	16	19	23	26	29
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1703	0.1732	3	6	9	12	15	18	21	24	27
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1987	0.2014	3	6	8	11	14	17	20	22	25
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.2279	3	5	8	11	13	16	18	21	24
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.2529	2	5	7	10	12	15	17	20	22
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2742	0.2765	2	5	7	9	12	14	16	19	21
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2967	0.2989	2	4	7	9	11	13	16	18	20
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3181	0.3201	2	4	6	8	11	13	15	17	19
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3385	0.3404	2	4	6	8	10	12	14	16	18
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.3598	2	4	6	8	10	12	14	15	17
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.3784	2	4	6	7	9	11	13	15	17
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3945	0.3962	2	4	5	7	9	11	12	14	16
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4116	0.4133	2	3	5	7	9	10	12	14	15
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4281	0.4298	2	3	5	7	8	10	11	13	15
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4440	0.4456	2	3	5	6	8	9	11	13	14
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4594	0.4609	2	3	5	6	8	9	11	12	14
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4742	0.4757	1	3	4	6	7	9	10	12	13
3.0	0.4771	0.4786	0.4800	0.4814	0.4829	0.4843	0.4857	0.4871	0.4886	0.4900	1	3	4	6	7	9	10	11	13
3.1	0.4914	0.4928	0.4942	0.4955	0.4969	0.4983	0.4997	0.5011	0.5024	0.5038	1	3	4	6	7	8	10	11	12
3.2	0.5051	0.5065	0.5079	0.5092	0.5105	0.5119	0.5132	0.5145	0.5159	0.5172	1	3	4	5	7	8	9	11	12
3.3	0.5185	0.5198	0.5211	0.5224	0.5237	0.5250	0.5263	0.5276	0.5289	0.5302	1	3	4	5	6	8	9	10	12
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.5428	1	3	4	5	6	8	9	10	11
3.5	0.5441	0.5453	0.5465	0.5478	0.5490	0.5502	0.5514	0.5527	0.5539	0.5551	1	2	4	5	6	7	9	10	11
3.6	0.5563	0.5575	0.5587	0.5599	0.5611	0.5623	0.5635	0.5647	0.5658	0.5670	1	2	4	5	6	7	8	10	11
3.7	0.5682	0.5694	0.5705	0.5717	0.5729	0.5740	0.5752	0.5763	0.5775	0.5786	1	2	3	5	6	7	8	9	10
3.8	0.5798	0.5809	0.5821	0.5832	0.5843	0.5855	0.5866	0.5877	0.5888	0.5899	1	2	3	5	6	7	8	9	10
3.9	0.5911	0.5922	0.5933	0.5944	0.5955	0.5966	0.5977	0.5988	0.5999	0.6010	1	2	3	4	5	7	8	9	10
4.0	0.6021	0.6031	0.6042	0.6053	0.6064	0.6075	0.6085	0.6096	0.6107	0.6117	1	2	3	4	5	6	8	9	10
4.1	0.6128	0.6138	0.6149	0.6160	0.6170	0.6180	0.6191	0.6201	0.6212	0.6222	1	2	3	4	5	6	7	8	9
4.2	0.6232	0.6243	0.6253	0.6263	0.6274	0.6284	0.6294	0.6304	0.6314	0.6325	1	2	3	4	5	6	7	8	9
4.3	0.6335	0.6345	0.6355	0.6365	0.6375	0.6385	0.6395	0.6405	0.6415	0.6425	1	2	3	4	5	6	7	8	9
4.4	0.6435	0.6444	0.6454	0.6464	0.6474	0.6484	0.6493	0.6503	0.6513	0.6522	1	2	3	4	5	6	7	8	9
4.5	0.6532	0.6542	0.6551	0.6561	0.6571	0.6580	0.6590	0.6599	0.6609	0.6618	1	2	3	4	5	6	7	8	9
4.6	0.6628	0.6637	0.6646	0.6656	0.6665	0.6675	0.6684	0.6693	0.6702	0.6712	1	2	3	4	5	6	7	7	8
4.7	0.6721	0.6730	0.6739	0.6749	0.6758	0.6767	0.6776	0.6785	0.6794	0.6803	1	2	3	4	5	5	6	7	8
4.8	0.6812	0.6821	0.6830	0.6839	0.6848	0.6857	0.6866	0.6875	0.6884	0.6893	1	2	3	4	4	5	6	7	8
4.9	0.6902	0.6911	0.6920	0.6928	0.6937	0.6946	0.6955	0.6964	0.6972	0.6981	1	2	3	4	4	5	6	7	8
5.0	0.6990	0.6998	0.7007	0.7016	0.7024	0.7033	0.7042	0.7050	0.7059	0.7067	1	2	3	3	4	5	6	7	8
5.1	0.7076	0.7084	0.7093	0.7101	0.7110	0.7118	0.7126	0.7135	0.7143	0.7152	1	2	3	3	4	5	6	7	8
5.2	0.7160	0.7168	0.7177	0.7185	0.7193	0.7202	0.7210	0.7218	0.7226	0.7235	1	2	2	3	4	5	6	7	7
5.3	0.7243	0.7251	0.7259	0.7267	0.7275	0.7284	0.7292	0.7300	0.7308	0.7316	1	2	2	3	4	5	6	6	7
5.4	0.7324	0.7332	0.7340	0.7348	0.7356	0.7364	0.7372	0.7380	0.7388	0.7396	1	2	2	3	4	5	6	6	7

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.5	0.7404	0.7412	0.7419	0.7427	0.7435	0.7443	0.7451	0.7459	0.7466	0.7474	1	2	2	3	4	5	5	6	7
5.6	0.7482	0.7490	0.7497	0.7505	0.7513	0.7520	0.7528	0.7536	0.7543	0.7551	1	2	2	3	4	5	5	6	7
5.7	0.7559	0.7566	0.7574	0.7582	0.7589	0.7597	0.7604	0.7612	0.7619	0.7627	1	2	2	3	4	5	5	6	7
5.8	0.7634	0.7642	0.7649	0.7657	0.7664	0.7672	0.7679	0.7686	0.7694	0.7701	1	1	2	3	4	4	5	6	7
5.9	0.7709	0.7716	0.7723	0.7731	0.7738	0.7745	0.7752	0.7760	0.7767	0.7774	1	1	2	3	4	4	5	6	7
6.0	0.7782	0.7789	0.7796	0.7803	0.7810	0.7818	0.7825	0.7832	0.7839	0.7846	1	1	2	3	4	4	5	6	6
6.1	0.7853	0.7860	0.7868	0.7875	0.7882	0.7889	0.7896	0.7903	0.7910	0.7917	1	1	2	3	4	4	5	6	6
6.2	0.7924	0.7931	0.7938	0.7945	0.7952	0.7959	0.7966	0.7973	0.7980	0.7987	1	1	2	3	3	4	5	6	6
6.3	0.7993	0.8000	0.8007	0.8014	0.8021	0.8028	0.8035	0.8041	0.8048	0.8055	1	1	2	3	3	4	5	5	6
6.4	0.8062	0.8069	0.8075	0.8082	0.8089	0.8096	0.8102	0.8109	0.8116	0.8122	1	1	2	3	3	4	5	5	6
6.5	0.8129	0.8136	0.8142	0.8149	0.8156	0.8162	0.8169	0.8176	0.8182	0.8189	1	1	2	3	3	4	5	5	6
6.6	0.8195	0.8202	0.8209	0.8215	0.8222	0.8228	0.8235	0.8241	0.8248	0.8254	1	1	2	3	3	4	5	5	6
6.7	0.8261	0.8267	0.8274	0.8280	0.8287	0.8293	0.8299	0.8306	0.8312	0.8319	1	1	2	3	3	4	5	5	6
6.8	0.8325	0.8331	0.8338	0.8344	0.8351	0.8357	0.8363	0.8370	0.8376	0.8382	1	1	2	3	3	4	4	5	6
6.9	0.8388	0.8395	0.8401	0.8407	0.8414	0.8420	0.8426	0.8432	0.8439	0.8445	1	1	2	2	3	4	4	5	6
7.0	0.8451	0.8457	0.8463	0.8470	0.8476	0.8482	0.8488	0.8494	0.8500	0.8506	1	1	2	2	3	4	4	5	6
7.1	0.8513	0.8519	0.8525	0.8531	0.8537	0.8543	0.8549	0.8555	0.8561	0.8567	1	1	2	2	3	4	4	5	5
7.2	0.8573	0.8579	0.8585	0.8591	0.8597	0.8603	0.8609	0.8615	0.8621	0.8627	1	1	2	2	3	4	4	5	5
7.3	0.8633	0.8639	0.8645	0.8651	0.8657	0.8663	0.8669	0.8675	0.8681	0.8686	1	1	2	2	3	4	4	5	5
7.4	0.8692	0.8698	0.8704	0.8710	0.8716	0.8722	0.8727	0.8733	0.8739	0.8745	1	1	2	2	3	4	4	5	5
7.5	0.8751	0.8756	0.8762	0.8768	0.8774	0.8779	0.8785	0.8791	0.8797	0.8802	1	1	2	2	3	3	4	5	5
7.6	0.8808	0.8814	0.8820	0.8825	0.8831	0.8837	0.8842	0.8848	0.8854	0.8859	1	1	2	2	3	3	4	5	5
7.7	0.8865	0.8871	0.8876	0.8882	0.8887	0.8893	0.8899	0.8904	0.8910	0.8915	1	1	2	2	3	3	4	4	5
7.8	0.8921	0.8927	0.8932	0.8938	0.8943	0.8949	0.8954	0.8960	0.8965	0.8971	1	1	2	2	3	3	4	4	5
7.9	0.8976	0.8982	0.8987	0.8993	0.8998	0.9004	0.9009	0.9015	0.9020	0.9025	1	1	2	2	3	3	4	4	5
8.0	0.9031	0.9036	0.9042	0.9047	0.9053	0.9058	0.9063	0.9069	0.9074	0.9079	1	1	2	2	3	3	4	4	5
8.1	0.9085	0.9090	0.9096	0.9101	0.9106	0.9112	0.9117	0.9122	0.9128	0.9133	1	1	2	2	3	3	4	4	5
8.2	0.9138	0.9143	0.9149	0.9154	0.9159	0.9165	0.9170	0.9175	0.9180	0.9186	1	1	2	2	3	3	4	4	5
8.3	0.9191	0.9196	0.9201	0.9206	0.9212	0.9217	0.9222	0.9227	0.9232	0.9238	1	1	2	2	3	3	4	4	5
8.4	0.9243	0.9248	0.9253	0.9258	0.9263	0.9269	0.9274	0.9279	0.9284	0.9289	1	1	2	2	3	3	4	4	5
8.5	0.9294	0.9299	0.9304	0.9309	0.9315	0.9320	0.9325	0.9330	0.9335	0.9340	1	1	2	2	3	3	4	4	5
8.6	0.9345	0.9350	0.9355	0.9360	0.9365	0.9370	0.9375	0.9380	0.9385	0.9390	1	1	2	2	3	3	4	4	5
8.7	0.9395	0.9400	0.9405	0.9410	0.9415	0.9420	0.9425	0.9430	0.9435	0.9440	0	1	1	2	2	3	3	4	4
8.8	0.9445	0.9450	0.9455	0.9460	0.9465	0.9469	0.9474	0.9479	0.9484	0.9489	0	1	1	2	2	3	3	4	4
8.9	0.9494	0.9499	0.9504	0.9509	0.9513	0.9518	0.9523	0.9528	0.9533	0.9538	0	1	1	2	2	3	3	4	4
9.0	0.9542	0.9547	0.9552	0.9557	0.9562	0.9566	0.9571	0.9576	0.9581	0.9586	0	1	1	2	2	3	3	4	4
9.1	0.9590	0.9595	0.9600	0.9605	0.9609	0.9614	0.9619	0.9624	0.9628	0.9633	0	1	1	2	2	3	3	4	4
9.2	0.9638	0.9643	0.9647	0.9652	0.9657	0.9661	0.9666	0.9671	0.9675	0.9680	0	1	1	2	2	3	3	4	4
9.3	0.9685	0.9689	0.9694	0.9699	0.9703	0.9708	0.9713	0.9717	0.9722	0.9727	0	1	1	2	2	3	3	4	4
9.4	0.9731	0.9736	0.9741	0.9745	0.9750	0.9754	0.9759	0.9763	0.9768	0.9773	0	1	1	2	2	3	3	4	4
9.5	0.9777	0.9782	0.9786	0.9791	0.9795	0.9800	0.9805	0.9809	0.9814	0.9818	0	1	1	2	2	3	3	4	4
9.6	0.9823	0.9827	0.9832	0.9836	0.9841	0.9845	0.9850	0.9854	0.9859	0.9863	0	1	1	2	2	3	3	4	4
9.7	0.9868	0.9872	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	0.9908	0	1	1	2	2	3	3	4	4
9.8	0.9912	0.9917	0.9921	0.9926	0.9930	0.9934	0.9939	0.9943	0.9948	0.9952	0	1	1	2	2	3	3	4	4
9.9	0.9956	0.9961	0.9965	0.9969	0.9974	0.9978	0.9983	0.9987	0.9991	0.9996	0	1	1	2	2	3	3	3	4

12_11th_BM-STAT_LogTable_EM.indd 276 21-04-2020 12:07:19 PM

ANTI LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.00	1.000	1.002	1.005	1.007	1.009	1.012	1.014	1.016	1.019	1.021	0	0	1	1	1	1	2	2	2
0.01	1.023	1.026	1.028	1.030	1.033	1.035	1.038	1.040	1.042	1.045	0	0	1	1	1	1	2	2	2
0.02	1.047	1.050	1.052	1.054	1.057	1.059	1.062	1.064	1.067	1.069	0	0	1	1	1	1	2	2	2
0.03	1.072	1.074	1.076	1.079	1.081	1.084	1.086	1.089	1.091	1.094	0	0	1	1	1	1	2	2	2
0.04	1.096	1.099	1.102	1.104	1.107	1.109	1.112	1.114	1.117	1.119	0	1	1	1	1	2	2	2	2
0.05	1.122	1.125	1.127	1.130	1.132	1.135	1.138	1.140	1.143	1.146	0	1	1	1	1	2	2	2	2
0.06	1.148	1.151	1.153	1.156	1.159	1.161	1.164	1.167	1.169	1.172	0	1	1	1	1	2	2	2	2
0.07	1.175	1.178	1.180	1.183	1.186	1.189	1.191	1.194	1.197	1.199	0	1	1	1	1	2	2	2	2
0.08	1.202	1.205	1.208	1.211	1.213	1.216	1.219	1.222	1.225	1.227	0	1	1	1	1	2	2	2	3
0.09	1.230	1.233	1.236	1.239	1.242	1.245	1.247	1.250	1.253	1.256	0	1	1	1	1	2	2	2	3
0.10	1.259	1.262	1.265	1.268	1.271	1.274	1.276	1.279	1.282	1.285	0	1	1	1	1	2	2	2	3
0.11	1.288	1.291	1.294	1.297	1.300	1.303	1.306	1.309	1.312	1.315	0	1	1	1	2	2	2	2	3
0.12	1.318	1.321	1.324	1.327	1.330	1.334	1.337	1.340	1.343	1.346	0	1	1	1	2	2	2	2	3
0.13	1.349	1.352	1.355	1.358	1.361	1.365	1.368	1.371	1.374	1.377	0	1	1	1	2	2	2	3	3
0.14	1.380	1.384	1.387	1.390	1.393	1.396	1.400	1.403	1.406	1.409	0	1	1	1	2	2	2	3	3
0.15	1.413	1.416	1.419	1.422	1.426	1.429	1.432	1.435	1.439	1.442	0	1	1	1	2	2	2	3	3
0.16	1.445	1.449	1.452	1.455	1.459	1.462	1.466	1.469	1.472	1.476	0	1	1	1	2	2	2	3	3
0.17	1.479	1.483	1.486	1.489	1.493	1.496	1.500	1.503	1.507	1.510	0	1	1	1	2	2	2	3	3
0.18	1.514	1.517	1.521	1.524	1.528	1.531	1.535	1.538	1.542	1.545	0	1	1	1	2	2	2	3	3
0.19	1.549	1.552	1.556	1.560	1.563	1.567	1.570	1.574	1.578	1.581	0	1	1	1	2	2	3	3	3
0.20	1.585	1.589	1.592	1.596	1.600	1.603	1.607	1.611	1.614	1.618	0	1	1	1	2	2	3	3	3
0.21	1.622	1.626	1.629	1.633	1.637	1.641	1.644	1.648	1.652	1.656	0	1	1	2	2	2	3	3	3
0.22	1.660	1.663	1.667	1.671	1.675	1.679	1.683	1.687	1.690	1.694	0	1	1	2	2	2	3	3	3
0.23	1.698	1.702	1.706	1.710	1.714	1.718	1.722	1.726	1.730	1.734	0	1	1	2	2	2	3	3	4
0.24	1.738	1.742	1.746	1.750	1.754	1.758	1.762	1.766	1.770	1.774	0	1	1	2	2	2	3	3	4
0.25	1.778	1.782	1.786	1.791	1.795	1.799	1.803	1.807	1.811	1.816	0	1	1	2	2	2	3	3	4
0.26	1.820	1.824	1.828	1.832	1.837	1.841	1.845	1.849	1.854	1.858	0	1	1	2	2	3	3	3	4
0.27	1.862	1.866	1.871	1.875	1.879	1.884	1.888	1.892	1.897	1.901	0	1	1	2	2	3	3	3	4
0.28	1.905	1.910	1.914	1.919	1.923	1.928	1.932	1.936	1.941	1.945	0	1	1	2	2	3	3	4	4
0.29	1.950	1.954	1.959	1.963	1.968	1.972	1.977	1.982	1.986	1.991	0	1	1	2	2	3	3	4	4
0.30	1.995	2.000	2.004	2.009	2.014	2.018	2.023	2.028	2.032	2.037	0	1	1	2	2	3	3	4	4
0.31	2.042	2.046	2.051	2.056	2.061	2.065	2.070	2.075	2.080	2.084	0	1	1	2	2	3	3	4	4
0.32	2.089	2.094	2.099	2.104	2.109	2.113	2.118	2.123	2.128	2.133	0	1	1	2	2	3	3	4	4
0.33	2.138	2.143	2.148	2.153	2.158	2.163	2.168	2.173	2.178	2.183	0	1	1	2	2	3	3	4	4
0.34	2.188	2.193	2.198	2.203	2.208	2.213	2.218	2.223	2.228	2.234	1	1	2	2	3	3	4	4	5
0.35	2.239	2.244	2.249	2.254	2.259	2.265	2.270	2.275	2.280	2.286	1	1	2	2	3	3	4	4	5
0.36	2.291	2.296	2.301	2.307	2.312	2.317	2.323	2.328	2.333	2.339	1	1	2	2	3	3	4	4	5
0.37	2.344	2.350	2.355	2.360	2.366	2.371	2.377	2.382	2.388	2.393	1	1	2	2	3	3	4	4	5
0.38	2.399	2.404	2.410	2.415	2.421	2.427	2.432	2.438	2.443	2.449	1	1	2	2	3	3	4	4	5
0.39	2.455	2.460	2.466	2.472	2.477	2.483	2.489	2.495	2.500	2.506	1	1	2	2	3	3	4	5	5
0.40	2.512	2.518	2.523	2.529	2.535	2.541	2.547	2.553	2.559	2.564	1	1	2	2	3	4	4	5	5
0.41	2.570	2.576	2.582	2.588	2.594	2.600	2.606	2.612	2.618	2.624	1	1	2	2	3	4	4	5	5
0.42	2.630	2.636	2.642	2.649	2.655	2.661	2.667	2.673	2.679	2.685	1	1	2	2	3	4	4	5	6
0.43	2.692	2.698	2.704	2.710	2.716	2.723	2.729	2.735	2.742	2.748	1	1	2	3	3	4	4	5	6
0.44	2.754	2.761	2.767	2.773	2.780	2.786	2.793	2.799	2.805	2.812	1	1	2	3	3	4	4	5	6
0.45	2.818	2.825	2.831	2.838	2.844	2.851	2.858	2.864	2.871	2.877	1	1	2	3	3	4	5	5	6
0.46	2.884	2.891	2.897	2.904	2.911	2.917	2.924	2.931	2.938	2.944	1	1	2	3	3	4	5	5	6
0.47	2.951	2.958	2.965	2.972	2.979	2.985	2.992	2.999	3.006	3.013	1	1	2	3	3	4	5	5	6
0.48	3.020	3.027	3.034	3.041	3.048	3.055	3.062	3.069	3.076	3.083	1	1	2	3	4	4	5	6	6
0.49	3.090	3.097	3.105	3.112	3.119	3.126	3.133	3.141	3.148	3.155	1	1	2	3	4	4	5	6	6

ANTI LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.50	3.162	3.170	3.177	3.184	3.192	3.199	3.206	3.214	3.221	3.228	1	1	2	3	4	4	5	6	7
0.51	3.236	3.243	3.251	3.258	3.266	3.273	3.281	3.289	3.296	3.304	1	2	2	3	4	5	5	6	7
0.52	3.311	3.319	3.327	3.334	3.342	3.350	3.357	3.365	3.373	3.381	1	2	2	3	4	5	5	6	7
0.53	3.388	3.396	3.404	3.412	3.420	3.428	3.436	3.443	3.451	3.459	1	2	2	3	4	5	6	6	7
0.54	3.467	3.475	3.483	3.491	3.499	3.508	3.516	3.524	3.532	3.540	1	2	2	3	4	5	6	6	7
0.55	3.548	3.556	3.565	3.573	3.581	3.589	3.597	3.606	3.614	3.622	1	2	2	3	4	5	6	7	7
0.56	3.631	3.639	3.648	3.656	3.664	3.673	3.681	3.690	3.698	3.707	1	2	3	3	4	5	6	7	8
0.57	3.715	3.724	3.733	3.741	3.750	3.758	3.767	3.776	3.784	3.793	1	2	3	3	4	5	6	7	8
0.58	3.802	3.811	3.819	3.828	3.837	3.846	3.855	3.864	3.873	3.882	1	2	3	4	4	5	6	7	8
0.59	3.890	3.899	3.908	3.917	3.926	3.936	3.945	3.954	3.963	3.972	1	2	3	4	5	5	6	7	8
0.60	3.981	3.990	3.999	4.009	4.018	4.027	4.036	4.046	4.055	4.064	1	2	3	4	5	6	6	7	8
0.61	4.074	4.083	4.093	4.102	4.111	4.121	4.130	4.140	4.150	4.159	1	2	3	4	5	6	7	8	9
0.62	4.169	4.178	4.188	4.198	4.207	4.217	4.227	4.236	4.246	4.256	1	2	3	4	5	6	7	8	9
0.63	4.266	4.276	4.285	4.295	4.305	4.315	4.325	4.335	4.345	4.355	1	2	3	4	5	6	7	8	9
0.64	4.365	4.375	4.385	4.395	4.406	4.416	4.426	4.436	4.446	4.457	1	2	3	4	5	6	7	8	9
0.65	4.467	4.477	4.487	4.498	4.508	4.519	4.529	4.539	4.550	4.560	1	2	3	4	5	6	7	8	9
0.66	4.571	4.581	4.592	4.603	4.613	4.624	4.634	4.645	4.656	4.667	1	2	3	4	5	6	7	9	10
0.67	4.677	4.688	4.699	4.710	4.721	4.732	4.742	4.753	4.764	4.775	1	2	3	4	5	7	8	9	10
0.68	4.786	4.797	4.808	4.819	4.831	4.842	4.853	4.864	4.875	4.887	1	2	3	4	6	7	8	9	10
0.69	4.898	4.909	4.920	4.932	4.943	4.955	4.966	4.977	4.989	5.000	1	2	3	5	6	7	8	9	10
0.70	5.012	5.023	5.035	5.047	5.058	5.070	5.082	5.093	5.105	5.117	1	2	4	5	6	7	8	9	11
0.71	5.129	5.140	5.152	5.164	5.176	5.188	5.200	5.212	5.224	5.236	1	2	4	5	6	7	8	10	11
0.72	5.248	5.260	5.272	5.284	5.297	5.309	5.321	5.333	5.346	5.358	1	2	4	5	6	7	9	10	11
0.73	5.370	5.383	5.395	5.408	5.420	5.433	5.445	5.458	5.470	5.483	1	3	4	5	6	8	9	10	11
0.74	5.495	5.508	5.521	5.534	5.546	5.559	5.572	5.585	5.598	5.610	1	3	4	5	6	8	9	10	12
0.75	5.623	5.636	5.649	5.662	5.675	5.689	5.702	5.715	5.728	5.741	1	3	4	5	7	8	9	10	12
0.76	5.754	5.768	5.781	5.794	5.808	5.821	5.834	5.848	5.861	5.875	1	3	4	5	7	8	9	11	12
0.77	5.888	5.902	5.916	5.929	5.943	5.957	5.970	5.984	5.998	6.012	1	3	4	5	7	8	10	11	12
0.78	6.026	6.039	6.053	6.067	6.081	6.095	6.109	6.124	6.138	6.152	1	3	4	6	7	8	10	11	13
0.79	6.166	6.180	6.194	6.209	6.223	6.237	6.252	6.266	6.281	6.295	1	3	4	6	7	9	10	11	13
0.80	6.310	6.324	6.339	6.353	6.368	6.383	6.397	6.412	6.427	6.442	1	3	4	6	7	9	10	12	13
0.81	6.457	6.471	6.486	6.501	6.516	6.531	6.546	6.561	6.577	6.592	2	3	5	6	8	9	11	12	14
0.82	6.607	6.622	6.637	6.653	6.668	6.683	6.699	6.714	6.730	6.745	2	3	5	6	8	9	11	12	14
0.83	6.761	6.776	6.792	6.808	6.823	6.839	6.855	6.871	6.887	6.902	2	3	5	6	8	9	11	13	14
0.84	6.918	6.934	6.950	6.966	6.982	6.998	7.015	7.031	7.047	7.063	2	3	5	6	8	10	11	13	15
0.85	7.079	7.096	7.112	7.129	7.145	7.161	7.178	7.194	7.211	7.228	2	3	5	7	8	10	12	13	15
0.86	7.244	7.261	7.278	7.295	7.311	7.328	7.345	7.362	7.379	7.396	2	3	5	7	8	10	12	13	15
0.87	7.413	7.430	7.447	7.464	7.482	7.499	7.516	7.534	7.551	7.568	2	3	5	7	9	10	12	14	16
0.88	7.586	7.603	7.621	7.638	7.656	7.674	7.691	7.709	7.727	7.745	2	4	5	7	9	11	12	14	16
0.89	7.762	7.780	7.798	7.816	7.834	7.852	7.870	7.889	7.907	7.925	2	4	5	7	9	11	13	14	16
0.90	7.943	7.962	7.980	7.998	8.017	8.035	8.054	8.072	8.091	8.110	2	4	6	7	9	11	13	15	17
0.91	8.128	8.147	8.166	8.185	8.204	8.222	8.241	8.260	8.279	8.299	2	4	6	8	9	11	13	15	17
0.92	8.318	8.337	8.356	8.375	8.395	8.414	8.433	8.453	8.472	8.492	2	4	6	8	10	12	14	15	17
0.93	8.511	8.531	8.551	8.570	8.590	8.610	8.630	8.650	8.670	8.690	2	4	6	8	10	12	14	16	18
0.94	8.710	8.730	8.750	8.770	8.790	8.810	8.831	8.851	8.872	8.892	2	4	6	8	10	12	14	16	18
0.95	8.913	8.933	8.954	8.974	8.995	9.016	9.036	9.057	9.078	9.099	2	4	6	8	10	12	15	17	19
0.96	9.120	9.141	9.162	9.183	9.204	9.226	9.247	9.268	9.290	9.311	2	4	6	8	11	13	15	17	19
0.97	9.333	9.354	9.376	9.397	9.419	9.441	9.462	9.484	9.506	9.528	2	4	7	9	11	13	15	17	20
0.98	9.550	9.572	9.594	9.616	9.638	9.661	9.683	9.705	9.727	9.750	2	4	7	9	11	13	16	18	20
0.99	9.772	9.795	9.817	9.840	9.863	9.886	9.908	9.931	9.954	9.977	2	5	7	9	11	14	16	18	20

EXPONENTIAL FUNCTION TABLE

	0	1	2	3	4	5	6	7	8	9
0.00	1.00000000	2.71828183	7.38905610	20.08553692	54.59815003	148.41315910	403.42879349	1096.63315843	2980.95798704	8103.08392758
0.01	1.01005017	2.74560102	7.46331735	20.28739993	55.14687056	149.90473615	407.48332027	1107.65450490	3010.91711288	8184.52127494
0.02	1.02020134	2.77319476	7.53832493	20.49129168	55.70110583	151.41130379	411.57859573	1118.78661775	3041.17733294	8266.77708126
0.03	1.03045453	2.80106583	7.61408636	20.69723259	56.26091125	152.93301270	415.71502938	1130.03061019	3071.74167327	8349.85957218
0.04	1.04081077	2.82921701	7.69060920	20.90524324	56.82634281	154.47001503	419.89303489	1141.38760663	3102.61319033	8433.77705601
0.05	1.05127110	2.85765112	7.76790111	21.11534442	57.39745705	156.02246449	424.11303004	1152.85874278	3133.79497129	8518.53792457
0.06	1.06183655	2.88637099	7.84596981	21.32755716	57.97431108	157.59051632	428.37543686	1164.44516577	3165.29013436	8604.15065402
0.07	1.07250818	2.91537950	7.92482312	21.54190268	58.55696259	159.17432734	432.68068157	1176.14803425	3197.10182908	8690.62380571
0.08	1.08328707	2.94467955	8.00446891	21.75840240	59.14546985	160.77405593	437.02919472	1187.96851851	3229.23323664	8777.96602703
0.09	1.09417428	2.97427407	8.08491516	21.97707798	59.73989170	162.38986205	441.42141115	1199.90780061	3261.68757023	8866.18605226
0.10	1.10517092	3.00416602	8.16616991	22.19795128	60.34028760	164.02190730	445.85777008	1211.96707449	3294.46807528	8955.29270348
0.11	1.11627807	3.03435839	8.24824128	22.42104440	60.94671757	165.67035487	450.33871517	1224.14754609	3327.57802989	9045.29489144
0.12	1.12749685	3.06485420	8.33113749	22.64637964	61.55924226	167.33536962	454.86469450	1236.45043347	3361.02074508	9136.20161642
0.13	1.13882838	3.09565650	8.41486681	22.87397954	62.17792293	169.01711804	459.43616068	1248.87696691	3394.79956514	9228.02196918
0.14	1.15027380	3.12676837	8.49943763	23.10386686	62.80282145	170.71576832	464.05357086	1261.42838910	3428.91786799	9320.76513183
0.15	1.16183424	3.15819291	8.58485840	23.33606458	63.43400030	172.43149032	468.71738678	1274.10595517	3463.37906548	9414.44037876
0.16	1.17351087	3.18993328	8.67113766	23.57059593	64.07152260	174.16445561	473.42807483	1286.91093291	3498.18660376	9509.05707757
0.17	1.18530485	3.22199264	8.75828404	23.80748436	64.71545211	175.91483748	478.18610609	1299.84460280	3533.34396362	9604.62469001
0.18	1.19721736	3.25437420	8.84630626	24.04675355	65.36585321	177.68281099	482.99195635	1312.90825825	3568.85466082	9701.15277293
0.19	1.20924960	3.28708121	8.93521311	24.28842744	66.02279096	179.46855293	487.84610621	1326.10320561	3604.72224646	9798.65097920
0.20	1.22140276	3.32011692	9.02501350	24.53253020	66.68633104	181.27224188	492.74904109	1339.43076439	3640.95030733	9897.12905874
0.21	1.23367806	3.35348465	9.11571639	24.77908622	67.35653981	183.09405819	497.70125129	1352.89226737	3677.54246627	9996.59685944
0.22	1.24607673	3.38718773	9.20733087	25.02812018	68.03348429	184.93418407	502.70323202	1366.48906071	3714.50238251	10097.06432815
0.23	1.25860001	3.42122954	9.29986608	25.27965697	68.71723217	186.79280352	507.75548350	1380.22250409	3751.83375209	10198.54151171
0.24	1.27124915	3.45561346	9.39333129	25.53372175	69.40785184	188.67010241	512.85851094	1394.09397087	3789.54030817	10301.03855791
0.25	1.28402542	3.49034296	9.48773584	25.79033992	70.10541235	190.56626846	518.01282467	1408.10484820	3827.62582144	10404.56571656
0.26	1.29693009	3.52542149	9.58308917	26.04953714	70.80998345	192.48149130	523.21894011	1422.25653720	3866.09410048	10509.13334045
0.27	1.30996445	3.56085256	9.67940081	26.31133934	71.52163562	194.41596245	528.47737788	1436.55045304	3904.94899215	10614.75188643
0.28	1.32312981	3.59663973	9.77668041	26.57577270	72.24044001	196.36987535	533.78866383	1450.98802511	3944.19438198	10721.43191645
0.29	1.33642749	3.63278656	9.87493768	26.84286366	72.96646850	198.34342541	539.15332908	1465.57069720	3983.83419453	10829.18409859
0.30	1.34985881	3.66929667	9.97418245	27.11263892	73.69979370	200.33680997	544.57191013	1480.29992758	4023.87239382	10938.01920817
0.31	1.36342511	3.70617371	10.07442466	27.38512547	74.44048894	202.35022839	550.04494881	1495.17718919	4064.31298371	11047.94812878
0.32	1.37712776	3.74342138	10.17567431	27.66035056	75.18862829	204.38388199	555.57299245	1510.20396976	4105.16000827	11158.98185341
0.33	1.39096813	3.78104339	10.27794153	27.93834170	75.94428657	206.43797416	561.15659385	1525.38177199	4146.41755226	11271.13148552
0.34	1.40494759	3.81904351	10.38123656	28.21912671	76.70753934	208.51271029	566.79631138	1540.71211367	4188.08974147	11384.40824018
0.35	1.41906755	3.85742553	10.48556972	28.50273364	77.47846293	210.60829787	572.49270901	1556.19652784	4230.18074313	11498.82344515
0.36	1.43332941	3.89619330	10.59095145	28.78919088	78.25713442	212.72494645	578.24635639	1571.83656296	4272.69476640	11614.38854204
0.37	1.44773461	3.93535070	10.69739228	29.07852706	79.04363170	214.86286770	584.05782889	1587.63378304	4315.63606270	11731.11508747
0.38	1.46228459	3.97490163	10.80490286	29.37077111	79.83803341	217.02227542	589.92770766	1603.58976783	4359.00892620	11849.01475419
0.39	1.47698079	4.01485005	10.91349394	29.66595227	80.64041898	219.20338555	595.85657969	1619.70611293	4402.81769423	11968.09933225
0.40	1.49182470	4.05519997	11.02317638	29.96410005	81.45086866	221.40641620	601.84503787	1635.98443000	4447.06674770	12088.38073022
0.41	1.50681779	4.09595540	11.13396115	30.26524426	82.26946350	223.63158768	607.89368106	1652.42634686	4491.76051155	12209.87097633
0.42	1.52196156	4.13712044	11.24585931	30.56941502	83.09628536	225.87912250	614.00311413	1669.03350774	4536.90345519	12332.58221972
0.43	1.53725752	4.17869919	11.35888208	30.87664275	83.93141691	228.14924542	620.17394801	1685.80757337	4582.50009296	12456.52673161
0.44	1.55270722	4.22069582	11.47304074	31.18695817	84.77494167	230.44218346	626.40679981	1702.75022115	4628.55498456	12581.71690655
0.45	1.56831219	4.26311452	11.58834672	31.50039231	85.62694400	232.75816591	632.70229281	1719.86314538	4675.07273551	12708.16526367
0.46	1.58407398	4.30595953	11.70481154	31.81697651	86.48750910	235.09742437	639.06105657	1737.14805735	4722.05799763	12835.88444790
0.47	1.59999419	4.34923514	11.82244685	32.13674244	87.35672301	237.46019276	645.48372697	1754.60668558	4769.51546949	12964.88723127
0.48	1.61607440	4.39294568	11.94126442	32.45972208	88.23467268	239.84670737	651.97094627	1772.24077593	4817.44989687	13095.18651418
0.49	1.63231622	4.43709552	12.06127612	32.78594771	89.12144588	242.25720686	658.52336322	1790.05209184	4865.86607325	13226.79532664
0.50	1.64872127	4.48168907	12.18249396	33.11545196	90.01713130	244.69193226	665.14163304	1808.04241446	4914.76884030	13359.72682966

EXPONENTIAL FUNCTION TABLE

	0	1	2	3	4	5	6	7	8	9
0.51	1.66529119	4.52673079	12.30493006	33.44826778	90.92181851	247.15112707	671.82641759	1826.21354282	4964.16308832	13493.99431650
0.52	1.68202765	4.57222520	12.42859666	33.78442846	91.83559798	249.63503719	678.57838534	1844.56729405	5014.05375679	13629.61121401
0.53	1.69893231	4.61817682	12.55350614	34.12396761	92.75856108	252.14391102	685.39821149	1863.10550356	5064.44583482	13766.59108401
0.54	1.71600686	4.66459027	12.67967097	34.46691919	93.69080012	254.67799946	692.28657804	1881.83002516	5115.34436165	13904.94762458
0.55	1.73325302	4.71147018	12.80710378	34.81331749	94.63240831	257.23755591	699.24417382	1900.74273134	5166.75442718	14044.69467150
0.56	1.75067250	4.75882125	12.93581732	35.16319715	95.58347983	259.82283632	706.27169460	1919.84551337	5218.68117245	14185.84619960
0.57	1.76826705	4.80664819	13.06582444	35.51659315	96.54410977	262.43409924	713.36984313	1939.14028156	5271.12979019	14328.41632413
0.58	1.78603843	4.85495581	13.19713816	35.87354085	97.51439421	265.07160579	720.53932925	1958.62896539	5324.10552531	14472.41930224
0.59	1.80398842	4.90374893	13.32977160	36.23407593	98.49443016	267.73561971	727.78086990	1978.31351375	5377.61367541	14617.86953434
0.60	1.82211880	4.95303242	13.46373804	36.59823444	99.48431564	270.42640743	735.09518924	1998.19589510	5431.65959136	14764.78156558
0.61	1.84043140	5.00281123	13.59905085	36.96605281	100.48414964	273.14423800	742.48301872	2018.27809772	5486.24867780	14913.17008727
0.62	1.85892804	5.05309032	13.73572359	37.33756782	101.49403213	275.88938323	749.94509711	2038.56212982	5541.38639368	15063.04993840
0.63	1.87761058	5.10387472	13.87376990	37.71281662	102.51406411	278.66211763	757.48217064	2059.05001984	5597.07825281	15214.43610708
0.64	1.89648088	5.15516951	14.01320361	38.09183673	103.54434758	281.46271848	765.09499302	2079.74381657	5653.32982444	15367.34373205
0.65	1.91554083	5.20697983	14.15403865	38.47466605	104.58498558	284.29146582	772.78432554	2100.64558942	5710.14673375	15521.78810420
0.66	1.93479233	5.25931084	14.29628910	38.86134287	105.63608216	287.14864256	780.55093713	2121.75742858	5767.53466250	15677.78466809
0.67	1.95423732	5.31216780	14.43996919	39.25190586	106.69774243	290.03453439	788.39560446	2143.08144525	5825.49934952	15835.34902351
0.68	1.97387773	5.36555597	14.58509330	39.64639407	107.77007257	292.94942992	796.31911202	2164.61977185	5884.04659134	15994.49692704
0.69	1.99371553	5.41948071	14.73167592	40.04484696	108.85317981	295.89362064	804.32225214	2186.37456223	5943.18224271	16155.24429358
0.70	2.01375271	5.47394739	14.87973172	40.44730436	109.94717245	298.86740097	812.40582517	2208.34799189	6002.91221726	16317.60719802
0.71	2.03399126	5.52896148	15.02927551	40.85380653	111.05215991	301.87106828	820.57063945	2230.54225819	6063.24248804	16481.60187677
0.72	2.05443321	5.58452846	15.18032224	41.26439411	112.16825267	304.90492296	828.81751148	2252.95958057	6124.17908811	16647.24472945
0.73	2.07508061	5.64065391	15.33288702	41.67910816	113.29556235	307.96926838	837.14726595	2275.60220079	6185.72811120	16814.55232047
0.74	2.09593551	5.69734342	15.48698510	42.09799016	114.43420168	311.06441098	845.56073585	2298.47238312	6247.89571226	16983.54138073
0.75	2.11700002	5.75460268	15.64263188	42.52108200	115.58428453	314.19066029	854.05876253	2321.57241461	6310.68810809	17154.22880929
0.76	2.13827622	5.81243739	15.79984295	42.94842598	116.74592590	317.34832892	862.64219579	2344.90460528	6374.11157799	17326.63167502
0.77	2.15976625	5.87085336	15.95863401	43.38006484	117.91924196	320.53773265	871.31189399	2368.47128836	6438.17246436	17500.76721836
0.78	2.18147227	5.92985642	16.11902095	43.81604174	119.10435004	323.75919042	880.06872411	2392.27482054	6502.87717335	17676.65285301
0.79	2.20339643	5.98945247	16.28101980	44.25640028	120.30136866	327.01302438	888.91356183	2416.31758219	6568.23217547	17854.30616767
0.80	2.22554093	6.04964746	16.44464677	44.70118449	121.51041752	330.29955991	897.84729165	2440.60197762	6634.24400628	18033.74492783
0.81	2.24790799	6.11044743	16.60991822	45.15043887	122.73161752	333.61912567	906.87080695	2465.13043529	6700.91926702	18214.98707751
0.82	2.27049984	6.17185845	16.77685067	45.60420832	123.96509078	336.97205363	915.98501008	2489.90540804	6768.26462527	18398.05074107
0.83	2.29331874	6.23388666	16.94546082	46.06253823	125.21096065	340.35867907	925.19081248	2514.92937342	6836.28681562	18582.95422504
0.84	2.31636698	6.29653826	17.11576554	46.52547444	126.46935173	343.77934066	934.48913473	2540.20483383	6904.99264036	18769.71601992
0.85	2.33964685	6.35981952	17.28778184	46.99306323	127.74038985	347.23438048	943.88090667	2565.73431683	6974.38897011	18958.35480204
0.86	2.36316069	6.42373677	17.46152694	47.46535137	129.02420211	350.72414402	953.36706749	2591.52037541	7044.48274457	19148.88943544
0.87	2.38691085	6.48829640	17.63701820	47.94238608	130.32091690	354.24898027	962.94856581	2617.56558819	7115.28097317	19341.33897375
0.88	2.41089971	6.55350486	17.81427318	48.42421507	131.63066389	357.80924171	972.62635979	2643.87255970	7186.79073580	19535.72266207
0.89	2.43512965	6.61936868	17.99330960	48.91088652	132.95357405	361.40528437	982.40141722	2670.44392068	7259.01918349	19732.05993893
0.90	2.45960311	6.68589444	18.17414537	49.40244911	134.28977968	365.03746787	992.27471561	2697.28232827	7331.97353916	19930.37043823
0.91	2.48432253	6.75308880	18.35679857	49.89895197	135.63941441	368.70615541	1002.24724229	2724.39046634	7405.66109828	20130.67399118
0.92	2.50929039	6.82095847	18.54128746	50.40044478	137.00261319	372.41171388	1012.31999453	2751.77104573	7480.08922969	20332.99062831
0.93	2.53450918	6.88951024	18.72763050	50.90697767	138.37951234	376.15451382	1022.49397962	2779.42680452	7555.26537625	20537.34058145
0.94	2.55998142	6.95875097	18.91584631	51.41860130	139.77024956	379.93492954	1032.77021496	2807.36050830	7631.19705565	20743.74428576
0.95	2.58570966	7.02868758	19.10595373	51.93536683	141.17496392	383.75333906	1043.14972818	2835.57495047	7707.89186111	20952.22238178
0.96	2.61169647	7.09932707	19.29797176	52.45732595	142.59379590	387.61012424	1053.63355724	2864.07295251	7785.35746218	21162.79571750
0.97	2.63794446	7.17067649	19.49191960	52.98453084	144.02688737	391.50567075	1064.22275054	2892.85736422	7863.60160548	21375.48535043
0.98	2.66445624	7.24274299	19.68781664	53.51703423	145.47438165	395.44036816	1074.91836700	2921.93106408	7942.63211550	21590.31254971
0.99	2.69123447	7.31553376	19.88568249	54.05488936	146.93642350	399.41460993	1085.72147619	2951.29695948	8022.45689535	21807.29879823

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