# Chapter 5

# Curves and Triangulation Survey

CHAPTER HIGHLIGHTS

- *Curves*
- *Triangulation survey*

## Curves

### Introduction

Curves are generally used on highways and railways where it is necessary to change the direction of motion. This chapter focuses on the elements of curves and methods for setting out of curves on ground.

Curve may be circular, parabolic or spiral and is always tangential to the two straight directions.

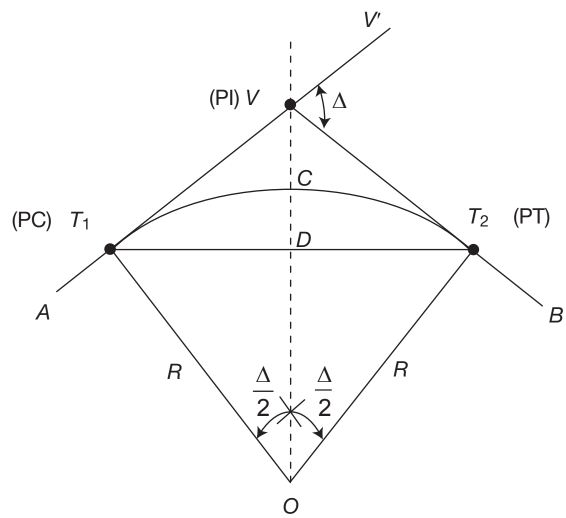
Circular curves are of three types:

1. Simple curves
2. Compound curves
3. Reverse curves

### Simple Curves

Simple curves consists of a single arc of a circle. It is tangential to both the straight lines.

1. **Back tangent ($AT_1$):** The tangent before curve is called back tangent or first tangent.
2. **Forward tangent ($BT_2$):** The tangent following the curve is called forward tangent or second tangent.
3. **Point of intersection, vertex ($V$):** It is the point of intersection of two tangents $AT_1$ and $BT_2$.




4. **Point of curve ($T_1$):** It is the beginning of the curve where the alignment changes from a tangent to a curve.
5. **Point of tangency ($T_2$):** It is the end of the curve where the alignment changes from a curve to a tangent.
6. **Deflection angle or Deviation angle ($\Delta$):** The difference between the slopes of the two tangents.

7. **Tangent distance:** It is the distance from PC to PI (also the distance from PI to PT)

$$\boxed{T_1 V = T_2 V = R \tan \frac{\Delta}{2}}$$

$R$ is the radius of the curve.

8. **External distance or apex distance (E):** The distance from vertex $V$ to the centre of the curve $C$.

$$\boxed{E = VC = R\left(\sec\frac{\Delta}{2} - 1\right)}$$

9. **Length of curve ($L$):** Distance from PC to PT,

$$\boxed{L = T_1CT_2 = R\,\Delta}\ \Delta \text{ in radians.}$$

10. **Length of long chord:** It is the chord joining PC to PT.

$$\boxed{T_1T_2 = 2R\sin\frac{\Delta}{2}}$$

11. **Mid-ordinate/versed sine:** The distance from mid-point of the long chord to mid-point of the curve.

$$\boxed{CD = R\left(1 - \cos\frac{\Delta}{2}\right)}$$

12. **Normal chord:** A chord between two successive regular stations on a curve.
13. **Sub-chord:** Any chord shorter than the normal chord.
14. **Right-hand curve:** If the curve deflects to the right of the direction of the progress of survey, it is called the right-hand curve.
15. **Left-hand curve:** If the curve deflects to the left of the direction of the progress of survey, it is called the left-hand curve.

## Chainages

Chainage of $T_1$ = Chainage of $V$ − Tangent length

Chainage of $T_2$ = Chainage of $T_1$ + Length of Curve

## Degree of a Curve (D)

- The central angle subtended by a chord of a fixed length.
- In India, the fixed length of chord is taken as 20 m to calculate the degree of curve.
  360 degrees = $2\pi R$
  $D$ degrees = 20 m
  ⇒ Therefore, $\boxed{R = \frac{1146}{D} \text{ metres}}$

## Methods of Setting out a Curve

### Linear Methods

In this method, only a chain or tape is used. Linear methods are used when:

- A high degree of accuracy is not required.
- The curve is short.

1. **By ordinates or offsets from the long chord:**
   Mid-ordinate,

$$\boxed{O_0 = R - \sqrt{R^2 - \left(\frac{L}{R}\right)^2}}$$

$$O_x = \sqrt{R_2 - X_2} - (R - O_0)$$

$$= \frac{X(K - X)}{2R} \text{ (approx)}$$

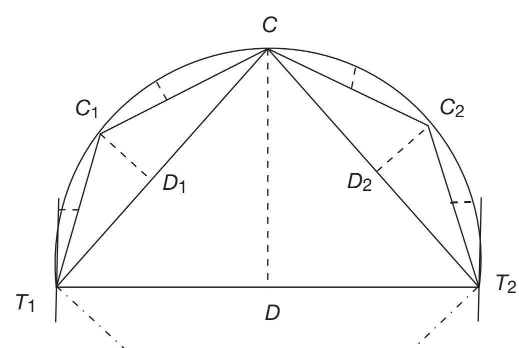
$O_x$ = Ordinate at distance of '$X$' from central ordinate.

To set out the curve, the long chord is divided into an even number of equal parts. Offsets calculated at each point are then set out at each of these points.

2. **By successive bisection of arcs or chords:**
   - Join tangent points $T_1$, $T_2$ and bisect them at $D$. Erect a perpendicular $DC$ whose length is equal to versed sine of the curve.

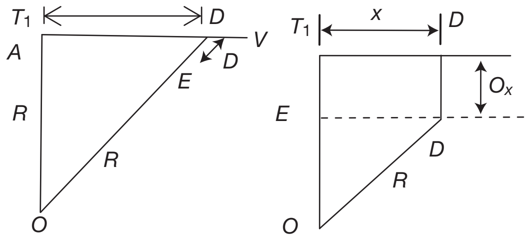
$$CD = R\left(1 - \cos\frac{\Delta}{2}\right) = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

   - Join $T_1C$ and $T_2C$ and bisect them at $D_1$ and $D_2$. Erect perpendiculars $D_1C_1$ and $D_2C_2$ which are equal to $R\left(1 - \cos\frac{\Delta}{4}\right)$
   - By successive bisection of these chords, more points may be obtained and by joining all $C$, $C_1$, $C_2$, … points, curve is obtained.




3. **By offsets from the tangents:** If the deflection angle and the radius of curvature are both small, the curves can be set out by offsets from the tangent. The offsets from tangent are of two types.

- Radial offsets
- Perpendicular offsets




**Radial offset:**

$$O_x = \sqrt{R^2 + x^2} - R \text{ (Exact)}$$

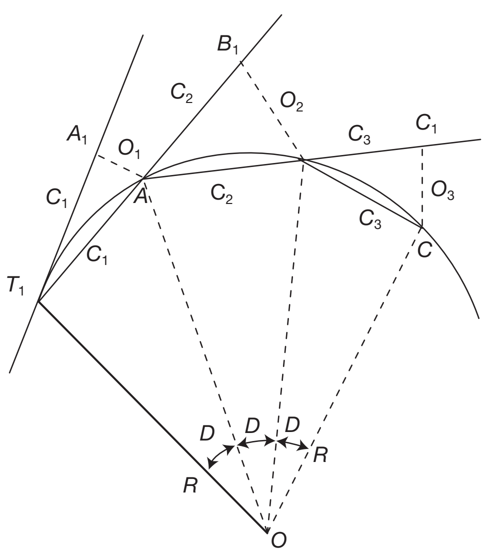
$$\approx \frac{x^2}{2R} \text{(approx)}$$

**Perpendicular offset:**

$$O_x = R - \sqrt{R^2 - x^2} \text{ (Exact)}$$

$$\approx \frac{x^2}{2R} \text{(approx)}$$

4. **By offsets produced from the chords (deflection distances)**
   - This method is very much useful for long curves and is generally used when a theodolite is not available.




- Assuming $C_1$, $C_2$, …, $C_n$ sub-chord lengths and calculating $O_1$, $O_2$, …, $O_n$.

  First offset, $O_1 = \dfrac{C_1^2}{2R}$

  Second offset, $O_2 = \dfrac{C_2^2}{2R}$

$$O_n = \frac{C_n(C_{n-1} + C_n)}{2R}$$

$O_1, O_2, \ldots O_n$ = 1st, 2nd, …, $n$th offset.

- Great disadvantage in this method is that the error in fixing a point is carried forward.

## Angular Methods

In this method, an instrument such as a theodolite is used with or without a chain (or tape).

1. **Rankine's method of deflection (tangential) angles:**

   A deflection angle ($\delta$) to any point on the curve is the angle at PC between the back tangent and the chord from the PC to that point.

   - $\boxed{\delta \text{ (in minutes)} = \dfrac{1718.9C}{R}}$

     $C$ is the length of the chord.
     $R$ is the radius of the curve.
   - One theodolite (to measure angles), one chain or tape (to measure distances) are used in this method.
   - The deflection angle for any chord is equal to the deflection angle for the previous chord plus the tangential angle for that chord.

     $\boxed{\Delta_1 = \delta_1;\ \Delta_2 = \Delta_1 + \delta_2;\ \Delta_n = \Delta_{n-1} + \delta_n}$

2. **Two theodolite method:**
   - In this method, two theodolites are used one at PC and the other at P.T.
   - This method is used when the ground is unsuitable for chaining.
   - Only angular measurements are used (no chain or tape).
   - It is based on the principle that the angle between the tangent and the chord is equal to the angle which that chord subtends in the opposite segment.

3. **Tacheometric method:**
   - Chaining is completely eliminated and the method is less accurate than Rankine's.
   - Setting of curve is done with stadia theodolite.
     **(a) Compound curves:** A curve with two or more simple curves turn in the same direction and join at common tangent points.
     **(b) Reverse curves:** Two curves turn in the opposite directions.
     [The characteristics, length and sight distance requirements of transition and vertical curves are discussed in transportation]

# Triangulation Survey

## Introduction

The triangulation is the system which consists of a number of inter-connected triangles in which length of one line and the angles of triangles are measured very precisely.

This chapter aims at the establishment of geodetic survey using triangulation.

## Geodetic Surveying

- To determine precisely the relative or absolute position on the earths' surface.
- The stations at which the astronomical observations for azimuth and longitude are also made are known as Laplace stations.

### *Objects of Geodetic Triangulation*

1. To provide the most accurate system of horizontal control points.
2. To assist in the determination of the size and shape of the earth.

## Classification of Triangulation System (Based on Accuracy)

### *First Order or Primary Triangulation*

This is of the highest order and is employed either to determine the earths figure or to furnish the most precise control points.

**General specifications:**

1. Average triangle closure < 1 second
2. Maximum triangle closure $\ngtr$ 3 seconds
3. Length of base line—5.5 km
4. Probable error in astronomic azimuth—0.5 seconds.

### *Second Order or Secondary Triangulation*

The stations are fixed at close intervals so that the sizes of the triangles formed are smaller than the primary triangulation.

**General specifications:**

1. Average triangle closure—3 seconds
2. Maximum triangle closure—8 seconds
3. Length of base line—1.5–5 km
4. Probable error in astronomic azimuth—2.0 seconds

### *Third-Order or Tertiary Triangulation*

**General specifications:**

1. Average triangle closure—6 seconds
2. Maximum triangle closure—12 seconds
3. Length of base line—0.5–3 km
4. Probable error in astronomic azimuth—5 seconds

## Triangulation Figures or Systems

1. **Single chain of triangles:** This figure is used where a narrow strip of terrain is to be covered. Though it is rapid and economical, it is not so accurate for primary work since the number of conditions to be fulfilled in the figure adjustment is relatively small.
2. **Double chain of triangles:** It is used to cover larger area.
3. **Centred figures:** These are used to cover area and give very satisfactory results in flat country. Centred figures may be quadrilaterals, pentagons or hexagons with central stations.
4. **Quadrilaterals:** Quadrilateral with four corner stations and observed diagonal forms the best figures. They are best suited for hilly country and most accurate.

## Criteria for Selection of the Figure

1. The figure should be such that the computations can be done through two independent routes.
2. The figure should be such that at least one, and preferably both routes should be well-conditioned.
3. All the lines in a figure should be of comparable length. Very long lines should be avoided.
4. The figure should be such that least work may secure maximum progress.
5. Complex figures should not involve more than about twelve conditions.
   - In very extensive survey, the primary triangulation laid in two series of chains usually in N–S and E–W respectively is filled by secondary and tertiary triangulation figures. This is known as the grid iron system and is adopted for France, Spain, Austria and India.
   - In another system called central system, which extends outward in all directions from base line and covered by a network of primary triangulation is adopted for United Kingdom.

## Well-conditioned Triangle

- The shape of the triangle should be such that any error in the measurement of angle shall have a minimum effect upon the lengths of the calculated side. Such a triangle is called well-conditioned triangle with base angles equal to 56°14′. Form practical considerations, an equilateral triangle is the most suitable. However triangle with an angle < 30° and >120° should be avoided.
- The accuracy attained in each figure depends on
  - The magnitude of the angles in each individual triangle and
  - The arrangement of triangles.

## Strength of Figure

- The strength of figure is to be considered in triangulation as the computations can be maintained within a desired degree of precision.
- The geodetic survey has developed a very rapid and convenient method of evaluating the strength of triangulation figure and is based on an expression for the square of the

probable error ($L^2$), that would occur in the sixth place of the logarithm of any side,

$$L^2 = \frac{4}{3} d^2 R$$

$$R = \frac{D-C}{D} \Sigma\left[\delta_A^2 + \delta_A \delta_B + \delta_A^2\right]$$

Where

$d$ = Probable error of an observed Direction (in seconds)

$D$ = Number of directions observed (forward/back)

$\delta_A$ = Difference per second in he sixth place of logarithm of the sine of the distance angle $A$ of each triangle.

$\delta_B$ = Same as $\delta_A$ but for the distance angle $B$.

$C$ = Number of angles and side conditions to be satisfied in the net from the known line to the side in equation.

$C = (n' - s' + 1) + (n - 2s + 3)$

$n$ = Total number of lines

$n'$ = Number of lines observed in both directions.

$s$ = Total number of stations.

$s'$ = Number of occupied stations.

$(n' - s' + 1)$ = Number of angle conditions.

$(n - 2s + 3)$ = Number of side conditions.

- The relative strength of figure can be computed in terms of factor $R$.
- Lower the value of $R$, stronger the figure.
- Value of $R$ computed for the strongest chain of triangles is called $R_1$ and that for the second strongest chain $R_2$. Generally, strength of a figure is almost equal to the strength of the strongest chain. Therefore $R_1$ is a measure of the strength of figure.
- For angles measured with the same precision, the strength of figure depends upon:
  - Number of directions observed.
  - The number of geometrical conditions imposed by the shape of the figures, together with the number of stations occupied in the field.
  - The size of distance angles used in computation.

## Signals and Towers

**Tower:** A tower is a structure erected over a station for the support of the instrument and observing party and is provided when station or the signal or both are to be elevated.

- A signal is a device erected to define the exact position of an observed station.

### Non-luminous Signals

Diameter of signal in cm = 1.3$D$ to 1.9$D$

Height of signal in cm = 13.3$D$

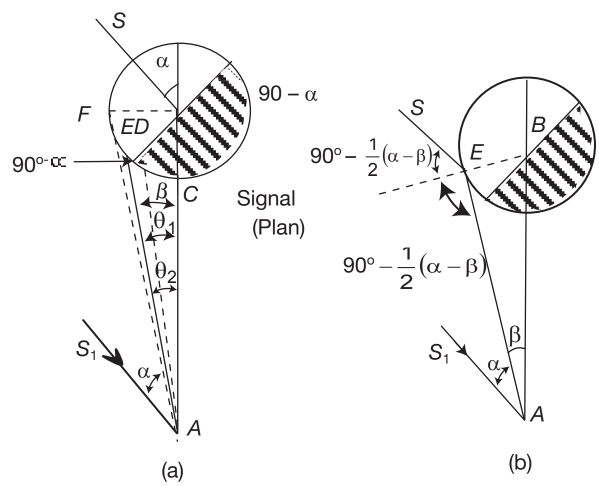
Where, $D$ = distance (length of sight) for non-luminous signals (km).

### Luminous or Sun Signals

Used when length of sight distance > 30 kms.

The heliotrope and heliograph and special instruments used as sun signals. The heliotrope consists of a plane mirror to reflect the sun's rays and a line of sight to enable the attendant to direct the reflected rays towards the observing stations. Another form of heliotrope is 'galton sun signal'.

***Phase of Signals*** It is the error of bisection which arises, when the signal is partly in light and partly in shade. The observer needs only illuminated portion and bisects it. It is thus apparent displacement of the signal. Thus the phase correction is necessary.




1. When observation is made on the bright portion,

   Phase correction,

$$\beta = \frac{r\cos^2\frac{\alpha}{2}}{D} \text{ radians}$$

   Where

   $\alpha$ = Angle which the direction of sun makes with line of sight.

   $r$ = Radius of the signal.

   $D$ = Distance of sight.

2. When the observation is made on the bright line,

$$\beta = \frac{r\cos\frac{\alpha}{2}}{D} \text{ radians}$$

   The phase correction is applied algebraically to the observed angle, according to the relative position of the sun and the signal.

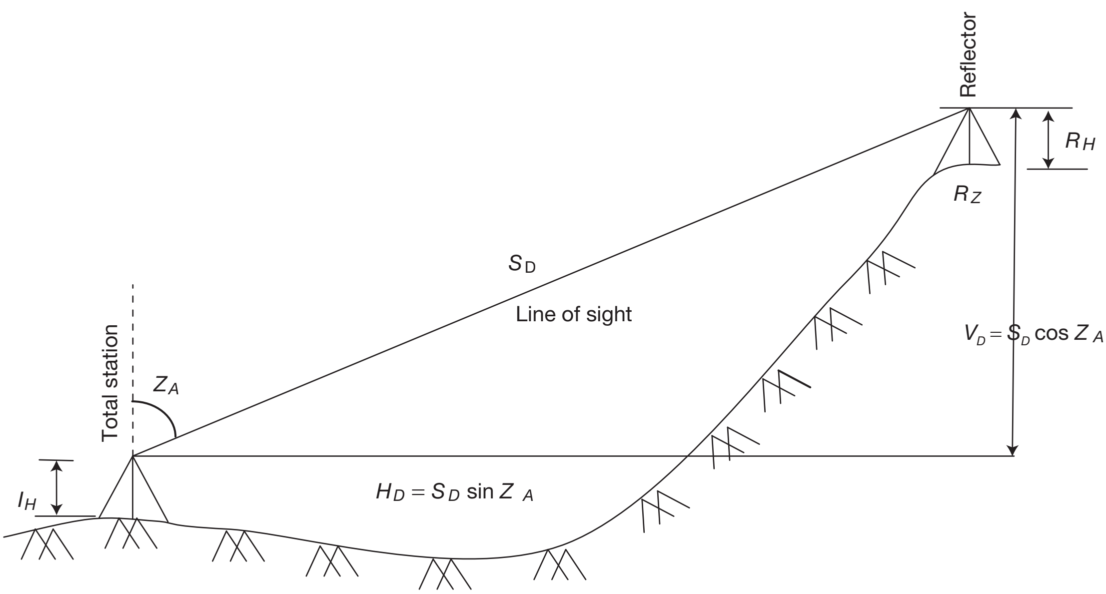
***Total Station*** There are three methods of measuring distance between any two given points.

1. Direct distance measurement (DDM), by chaining or taping.
2. Optical distance measurement (ODM), by tacheometry.
3. Electro-magnetic distance measurement (EDM), by geodimeter, tellurometer or distomat, etc.

A total station is a combination of an electronic theodolite and an electronic distance meter (EDM). This combination makes it possible to determine the coordinates of a reflector by aligning the instruments cross-hairs on the reflector and simultaneously measuring the vertical and horizontal angles and slope distances.

## Fundamental Measurements

1. The rotation of the instrument's optical axis from the instrument north in a horizontal plane, i.e., horizontal angle.
2. The inclination of the optical axis from the local vertical, i.e., vertical angle.
3. The distance between the instrument and the target, i.e., slope distance.




## Geometry of the Instrument (Total Station) and Reflector

Horizontal distance $(H_D) = S_D \sin Z_A$

$S_D$ = Slope distance

$Z_A$ = Zenith angle

Elevation difference between two points on the ground.

$$d_z = V_D + (I_H - R_H)$$

- If the instrument is at known elevation $I_z$, then the elevation of the ground beneath the reflector, $R_z$ is

$$R_Z = I_Z + S_D \cos Z_A + (I_H - R_H)$$

## Exercises

1. Match List I with List II and select the correct answer using the codes given: (adopting standard notations)

| | List I | | List II |
|---|---|---|---|
| a. | Cubic parabola equation | 1. | $\frac{NS^2}{4.4}$ |
| b. | Shifting transition curve | 2. | $\frac{L^2}{24R}$ |
| c. | Valley curve | 3. | $\frac{NS^2}{1.50 + 0.035S}$ |
| d. | Summit curve | 4. | $\frac{x^3}{6RL}$ |
| | | 5. | $\frac{V^2}{gR}$ |

**Codes:**

| | a | b | c | d | | a | b | c | d |
|---|---|---|---|---|---|---|---|---|---|
| (A) | 1 | 2 | 3 | 4 | (B) | 3 | 4 | 1 | 2 |
| (C) | 1 | 5 | 2 | 4 | (D) | 4 | 2 | 3 | 1 |

**2.** A parabolic vertical curve is to be set out connecting two uniform grades of +0.6% and +1.0%. The rate of changes of grade is to be 0.06% per 30 m. The length of the curve will be

(A) $66\frac{2}{3}$ m (B) $133\frac{1}{3}$ m
(C) 200 m (D) $266\frac{2}{3}$ m

**3.** For a chord of 60 m, the mid-ordinate for a circular curve of 50 m radius will be
(A) 10 m (B) 12.5 m
(C) 15 m (D) 18.75 m

**4.** If the radius of a simple curve is $R$, then the length of the chord for calculating the offsets by the 'method of chords produced' should not exceed
(A) $R/5$ (B) $R/10$
(C) $R/20$ (D) $R/25$

**5.** The observation ray between two triangulation stations A and B just grazes the sea. If the heights of A and B are 6000 m and 2000 m respectively, the distance between A and B is (Let radius of earth $R = 6440$ km)
(A) 432.4 km
(B) 438.3 km
(C) 450.2 km
(D) 442.4 km

**6.** A Circular curve has 300 m radius and 55° deflection angle, then the apex distance is
(A) 38.23 m (B) 38.21 m
(C) 39.23 m (D) 40.24 m

**7.** The length of mid-ordinate in the above question is
(A) 37.23 m (B) 38.62 m
(C) 33.89 m (D) 32.43 m

**8.** A circular curve of radius is to be set with a long chord 60 m, the length of middle ordinate in 'm' is
(A) 7.62 (B) 9.3
(C) 10.23 (D) 2.52

**9.** The degree of curve for 860 m radius using standard chain of 30 m is
(A) 1° (B) 1.5°
(C) 2° (D) 2.5°

**10.** What is the volume of a 6 m deep tank having rectangular shaped top 6 m × 4 m and bottom 4 m × 2 m? (Computed through prismoidal formula)
(A) 96 $m^3$
(B) 94 $m^3$
(C) 92 $m^3$
(D) 90 $m^3$

## Previous Years' Questions

**1.** Two straight lines intersect at an angle of 60°. The radius of a curve joining the two straight lines is 600 m. The length of long chord and mid-ordinates (in metres) of the curve are **[GATE, 2007]**
(A) 80.4, 600.0 (B) 600.0, 80.4
(C) 600.0, 39.89 (D) 49.89, 300.0

**2.** A road is provided with a horizontal circular curve having deflection angle of 55° and centre line radius of 250 m. A transition curve is to be provided at each end of the circular curve of such a length that the rate of gain of radial acceleration is 0.3 m/s$^3$ at a speed of 50 km/h. Length of the transition curve required at each of the ends is **[GATE, 2008]**
(A) 2.57 m (B) 33.33 m
(C) 35.73 mm (D) 1666.67 m

**3.** The chainage of the intersection point of two straights is 1585.60 m and the angle of intersection is 140°. If the radius of a circular curve is 600.00 m, the tangent distance (in m) and length of the curve (in m), respectively are **[GATE, 2014]**
(A) 418.88 and 1466.08
(B) 218.38 and 1648.49
(C) 218.38 and 418.88
(D) 418.88 and 218.38

**4.** A circular curve of radius $R$ connects two straights with a deflection angle of 60°. The tangent length is **[GATE, 2016]**
(A) 0.577 $R$
(B) 1.155 $R$
(C) 1.732 $R$
(D) 3.464 $R$

## Answer Keys

### Exercises

**1.** D **2.** C **3.** A **4.** C **5.** B **6.** B **7.** C **8.** C **9.** C **10.** C

### Previous Years' Questions

**1.** B **2.** C **3.** C **4.** A