Theme 6: Surface Areas and Volumes

Prior Knowledge

It is recommended that you revise the following topics before you start working on this unit.

- Formulae for surface area and volume of cube, cuboid, cone, cylinder, and sphere.
- Knowledge about when to consider lateral/curved surface area and when to go for total surface area
- Relation between volume of cylinder, volume of cone and volume of sphere for known radius and height.

Case Study A - Juice Vendor

Vittal runs a small refreshment store in which he sells buns, coffee, tea and a special type of fruit juice. His juice dispenser is of cuboidal shape. The base has an area of 500 cm^2 and the height of the dispenser is 25 cm. Every morning he prepares this juice using water and concentrated fruit syrup in the ratio of 3:1. He uses 2.5 litres of fruit syrup daily (1 litre = 1000 cm^3).

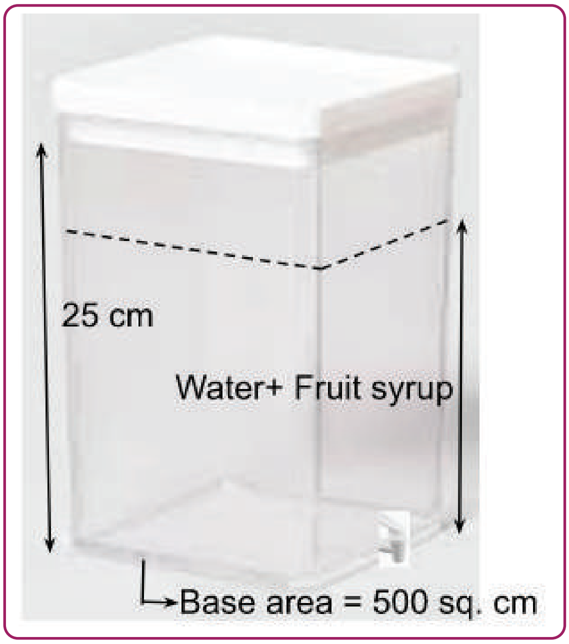


Fig. 6.1, Juice dispenser filled with juice (water and fruit syrup)

Question 1

Once the juice is made, how much of the dispenser will be filled?

a. 75%	b. 85%	Answer
c. 80%	d. 90%	

Question 2

As the consumption of juice increases during summer, Vittal fills his juice dispenser to its maximum capacity. How much fruit syrup is used to prepare a dispenser full of juice?

Answer

Question 3

Vittal introduced a token/coin operated juice dispenser. The customer gets a token and an empty glass from Vittal after paying the requisite amount, which when inserted in the machine fills the glass. For the entire dispenser to get emptied, 60 coins are needed. What is the approximate volume of juice each cup will hold?

a. Slightly more than 200 mL	b. Exact 200 mL	Answer
c. Slightly less than 200 mL	d. Around 150 mL	

Case Study B - Zara's Rubik's Cube

One of the most popular puzzles globally, a Rubik's cube is a 3x3x3 cu. unit cube, where each face is tiled with 9 identically coloured stickers. The 6 different faces, each with a different colour, can be mixed around by independently rotating each face of the cube. Once the colours are mixed, the challenge, of course, is to keep playing and make each face a single colour once again! As you can imagine, the combinations seem endless, and to the uninitiated, virtually impossible to solve! However, its apparent simplicity keeps even the novice hooked. The mathematical nature of the puzzle allows one to make several conceptual connections with it: e.g. to study surface area, volume, ratios, fractions etc., as you yourself will do by answering several of the questions below.

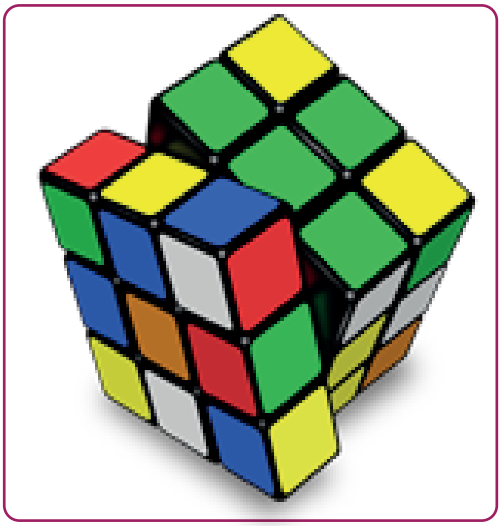


Fig. 6.2, Rubik's cube; Image by Booyabazooka via Wikimedia Commons

Question 4

Zara was playing with her 3 x 3 Rubik's cube and it fell down from her hand and all the small cubes from which it was made fell apart. She rearranged them, but the central piece on the top surface was missing.

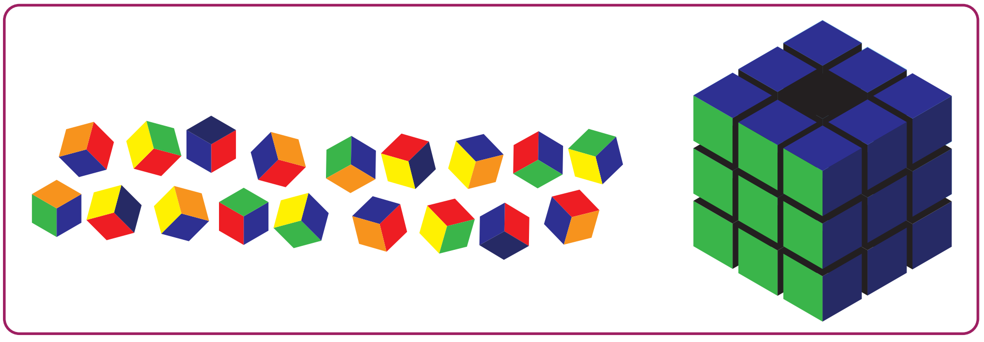


Fig. 6.3, Rubik's cube with missing piece

What will be the new surface area if each smaller cube is made from a cube of side 1 unit?

		Answer
a. 58 sq.units	b. 57 sq.units	
c. 53 sq.units	d. 54 sq.units	

Question 5

Which of the following is a correct statement with respect to the comparison among the non-broken cube (Fig. 6.2) and the broken cube (Fig. 6.3)? Note that 1 cm^3 = 1 ml.

a. Both surface area and volume decrease.
b. Both surface area and volume increase.
c. Surface area increases but volume decreases.
d. Surface area decreases but volume increases.

Answer

Question 6

Knowing Zara's love for her Rubik's cube, her father gifted her a pen holder, which is in the shape of a Rubik's cube from outside and cylindrical from inside, as shown in Fig. 6.4. By using a ruler and a thread, Zara measured the circumference of the circular opening portion of the pen holder as 22 cm. She also measured its depth as 7 cm.

Fig. 6.4, Pen holder

To avoid the sharpened points of the pencil from breaking by coming in contact with the hard surface of the base, she decided to use a thin piece of foam for it and also pasted a colour paper layer on the inside of the cylindrical holder to make it more attractive.

i. Find the area of the foam piece required. ($\pi = \frac{22}{7}$)

Answer

ii. What should be the area of the rectangular colour paper that can be used to cover the inner surface of the pen holder, without any wastage?

Answer

Question 7

For her school craft exhibition, Zara prepared a scaled up static model of a Rubik's cube, using thick cardboard (45 cm x 45 cm x 45 cm). She observed that each face of the Rubik's cube has multiple squares. As seen in Fig. 6.5, she used square-shaped paper sheets (15 cm x 15 cm) of multiple colours. She covered all the faces of the cube except the base, since it won't be visible.

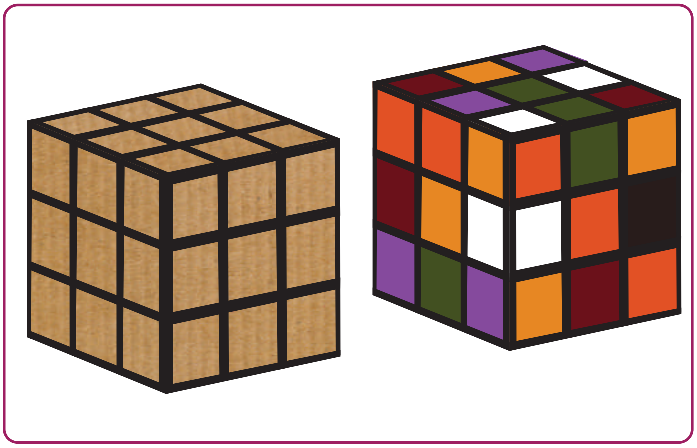


Fig. 6.5, Rubik's cube made using thick cardboard

There was no overlapping or wastage while pasting and she drew thick black lines along the edges of the colour sheets. What is the total number of colour paper sheets she used for making this Rubik's cube model?

Answer

Case Study C - Fragrances of India

Fragrances of India, a famous perfume manufacturing company, sells its 3 perfumes in 3 different types of glass bottles, as shown in Fig. 6.6. The cost of the bottle is decided by the quantity and type of perfume it contains.

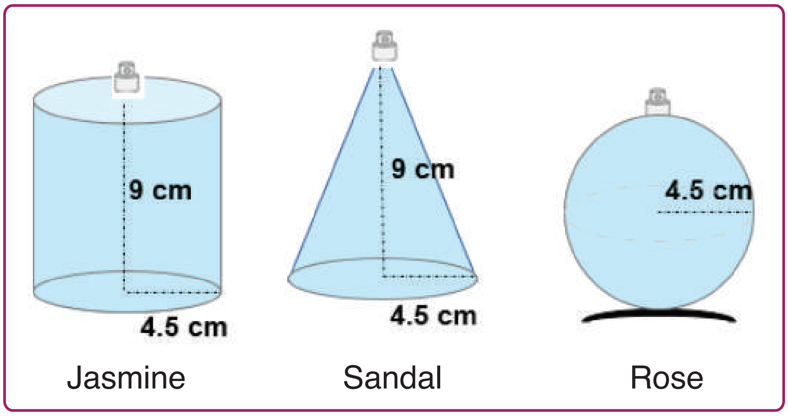


Fig. 6.6, Perfume bottles

Question 8

If all the bottles in Fig. 6.6 cost the same, which fragrance is the most expensive per unit volume? Choose the correct option which shows the price in increasing order.

a. Rose, Jasmine, Sandal	b. Sandal, Jasmine, Rose
c. Jasmine, Sandal, Rose	d. Jasmine, Rose, Sandal

Answer

Question 9

For the data in Question 8, check the correctness of the assertion and reason.

Assertion (A): Without doing any calculation, we can arrange these bottles in the increasing order of cost per unit volume by comparing the relationship between their dimensions.

Reason (R): The base radius of all the three bottles is the same and the height is twice this radius for the cone and cylinder. So the cone will have one-third the volume of the cylinder; the sphere will have two-third the volume of the cylinder. Or, in other words, the cylinder's volume will be three times that of the cone and 3/2 times that of the sphere

a. A and R are true and R is the correct reason for A

b. A is true, but R is the wrong reasoning for it

c. A is true and R is false

d. A is false; but R is the correct reasoning in itself.

Answer

Question 10

The manufacturer is looking at having different shaped bottles, but with the same volume. He decided to change the dimensions of the cone and the cylinder shaped bottles shown in Fig. 6.6, so that all the three will have the same volume. The vendor who supplies these bottles comes with four proposals, which he can implement to alter the size of the bottles while manufacturing the bottles.

Solution 1: Double the height of the cone keeping the same radius.

Solution 2: Decrease the height of the cylinder to two-third of its height, keeping the same radius.

Solution 3: Double the base radius of the cone without changing its height.

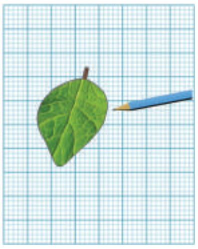
Solution 4: Double the base radius of the cylinder without changing its height.

Which of the above solutions should be undertaken, individually or together, to match the requirement that all three will have the same volume?

a. Both solution 2 and solution 3
b. Both solution 1 and solution 3
c. Both solution 1 and solution 2
d. Only solution 3

Answer

Exploration Pathway

 Cylinder - Area and Volume	Make cardboard cylinders of different sizes. Find their curved surface area and volumes, and then compare. Also, make a right circular cone and compare its volume with the cylinder's volume.
 Sound - Wine Glass	Singing Glasses or a 'Glass harp' is a musical instrument played by running moist fingers around the rim of the glasses. As we move our moistened fingers around the rim of the glass, the friction between the fingers and the glass makes the glass vibrate, pushing air back and forth, thereby creating sound vibrations.
 Measure - Leaf Area	A simple, effective and enjoyable way to measure the area of irregular shapes, such as leaves, is by tracing out the shape on a graph paper.