Maharashtra State Board
Class X Mathematics - Algebra
Board Paper – 2017

Time: 2 hours **Maximum Marks: 40**

Note: - (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1. Attempt any five of the following sub questions : **5**

(i) State whether the following sequence is an Arithmetic Progression or not :

3, 6, 12, 24,…….

(ii) If one root of the quadratic equation is $3 - 2\sqrt{5}$, then write another root of the equation.

(iii) There are 15 tickets bearing the numbers from 1 to 15 in a bag and one ticket is drawn from this bag at random. Write the sample space (S) and n(S).

(iv) Find the class mark of the class 35-39.

(v) Write the next two terms of the A.P. whose first term is 3 and the common difference is 4.

(vi) Find the values of a,b,c for the quadratic equation $2x^2 = x + 3$ by comparing with standard form $ax^2 + bx + c = 0$

2. Attempt any four of the following sub questions : **8**

(i) Find the first two terms of the sequence for which S_n is given below :

$$S_n = n^2 (n + 2)$$

(ii) Find the value of discriminant (Δ) for the quadratic equation :

$$X^2 + 5x + 1 = 0$$

(iii) Write the equation of X-axis. Hence find the point of intersection of the graph of the equation $x + y = 3$ with the X-axis.

(iv) For a certain frequency distribution the values of Assumed mean (A) = 1200, $\sum f_i d_i$=700 and $\sum f_i$. Find the value of mean $(\overline{X})$.

(v) Two coins are tossed simultaneously. Write the sample space (S). n(S), the following event A using set notation and n(A), Where 'A' is the event of getting at the most one tail.'

(vi) Find the value of k for which the given simultaneous equation have infinity many solution :

$$kx + 2y = 6$$
$$9x + 6y = 18$$

3. Attempt any three of the following sub questions : 9

(i) How many three digit natural numbers are divisible by 2?

(ii) Solve the following quadratic equation by factorization method :

$$3x^2 - 22x + 40 = 0$$

(iii) Solve the following simultaneous equation by using Cramer's rule :

$$x + 2y = 4;$$
$$3x + 4y = 6$$

(iv) The following is the frequency distribution of waiting time at ATM centre; draw histogram to represent the data :

Waiting time (In seconds)	**Number of Customers**
0 – 30	10
30 – 60	54
60-90	68
90-120	28
120-150	20

4. Attempt any two of the following sub equations : **8**

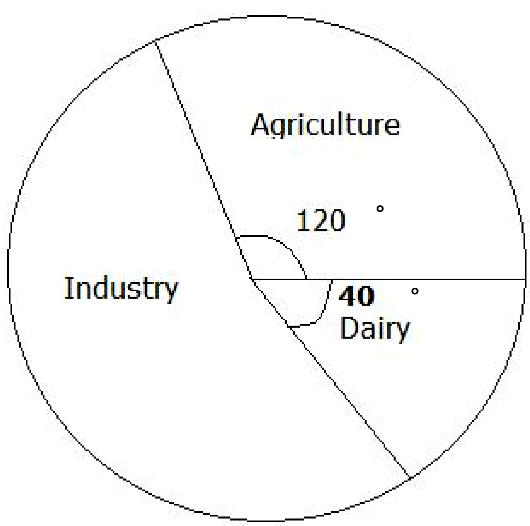
(i) Three horses A, B and C are in a race, A is twice as likely to win as B and B is twice as likely to win as C. What are their probabilities of winning?

(ii) The following is the distribution of the size of certain farms from a taluka (tehasil) :

Size of Farms (in acres)	**Number of Farms**
5 – 15	7
15 -25	12
25 – 35	17
35 – 45	25
45 - 55	31
55 – 65	5
65 – 75	3

Find the median size of farms.

(iii) The following pie diagram represents the sectorwise loan amount in crores of rupees distributed by a bank. From the information answer the following questions :



(a) If the dairy sector receives `20 crores, then find the total loan disbursed.

(b) Find the loan amount for agriculture sector and also for industrial sector.

(c) How much additional amount did industrial sector receive than agriculture sector?

5. Attempt any two of the following sub questions : 10

(i) If the cost of bananas in increased by 10 per dozen, one can get 3 dozen less for 600. Find the original cost of one dozen of bananas.

(ii) If the sum of first p terms of an A.P. is equal to the sum of first q terms, then show that the sum of its first (p + q) terms is zero where $p \neq q$.

(iii) Solve the following simultaneous equations :

$$\frac{1}{3x} - \frac{1}{4y} + 1 = 0$$

$$\frac{1}{5x} + \frac{1}{2y} = \frac{4}{15}$$

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Note: - (1) All questions are compulsory.
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1.

i. 3, 6, 12, 24....

Let, 'a' be the first term of the given sequence and 'd' be the common difference.

Also, t_2, t_3, t_4 be the $2^{nd}, 3^{rd}, 4^{th}$ terms respectively.

Consider,

$t_2 - a = 6 - 3$

$= 3$

and

$t_3 - t_2 = 12 - 6$

$= 6$

Here, we can see that difference between two successive terms is not constant.

Hence, it is not an Arithmetic Progression.

ii. One root of the quadratic equation is given to be $3-2\sqrt{5}$.

The other root will be the conjugate of $3-2\sqrt{5}$.

Conjugate of $3-2\sqrt{5} = 3+2\sqrt{5}$

iii. Given that there are 15 tickets bearing the numbers from 1 to 15 in a bag.

Hence, sample space can be written as:

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$n(S) = 15$

iv. Class mark for the given class 35 - 39 is :

$\frac{35+39}{2} = \frac{74}{2}$

$= 37$

v. The first term, $a = 3$

common difference, $d = 4$

So, the next two terms would be $a + d, a + 2d$.

That is, the next two terms are 7 and 11.

vi. Given quadratic equation is $2x^2 = x + 3$.

Writing this equation in standard form, we get

$2x^2 - x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 2, b = -1, c = -3$

2.

i. To find the first term, substitute n = 1 in $S_n = n^2(n+2)$

$\Rightarrow S_1 = 1^2(1+2) = 1(3) = 3$

Now S_1 is the sum of the first term itself, which is also the first term.

So, the first term = 3

To find the second term, substitute n = 2 in $S_n = n^2(n+2)$

$\Rightarrow S_2 = 2^2(2+2) = 4(4) = 16$

Now S_2 is the sum of the first two terms.

$\Rightarrow$ the first term + the second term =16

$\Rightarrow$ 3 + the second term =16

$\Rightarrow$ the second term = 13

So, the first term = 3 and the second term = 13.

ii. Given quadratic equation is $x^2 + 5x + 1 = 0$.

$a = 1, b = 5, c = 1$

The discriminant $(\Delta) = b^2 - 4ac = 5^2 - 4(1)(1) = 25 - 4 = 21$

iii. The equation of the X-axis is y = 0.

To find the point of intersection of the equation $x + y = 3$ with the X - axis,

substitute $y = 0$ in $x + y = 3$.

$\Rightarrow x + 0 = 3$

$\Rightarrow x = 3$

So, the point of intersection will be (3,0.)

iv. We have,

$A = 1200, \sum f_i d_i = 700$ and $\sum f_i = 100 = N$

$$\overline{X} = A + \frac{1}{N}\sum_{i=1}^{n} f_i d_i$$

$$\Rightarrow \overline{X} = 1200 + \frac{1}{100}(700)$$

$$\Rightarrow \overline{X} = 1200 + 7$$

$$\Rightarrow \overline{X} = 1207$$

v. Since two coins are tossed simultaneously,

$S = \{HT, TH, HH, TT\}$

$\Rightarrow n(S) = 4$

$A = \{HT, TH, HH\}$

$\Rightarrow n(A) = 3$

vi. The given equations are

$kx + 2y = 6$ ……(i)

and $9x + 6y = 18$

$\Rightarrow 3\ (3x + 2y) = 18$

$\Rightarrow 3x + 2y = 6$ …..(ii)

From (i) and (ii), we get $k = 3$

3.

i. The first three digit natural number divisible by 2 is 100.

Common difference $d = 2$

Last three digit natural number divisible by 2 is 998

We know that,

$t_n = a + (n - 1)\ d$

$\Rightarrow 998 = 100 + (n - 1)\ 2$

$\Rightarrow 898 = 2\ (n - 1)$

$\Rightarrow 449 = n - 1$

$\Rightarrow n = 450$

Hence, there are 450 three digit natural numbers divisible by 2.

ii. $3x^2 - 22x + 40 = 0$

$\Rightarrow 3x^2 - 12x - 10x + 40 = 0$

$\Rightarrow 3x\ (x - 12) - 10(x - 12) = 0$

$\Rightarrow (x - 12)(3x - 10) = 0$

$\Rightarrow x - 12 = 0$ or $3x - 10 = 0$

$$\Rightarrow x = 12 \text{ or } x = \frac{10}{3}$$

iii. Consider,

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2 \neq 0$$

$$x = \frac{\begin{vmatrix} 4 & 2 \\ 6 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{16 - 12}{-2} = -2$$

$$y = \frac{\begin{vmatrix} 1 & 4 \\ 3 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{6 - 12}{-2} = 3$$

So, $x = -2$ and $y = 3$.

iv. S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\Rightarrow$n(S) = 36

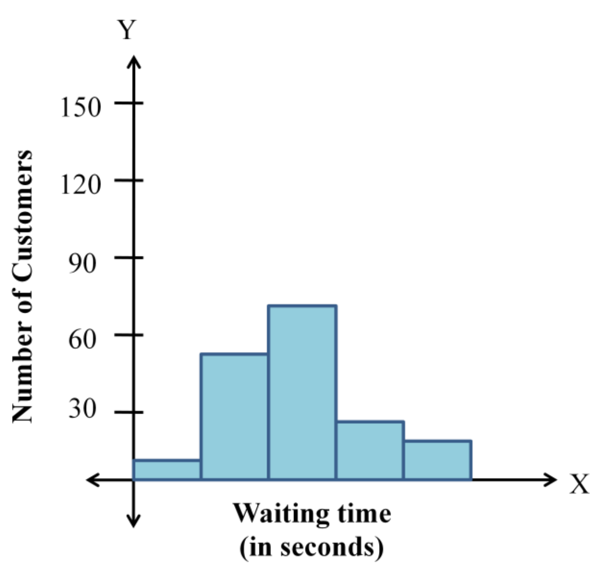
Let A = event that the product of numbers on their upper faces is 12

$\Rightarrow$A = {(2,6), (3, 4), (4,3), (6,2)}

$\Rightarrow$n(A) = 4

$$\Rightarrow \text{Probability of the event A} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{36} = \frac{1}{9}$$

v.



4.

i.

$\Pr(A) = 2\Pr(B)$, and $\Pr(B) = 3\Pr(C)$

Hence, $\Pr(A) = 6\Pr(C)$

But $\Pr(A) + \Pr(B) + \Pr(C) = 1$

Consequently, $6\Pr(C) + 3\Pr(C) + \Pr(C) = 1$

So, $\Pr(C) = \frac{1}{10}$;

Since $\Pr(B) = 3\Pr(C)$

$\Rightarrow \Pr(B) = \frac{3}{10}$

and since $\Pr(A) = 6\Pr(C)$

$\Rightarrow \Pr(A) = \frac{6}{10}$

ii. Calculation of the Median size of farms.

Size of Farms	**f**	**cf**
5 - 15	7	7
15 – 25	12	19
25 - 35	17	36
35 - 45	25	61
45 – 55	31	92
55 - 65	5	97
65 - 75	3	100

We have $N = 100 \Rightarrow \frac{N}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 61 and the corresponding class is 35 - 45.

Thus, 35 - 45 is the median class such that $l = 35, f = 25, cf = 36, h = 10$.

$$\Rightarrow \text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 35 + \frac{50 - 36}{25} \times 10$$

$$= 35 + 5.6$$

$$= 40.6$$

iii. We compute the central angle for each crop as shown in the following table.

Sector	**Measure of central angle**	**Amount (in crores)**
Agriculture	120°	$\frac{120}{360} \times 180 = \text{Rs.}60$
Dairy	40°	Rs. 20
Industry	360 - (120° + 40°) = 200°	$\frac{200}{360} \times 180 = \text{Rs.}100$
Total	360°	Rs. 180

(a)

$$\frac{40^\circ}{360^\circ} \times \text{Total} = 20$$

$$\Rightarrow \text{Total} = \frac{20 \times 360}{40}$$

$$\Rightarrow \text{Total} = \text{Rs.}180 \text{ crores}$$

Hence, the total loan disbursed is Rs. 180 crores.

(b)

Total loan for the agriculaure sector :

$$\frac{120}{360} \times 180 = \text{Rs. } 60$$

(c)

Total loan for industrial sector :

$$\frac{200}{360} \times 180 = \text{Rs.}100$$

The additional amount the industrial sector received than the agriculture sector

= Rs. 100 - Rs. 60

= Rs. 40

5.

(i)

Let x be the original cost of a dozen bananas.

For Rs.600 let us one gets y dozens.

$xy = 600 \quad .....(1)$

$\Rightarrow y = \frac{600}{x}$

$(x+10)(y-3) = 600 \quad .....(2)$

Substituting the y value in (2), we get,

$(x+10)\left(\frac{600}{x} - 3\right) = 600$

$\Rightarrow (x+10)\left(\frac{600-3x}{x}\right) = 600$

$\Rightarrow (10+x)(600-3x) = 600x$

$\Rightarrow 6000 + 570x - 3x^2 = 600x$

$\Rightarrow 6000 - 30x - 3x^2 = 0$

$\Rightarrow 2000 - 10x - x^2 = 0$

$\Rightarrow x^2 + 10x - 2000 = 0$

$\Rightarrow (x+50)(x-40) = 0$

$\Rightarrow x = -50 \text{ or } 40$

Since cost of bananas cannot be negative, x = 40.

So, the original cost of one dozen of bananas is Rs. 40.

(ii)

To show : $S_{p+q} = 0$

that is, to show : $\frac{p+q}{2}\left(2a + (p+q-1)d\right) = 0$

Given that $S_p = S_q$

Let a be the first term of the AP and d be the common difference.

$\Rightarrow \frac{p}{2}\left(2a + (p-1)d\right) = \frac{q}{2}\left(2a + (q-1)d\right)$

$\Rightarrow p\left(2a + (p-1)d\right) = q\left(2a + (q-1)d\right)$

$\Rightarrow 2ap + (p-1)dp = 2aq + (q-1)dq$

$\Rightarrow 2ap - 2aq + (p-1)dp - (q-1)dq = 0$

$\Rightarrow 2ap - 2aq + (p-1)dp - (q-1)dq = 0$

$\Rightarrow 2a(p-q) + d\left[p^2 - p - q^2 + q\right] = 0$

$\Rightarrow 2a(p-q) + d\left[p^2 - q^2 - p + q\right] = 0$

$\Rightarrow 2a(p-q) + d\left[(p-q)(p+q) - (p-q)\right] = 0$

$\Rightarrow 2a(p-q) + d\left[(p-q)[(p+q)-1]\right] = 0$

Dividing throughout by $p-q$, since $p \neq q$.

$\Rightarrow 2a + ((p+q)-1)d = 0$

$\Rightarrow 2a + (p+q-1)d = 0$

$S_{p+q} = \frac{p+q}{2}\left(2a + (p+q-1)d\right) = \frac{p+q}{2}(0) = 0$

Hence proved.

(iii)

$\frac{1}{3x} - \frac{1}{4y} + 1 = 0$ and $\frac{1}{5x} + \frac{1}{2y} = \frac{4}{15}$

$\frac{1}{3x} - \frac{1}{4y} = -1$ and $\frac{1}{5x} + \frac{1}{2y} = \frac{4}{15}$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$\Rightarrow \frac{a}{3} - \frac{b}{4} = -1$ and $\frac{a}{5} + \frac{b}{2} = \frac{4}{15}$

$\Rightarrow 4a - 3b = -12$ and $2a + 5b = \frac{40}{15}$

$\Rightarrow 4a - 3b = -12$ and $2a + 5b = \frac{8}{3}$

$\Rightarrow 4a - 3b = -12$ and $3a + 15b = 8$

Solving the two equations, we get $a = -2$ and $b = \frac{4}{3}$.

Resubstituting $\frac{1}{x} = a$ and $\frac{1}{y} = b$,

$\Rightarrow x = -\frac{1}{2}$ and $y = \frac{3}{4}$