#### **Properties of Atoms. Spectra (Part - 1)**

Q.97. The binding energy of a valence electron in a Li atom in the states 2S and 2P is equal to 5.39 and 3.54 eV respectively. Find the Rydberg corrections for S and P terms of the atom.

Ans. From the Rydbeig formula we write

$$E_n = -\frac{\hbar R}{\left(n+\alpha_l\right)^2}$$

we use  $\hbar R = 13.6 \text{ eV}$ . Then for n = 2 state

$$5.39 = -\frac{13.6}{(2+\alpha_0)^2}$$
,  $l = 0(S)$  state

 $\alpha_1 = -0.41$ 

for p state

$$3.54 - \frac{13.6}{(2+\alpha_1)^2}$$
  
$$\alpha_1 = -0.039$$

Q.98. Find the Rydberg correction for the 3P term of a Na atom whose first excitation potential is 2.10 V and whose valence electron in the normal 3S state has the binding energy 5.14 eV.

**Ans.** The energy of the 3p state must be  $-(E_0 - e\varphi)$  where  $-E_0$  is the energy of the 3S state.

Then

$$E_0 - e \varphi_1 = \frac{\hbar R}{(3 + \alpha_1)^2}$$
$$\alpha_1 = \sqrt{\frac{\hbar R}{E_0 - e \varphi_1}} - 3 =$$
so

Q.99. Find the binding energy of a valence electron in the ground state of a Li atom if the wavelength of the first line of the sharp series is known to be equal to  $\lambda_1 = 813$  nm and the short-wave cutoff wavelength of that series to  $\lambda_2 = 350$  nm.

Ans. For the first line of the sharp series  $(3S \rightarrow 2P)$  in a Li atom

$$\frac{2\pi\hbar c}{\lambda_1} = -\frac{\hbar R}{\left(3+\alpha_0\right)^2} + \frac{\hbar R}{\left(2+\alpha_1\right)^2}$$

For the short wave cut-off wave-length of the same series

$$\frac{2\pi\hbar c}{\lambda_2} = \frac{\hbar R}{\left(2+\alpha_1\right)^2}$$

From these two equations we get on subtraction

$$3 + \alpha_0 = \sqrt{\frac{\hbar R}{2\pi \hbar c (\lambda_1 - \lambda_2)}}$$
$$= \sqrt{\frac{R \lambda_1 \lambda_2}{2\pi c \Delta \lambda}}, \ \Delta \lambda = \lambda_1 - \lambda_2$$

Thus in the ground state, the binding energy of the electron is

$$E_{b} = \frac{\hbar R}{(2 + \alpha_{0})^{2}}$$
$$= \hbar R / \left( \sqrt{\frac{R \lambda_{1} \lambda_{2}}{2 \pi c \Delta \lambda}} - 1 \right)^{2} = 5.32 \text{ eV}$$

Q.100. Determine the wavelengths of spectral lines appearing on transition of excited Li atoms from the state 3S down to the ground state 2S. The Rydberg corrections for the S and P terms are -0.41 and -0.04.

**Ans.** The energy of the 3 S state is

$$E(3S) = -\frac{\hbar R}{(3-0.41)^2} = -2.03 \text{ eV}$$

The energy of a 2 S state is

$$E(2S) = -\frac{\hbar R}{(2-0.41)^2} = -5.39 \text{ eV}$$

The energy of a 2 P state is

$$E(2P) = -\frac{\hbar R}{(2-04)^2} = -3.55 \text{ eV}$$

We see that

E (2S) <E (2P)< E (3S) The transitions are  $3S \rightarrow 2P$  and  $2P \rightarrow 2S$ . Direct  $3S \rightarrow 2S$  transition is forbidden by selection rules. The wavelengths are determined by

$$E_2 - E_1 = \Delta E = \frac{2 \pi \hbar c}{\lambda}$$

Substitution gives

 $\lambda = 0.816 \ \mu m (3S \rightarrow 2P)$ 

and  $\lambda = 0.674 \ \mu m \ (2P \rightarrow 2S)$ 

Q.101. The wavelengths of the yellow doublet components of the resonance Na line caused by the transition  $3P \rightarrow 3S$  are equal to 589.00 and 589.56 nm. Find the splitting of the 3P term in eV units.

**Ans.** The splitting of the Na lines is due to the fine structure splitting of 3 p lines (The 3 s state is nearly single except for possible hyperfine effects.) The splitting of the 3 p level then equals the energy difference

$$\Delta E = \frac{2 \pi \hbar c}{\lambda_1} - \frac{2 \pi \hbar c}{\lambda_2} = \frac{2 \pi \hbar c (\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \approx \frac{2 \pi \hbar c \Delta \lambda}{\lambda^2}$$

Here  $\Delta \lambda$  = wavelength difference &  $\lambda$  = average wavelength. Substitution gives  $\Delta E$  = 2.0 meV

## Q.102. The first line of the sharp series of atomic cesium is a doublet with wavelengths 1358.8 and 1469.5 nm. Find the frequency intervals (in rad/s units) between the components of the sequent lines of that series.

Ans. The sharp series arise from the transitions  $ns \rightarrow mp$ . The s lines are unsplit so the splitting is due entirely to the p level. The frequency difference between sequent lines

is  $\frac{\Delta E}{\hbar}$  and is the same for all lines of the sharp series. It is

$$\frac{1}{\hbar} \left( \frac{2 \pi \hbar c}{\lambda_1} - \frac{2 \pi \hbar c}{\lambda_2} \right) = \frac{2 \pi c \Delta \lambda}{\lambda_1 \lambda_2}$$

Evaluation gives 1.645 x 10<sup>14</sup> rad/s

### Q.103. Write the spectral designations of the terms of the hydrogen atom whose electron is in the state with principal quantum number n = 3.

Ans. We shall ignore hyperfine interaction. The state with principal quantum number n = 3 has orbital angular momentum quantum number / = 0, 1, 2

The levels with these terms are 3 5, 3 P, 3 D. The total angular momentum is obtained by combining spin and angular momentum. For a single electron this leads to

$$J = \frac{1}{2}, \text{ if } L = 0$$
$$J = L - \frac{1}{2} \text{ and } L + \frac{1}{2} \text{ if } L \neq 0$$

We then get the final designations

$$3S_{\frac{1}{2}}, 3P_{\frac{1}{2}}, 3P_{3/2}, 3D_{3/2}, 3D_{5/2}.$$

Q.104. How many and which values of the quantum number J can an atom possess in the state with quantum numbers S and L equal respectively to

(a) 2 and 3;
(b) 3 and 3;
(c) 5/2 and 2?

Ans. The rule is that if  $\vec{J} = \vec{L} + \vec{S}$  then J takes the values

0

|L-S| to L+Sin step of 1. Thus :

(a) The values are 1, 2, 3, 4, 5

(b) The values are 0, 1, 2, 3, 4, 5, 6

(c) The values are  $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$ .

### Q.105. Find the possible values of total angular momenta of atoms in the states 4P and 6D.

**Ans.** For the state 4 p, L = 1, S =  $\frac{3}{2}$  (since 2s + 1 = 4). For the state 5 d, Z, = 2, s = 2

The possible values of J are

$$J:\frac{5}{2},\frac{3}{2},\frac{1}{2}$$
 for  $4p$ 

The value of the magnitude of angular momentum is  $\frac{\pi\sqrt{J(J+1)}}{J(J+1)}$ . Substitution gives the values 4 P:

$$\hbar \sqrt{\frac{1}{2} \cdot \frac{3}{2}} = \frac{\hbar \sqrt{3}}{2}, \ \hbar \sqrt{\frac{3}{2} \cdot \frac{5}{2}} = \frac{\hbar \sqrt{15}}{2}$$

and  $\hbar \sqrt{\frac{5}{2} \cdot \frac{7}{2}} = \frac{\hbar \sqrt{35}}{2}$ 

$$5 D: 0, \pi\sqrt{2}, \pi\sqrt{6}, \pi\sqrt{12}, \pi\sqrt{20}$$

Q.106. Find the greatest possible total angular momentum and the corresponding spectral designation of the term

(a) of a Na atom whose valence electron possesses the principal quantum number n = 4;

(b) of an atom with electronic configuration  $1s^22p3d$ .

**Ans.** (a) For the Na atoms the valence electron has principal quantem number n = 4, and the possible values of orbital angular momentum are l = 0, 1, 2, 3 so  $l_{max} = 3$ . The state

is <sup>2</sup>F, maximum value of J is 
$$\frac{7}{2}$$
.

Thus the state with maximum angular momentum will be

$$M_{\rm max} = \frac{\pi}{h} \sqrt{\frac{7}{2} \cdot \frac{9}{2}} = \frac{\pi \sqrt{63}}{2}$$

For this state

(b) For the atom with electronic configuration  $1 \text{ s}^2 2 \text{ p} 3 \text{ d}$ . There are two inequivalent valence electrons. The total orbital angular moments will be 1, 2, 3 so we pick l = 3. The total spin angular momentum will be s = 0, 1 so we pick up s = 1. Finally 7 will be 2, 3, 4 so we pick up 4. Thus maximum angular momentum state is

For this state  $M_{\text{max}} = \hbar \sqrt{4 \times 5} = 2\hbar \sqrt{5}$ .

# Q.107. It is known that in F and D states the number of possible values of the quantum number J is the same and equal to five. Find the spin angular momentum in these states.

Ans. For the f state L = 3, For the d state L = 2. Now if the state has spin s the possible angular momentum are

$$|L-S|$$
 to  $L+S$ 

The number of J angular momentum values is 2 S + 1 if  $L \ge S$  and 2 L + 1 if L < S. Since the number of states is 5, we must have  $S \ge L = 2$  for D state while  $S \le 3$  and 2 S + 1 = 5 in ply S = 2 for F state. Thus for the F state total spin angular momentum

 $M_s = \hbar \sqrt{2 \cdot 3} = \hbar \sqrt{6}$ 

while for D state  $M_s \ge \pi \sqrt{6}$ .

Q.108. An atom is in the state whose multiplicity is three and the total angular momentum is  $\hbar \sqrt[7]{20}$ . What can the corresponding quantum number L be equal to?

**Ans.** Multiplicity is 2S + 1 so S = 1.

Total angular m om entum is  $\hbar \sqrt{J(J+1)}$  so J = 4. Then L must equal 3,4,5

in order that J = 4 may be included in |L-S| to L+S.

Q.109. Find the possible multiplicities x of the terms of the types (a)  $xD_2$ ; (b)  $xHP_{3/2}$ ; (c)  $xF_1$ .

Ans. (a) Here J = 2, L = 2. Then S = 0, 1, 2, 3, 4and the multiplicities(2S - 1) are 1, 3, 5, 7, 9.

(b) Here J = 3 / 2, L = 1 Then

 $s = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$  and the multiplicities are 6, 4, 2

(c) Here J = 1, L = 3. Then S = 2, 3, 4 and the multiplicities are 5, 7, 9

Q.110. A certain atom has three electrons (s, p, and d), in addition to filled shells, and is in a state with the greatest possible total mechanical moment for a given configuration. In the corresponding vector model of the atom find the angle between the spin momentum and the total angular momentum of the given atom.

Ans. The total angular momentum is greatest when L, S are both greatest and add to

form J. Now for a triplet of s, p> d electrons Maximum spin  $\rightarrow s - \frac{3}{2}$  corresponding to

$$M_{s} = \hbar \sqrt{\frac{3}{2} \cdot \frac{5}{2}} = \frac{\hbar \sqrt{15}}{2}$$

Maximum oibital angular momentum  $\rightarrow$  L = 3

$$M_L = \hbar \sqrt{\frac{3}{2} \cdot \frac{5}{2}} = \frac{\hbar \sqrt{15}}{2}$$

corresponding to

 $J = \frac{9}{2}$ 

Maximum total angular momentum

corresponding to 
$$M = \frac{\hbar}{2}\sqrt{99}$$

In vector model  $\vec{L} - \vec{J} - \vec{s}$  or in magnitude squared

$$L\left(L+1\right)\hbar^{2} = J\left(J+1\right)\hbar^{2} + S\left(S+1\right)\hbar^{2} - 2\vec{J}\cdot\vec{S}$$

Thus

$$\cos(\checkmark \vec{J}, \vec{S}) = \frac{J(J+1) + S(S+1) - L(L+1)}{2\sqrt{J(J+1)}\sqrt{S(S+1)}}$$

Substitution gives  $\langle \vec{J}, \vec{S} \rangle = 31.1^{\circ}$ .

Q.111. An atom possessing the total angular momentum  $\hbar \sqrt{6}$  is in the state with spin quantum number S = 1. In the corresponding vector model the angle between the spin momentum and the total angular momentum is  $\theta = 73.2^{\circ}$ . Write the spectral symbol for the term of that state.

Ans. Total angular momentum  $\hbar\sqrt{6}$  means J = 2. It is gives that S = 1.

This means that L = 1, 2, or 3. From vector model relation

 $L(L+1)\hbar^2 = 6\hbar^2 + 2\hbar^2 - 2\hbar^2\sqrt{6}\sqrt{2} \cos 73.2^\circ$ 

 $= 5.998 \hbar^2 \approx 6 \hbar^2$ 

Thus E = 2 and the spectral symbol of the state is  ${}^{3}D_{2}$ .

Q.112. Write the spectral symbols for the terms of a two-electron system consisting of one p electron and one d electron.

Ans. In a system containing a p electron and a d electron S = 0,1L = 1,2,3 For S = 0 we have the terms

 ${}^{1}P_{1}$ ,  ${}^{1}D_{2}$ ,  ${}^{1}F_{3}$ 

For S = 1 we have the terms

 ${}^{3}P_{0}, {}^{3}P_{1}, {}^{3}P_{2}, {}^{3}D_{1}, {}^{3}D_{2}, {}^{3}D_{3}, {}^{3}F_{2}, {}^{3}F_{3}, {}^{3}F_{4}$ 

Q.113. A system comprises an atom in  ${}^{2}P_{_{3/2}}$  state and a d electron. Find the possible spectral terms of that system.

Ans. The atom has

 $S_1 = 1/2, l_1 = 1, j_1 = \frac{3}{2}$ 

The electron has  $s_2 = \frac{1}{2}$ ,  $l_2 = 2$  so the total angular momentum quantum number must be

$$j_2 = \frac{3}{2}$$
 or  $\frac{5}{2}$ 

In L - S compling we get S = 0, 1. L = 1,2, 3 and the terms that can be formed are the same as written in the problem above. The possible values of angular momentum are

consistant

with the addition  $j_1 = \frac{3}{2}$  to  $j_2 = \frac{3}{2}$  or  $\frac{5}{2}$ .

The latter gives us J = 0, 1, 2, 3; 1, 2, 3, 4

All these values are reached above.

Q.114. Find out which of the following transitions are forbidden by the selection rules:  ${}^{2}D_{3/2} \rightarrow {}^{2}P_{1/2}, {}^{3}P_{1} \rightarrow {}^{2}S_{1/2}, {}^{3}F_{3} \rightarrow {}^{3}P_{2}, {}^{4}F_{7/2} \rightarrow {}^{4}D_{5/2}.$ 

**Ans.** Selection rules are  $\Delta S = 0$ 

 $\Delta L = \pm 1$ 

 $\Delta J = 0, \pm 1 (\text{no } 0 \rightarrow 0).$ 

Thus  ${}^{2}D_{3/2} \rightarrow {}^{2}P_{1/2}$  is allowed

 ${}^{3}P_{1} \rightarrow {}^{2}S_{1/2}$  not allowed

$${}^{3}F_{3} \rightarrow {}^{3}P_{2}$$
 is not allowed ( $\Delta L = 2$ )

 ${}^{4}F_{7/2} \rightarrow {}^{4}D_{5/2}$  is allowed

Q.115. Determine the overall degeneracy of a 3D state of a Li atom. What is the physical meaning of that value?

Ans. For a 3 d state of a Li atom,  $s - \frac{1}{2}$  because there is only one electron and L = 2.

The total degeneracy is

g = (2L + 1) (2S + 1) = 5 x 2 = 10.

The states are  ${}^{2}D_{3}$  and  ${}^{2}D_{5^{\prime}2}$  and we check that

$$g = 4 + 6 = \left(2 \times \frac{3}{2} + 1\right) + \left(2 \times \frac{5}{2} + 1\right)$$

#### **Properties of Atoms. Spectra (Part - 2)**

Q.116. Find the degeneracy of the states <sup>2</sup>P, <sup>3</sup>D, and <sup>4</sup>F possessing the greatest possible values of the total angular momentum.

Ans. The state with greatest possible total angular momentum are

For a<sup>2</sup>P state 
$$J = \frac{1}{2} + 1 = \frac{3}{2}$$
 i.e.  ${}^{2}P_{3/2}$ 

Its degeneracy is 4.

For a <sup>3</sup>D state J = 1 + 2 = 3 i.e. <sup>3</sup>D<sub>3</sub>

Its degeneracy is  $2 \times 3 + 1 = 7$ 

 $J = \frac{3}{2} + 3 = \frac{9}{2}$  i.e.  ${}^{4}F_{\frac{4}{2}}$ . For a  ${}^{4}F$  state

Its degeneracy is 
$$2 \times \frac{9}{2} + 1 = 10$$

### Q.117. Write the spectral designation of the term whose degeneracy is equal to seven and the quantum numbers L and S are interrelated as L = 3S.

Ans. The degeneracy is 2J + 1. So we must have J = 3. From L = 3 S, we see that S must be an integer since L is integral and S can be either integral or half integral. If S = 0 then L = 0 but this is consistent with J = 3. For  $S \ge 2$ ,  $L \ge 6$  and then J = 3. Thus the state is

<sup>3</sup>F<sub>3</sub>

### Q.118. What element has the atom whose K, L, and M shells and 4s subshell are filled completely and 4p subshell is half-filled?

Ans. The order of filling is K , L , M shells, then 4  $s^2$  , 3d^{10} then 4p^3. The electronic configuration of the element will be

 $1 s^2 2 s^2 2 p^6 3 s^2 3 p^6 4 s^2 3 d^{10} 4 p^3$ 

(There must be three 4p electrons)

The number of electrons is Z = 33 and the element is As. (The 3d subshell must be filled before 4p fills up.)

### Q.119. Using the Hund rules, find the basic term of the atom whose partially filled subshell contains (a) three p electrons; (b) four p electrons.

**Ans.** (a) when the partially filled shell contains three p electrons, the total spin S must equal

$$s = \frac{1}{2}$$
 or  $\frac{3}{2}$ . The state  $s = \frac{3}{2}$  has maximum spin and is totally symmetric under

exchange of spin lables. By Pauli's exclusion principle this implies that the angular part

of the wavefunction must be totally anti symmetric. Since the angular part of the wave

function a p electron is  $\vec{r}$ , the total wavefunction of three p electrons is the

totally antisymmetric combination  $\vec{r_1}, \vec{r_2}, \vec{r_3}$ . The only such combination is

$$\vec{r_1} \cdot \left(\vec{r_2} \times \vec{r_3}\right) = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ x_1 & x_2 & x_3 \end{vmatrix}$$

This combination is a scalar and hence has L = 0. The spectral term of the ground state is then

$${}^{4}S_{\frac{3}{2}}$$
 since  $J = \frac{3}{2}$ .

(b) We can think of four p electrons as consisting of a full p shell with two p holes. The state of maximum spin S is then S = 1. By Pauli's principle the orbital angular momentum part must be antisymmetric and can only have the form  $\vec{r_1} \times \vec{r_2}$ 

where  $\vec{r_1}, \vec{r_2}$  are the coordinates of holes. The result is harder to see if we do not use the concept of holes. Four p electrons can have S = 0, 1, 2 but the S = 2 state is totally symmetric. The corresponding angular wavefunction must be totally antisymmetric. But this is impossible : there is no quantity which is antisymmetric in four vectors. Thus the maximum allowed S is S = 1. We can construct such a state by coupling the spins of electrons 1 & 2 to 5 = 1 and of electrons 3 & 4 to S = 1 and then coupling the resultant spin states to S = 1. Such a state is symmetric under the exchange of spins of 1 & 2nd 3

and 4 but antisymmetric under the simultaneous exchange of (1, 2) & (3, 4). the conjugate angular wavefinction must be antisymmetric under the exchange of (1, 2) and under the exchange of (3, 4) by Pauli principle. It must also be antisymmetric under the simultaneous exchange of (1, 2) and (3, 4). (This is because two exchanges of electrons are involved.) The required angular wavefunction then has the form

$$\left(\vec{r_{1}}\times\vec{r_{2}}\right)\times\left(\vec{r_{3}}\times\vec{r_{4}}\right)$$

and is a vector, L = 1. Thus, using also the fact that the shell is more than half full, we find the spectral term  ${}^{3}P_{2}$  (J = L + S).

## Q.120. Using the Hund rules, find the total angular momentum of the atom in the ground state whose partially filled subshell contains (a) three d electrons; (b) seven d electrons.

Ans. (a) The maximum spin angular momentum of three electrons can be  $s - \frac{3}{2}$ . This state is totally symmetric and hence the conjugate angular wave function must be antisymmetric By Pauli's exclusion principle the totally antisymmetric state must have different magnetic quantum numbers. It is easy to see that for d electrons the maximum value of the magnetic quantum number for oibital angular momentum  $|M_{L_*}| = 3$  (from 2 + 1 + 0). Higher values violate Pauli's principle. Thus the state of highest orbital angular momentum consistent with Pauli's principle is L = 3.

The state of the atom is then  $F_J$  where J = L - S by Hund's rule. Thus we get

The magnitude of the angular momentum is

$$\frac{1}{h}\sqrt{\frac{3}{2}\cdot\frac{5}{2}}=\frac{\hbar}{2}\sqrt{15}.$$

(b) Seven d electrons mean three holes. Then  $s - \frac{3}{2}$  and L = 3 as before. But

$$J - L + S - \frac{9}{2}$$
 by Hund's rule for more than half filled shell. Thus the state is

#### 4F9/2

Total angular momentum has the magnitude

$$\hbar \sqrt{\frac{9}{2} \cdot \frac{11}{2}} - \frac{3\hbar}{2} \sqrt{11}.$$

### Q.121. Making use of the Hund rules, find the number of electrons in the only partially filled subshell of the atom whose basic term is (a) ${}^{3}F_{2}$ ; (b) ${}^{2}P_{3/2}$ ; (c) ${}^{6}S_{5/2}$ .

Ans. (a)  ${}^{3}F_{2}$ : The maximum value of spin is S = 1 here. This means there are 2 electrons.

L - 3 so s and p electrons are ruled out. Thus the simplest possibility is d electrons. This is the correct choice for if we were considering f electrons, the maximum value of L allowed by Pauli principle will be L = 5 (maximum value of the magnitude of magnetic quantum number will be 3 + 2 - 5.)

Thus the atom has two d electrons in the unfilled shell.

(b) 
$${}^{2}P_{3/2}$$
 Here  $L = 1, S = \frac{1}{2}$  and  $J = \frac{3}{2}$ 

Since J = L + S, Hund's rule im plies the shell is more than halffull. This means one electron less than a full shell. On the basis of hole picture it is easy to see that we have p electrons. Thus the atom has 5 p electrons.

(c)  ${}^{6}S_{5/2}$  Here  ${}^{5}S_{2} = \frac{5}{2}L = 0$ . We either have five electrons or five holes. The angular part is antisymmetric. For five d electrons, the maximum value of the quantum number consistent with Pauli exclusion principle is (2 + 1 + 0 - 1 - 2) = 0 so L = 0. For f or g electrons L > 0 whether the shell has five electrons or five holes. Thus the atom has five d electrons.

# Q.122. Using the Hund rules, write the spectral symbol of the basic term of the atom whose only partially filled subshell (a) is filled by 1/3, and S = 1; (b) is filled by 70%, and S = 3/2.

Ans. (a) If S = 1 is the maximum spin then there must be two electrons (If there are two holes then the shell will be more than half full.). This means that there are 6 electrons in the full shell so it is a p shell. By Paul's principle the only antisymmetric combination of two electrons has L = 1 Also J = L - S as the shell is less than half full. Thus the term is  ${}^{3}P_{0}$ 

(b)  $s = \frac{3}{2}$  means either 3 electrons or 3 holes. As the shell is more than half full the former possibility is ruled out. Thus we must have seven d electrons. Then as in

problem 6.120 we get the term  ${}^{4F_{9/2}}$ 

# Q.123. The only partially filled subshell of a certain atom contains three electrons, the basic term of the atom having L = 3. Using the Hund rules, write the spectral symbol of the ground state of the given atom.

Ans. With three electrons  $s = \frac{3}{2}$  and the spin part is totally symmetric. It is given that the basic term has L = 3 so L = 3 is the state of highest orbital angular momentum.

This is not possible with p electron so we must have d electrons for which L = 3 for 3 electrons. For three f, g electrons L > 3. Thus we have 3 d electrons. Then as in (6.120) the ground state is

 ${}^{4}F_{\frac{3}{2}}$ 

### Q.124. Using the Hund rules, find the magnetic moment of the ground state of the atom whose open subshell is half-filled with five electrons.

Ans. We have 5d electrons in the only unfilled shell. Then  $s = \frac{5}{2}$  aximum value of L consistent with Pauli's principle is L = 0. Then

$$J = \frac{5}{2}$$

So by Lande's formula

$$g = 1 + \frac{\frac{5}{2}\left(\frac{7}{2}\right) + \frac{5}{2}\left(\frac{7}{2}\right) - 0}{2\frac{5}{2}\left(\frac{7}{2}\right)} = 2$$

Thus  $\mu = g\sqrt{J(J+1)} \ \mu_B = 2 \frac{\sqrt{35}}{2} \mu_B = 2\sqrt{35} \ \mu_B$ .

The ground state is  ${}^{6}S_{5/2}$ .

Q.125. What fraction of hydrogen atoms .is in the state with the principal quantum number n = 2 at a temperature T = 3000 K?

Ans. By Boltzmann formula

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\Delta E/KT}$$

Here  $\Delta E$  = energy difference between n = 1 and n = 2 states

$$= 13.6 \left( 1 - \frac{1}{4} \right) eV = 10.22 eV$$

$$\frac{N_2}{N_1} = 4 e^{-10.22 \times 1.602 \times 10^{-19} / 1.38 \times 10^{-23} \times 3000} = 2.7 \times 10^{-17}$$
Explicitly  $\eta = \frac{N_2}{N_1} = n^2 e^{-\Delta E / KT}, \ \Delta E_n = \pi R \left( 1 - \frac{1}{n^2} \right)$ 

for the nth excited state because the degeneracy of the state with principal quantum number n is  $2 n^2$ .

Q.126. Find the ratio of the number of atoms of gaseous sodium in the state 3P to that in the ground state 3S at a temperature T 2400 K. The spectral line corresponding to the transition  $3P \rightarrow 3S$  is known to have the wavelength  $\lambda = 589$  nm.

Ans. We have

$$\frac{N}{N_0} = \frac{g}{g_0} e^{-\hbar\omega/kT} = \frac{g}{g_0} e^{-2\pi\hbar c/\lambda kT}$$

Here g = degeneracy of the 3P sate = 6,  $g_0 = degeneracy$  of the 3S state = 2

and  $\lambda$  wavelength of the  $3P \rightarrow 3S_{\text{line}} \left(\frac{2\pi\hbar c}{\lambda} = \text{energy difference between 3P \& 3S levels.}\right)$ 

Substitution gives 
$$\frac{N}{N_0} = 1.13 \times 10^{-4}$$

Q.127. Calculate the mean lifetime of excited atoms if it is known that the intensity of the spectral line appearing due to transition to the ground state diminishes by a factor  $\eta = 25$  over a distance l = 2.5 mm along the stream of atoms whose velocity is v = = 600 m/s.

Ans. Let T = mean life time of the excited atoms. Then the number of excited atoms will decrease with time as  $e^{-\nu T}$ . In time t the atom travels a distance vt so  $t - \frac{l}{v}$ . Thus the number of excited atoms in a beam that has traversed a distance 1 has decreased by  $e^{-\nu v T}$ 

The intensity of the line is proportional to the number of excited atoms in the, beam.

Thus

$$e^{-l/v\tau} = \frac{1}{\eta}$$
 or  $\tau = \frac{l}{v \ln \eta} = 1.29 \text{ x } 10^{-6} \text{ second.}$ 

Q.128. Rarefied Hg gas whose atoms are practically all in the ground state was lighted by a mercury lamp emitting a resonance line of wavelength  $\lambda = 253.65$  nm. As a result, the radiation power of Hg gas at that wavelength turned out to be P = 35 mw. Find the number of atoms in the state of resonance excitation whose mean lifetime is  $\zeta = 0.15\mu s$ .

Ans. As a result of the lighting by the mercury lamp a number of atoms are pumped to the excited state. In equilibrium the number of such atoms is N. Since the mean life N

time of the atom is T, the number decaying per unit time is  $\overline{\tau}$  Since a photon of

energy  $\frac{2\pi\hbar c}{\lambda}$  results from each decay, the total radiated power will be  $\frac{2\pi\hbar c}{\lambda} \cdot \frac{N}{\tau}$ . This

must equal P. Thus

$$N = P\tau \left/ \frac{2\pi\hbar c}{\lambda} = \frac{P\tau\lambda}{2\pi\hbar} = 6.7 \times 10^9$$

Q.129. Atomic lithium of concentration  $n = 3.6.10^{16}$  cm<sup>-3</sup> is at a temperature T = 1500 K. In this case the power emitted at the resonant line's wavelength  $\lambda = 671$  nm (2P  $\rightarrow$  2S) per unit volume of gas is equal to P = 0.30 W/cm<sup>3</sup>. Find the mean lifetime of Li atoms in the resonance excitation state.

Ans. The number of excited atoms per unit volume of the gas in 2P state is

$$N = n \frac{g_p}{g_s} e^{-2\pi h c/\lambda kT}$$

Here  $g_p =$  degeneracy of the 2p state = 6,  $g_r =$  degeneracy of the 2s state = 2 and  $\lambda = \frac{N}{\tau}$ wavelength of the resonant line 2 p  $\rightarrow$  2 s. The rate of decay of these atoms is  $\frac{N}{\tau}$  per sec. per unit volume. Since each such atom emits light of wavelength X, we must

 $\frac{1}{\tau} \frac{2\pi\hbar c}{\lambda} n \frac{g_p}{g_s} e^{-2\pi\hbar c/\lambda kT} = P$  $\tau = \frac{1}{P} \frac{2\pi\hbar c}{\lambda} n \frac{g_p}{g_s} e^{-2\pi\hbar c/\lambda kT} = 65.4 \times 10^{-9} \text{ s} = 65.4 \text{ ns}$ Thus

Q.130. Atomic hydrogen is in thermodynamic equilibrium with its radiation. Find: (a) the ratio of probabilities of induced and spontaneous radiations of the atoms from the level 2P at a temperature T = 3000 K; (b) the temperature at which these probabilities become equal.

**Ans.** (a) We know that

have

 $P_{21}^{sp} = A_{21}$   $P_{21}^{ind} = B_{21} u_{ar}$   $= \frac{\pi^2 c^3}{\hbar \omega^3} A_{21} \cdot \frac{\hbar \omega^3}{\pi^2 c^3} \frac{1}{e^{-\hbar/kT} - 1} = \frac{A_{21}}{e^{-\hbar \omega/kT} - 1}$   $\frac{P_{21}^{ind}}{P_{21}^{sp}} = \frac{1}{e^{-\hbar \omega/kT} - 1}$ Thus
For the transition 2P $\rightarrow$ 1S  $\hbar \omega = \frac{3}{4} \hbar R \text{ and}$ 

$$\frac{P_{21}^{1in}}{P_{22}^{sp}} = e^{-h\omega kT}$$

we get  $P_{21}^{sp}$ 

substitution gives 7 x 10<sup>-18</sup>

(b) The two rates become equal when  $e^{i\hbar\omega/kT} = 2$ 

or 
$$T = (\pi \omega / k \ln 2) = 1.71 \times 10^5 \text{ K}$$

#### **Properties of Atoms. Spectra (Part - 3)**

Q.131. A beam of light of frequency  $\omega$ , equal to the resonant frequency of transition of atoms of gas, passes through that gas heated to temperature T. In this case  $h\omega \gg kT$ . Taking into account induced radiation, demonstrate that the absorption coefficient of the gas x varies as  $x = x_0$  (1. —  $e^{-h\omega/kT}$ ), where  $x_0$  is the absorption coefficient for  $T_0$ .

**Ans.** Because of the resonant nature of the processes we can ignore nonresonant processes. We also ignore spontaneous emission since it does not contribute to the absorption coefficient and is a small term if the beam is intense enough.

Suppose I is the intensity of the beam at some point. The decrease in the value of this intensity on passing through the layer of the substance of thickness d x is equal to

$$-\,d\,I\,=\,X\,I\,d\,x\,=\,(\,N_1\,B_{12}\,-\,N_2\,B_{21}\,)\,\Big(\frac{I}{c}\Big)\hbar\,\omega\,\,dx$$

Here  $N_1 = N_0$ . of atoms in lower level  $N_2 = N_0$  of atoms in the upper level per unit volume.  $B_{12}$ ,  $B_{21}$  are E in stein coefficients and  $I_c$  = energy density in the beam , c = velocity of light

A factor  $^{\dagger \omega}$  arises because each transition result in a loss or gain of energy  $^{\dagger \omega}$ 

$$x = \frac{\pi \omega}{c} N_1 B_{12} \left( 1 - \frac{N_2 B_{21}}{N_1 B_{12}} \right)$$

Hence

But  $g_1B_{12} = g_2B_{21}$  so

$$x = \frac{\hbar \omega}{c} N_1 B_{12} \left( 1 - \frac{g_1}{g_2} \frac{N_2}{N_1} \right)$$

By Boltzman factor  $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-h \omega/kT}$ 

When  $h \to kT$  we can put  $N_1 = N_0$  the total number of atoms per unit volume.

Then  $x = x_0 \left(1 - e^{-h \omega/kT}\right)$ 

where  $x_0 = \frac{\hbar \omega}{c} N_0 B_{12}$  is the absorption coefficient for  $T \to 0$ .

Q.132. The wavelength of a resonant mercury line is  $\lambda = 253.65$  nm. The mean lifetime of mercury atoms in the state of resonance excitation is  $\zeta = 0.15 \mu s$ . Evaluate the ratio of the Doppler line broadening to the natural linewidth at a gas temperature T = 300 K.

Ans. A short lived state of mean life T has an uncertainty in energy of  $\Delta E_{-}\frac{\hbar}{T}$  which is transmitted to the photon it emits as natural broadening. Then

$$\Delta \omega_{nat} = \frac{1}{T} \quad \text{so} \quad \Delta \lambda_{nat} = \frac{\lambda^2}{2 \pi c \tau}.$$

The Doppler broadening on the other hand arises from the thermal motion of radiating atoms.

The effect is non-relativistic and the maximum broadening can be written as

$$\frac{\Delta \lambda_{Dop}}{\lambda} = 2\beta = \frac{2v_{pr}}{c}$$

Thus  $\frac{\Delta \lambda_{Dopp}}{\Delta \lambda_{max}} = \frac{4 \pi v_{pr} \tau}{\lambda}$ 

$$\int_{\Omega} v_{pr} = \sqrt{\frac{2RT}{M}} = 157 \,\mathrm{m/s},$$

Substitution gives using

$$\frac{\Delta \lambda_{Dopp}}{\Delta \lambda_{nat}} \approx 1.2 \times 10^3$$

Note: Our formula is an order of magnitude estimate.

Q.133. Find the wavelength of the  $K_{\alpha}$  line in copper (Z = 29) if the wavelength of the  $K_{\alpha}$  line in iron (Z = 26) is known to be equal to 193 pm.

Ans. From Moseley's law

$$\omega_{K_{\alpha}} = \frac{3}{4}R(Z-1)^{2}$$
or
$$\lambda_{K_{\alpha}} = \frac{4}{3R}\frac{1}{(Z-1)^{2}}$$

$$\frac{\lambda_{K_{\alpha}}(Cu)}{\lambda_{K_{\alpha}}(Fe)} = \left(\frac{25}{28}\right)^{2} = \left(\frac{Z_{Fe}-1}{Z_{cu}-1}\right)^{2}$$
Thus

Substitution gives

 $\lambda_{K_{\bullet}}(Cu) = 153.9 \text{ pm}$ 

Q.134. Proceeding from Moseley's law find:
(a) the wavelength of the K<sub>a</sub> line in aluminium and cobalt:
(b) the difference in binding energies of K and L electrons in vanadium.

Ans. (a) From Moseley's law

$$\omega_{k\alpha} = \frac{3}{4}R(Z-\sigma)^2$$
or
$$\lambda_{K_{\alpha}} = \frac{2\pi c}{\omega_{K_{\alpha}}} = \frac{8\pi c}{3R}\frac{1}{(Z-\sigma)^2}$$

We shall take  $\sigma = 1$ . For Aluminium (2 - 13)

 $\lambda_{K_a}(Al) = 843.2 \text{ pm}$ 

and for cobalt  $(\frac{2}{2} = 27)$ 

$$\lambda_{K_a}$$
 (Co) = 179-6 pm

(b) This difference is nearly equal to the energy of the  $K_{\alpha}$  line which by Moseley's law is equal to (= 23 for vanadium)

$$\Delta E = \hbar \omega_{K_{a}} = \frac{3}{4} \times 13.62 \times 22 \times 22 = 4.94 \, \text{keV}$$

Q.135. How many elements are there in a row between those whose wavelengths of  $K_{\alpha}$  lines are equal to 250 and 179 pm?

Ans. We calculate the <sup>2</sup> values corresponding to the given wavelengths using Moseley's law. See problem (134). Substitution gives that

$$\mathbf{z} = 23$$
 corresponding to  $\lambda = 250$  pm

and  $\mathbf{z} = 27$  corresponding to  $\lambda = 179$  pm

There are thus three elements in a row between those whose wavelengths of  $K_{\alpha}$  lines are equal to 250 pm and 179 pm.

## Q.136. Find the voltage applied to an X-ray tube with nickel anticathode if the wavelength difference between the $K_{\alpha}$ line and the short-wave cut-off of the continuous X-ray spectrum is equal to 84 pm.

Ans. From Moseley's law

$$\lambda_{K_{\alpha}}(Ni) = \frac{8\pi c}{3R} \frac{1}{(Z-1)^2}$$

where  $\mathbf{z} = 28$  for N<sub>i</sub>. Substitution gives

 $\lambda_{K_{-}}(Ni) = 166.5 \text{ pm}$ 

Now the short wave cut of off the continuous spectrum must be more energetic (smaller wavelength) otherwise  $K_{\alpha}$  lines will not emerge. Then since  $\lambda = \lambda_{K_{\alpha}} - \lambda_0 = 84 \text{ pm}$  we get

 $\lambda = 82.5 \text{ pm}$ This corresponds to a voltage of

$$V = \frac{2\pi\hbar c}{e\lambda_0}$$

Substitution gives V = 15.0 kV

Q.137. At a certain voltage applied to an X-ray tube with aluminium anticathode the short-wave cut-off wavelength of the continuous X-ray spectrum is equal to 0.50 nm. Will the K series of the characteristic spectrum whose excitation potential is equal to 1.56 kV be also observed in this case?

Ans. Since the short wavelength cut off of the continuous spectrum is

 $\lambda_{\rm o}=0.50nm$ 

$$V = \frac{2\pi\hbar c}{e\lambda_0} = 2.48 \, k \, \text{V}$$

the voltage applied m ust be

since this is greater than the excitation potential of the K series of the characteristic spectrum (which is only 1.56 k V) the latter will be observed.

Q.138. When the voltage applied to an X-ray tube increased from  $V_1 = 10kV$  to  $V_2 = 20 kV$ , the wavelength interval between the  $K_{\alpha}$  line and the short-wave cut-off of the continuous X-ray spectrum increases by a factor n = 3.0. Find the atomic number of the element of which the tube's anticathode is made.

**Ans.** Suppose  $\lambda_{o}$  = wavelength of the characteristic X-ray line. Then using the formula for short wavelength limit of continuous radiation

$$\frac{\lambda_0 - \frac{2\pi\hbar c}{eV_1}}{\lambda_0 - \frac{2\pi\hbar c}{eV_2}} = \frac{1}{n}$$
  
Hence  $\lambda_0 = \frac{2\pi\hbar c}{eV_1} \frac{\left(n - \frac{V_1}{V_2}\right)}{n - 1}$ 

Using also Moseley's law, we get

$$Z = 1 + \sqrt{\frac{8\pi c}{3R\lambda}} = 1 + 2\sqrt{\frac{n-1}{3\hbar R}} \frac{eV_1}{n - \frac{V_1}{V_2}} = 29$$

#### Q.139. What metal has in its absorption spectrum the difference between the frequencies of X-ray K and L absorption edges equal to $\Delta \omega = 6.85.10^{18} \text{ s}^{-1}$ ?

**Ans.** The difference in frequencies of the K and L absorption edges is equal, according to the Bohr picture, to the frequency of the  $K_{\alpha}$  line (see the diagram below). Thus by Moseley's formule



$$\Delta \omega = \frac{3}{4} R \left( \mathbb{Z} - 1 \right)^2$$

Or

$$Z = 1 + \sqrt{\frac{4\Delta\omega}{3R}} = 22$$

The metal is titanium,

### Q.140. Calculate the binding energy of a K electron in vanadium whose L absorption edge has the wavelength $\lambda_L = 2.4$ nm.

**Ans.** From the diagram above w e see that the binding energy  $E_b$  of a K electron is the sum of the energy of a  $K_{\alpha}$  line and the energy corresponding to the L edge of absorption spectrum

$$E_b = \frac{2 \pi \hbar c}{\lambda_L} + \frac{3}{4} \hbar R (Z1)^2$$

For vanadium = 23 and the energy of  $K_{\alpha}$  line of vanadium has been calculated in problem 134 (b). Using

 $\frac{2\pi\hbar c}{\lambda_L} = 0.51 \,k\,\text{eV} \text{ for } \lambda_L = 2.4\,\text{nm}$ 

we get  $E_b = 5.46 \text{ k eV}$ 

Q.141. Find the binding energy of an L electron in titanium if the wavelength difference between the first line of the K series and its short-wave cut-off is  $\Delta \lambda = 26$  pm.

Ans. By Moseley's law

$$\hbar \omega = \frac{2 \pi \hbar c}{\lambda} E_K - E_L = \frac{3}{4} \hbar R (Z - 1)^2$$

where -  $E_{\kappa}$  is the energy of the K electron and -  $E_{L}$  of the L electron. Also the energy of the line corresponding to the short wave cut off of the K series is

$$E_{K} = \frac{2\pi\hbar c}{\lambda - \Delta\lambda} = \frac{2\pi\hbar c}{\frac{2\pi c}{\omega} - \Delta\lambda}$$
$$= \frac{\hbar}{\frac{1}{\omega} - \frac{\Delta\lambda}{2\pi c}} = \frac{\hbar\omega}{1 - \frac{\omega\Delta\lambda}{2\pi c}}$$
$$E_{L} = \frac{\hbar\omega}{1 - \frac{\omega\Delta\lambda}{2\pi c}} - \hbar\omega = \frac{\hbar\omega}{\frac{2\pi c}{\omega\Delta\lambda} - 1}$$
Hence

Substitution gives for titanium ( ${}^{2} = 22$ )

$$\label{eq:second} \begin{split} \omega &= 6.85 \ x \ 10^{\scriptscriptstyle 18} \ s^{\scriptscriptstyle -1} \\ \text{and hence} \ E_{\scriptscriptstyle L} &= 0.47 keV \end{split}$$

## Q.142. Find the kinetic energy and the velocity of the photoelectrons liberated by $K_{\alpha}$ radiation of zinc from the K shell of iron whose K band absorption edge wavelength is $\lambda_{\kappa}$ = 174 pm.

Ans. The energy of the K a radiation of  $\frac{2}{3}$  is

$$\hbar\omega=\frac{3}{4}\hbar R\left(\mathcal{Z}-1\right)^2$$

where E = atomic number of z inc = 30. The binding energy of the K electrons in

iron is obtained from the wavelength of K absorption edge as  $E_{K} = 2\pi \hbar c/\lambda_{K}$ . Hence by

Einstein equation

$$T = \frac{3}{4} \hbar R \left( \mathcal{Z} - 1 \right)^2 - \frac{2 \pi \hbar c}{\lambda_K}$$

Substitution gives T = 1.463 k eVThis corresponds to a velocity of the photo electrons of  $v = 2.27 \times 10^6 \text{m/s}$ 

#### Q.143. Calculate the Lande g factor for atoms (a) in S states; (b) in singlet states.

Ans. From the Lande formula  $g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$ 

(a) For S states L = 0. This implies J = S. Then, if S = 0g = 2 (For singlet states g is not defined if L = 0)

(b) For singlet states, J = L

$$g = 1 + \frac{J(J+1) - L(L+1)}{2J(J+1)} = 1$$

Q.144. Calculate the Lande g factor for the following terms: (a)  ${}^{6}F_{1/2}$ ; (b)  ${}^{4}D_{1/2}$ ; (c)  ${}^{5}F_{2}$ ; (d)  ${}^{5}P_{1}$ ; (e)  ${}^{3}P_{0}$ .

Ans. (a) <sup>6</sup>F<sub>1</sub>/<sub>2</sub> Here 
$$S = \frac{5}{2}, L = 3, J = \frac{1}{2}$$
  
 $g = 1 + \frac{\frac{3}{4} + \frac{35}{4} - 12}{2 \times \frac{3}{4}} = 1 + \frac{38 - 48}{6} = -\frac{2}{3}$   
(b) <sup>4</sup>D<sub>1/2</sub>: Here  $S = \frac{3}{2}, L = 2, J = \frac{1}{2}$   
 $g = 1 + \frac{\frac{3}{4} + \frac{15}{4} - 6}{2 \times \frac{3}{4}} = 1 + \frac{18 - 24}{6} = 0$   
(c) <sup>5</sup>F<sub>2</sub> Here S = 2, L = 3, J = 2  
 $g = 1 + \frac{6 + 6 - 12}{2 \times 6} = 1$   
(d) <sup>5</sup>P<sub>1</sub> Here S = 2, L = 1, J = 1  
 $g = 1 + \frac{2 + 6 - 2}{2 \times 2} = \frac{5}{2}$ 

(e)  ${}^{3}P_{0}$ . For states with J = 0, L = S the g gactor is indeterminate.

Q.145. Calculate the magnetic moment of an atom (in Bohr magnetons) (a) in <sup>1</sup>F state; (b) in  ${}^{2}D_{3/2}$  state; (c) in the state in which S = 1, L = 2, and Lande factor g = 4/3.

**Ans.** (a) For the lF state S = 0, L = 3 = J

$$g = 1 + \frac{3 \times 4 - 3 \times 4}{2 \times 3 \times 4} = 1$$

Hence  $\mu = \sqrt{3 \times 4} \mu_B = 2\sqrt{3} \mu_B$ 

(b) For the 
$${}^{2}D_{3/2}$$
 state  $S = \frac{1}{2}, L = 2, J = \frac{3}{2}$ 

$$g = 1 + \frac{\frac{15}{4} + \frac{3}{4} - 6}{2 \times \frac{15}{4}} = 1 + \frac{18 - 24}{30} = \frac{4}{5}$$

$$\mu = \frac{4}{5}\sqrt{15/4} \ \mu_B = \frac{2}{5}\sqrt{15} \ \mu_B = 2\sqrt{\frac{3}{5}} \ \mu_B.$$
 Hence

(c) We have 
$$\frac{4}{3} = 1 + \frac{J(J+1)+2-6}{2J(J+1)}$$

or 
$$\frac{4}{3}J(J+1) = J(J+1) - 4$$

or 
$$J(J+1) = 12 \implies J = 3$$

$$\mu = \frac{4}{3}\sqrt{12} \mu_B = \frac{8}{\sqrt{3}} \mu_B.$$

He

#### Q.146. Determine the spin angular momentum of an atom in the state D<sub>2</sub> if the maximum value of the magnetic moment projection in that state is equal to four Bohr magnetons.

**Ans.** The expression for the projection of the magnetic moment is

 $\mu_Z = g m_J \mu_B$ 

where mj is the projection of  $\vec{J}$  on the Z-axis.

Maximum value of the m<sub>j</sub> is J. Thus

gJ = 4Since J = 2, we get g = 2. Now  $2 = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$  $= 1 + \frac{6 + S(S+1) - 6}{2 \times 6}, \text{ as } L = 2$  $= 1 + \frac{S(S+1)}{12}$ Hence S(S+1) = 12 or S = 3 Thus  $M_S = \pi \sqrt{3 \times 4} = 2\sqrt{3}\pi$ 

# Q.147. An atom in the state with quantum numbers L = 2, S = 1. is located in a weak magnetic field. Find its magnetic moment if the least possible angle between the angular momentum and the field direction is known to be equal to $30^{\circ}$ .

**Ans.** The angle between the angular momentum vector and the field direction is the least when the angular momentum projection is maximum i.e. *In*.

Thus 
$$J\hbar = \sqrt{J(J+1)} \hbar \cos 30^\circ$$

or 
$$\sqrt{\frac{J}{J+1}} - \frac{\sqrt{3}}{2}$$

Hence J = 3

Then 
$$g = 1 + \frac{3 \times 4 + 1 \times 2 - 2 \times 3}{2 \times 3 \times 4} = 1 + \frac{8}{24} = \frac{4}{3}$$

$$\mu = \frac{4}{3}\sqrt{3\times 4} \ \mu_B = \frac{8}{\sqrt{3}} \ \mu_B. \label{eq:multiple}$$

and

# Q.148. A valence electron in a sodium atom is in the state with principal quantum number n = 3, with the total angular momentum being the greatest possible. What is its magnetic moment in that state?

Ans. For a state with n = 3, 1 = 2. Thus the state with maximum angular momentum is

<sup>2</sup>D<sub>5/2</sub>

$$g = 1 + \frac{\frac{5}{2} \times \frac{7}{2} + \frac{1}{2} \times \frac{3}{2} - 2 \times 3}{2 \times \frac{5}{2} \times \frac{7}{2}}$$

Then

$$= 1 + \frac{35 + 3 - 24}{70} = 1 + \frac{1}{5} = \frac{6}{5}.$$

Hence 
$$\mu = \frac{6}{5} \sqrt{\frac{5}{2} \times \frac{7}{2}} \mu_B = 3 \sqrt{\frac{7}{5}} \mu_B.$$

#### Properties of Atoms. Spectra (Part - 4)

Q.149. An excited atom has the electronic configuration 1s<sup>2</sup>2s<sup>2</sup>2p<sup>3</sup>d being in the state with the greatest possible total angular momentum. Find the magnetic moment of the atom in that state.

Ans. To get the greatest possible angular momentum we must have  $S=S_{\text{max}}=1$   $L=L_{\text{max}}=1+2=3$  and J=L+S=4

Then  $g = 1 + \frac{4 \times 5 + 1 \times 2 - 3 \times 4}{2 \times 4 \times 5} = 1 + \frac{10}{40} = \frac{5}{4}$ 

and  $\mu = \frac{5}{4}\sqrt{4\times 5} \mu_B = \frac{5\sqrt{5}}{2}\mu_B$ 

Q.150. Find the total angular momentum of an atom in the state with S = 3/2 and L = 2 if its magnetic moment is known to be equal to zero.

Ans. Since  $\mu = 0$  we must have either J = 0 or g = 0. But J = 0 is incompatible with L

$$= 2 \text{ and } S = \frac{3}{2} \text{ Hence } g = 0. \text{ Thus}$$

$$0 = 1 + \frac{J(J+1) + \frac{3}{2} \times \frac{5}{2} - 2 \times 3}{2J(J+1)}$$
or
$$-3J(J+1) = \frac{15}{4} - 6 = -\frac{9}{4}$$
Hence
$$J = \frac{1}{2}$$
Hence
$$M = \pi \sqrt{\frac{1}{2} \times \frac{3}{2}} = \frac{\pi \sqrt{3}}{2}$$
Thus

Q.151. A certain atom is in the state in which S = 2, the total angular momentum  $M = \sqrt{2}\hbar_1$ , and the magnetic moment is equal to zero. Write the spectral symbol of the corresponding term.

Ans. From  $M = \hbar \sqrt{J+1} = \sqrt{2}\hbar$ 

we find J = 1. From the zero value of the magnetic moment we find g = o

or  

$$1 + \frac{1 \times 2L(L+1) + 2 \times 3}{2 \times 1 \times 2} = 0$$

$$1 + \frac{-L(L+1) + 8}{4} = 0$$
or  

$$12 = L (L+1)$$

Hence L = 3. The state is

<sup>5</sup>*F*<sub>1</sub>.

Q.152. An atom in the state  ${}^{2}P_{3/2}$  is located in the external magnetic field of induction B = 1.0 kG. In terms of the vector model find the angular precession velocity of the total angular momentum of that atom.

Ans. If  $\vec{M}$  is the total angular momentum vector of the atom then there is a magnetic moment

$$\vec{\mu}_m = g \mu_B \vec{M} / \hbar$$

associated with it; here g is the Lande factor. In a magnetic field of induction  $\vec{B}$ , an energy

$$H' = -g \mu_B \vec{M} \cdot \vec{B}/\hbar$$

is associated with it. This interaction term corresponds to a presession of the angular momentum vector because if leads to an equation of motion of the angular momentum vector of the form

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times M$$
  
where 
$$\vec{\Omega} = \frac{g\mu_B\vec{B}}{h}$$

Using Gaussian unit expression of  $\mu_B \mu_B = 0.927 \times 10^{-20}$  eig/gauss, B = 10<sup>3</sup> gauss

 $h = 1.054 \times 10^{-27}$  erg sec and for the  ${}^{2}P_{3/2}$  state

$$g = 1 + \frac{\frac{3}{2} \times \frac{5}{2} + \frac{1}{2} \times \frac{3}{2} - 1 \times 2}{2 \times \frac{3}{2} \times \frac{5}{2}} = 1 + \frac{1}{3} = \frac{4}{3}$$

and  $\Omega = 1.17 \text{ x } 10^{10} \text{ rad/s}$ 

The same formula is valid in MKS units also But  $\mu_B = 0.927 \ x \ 10^{-23} \ A.m^2$ ,  $B = 10^{-1} T$  and

 $h = 1.054 \text{ x } 10^{-34}$  Joule sec. The answer is the same.

Q.153. An atom in the state  ${}^{2}P_{1/2}$  is located on the axis of a loop of radius r = 5 cm carrying a current I = 10 A. The distance between the atom and the centre of the loop is equal to the radius of the latter. How great may be the maximum force that the magnetic field of that current exerts on the atom?

Ans. The force on an atom with magnetic moment  $\vec{\mu}$  in a magnetic Geld of induction  $\vec{B}$  is given by

$$\vec{F} = (\vec{\mu} \cdot \vec{\nabla}) \vec{B}$$

In the present case, the maximum force arise when  $\vec{\mu}$  is along the axis or close to it.

Then  $F_{Z} = (\mu_{Z})_{\max} \frac{\partial B}{\partial Z}$ 

Here  $(\mu_z)_{max} = g \mu_B J$ . The Lande factor g is for  ${}^2P_{1/2}$ 

$$g = 1 + \frac{\frac{1}{2} \times \frac{3}{2} + \frac{1}{2} \times \frac{3}{2} - 1 \times 2}{2\frac{1}{2} \times \frac{3}{2}} = 1 - \frac{1/2}{3/2} = \frac{2}{3}.$$

and  $J = \frac{1}{2}$  so  $(\mu_Z)_{max} = \frac{1}{3}\mu_B$ .

The magnetic field is given by

$$B_{Z} = \frac{\mu_0}{4\pi} \cdot \frac{2 I \pi r^2}{(r^2 + Z^2)^{3/2}}$$

or 
$$\frac{\partial B_z}{\partial \mathcal{Z}} = -\frac{\mu_0}{4\pi} 6 I \pi r^2 \frac{\mathcal{Z}}{(r^2 + \mathcal{Z}^2)^{5/2}}$$

Thus  $\left(\frac{\partial B_z}{\partial Z}\right)_{z=r} = \frac{\mu_0}{4\pi} \frac{3I\pi}{\sqrt{8}r^2}$ .

Thus the maximum force is

$$F = \frac{1}{3} \mu_B \frac{\mu_0}{4\pi} \frac{3\pi}{\sqrt{8}} \frac{I}{r^2}$$

Substitution gives (using data in M K S units)

 $F = 4.1 \text{ x } 10^{-27} \text{N}$ 

### Q.154. A hydrogen atom in the normal state is located at a distance r = 2.5 cm from a long straight conductor carrying a current I = 10 A. Find the force acting on the atom.

**Ans.**The magnetic field at a distance r from a long current carrying wire is mostly tangential and given by

$$B_{\varphi}=\frac{\mu_0I}{2\,\pi\,r}=\frac{\mu_0}{4\,\pi}\frac{2\,I}{r}.$$

The force on a magnetic dipole of moment  $\vec{\mu}$  due to this magnetic field is also tangential and has a magnitude

#### $(\vec{\mu} \nabla) B_{\varphi}$

This force is nonvanishing only when the component of  $\vec{\mu}$  along  $\vec{r}$  non zero. Then

$$F = \mu_r \frac{\partial}{\partial r} B_{\varphi} = -\mu_r \frac{\mu_0}{4\pi} \frac{2I}{r^2}$$

Now the maxim um value of  $\mu_r = \pm \mu_B$ . Thus the force is

$$F_{\text{max}} = \mu_B \frac{\mu_0}{4\pi} \frac{2I}{r^2} = 2.97 \times 10^{-26} \text{ N}$$

Q.155. A narrow stream of vanadium atoms in the ground state  ${}^{4}F_{3/2}$  is passed through a transverse strongly inhomogeneous magnetic field of length  $l_{1} = 5.0$  cm as in the Stern-Gerlach experiment. The beam splitting is observed on a screen

located at a distance  $l_2 = 15$  cm from the magnet. The kinetic energy of the atoms is T = 22 MeV. At what value of the gradient of the magnetic field induction B is the distance between the extreme components of the split beam on the screen equal to  $\delta = 2.0 \text{ mm}$ ?

**Ans.** In the homogeneous magnetic field the atom experinces a force

$$F = g J \mu_B \frac{\partial B}{\partial Z}$$

Depending on the sign of /, this can be either upward or downward. Suppose the latter is true. The atom then traverses first along a parabola inside the field and, once outside, in a straight line. The total distance between extreme lines on the screen will be

$$\delta = 2 g J \mu_B \frac{\partial B}{\partial Z} \left\{ \frac{1}{2} \left( \frac{l_1}{v} \right)^2 + \frac{l_1}{v} \cdot \frac{l_2}{v} \right\} / m_V$$

Here my is the mass of the vanadium atom. (The first term is the displacement within the field and the second term is the displacement due to the transverse velocity acquired in the magnetic field).



For vanadium atom in the ground state  ${}^{4}F_{3/2}$ 

$$g = 1 + \frac{\frac{3 \times 5}{4} + \frac{3 \times 5}{4} - 3 \times 4}{2 \times \frac{3 \times 5}{4}} = 1 + \frac{30 - 48}{30} = 1 - \frac{18}{30} = \frac{2}{5}$$

$$J = \frac{3}{2}$$
 using other data, and substituting

we get 
$$\frac{\partial B}{\partial Z} = 1.45 \times 10^{13} \text{ G/cm}$$

This value differs from the answer given in the book by almost a factor of  $10^{\circ}$ . For neutral atoms in stem Gerlach experiments, the value T = 22 MeV is much too laige. A more appropriate value will be T = 22 meV i.e.  $10^{\circ}$  times smaller. Then one gets the right answer.

Q.156. Into what number of sublevels are the following terms split in a weak magnetic field: (a)  ${}^{3}P_{0}$ ; (b)  ${}^{2}F_{5/2}$ ; (c)  ${}^{4}P_{1/2}$ ?

Ans. (a) The term  $3P_0$  does not split in weak magnetic field as it has zero total angular momentum.

(b) The term  ${}^{2}F_{5/2}$  will split into  $2 \times \frac{5}{2} + 1 = 6$  sublevels. The shift in each sublevel is given by

 $\Delta E = -g \,\mu_B M_Z B$ 

where  $M_j = -J(J-1), \dots, J$  and g is the Landi factor

$$g = 1 + \frac{\frac{5 \times 7}{4} + \frac{1 \times 3}{4} - 3 \times 4}{2 \times \frac{5 \times 7}{4}} = 1 + \frac{38 - 48}{70} = \frac{6}{7}$$

(c) In this case for the 
$${}^{4}D_{1/2}$$
 term

$$g = 1 + \frac{\frac{1 \times 3}{4} + \frac{3 \times 5}{4} - 2 \times 3}{2 \times \frac{1 \times 3}{4}} = 1 + \frac{3 + 15 - 24}{6} = 1 - 1 = 0$$

Thus the energy differences vanish and the level does not split.

Q.157. An atom is located in a magnetic field of induction B = 2.50 kG. Find the value of the total splitting of the following terms (expressed in eV units): (a) <sup>1</sup>D; (b) <sup>3</sup>F<sub>4</sub>.

**Ans.** (a) for the  ${}^{1}D_{2}$  term

$$g = 1 + \frac{2 \times 3 + 0 - 2 \times 3}{2 \times 2 \times 3} = 1$$

and  $\Delta E = -\mu_B M_J B$ 

 $M_J = -2, -1, 0, +1, +2$ . Thus the splitting is

#### $\delta E = 4 \mu_B B$

Substitution gives  $\delta E = 57.9 \,\mu eV$ 

(b) For the  ${}^{3}F_{4}$  teim  $g = 1 + \frac{4 \times 5 + 1 \times 2 - 3 \times 4}{2 \times 4 \times 5} = 1 + \frac{10}{40} = \frac{5}{4}$ .

and 
$$\Delta = -\frac{5}{4}\mu_B B M_J$$

where Mj = -4 to + 4. Thus

$$\delta E \,=\, \frac{5}{4}\,\mu_B\,B \times 8 \,=\, 10\,\mu_B\,B\,(\,=\, 2\,g\,J\,\mu_B\,)$$

Substitution gives  $\delta E = 144.7 \,\mu eV$ 

Q.158. What kind of Zeeman effect, normal or anomalous, is observed in a weak magnetic field in the case of spectral lines caused by the following transitions: (a)  ${}^{1}P \rightarrow {}^{1}S$  (b)  ${}^{2}D_{5/2} \rightarrow {}^{2}P_{3/2}$ ; (c)  ${}^{3}D_{1} \rightarrow {}^{3}P_{0}$ ; (d)  ${}^{5}l_{5} \rightarrow {}^{5}H_{4}$ ?

Ans. (a) The term  ${}^{1}P_{1}$  splits into 3 lines with  $M_{z} = \pm 1, 0$  in accordance with the formula

 $\Delta E = -g \mu_B B M_Z$ where  $g = 1 + \frac{1 \times 2 + 0 - 1 \times 2}{2 \times 1 \times 2} = 1$ 

The term  ${}^{1}S_{0}$  does not split in weak magnetic field. Thus the transitions between  ${}^{1}P_{1}$  &  ${}^{1}S_{0}$  will result in 3 lines i.e. a normal Zeeman triplet.

(b) The term  ${}^{2}D_{5/2}$  will split ot in 6 terms in accordance with the formula





$$M_z = \pm \frac{5}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}, \text{ and}$$
  
 $g = 1 + \frac{5 \times 7 + 1 \times 3 - 4 \times 2 \times 3}{2 \times 5 \times 7} = \frac{6}{5}$ 

Ther term  ${}^{2}P_{3/2}$  will also split into 4 lines in accordance with the above formula with

$$M_z = \pm \frac{3}{2}, \pm \frac{1}{2}$$
 and  $g = 1 + \frac{3 \times 5 + 1 \times 3 - 4 \times 1 \times 2}{2 \times 3 \times 5} = \frac{4}{3}$ 

It is seen that the Z eeman splitting is auomalous as g factors are different

$$(c)^{3}D_{1} \rightarrow {}^{3}P_{0}$$

The term  ${}^{3}D_{1}$  splits into 3 levels (g = 5 /2) The term  ${}^{3}P_{0}$  does not split. Thus the Z eeman spectrum is normal.

(d) For the 5I5 term

$$g = 1 + \frac{5 \times 6 + 2 \times 3 - 6 \times 7}{2 \times 5 \times 6}$$
$$= 1 + \frac{36 - 42}{60} = 1 - \frac{1}{10} = \frac{9}{10}$$

For the <sup>5</sup>H<sub>4</sub> term

$$g = 1 + \frac{4 \times 5 + 2 \times 3 - 5 \times 6}{2 \times 4 \times 5} = 1 + \frac{26 - 30}{40} = \frac{9}{10}$$

We see that the splitting in the two levels given by  $\Delta E - g \mu_B B M_2$  is the same though the number of levels is different (11 and 9). It is then easy to see that only the lines with following energies occur

 $\pi\omega_0,\pi\omega_0\pm g\mu_BB.$ 

The Z eeman pattern is normal

Q.159. Determine the spectral symbol of an atomic singlet term if the total splitting of that term in a weak magnetic field of induction B = 3.0 kG amounts to  $\Delta E = 104 \mu eV$ 

**Ans**. For a singlet term S = 0, L = J, g = 1

Then the total splitting is  $\delta E = 2J \mu_B B$ 

Substitution gives  $J = 3(-\delta E/2 \mu_B B)$ 

The term is  ${}^{1}F_{3}$ .

## Q.160. It is known that a spectral line $\lambda = 612$ nm of an atom is caused by a transition between singlet terms. Calculate the interval $\Delta\lambda$ between the extreme components of that line in the magnetic field with induction B = 10.0 kG.

**Ans.** As the spectral line is caused by transition between singlet terms, the Z eeman effect will be normal (since g = 1 for both terms). The energy difference between extreme components of the line will be 2  $\mu_B B$ . This must equal

$$-\Delta \left(\frac{2 \pi \hbar c}{\lambda}\right) = \frac{2 \pi \hbar c \Delta \lambda}{\lambda^2}$$
  
Thus 
$$\Delta \lambda = \frac{\mu_B B \lambda^2}{\pi \hbar c} = 35 \text{ pm}.$$

Q.161. Find the minimum magnitude of the magnetic field induction B at which a spectral instrument with resolving power  $\lambda/\delta\lambda = = 1.0.10^5$  is capable of resolving the components of the spectral line  $\lambda = 536$  nm caused by a transition between singlet terms. The observation line is at right angles to the magnetic field direction.

Ans. From the previous problem, if the components are  $\lambda$ ,  $\lambda \pm \Delta \lambda$ , then

$$\frac{\lambda}{\Delta\lambda} = \frac{2\pi\hbar c}{\mu_B B\lambda}$$

For resolution  $\frac{\lambda}{\Delta\lambda} \leq R = \frac{\lambda}{\delta\lambda}$  of the instrument

Thus

$$\frac{2\pi\hbar c}{\mu_B B \lambda} \le R \quad \text{or} \quad B \ge \frac{2\pi\hbar c}{\mu_B \lambda R}$$

Hence the minimum megnetic induction is

$$B_{\min} = \frac{2\pi\hbar c}{\mu_B\lambda R} = 4kG = 0.4 \mathrm{T}$$

Q.162. A spectral line caused by the transition  ${}^{3}D_{1} \rightarrow {}^{3}P_{0}$  experiences the Zeeman splitting in a weak magnetic field. When observed at right angles to the magnetic field direction, the interval between the neighbouring components of the split line is  $\Delta \omega = 1.32.10^{10}$  s<sup>-1</sup> Find the magnetic field induction B at the point where the source is located.

**Ans.** The  ${}^{3}P_{0}$  term does not split The  ${}^{3}D_{1}$  term splits into 3 lines corresponding to the shift

 $\Delta E = -g \,\mu_B B \,M_Z$ 

with  $M_z = \pm 1, 0$ . The interval between neighbouring components is then given by

 $\hbar \Delta \omega = g \mu_B B$ 

Hence

$$B = \frac{\hbar \Delta \omega}{g \mu_B}$$

Now for the  ${}^{3}D_{1}$  term

 $g = 1 + \frac{1 \times 2 + 1 \times 2 - 2 \times 3}{2 \times 1 \times 2} = 1 + \frac{4 - 6}{4} = \frac{1}{2}.$ 

Substitution gives B = 3.00 kG = 0.3 T.

Q.163. The wavelengths of the Na yellow doublet ( ${}^{2}P \rightarrow {}^{2}S$ ) are equal to 589.59 and 589.00 nm. Find:

(a) the ratio of the intervals between neighbouring sublevels of the Zeeman splitting of the terms  ${}^{2}P_{3/2}$  and  ${}^{2}P_{1/2}$  in a weak magnetic field;

(b) the magnetic field induction B at which the interval between neighbouring sublevels of the Zeeman splitting of the term  ${}^{2}P_{3/2}$  is  $\eta = 50$  times smaller than the natural splitting of the term  ${}^{2}P$ .

**Ans.** (a) For the  ${}^{2}P_{3/2}$  term

$$g = 1 + \frac{\frac{3}{2} \times \frac{5}{2} + \frac{1}{2} \times \frac{3}{2} - 1 \times 2}{2 \times \frac{3}{2} \times \frac{5}{2}} = 1 + \frac{10}{30} = \frac{4}{3}$$

and the energy of the  ${}^{2}P_{3/2}$  sublevels will be

$$E(M_Z) = E_0 - \frac{4}{3} \mu_B B M_Z$$

 $M_z = \pm \frac{3}{2}, \pm \frac{1}{2}$ . where *M<sub>z</sub>* = ±  $\frac{3}{2}$ , ±  $\frac{1}{2}$ .

$$\delta E \left( {}^{2}P_{3/2} \right) = \frac{4}{3} \mu_{B} B$$

For the  ${}^{2}P_{1/2}$  terms

$$g = 1 + \frac{\frac{1}{2} \times \frac{3}{2} + \frac{1}{2} \times \frac{3}{2} - 1 \times 2}{2 \times \frac{1}{2} \times \frac{3}{2}}$$
$$= 1 + \frac{6 - 8}{6} = 1 - \frac{1}{3} = \frac{2}{3}$$

and the separation between the two sublevels into which the  ${}^{2}P_{1/2}$  term will split is

$$\delta E \left( {}^{2}\!P_{1/2} \right) = \frac{2}{3} \mu_{B} B$$

The ratio of the two splittings is 2 : 1.

(b) The interval between neighbouring Zeem an sublevels of the  ${}^{2}P_{3/2}$  term

is  $\frac{4}{3}\mu_B B$  The energy separation between D<sub>1</sub> and D<sub>2</sub> lines is  $\frac{2\pi\hbar c}{\lambda^2}\Delta\lambda$  (this is the natural separation of the <sup>2</sup>P them)

Thus 
$$\frac{\frac{4}{3}\mu_BB}{\lambda^2\eta}=\frac{2\pi\hbar c\,\Delta\lambda}{\lambda^2\eta}$$

$$B = \frac{3\pi\hbar c\,\Delta\lambda}{2\,\mu_B\,\lambda^2\,\eta}$$

Substitution gives

B = 5-46kG

or

## Q.164. Draw a diagram of permitted transitions between the terms ${}^{2}P_{3/2}$ and ${}^{2}S_{1/2}$ in a weak magnetic field. Find the displacements (in rad/s units) of Zeeman components of that line in a magnetic field B = 4.5 kG.

Ans. for the  ${}^{2}P_{3/2}$  level g = 4 / 3 (see above) and the energies of sublevels are

$$E' = E'_0 - \frac{4}{3} \mu_B B M'_z$$

where  $M'_{\mathcal{Z}} = \pm \frac{3}{2}, \pm \frac{1}{2}$  for the four sublevels

For the  $\frac{{}^{2}S_{\frac{1}{2}}$  level, g = 2 (since L = 0) and

 $E = E_0 - 2 \,\mu_B \, B \, M_Z$ 

where  $M_z = \pm \frac{1}{2}$ 

Permitted transitions must have  $\Delta M_{z=}$  0,  $\pm 1$ Thus only the following transitions occur

$$\frac{3}{2} \rightarrow \frac{1}{2}$$

$$-3/2 \rightarrow -1/2$$

$$\Delta \omega = \pm \mu_B B/\hbar = 3.96 \times 10^{10} \text{ rad/s}$$

$$\frac{1}{2} \rightarrow \frac{1}{2}$$

$$-\frac{1}{2} \rightarrow -\frac{1}{2}$$

$$\Delta \omega = \pm \frac{1}{3} \mu_B B/\hbar = 1.32 \times 10^{10} \text{ rad/s}$$

$$\frac{1}{2} \rightarrow -\frac{1}{2}$$

$$\Delta \omega = \pm \frac{5}{3} \frac{\mu_B B}{\hbar} = 6.6 \times 10^{10} \text{ rad/s}$$

These six lines are shown below



Q.165. The same spectral line undergoing anomalous Zeeman splitting is observed in direction 1 and, after reflection from the mirror M (Fig. 6.9), in direction



### 2. How many Zeeman components are observed in both directions if the spectral line is caused by the transition (a) ${}^{2}P_{3/2} \rightarrow {}^{2}S_{1/2}$ ; (b) ${}^{3}P_{2} \rightarrow {}^{3}S_{1}$ ?

Ans.The difference arises because of different selection rules in the two cases. In (1) the line is emitted perpendicular to the field. The selection rules are then  $\Delta M_{z=} 0, \pm 1$ 

In (2) the light is emitted along die direction of die field. Then the selection rules are  $\Delta M_{z=} \pm 1$  $\Delta M_{z=} 0$  is forbidden.

(a) In the transition  ${}^{2}P_{3/2} \rightarrow {}^{2}S_{1/2}$ 

This has been considered above. In (1) we get all the six lines shown in the problem above

In (2) the line corresponding to 
$$\frac{1}{2} \rightarrow \frac{1}{2}$$
 and  $-\frac{1}{2} \rightarrow -\frac{1}{2}$  is forbidden.

Then we get four lines

(b)  ${}^{3}P_{2} \rightarrow {}^{3}S_{1}$ For the  ${}^{3}P_{2}$  level,  $g = 1 + \frac{2 \times 3 + 1 \times 2 - 1 \times 2}{2 \times 2 \times 3} = \frac{3}{2}$ 

so the energies of the sublevels are

where 
$$M'_{z} = \pm 2, \pm 1, 0$$

For the  ${}^{3}S_{1}$  line, g = 2 and the energies of the sublevels are

 $E(M_z) = E_0 - 2 \mu_B B M_z$ 

where  $M_{z=}$  1,  $\pm 0$  The lines are

 $\Delta M_z = M_z - M_z = +1$  :  $-2 \rightarrow -1$ ,  $-1 \rightarrow 0$  and  $0 \rightarrow 1$ 

 $\Delta M_z = 0 - 1 \rightarrow -1, 0 \rightarrow 0, 1 \rightarrow 1$ 

 $\Delta M_z = 1, 2 \rightarrow 1, 1 \rightarrow 0, 0 \rightarrow -1$ 

All energy differences are unequal because die two g values are unequal. There are then nine lines if viewed along (1) and Six lines if viewed along (2).

### Q.166. Calculate the total splitting Ae) of the spectral line ${}^{3}D_{3} \rightarrow {}^{3}P_{2}$ in a weak magnetic field with induction B = 3.4 kG.

**Ans.** For the two levels

 $E'_{0} = E_{0} - g' \mu'_{B} M'_{z} B$  $E_{0} = E_{0} - g \mu_{B} M_{z} B$ 

and hence the shift of the component is the value of

$$\Delta \omega = \frac{\mu_B B}{\hbar} \left[ g' M'_z - g M_z \right]$$

subject to the selection rule  $\Delta M_z = 0$ ,  $\pm 1$ . For  ${}^3D_3$ 

$$g' = 1 + \frac{3 \times 4 + 1 \times 2 - 2 \times 3}{2 \times 3 \times 4} = 1 + \frac{8}{24} = \frac{4}{3}$$

For  ${}^{3}P_{2}$ ,

$$g = 1 + \frac{2 \times 3 + 1 \times 2 - 1 \times 2}{2 \times 2 \times 3} = \frac{3}{2}$$
  
Thus 
$$\Delta \omega = \frac{\mu_B B}{\hbar} \left| \frac{4}{3} M'_z - \frac{3}{2} M_z \right|$$

For the different transition we have the following table

		0 → 1	$-\frac{3}{2}\mu_B B$
		0→0	0
$3 \rightarrow 2$	μ <sub>B</sub> B		
		0 → - 1	3/2 μ <sub>B</sub> B
2 <b>→</b> 2	$-\frac{1}{2}\mu_B B$	- 1 → 0	– 4/3 μ <sub>B</sub> B
	3.2	-1→-1	1/6 μ <sub>B</sub> B
2→1	7/6 μ <sub>B</sub> B	$-1 \rightarrow -2 \rightarrow$	$5/3 \mu_B B$
1→2	– 5/3 μ <sub>B</sub> B	$-2 \rightarrow -1 \rightarrow$	- 7/6 µ. B
1→1	– 1/6 μ <sub>B</sub> B	$-2 \rightarrow -2 \rightarrow$	$1/3 \mu_{B} B$
1 → 0	4/3 μ <sub>B</sub> B	$-3 \rightarrow -2 \rightarrow$	$-\mu_B B$

There are 15 lines in all.

The lines farthest out are  $1 \rightarrow 2$  and  $-1 \rightarrow -2$ .

The splitting between them is the total splitting. It is

$$\Delta \omega = \frac{10}{3} \mu_B B / \hbar$$

Substitution gives  $\Delta \omega = 7.8 \times 10^{10} \text{ rad/sec}$ .