Chapter 9

TRIANGLE AND QUADRILATERAL

You have seen many objects around that are shaped like the flag on the temple, paper flags that you use to decorate your school on the Independence day and the paratha that you eat! Let us observe some more such shapes:

- (1) The set-square in the compass box (fig1).
- (2) A folded corner of your notebook (fig2).



- Fig 1
- (3) The figure obtained by joining the 3 points in fig 3 with the



help of line segments.

What is the similarity in these figures?

Did you notice more shapes like these around yourself? Write down where have you seen them? Find out these kind of shapes in the figures given below.





On what basis have you selected/categorised the figures?

You must have noticed that the similarity in all these figures selected. There are 3 arms in all of them that meet at 3 points. So, they are called triangles.

All the figures from 17 to 23 given below are made up of 3 arms but all of them are not triangles. Why do you think some of them are not triangles?





You have read about closed and open figures in the lesson on line segments. In the figures given above fig no. 18 & 20 are closed figures. Others are all open figures. The closed figures are made up of 3 line segments that make three angles also. In the open figures, also there are 3 arms but these arms are not making 3 angles. Therefore, all the figures that are made up of three line segments are not triangles. **The closed figures made up of three line segments are known as triangles**.

ACTIVITY 1

In each of the figures given below. There are 3 points. Can you make triangles by joining them at the three given points.



In the figures above you must have noticed that whenever the three given points are in a straight line, they cannot be joined by three line segments. Therefore, they are not making triangles. This means three points that are not in a straight line and can be joined by line segments can make a slope that is called triangle.

The Parts of A Triangle

The triangle ABC has three angle \angle ABC, \angle CAB and \angle BCA. A, B and C are the vertices AB, BC & CA are the arms. \angle ABC, \angle BCA and \angle CAB are the angles.



In the figure above, two arms meet at each vertex and make an angle. For the triangles given below. Write the names of the vertices, arms and the angles.



The Internal Angles of a Triangle

The angles made by the arms of a triangle are all in a closed area, and so are interior angles. All the names of angles that you have listed in figures 33, 34, 35 & 36 are all interior angles.

ACTIVITY 3

Now measure the internal angles of the triangles in figures 37, 38, 39 & 40 with the help of your protractor and write them in the given table. Find the sum of the angles that you have measured for each triangle.



Fig No.	ZA	∠B	∠C	$\angle A + \angle B + \angle C$ The sum of all the triangles
37				
38				
39				
40				

On measuring the interior angles of a triangles, we find that the sum of the three angles is approximately 180° . The sum of the three internal angles of a triangle is equal to 180° . We will learn how to prove this statement in our higher classes. Presently, let us try to find out the value of one angle of a triangle on the basis of values of two given angles.

S. No.	Measure of the 1 st angle	Measure of the 2 nd angle	Value of the 3^{rd} angle = 180° – $(1^{st}$ angle + 2^{nd} angle)
01	40°	60°	$180^{\circ} - (40^{\circ} + 60^{\circ}) = 80^{\circ}$
02	40°	30°	
03	45°	95°	
04	70°	50°	

The External Angles of a Triangle

If one of the arms of a triangle is extended out of the closed area, it makes an angle, which is called the exterior angle. $\checkmark D$



In all the above figures, one of the arms of the given triangles have been extended to point D which show $\angle ACD$, $\angle CAD$ and $\angle ABD$ respectively. These are all *exterior angles*. Each exterior angle is attached to an interior angle, which is complementary angle to the exterior angle. This interior angle which is joined to the exterior angle is kown as the *adjacent interior*

angle. For example:

Fig no.	Exterior angle	Interior angle
41	∠ACD	∠ACB
42	∠CAD	∠CAB
43	∠ABD	∠ABC

The interior angle attached to the external angle is called the adjacent interior or adjoining interior angle and the other two interior angles are known as distant interior angles. Thus in fig 41 \angle BAC & \angle CBA, in fig42 \angle ABC & \angle BCA and in fig43 \angle BCA & \angle CAB are distant interior angles.

Point out the exterior angles. Then find out adjoining interior angles for the exterior angles in the given figures. If no exterior angles are being formed in any figure, what do you think is the reason behind this?



In fig45 & 46, no exterior angles are being formed because QRS & UVW are not straight lines.

ACTIVITY 4

In the figures given below, fill in the table with the names of adjoining interior angles, distant angles and the exterior angles of the triangles drawn.



You have now learnt to identify the exterior, adjoining and distant interior angles of a triangle. Let us now take up another activity with angles.

Fig No.	Name of the	Name of the	Adjoining	Distant interior angle	
triangle		angle	angle	Ι	П
47					
48					
49					

ACTIVITY 5

Find out the values of $\angle X$, $\angle Y$ and $\angle Z$ in the triangles given below and complete the table.



What is the relationship between $\angle X$ and $\angle Y + \angle Z$? It is clear from the table below that the measure of an exterior angle of a triangle is equal to the sum of the distant interior angles of a triangle.

Fig No.	∠X	∠Y	∠Z	$\angle Y + \angle Z$
50				
51				
52				

Classification of Triangles

Till now you have seen different shapes of triangles. On the basis of arms & angles of triangles, they can be classified into the following types:

1. Classification of triangles according to measures of arm length

- (a) Triangles in which all the three arms are of different lengths are called **scalene triangles**.
- (b) Triangles in which two arms are of equal length and the third is of a different measure are known as **isoceles triangles**.
- (c) Triangles in which all the three arms are of equal length are called **equilateral triangles.**

Practice 1

In the table given below, measures of the arms (sides) of triangles are shown. Classify the triangles on the basis of the given measures.

S. No.	Measures of arms	Types of triangles
1	4cm, 5cm, 6cm	
2	7cm, 7cm, 7cm	
3	6cm, 5cm, 6cm	
4	7.2cm, 7.2cm, 6cm	

ACTIVITY 6

Measure the sides or arms of the triangles in the figures given below and classify the triangles into three types.



Fig 56





Fig No.	LENG	Types of triangle		
	1	2	3	- , ,
53				
54				
55				
56				
57				

2. Classification of triangles on the basis of angles

- (a) A triangle in which all the three angles are acute angles is called a **Acute angled triangle.**
- (b) A triangle in which one of the angles is right angle is known as a **right angled triangle.**
- (c) A triangle in which one of the angles is obtuse is an **obtuse angled triangle.**

ACTIVITY 7

Classify the triangles on the basis of the measures of angles given:

S. No.	Angles of a triangle	Types of triangles
1	30°, 30°, 120°	
2	60°, 90°, 30°	
3	45°, 40°, 95°	
4	30°, 70°, 80°	
5	60°, 60°, 60°	

ACTIVITY 8

Measure the angles of the triangles in the given figures and write the type of triangle in the space given below each figure.



Now make some triangles and classify them according to their angles and arm lengths. **3. Classification of triangles on the basis of both arm as well as angles**

In the given figures measure the lengths of arms and the angles of the triangles and write them separately in the table below. Classify the triangles according to the arm lengths and angles.



ACTIVITY 9

Fig.	Fig. Measures of the 3 arms		Measures of the 3 angles			Type of triangle		
No.	1	2	3	1	2	3	On the basis of angle	On the basis of arm length
64	3cm	3cm	3cm	60°	60°	60°	Acute angled triangle	Equilateral triangle
65								
66								
67								
68								
69								

The following conclusions can be drawn from the observations made above:

- (1) In a scalene triangle, the measures of all the three sides of the triangle are different and all the three angles also are of different measures.
- (2) In an isoceles triangle, two arms and two angles are equal.
- (3) In an equilateral triangle, all the three arms & all the three angles are of equal measures.

ACTIVITY 10

Take sticks of different lengths and make triangles.

for e.g. A triangle whose arm lengths are 8cm, 10cm & 12cm long.



Fig 70

You find that triangles of some given measures can be made.

Make triangles in the same way with sticks of the measures given below. Note whether triangles can be made in all the situations. If not, find out reasons for it.

- (1) 8cm, 10cm & 12cm
- (2) 5cm, 9cm & 3cm
- (3) 6cm, 8cm & 9cm
- (4) 5cm, 7cm & 12cm
- (5) 15cm, 5cm & 12cm

Now verify the conclusions that you have drawn

- (1) If the sum of lengths of two sides (arms) of a triangle is greater than the length of the third side, only then can we make a triangle.
- (2) If the sum of lengths of two arms of the triangle is less than the given length of the third arm, no triangle can be formed.

The situation in example 2, shows that when measures of arm lengths are as follows: Sum of lengths of 2 arms of the triangle

5cm + 3cm = 8cm

which is less than the given measure of the third arm i.e. 9cm.

So, this triangle cannot be formed.

In example 4, sum of the two arm lengths of the triangle given is 5cm + 7cm = 12cm, which is equal to the given measure 12cm of the third arm.

In this situation also the triangle cannot be made.

Make triangles of similar measures yourself and help your friends to make the angles of different measures.

EXERCISE 9.1

- 1. Julie has made the following statements, Identify whether they are true or false. Select the false statements & correct them.
 - (i) One arm or side of a triangle cannot be smaller than the sum of the other two arm lengths or sides.
 - (ii) A triangle has 3 arms, 3 vertices and 3 internal angles.
 - (iii) The length of one arm of a triangle is equal to the sum of the other two arm-lengths.
 - (iv) When one angle of a triangle is obtuse, then the triangle is known as an obtuse angled triangle.
 - (v) A triangle can have two angles of 90° .
 - (vi) All the 3 angles of an acute angled triangle need not be acute.
 - (vii) In the measure of two angles of a triangle are given, the measure of the third angle can be determined.
 - (viii) All the three sides (arms) of an equilateral triangle are equal but all its three angles are not equal.
 - (ix) The angles infront of the two equal arms of an isoceles triangle are also equal.
 - (x) An equilateral triangle is always an acute angled triangle.
- 2. If the two angles of a triangle are 65° and 75° , find out the measure of the third angle ?
- 3. One angle of right angled triangle is 45° , find the other angle ?
- 4. What is the measure of each angle in an equilateral triangle?
- 5. If one angle of a triangle is equal to the measure of the other two angles, will the triangle be a right angled triangle?
- 6. Can the following situations lead to construction of triangles? Say yes or no.
 - (i) If two angles are right angles
 - (ii) When two angles are obtuse angles
 - (iii) Sum of all the 3 angles measure 60° .

- (iv) All the three angles are acute angles
- (v) All the angles are less than 60° .
- (vi) All the angles are greater than 60° .

Quadrilateral

You know about triangles. Everyday you see shapes like a black board, football playground, Kabaddi playground and pages of your copies, books etc. How many sides do each of these have? Where else have you seen shapes like these? Write down more names.

Choose figures like these from among the following: -



You have selected the four cornered shapes from the figures above. These shapes have 4 sides and are therefore called Quadrilaterals.

Some more shapes each of which, is formed by joining four sides are given below. Are all these quadrilaterals? If not, then think of the reason for each of your answers?





Fig 83

ou can observe that figures 78, 89 and 80 are closed shapes enclosed by four sides and the enclosed region has four angles. All these are therefore Quadrilaterals.

Figures 81, 82 and 83 are not closed shapes and, therefore are not quadrilaterals. In this way, we say "Closed shapes having four sides where four angles are formed are called quadrilaterals".

Parts of A Quadrilateral

In the quadrilateral ABCD, AB, BC, CD and DA are the four sides and A, B, C, D are the four vertices. Every vertex is formed by the joining of two sides and at every vertex the two sides form one interior angle. In this way four interior angles are formed. These are \angle BAD, \angle ADC, \angle DCB and \angle CBA respectively.



ACTIVITY 11

In the figures below, identify the sides, angles & vertices and write them at the appropriate places.

Figure No.	Figure	Vertices	Sides	Angles
85	D C	(i) A (ii) B (iii) C (iv) D	(i) AB (ii) BC (iii) CD (iv) DA	 (i) ∠ADC or ∠CDA (ii) ∠DCB or ∠BCD (iii) ∠CBA or ∠ABC (iv) ∠BAD or ∠DAB
86	A D C B	(i) (ii) (iii) (iv)	(i) (ii) (iii) (iv)	(i) (ii) (iii) (iv)
87	P Q S R	(i) (ii) (iii) (iv)	(i) (ii) (iii) (iv)	(i) (ii) (iii) (iv)

Interior Region and Exterior egions of a quadrilateral



We are familiar with Kabaddi grounds. The adjacent figure shows players playing Kabaddi. Can you tell the number of players in the ground?

We can see in the picture that some players are outside the ground. They are 3 in number.

Is the Kabaddi ground ABCD a quadrilateral?

In the adjacent figure, the region inside the boundary of the quadrilateral is called the interior region of the quadrilateral. In figure 88, points P and Q are shown in the interior region of the quadrilateral.

The part of the plane (ground), outside the quadrilateral, is called exterior region of the The part of the plane





(ground), outside the quadrilateral, is called exterior region of the quadrilateral. In figure 89 points R and S are in the exterior region of the quadrilateral. Numbers, letters etc written on any page

of your book, are located in which region of the page?



Adjacent Sides and Opposite Sides

In figure 90, you can see that the sides SP and QP are meeting at the vertex P. Similarly, the sides PQ and RQ meet at the vertex Q.

The sides of a quadrilateral, that meet each other at a point (vertex), are called adjacent sides. Here RS and PS are adjacent sides, which meet at the vertex S. Write the name of the adjacent S sides, which meet at the vertices Q and R.



In figure 90, sides PQ and RS do not meet; therefore these sides are known as opposite sides. In figure 90 write the second pair of opposite sides.



Identify the pairs of adjacent sides in the following figures and write them along with their vertices in the following table-



Figure No.	Adjacent sides	Vertices	Opposite side
91	(i)	(i)	(i)
	(ii)	(ii)	(ii)
	(iii)	(iii)	(iii)
	(iv)	(iv)	(iv)
92	(i)	(i)	(i)
	(ii)	(ii)	(ii)
	(iii)	(iii)	(iii)
	(iv)	(iv)	(iv)
93	(i)	(i)	(i)
	(ii)	(ii)	(ii)
	(iii)	(iii)	(iii)
	(iv)	(iv)	(iv)

Adjacent Angles and Opposite Angles

We have studied that a quadrilateral has four interior angles. Out of these, two such angles, which are formed by one common side, are called adjacent angles.

In figure 94, $\angle A$ is formed by sides DA and AB, and $\angle B$ is formed by sides AB and BC. Here AB is the common side therefore $\angle A$ and $\angle B$ are adjacent angles.



Is there any other angle adjacent to $\angle A$?

In the same manner, write the adjacent angles of $\angle B$, $\angle C \& \angle D$.

In the above figure 94, $\angle B$ has 2 adjacent angles $\angle A & \angle C$ but $\angle B & \angle D$ are not adjacent angles.

Therefore any two angles of a quadrilateral, which are not adjacent are called opposite angles. In figure 94, $\angle D$ is the opposite angle of $\angle B$, and $\angle A$ is the opposite angle of $\angle C$. Opposite angles face each other.

Diagonal of a Quadrilateral and the Sum of the Interior Angles

ABCD is a quadrilateral. If two opposite vertices of this quadrilateral are joined by a line segment then it gets divided into two triangles. Line segment AC is the diagonal of the quadrilateral ABCD. This is formed by joining the opposite vertices A and C.



Similarly, the line segment BD will also be a diagonal.

The sum of the interior angles of a quadrilateral is equal to 360°.

Types of Quadrilaterals

Use a scale to take broom sticks of lengths mentioned below. Join them head to head and form quadrilaterals of different shapes.

(i) 8cm, 4cm, 8cm and 4cm.

Following are some of the quadrilaterals so formed by these: -



Among these figures, 96, 97, 99 and 100 have both pairs of opposite sides parallel to each other and equal in length. These are known as parallelograms.

Therefore, those quadrilaterals in which opposite sides are parallel and equal to each other are called parallelograms.

Figures 96, 97 are parallelograms with all angles of 90° each. These are called rectangles. **Thus, those parallelograms, which have all angles as right angles, are called rectangles.**

In figure 98, the opposite sides are neither parallel nor equal. Therefore it is not a parallelogram.

(ii) Take 4 sticks of 4 cm length each and make quadrilaterals: -



Some of the quadrilaterals made by you would be similar to the figures drawn above. Are these quadrilaterals parallelograms?

You will observe that, all the pairs of opposite sides in these figures are parallel and equal. Thus, all these are parallelograms. Since all sides of these quadrilaterals are equal, so they are a special type of parallelogram.

Those parallelograms, which have all sides equal are known as Rhombus.

Figure 101 is also a Rhombus. Apart from having all equal sides, this parallelogram has another specialty too. Each angle of this quadrilateral is of 90°.

Such a quadrilateral, which has all equal sides and all angles as right angles, is known as a square. Thus, square is a special type of Rhombus.

(iii) Now take sticks of lengths 3 cm, 4cm, 5 cm and 6 cm respectively, join them head to head and form many - different quadrilaterals. Some of the quadrilaterals formed by you may be of the following types: -



With the help of the sticks of specified lengths try and form some more quadrilaterals.

Each side of figure 104 is of a different length and opposite sides are not parallel. This is a quadrilateral having all sides of different lengths.

Quadrilaterals shown in figures 105 and 106 have two of their opposite sides (AB and DC) parallel but of different lengths. These are called Trapeziums. In a trapezium, perpendiculars drawn from the vertex on the opposite parallel side are of equal lengths.

Thus those quadrilaterals, in which one pair of opposite sides is parallel, are called Trapeziums.

ACTIVITY 15

Classify the following figures as rectangles, squares, rhombuses, trapezium and quadrilaterals with all sides of different lengths and fill the table given below:



EXERCISE 9.2

Q1. Fill in the blanks-

- (i) A quadrilateral has <u>diagonals</u>.
- (ii) The diagonal of a quadrilateral divides the quadrilateral into two_____.
- (iii) The sum of all the interior angles of a quadrilateral is ______ degrees.
- (iv) ______pair(s) of opposite angle(s) are/is formed in a quadrilateral.
- (v) Every quadrilateral has ______ vertices, among which more than ______ vertices cannot lie on a straight line.
- Q2. (i) If in a quadrilateral, only one pair of opposite sides is parallel, then such a quadrilateral is called ______

(iii) In a rhombus, opposite sides are _____ and all four sides are ______to each other.

(iv) A parallelogram in which each angle is of 90° and all sides are of equal lengths is called ______.

(v) A quadrilateral whose all sides are equal is called ______.

- Q3. State true or false and correct the false statements: -
 - (i) A Rectangle is a parallelogram.
 - (ii) Every parallelogram is a rectangle.
 - (iii) Every Rhombus is a square
 - (iv) Opposite sides of a trapezium are parallel.

What Have We Learnt?

- 1. Triangle is an area circumscribed by 3 arms.
- 2. A triangle is a closed figure. If the 3 arms together do not form a closed figure, it doesn't make a triangle.
- 3. The vertex, arms and angles are parts of a triangle.

4.	Triangles have 3 angles.
5.	The measures of the sum of three internal angles of a triangle is equal to two right angles.
6.	The exterior angle formed on extending the length of one of the arms of a triangle is equal to the sum of the two distant interior angles of the triangle.
7.	Triangles can be classified into equilateral, isoceles & scalene triangles on the basis of the measures of arm length of these triangles.
8.	On the basis of angles, triangles can be classified into acute angled, right angled and obtuse angled triangle.
9.	The measures of the three arm lengths and the three angles of a scalene triangle are different from each other.
10.	In an isoceles triangle, two arms and two angles are equal.
11.	In an euqilateral triangle, the three arm lengths and three angles are equal.
12.	A triangle can be constructed only when the sum of two arm lengths is greater than the length of the third arm.
13.	A closed shape formed by four sides having four interior angles is known as a quadrilateral.
14.	There are four vertices, four sides and four angles in a quadrilateral.
15.	The line, joining opposite vertices of a quadrilateral is called a diagonal. There are two diagonals in a quadrilateral.
16.	Sides of a quadrilateral having one common vertex are called adjacent sides.
17.	Sides of a quadrilateral, which do not have any common vertex, are called opposite sides.
18.	Interior of the quadrilateral ABCD together with the boundary of the quadrilateral forms the region of quadrilateral ABCD.
19.	Sum of all the angles of a quadrilateral is 360°.
20.	Opposite sides of a parallelogram are equal and parallel to each other.
21.	A parallelogram each of whose angles is of 90° is called a rectangle.
22.	A parallelogram in which all the sides are equal is called a rhombus.
23.	A quadrilateral in which one pair of opposite sides is parallel is called a trapezium.
24.	A parallelogram each of whose angle is of 90° and all sides are equal, is called a square.