Electromagnetics Test 3

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The lines of force due to charged particles are
(A) always curved (B) always straight
(C) sometimes curved (D) None of the above

2. If the sheet of Bakelite is inserted between the plates of an air capacitor, the capacitance will
(A) decrease (B) remains unchanged
(C) increase (D) become zero

3. The dissipation factor of a good dielectric is of the order of
(A) 0.0001 (B) 0.001
(C) 0.01 (D) 0.1

4. Which one of the following relation is correct?
(A) $\oint_l A.dl = \int_s \nabla.A\,ds$
(B) $\int_l \nabla\times A\,dl = \oint_s (A)\,ds$
(C) $\oint_S A.ds = \int_V \nabla\times A\,dv$
(D) $\oint_S A.ds = \int_V \nabla.A\,dv$

5. "The total electric flux through any closed surface is equal to amount of charge enclosed".
The above statement is
(A) Ampere's law (B) Coloumb's law
(C) Gauss's law (D) Maxwell's first law

6. A field F is said to be conservative if
(A) $\nabla\times\vec{F} = 0$ (B) $\nabla.F = 0$
(C) $\nabla.\nabla F = 0$ (D) All the above

7. The electric field $E = 6\cos(\omega t - \beta z)\,\hat{a}_x + 6\cos(\omega t - \beta z + 60^o)\,\hat{a}_y$ has
(A) linear polarization
(B) left circular polarization
(C) elliptical polarization
(D) right circular polarization

8. A dielectric material must be
(A) good conductor (B) resistor
(C) semi conductor (D) insulator

9. The work done by a force $E = 3\hat{a}_x - 4\hat{a}_y + 2\hat{a}_z$ N/C in giving a $2nC$ charge a displacement of $10\,\hat{a}_x + 2\,\hat{a}_y - 7\,\hat{a}_z$ meter is
(A) 4 J (B) 24 nJ
(C) 16 nJ (D) 8 J

10. EMW travelling in high loss medium at frequency f_1 has attained wavelength of λ_1 when frequency became four times what is the corresponding wavelength?
(A) $\frac{\lambda_1}{2}$ (B) $2\lambda_1$
(C) $\frac{\lambda_1}{4}$ (D) $4\lambda_1$

11. In a non-magnetic medium $E = 10\cos(2\pi\times10^8\,t - 2x)\,\hat{a}_z$ V/m then total power crossing 40 cm^2 of plane $3x + y = 7$ is
(A) 2.53 W (B) 69.21 W
(C) 96.12 W (D) 52.3 W

12. Determine the energy density stored in free space by the field $2\times10^{-3}\,\hat{a}_x + 4\times10^{-3}\,\hat{a}_y$ T.
(A) $2\sqrt{5}\times10^{-3}$ J/m^2
(B) 7.9 J/m^3
(C) 4.8 J/m^3
(D) 1.3 J/m^2

13. Transform a vector $A = y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$ into cylindrical co-ordinates
(A) $\hat{a}_\rho + \rho\hat{a}_\varphi + z\hat{a}_z$ (B) $-\rho\hat{a}_\varphi + z\hat{a}_z$
(C) $\rho\hat{a}_\rho + z\hat{a}_z$ (D) $\rho\hat{a}_\rho - \hat{a}_\varphi + z\hat{a}_z$

14. A potential field is $V = 6x^2y - xyz$. The electric field at $P(-2, 1, 4)$ shall be
(A) $8\hat{a}_x + 32\hat{a}_y + 2\hat{a}_y$ (B) 0
(C) $32\hat{a}_y$ (D) $-28\hat{a}_x - 32\hat{a}_y - 2\hat{a}_z$

15. In a certain region current density is $= (4z\,\hat{a}_x + 2x^2z\,\hat{a}_y + 4z^2\,\hat{a}_z)\sin(10^6\,t)$ A/m. Then volume charge density is
(A) $(4z + 2x^2z + 4z^2)\sin(10^6\,t)$ μC /m^3
(B) $-18x\cos(10^6\,t)$ μC/m^3
(C) $4xz\sin(10^6 t)$ mC/m^3
(D) $-8z\cos(10^6\,t)$ μC/m^3

16. The curl of vector field $A = \rho z\sin\phi\,\hat{a}_\rho + 3\rho z^2\,\hat{a}_z$ at point (3, 90°, 2) is
(A) $9\hat{a}_\varphi - 2\hat{a}_z$ (B) $2\hat{a}_\rho - 9\hat{a}_\varphi + 2\hat{a}_z$
(C) $-9\hat{a}_\varphi$ (D) $4\hat{a}_z$

17. Two free charges q and $9q$ are placed at a distance d apart a third charge Q is placed between then at a distance x from charge q such that the system is in equilibrium. Then Q is equal to

(A) $9q$ (B) $\frac{9q}{16}$
(C) $\frac{9q}{4}$ (D) $\frac{3q}{2}$

18. An electric charge Q is placed in a dielectric medium. Which of the following quantities are independent of the dielectric constant ε of the medium?
(A) Electric potential and displacement
(B) Electric field intensity and displacement density
(C) Displacement density and displacement
(D) Electric field intensity and displacement density

19. When a plane is incident normally from medium 1 (μ_o, ε_1) into medium 2 ($4\mu_o$, ε_2), the electric field of the transmitted wave is –0.5 times the electric field of the reflected wave. Then the ratio of $\frac{\varepsilon_1}{\varepsilon_2}$ is

(A) 10 (B) 0.1
(C) 100 (D) 0.01

20. The xy-plane serves as the interface between two different media. Medium$_1$ ($z < 0$) is filled with a material whose $\mu_r = 3$ and medium$_2$ ($z > 0$) is filled with a material whose $\mu_r = 2$. If the interface carries current $2/\mu_o$ $\hat{a}_y$ mA/m and $B_2 = 3\hat{a}_x + 7\hat{a}_z$ mwb/m^2. Then H_1 is

(A) $\frac{14}{\mu_o}\hat{a}_x + \frac{6}{\mu_o}\hat{a}_z$ (B) $\frac{14}{\mu_o}\hat{a}_x$
(C) $\frac{-14}{\mu_o}\hat{a}_x + \frac{6}{\mu_o}\hat{a}_z$ (D) $\frac{-0.5}{\mu_o}\hat{a}_x$

21. An EMW working in free space strikes a block of brass ($\sigma = 1.8 \times 10^7$ S/m) along its normal surface impedance of brass is $\eta_s = 0.08\ \angle 45^\circ\ \Omega$. Calculate skin depth in brass.
(A) 0.49 μm (B) 0.04 μm
(C) 0.98 μm (D) 0.69 μm

22. In a lossless dielectric for $\eta = 30\pi\Omega$, $\mu_r = 1$ and $H = 0.5 \cos(\omega t - z)\ \hat{a}_x + 0.1 \sin(\omega t - z)\ \hat{a}_y$ A/m then $\vec{E}$ is

(A) $15\pi \cos(75 \times 10^6 t - z)\ \hat{a}_x$ + $3\pi \sin(75 \times 10^6 t - z)\ \hat{a}_y$

(B) $3\pi \sin(300 \times 10^6 t - z)\ \hat{a}_x$ + $15\pi \cos(300 \times 10^6 t - z)\ \hat{a}_y$

(C) $3\pi \sin(75 \times 106t - z)\ \hat{a}_x$ – $15\pi \cos(75 \times 10^6 t - z)\ \hat{a}_y$

(D) $-15\pi \cos(300 \times 10^6 t - z)\ \hat{a}_x$ + $3\pi \sin(300 \times 10^6 t - z)\ \hat{a}_y$

23. A uniform plane wave propagating in a medium has $E = 4e^{-\alpha z} \cos(2 \times 10^8\ t - \beta z)\ \hat{a}_x$ V/m. If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 16$ and $\sigma = 5$ S/m. Then phase constant is
(A) 2.67 rad/m (B) 76.2 rad/m
(C) 20 rad/m (D) 100.2 rad/m

24. A point charge $Q_A = 2\ \mu C$, is at A (0, 0, 1) and $Q_B = -2\ \mu C$ is at B(0, 0, –1). Find electric field intensity along r co-ordinate [E_r] at p(1, 2, 3).
(A) 2070 (B) –206.81
(C) –690 (D) –720

25. A uniform line charge $\rho_\ell = 30\ \mu C/m^2$ on the z-axis. Then find $\overline{D}$ at (3, 4, –5)

(A) $9\hat{a}_x + 12\hat{a}_y$ μC/m^2

(B) $1.5\hat{a}_x - 2.2\hat{a}_y$ μC/m^2

(C) $3.4\hat{a}_x - 4.7\hat{a}_y$ μC/m^2

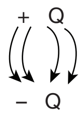
(D) $0.56\hat{a}_x + 0.76\hat{a}_y$ μC/m^2

Answer Keys

1. A	**2.** C	**3.** A	**4.** D	**5.** C	**6.** A	**7.** B	**8.** D	**9.** C	**10.** A
11. C	**12.** B	**13.** B	**14.** D	**15.** D	**16.** C	**17.** B	**18.** C	**19.** D	**20.** D
21. C	**22.** C	**23.** D	**24.** B	**25.** D					

HINTS AND EXPLANATIONS

1. Lines of force due to charged particles are always curved.



Choice (A)

2. C α ε_r
ε_r = 1 for air
= 5 for bakelite
so capacitance will increase. Choice (C)

3. Dissipation factor is $\frac{\sigma}{\omega\varepsilon}$

for dielectric $\sigma \to 0$ Choice (A)

4. $\int_l A.dI = \int \nabla \times A \,.\, ds$

$\oint_s A.ds = \int_v \nabla . A.dv$ Choice (D)

5. $\Psi = \int D.da = Q \to$ Gauss's law. Choice (C)

6. $\nabla . \vec{F} = 0$ is said to be solenoidal
$\nabla \times \vec{F} = 0$ is said to be conservative (or) irrotational.
Choice (A)

7. $E = 6\cos(\omega t - \beta z)\,\hat{a}_x + 6\cos(\omega t - \beta z + 60^o)\,\hat{a}_y$

If $\omega t - \beta z = 0 \Rightarrow E_1 = 6\,\hat{a}_x + 3\,\hat{a}\,y$

If $\omega t - \beta z = 30 \Rightarrow E_2 = \frac{6\sqrt{3}}{2}\hat{a}_x$

E has left circular polarization. Choice (B)

8. For dielectric material (insulator) σ = 0. Choice (D)

9. $W = Q\, E.dl = F.dl$
$= [2 \times 10^{-9}[30 - 8 - 14]$
$= 16nJ.$ Choice (C)

10. For high loss media

$\frac{2\pi}{\lambda} = \beta\, \alpha \sqrt{f} \Rightarrow \lambda_1 \sqrt{f_1} = \lambda_2 \sqrt{f_2}$

$\Rightarrow \lambda_1 \sqrt{f_1} = \lambda_2 \sqrt{4 f_1}$

$\Rightarrow \lambda_2 = \lambda_{\frac{1}{2}}.$ Choice (A)

11. $P = E \times H = \frac{E_0^2}{\eta} \cos^2(2\pi \times 10^8 t - 2x)\ \hat{a}_x$

$P_{avg} = \frac{100}{\eta}\ [\beta\, \omega \sqrt{\mu\varepsilon} \Rightarrow 2 = \frac{2\pi \times 10^7}{3 \times 10^8}\sqrt{\varepsilon_r}$

$\eta = \frac{120\pi}{\sqrt{\varepsilon_r}} = 4\pi \quad \sqrt{\varepsilon_r} = \frac{30}{\pi}]$

$P_{avg} = \frac{25}{\pi r^2}\frac{W}{m^2}\hat{a}_x$

On plane $3x + y = 7$ is

$P = \int P_{avg}.ds = \left(\frac{25}{\pi^2} \times 40\right)$

$\hat{a}_x \left(\frac{3\hat{a}_x + \hat{a}_y}{\sqrt{10}}\right)$

$= \frac{75 \times 40}{\pi^2 \sqrt{10}} = 96.12\,\text{w}.$ Choice (C)

12. Given that $B = 2 \times 10^{-3}\ \hat{a}_x + 4 \times 10^{-3}\ \hat{a}_y$

Energy density $= \frac{B^2}{2\mu_0}$

$= \frac{4 \times 10^{-6} + 16 \times 10^{-6}}{2 \times 4\pi \times 10^{-7}} = 7.9 \frac{J}{m^3}.$ Choice (B)

13. $\hat{a}_x = \hat{a}_\rho \cos\phi - \hat{a}_\phi \sin\phi$

$\hat{a}_y = \sin\phi\ \hat{a}_\rho + \hat{a}_\varphi\ \cos\phi$

$\Rightarrow\ y = \rho \sin\phi$
$x = \rho\cos\phi$

$\Rightarrow\ \rho\sin\phi\ (\hat{a}_\rho\ \cos\phi - \hat{a}_\varphi\ \sin\phi) - \rho\cos\phi\ (\sin\phi\ \hat{a}_\rho + \cos\phi\ \hat{a}_\varphi) + z\ \hat{a}_z = A$

$\Rightarrow\ A = \hat{a}_\rho\ (\rho\sin\phi\ \cos\phi - \rho\sin\phi\cos\phi) - \hat{a}_\varphi\ \rho(\sin^2\phi + \cos^2\phi) + z\,\hat{a}_z$

$\Rightarrow\ -\rho\,\hat{a}_\varphi + z\,\hat{a}_z\ a_z = A.$ Choice (B)

14. $E = -\nabla V$

$$= -\left[\frac{\partial}{\partial x}(6x^2 y - xyz) + \frac{\partial}{\partial y}(6x^2 y - xyz) + \frac{\partial}{\partial z}(6x^2 y - xyz)\right]$$

$= -[(12xy - yz)\,\hat{a}_x + (6x^2 - xz)\,\hat{a}_y + (-xy)\,\hat{a}_z\,]$

at (–2, 1, 4)

$= -[28\,\hat{a}_x + 32\,\hat{a}_y + 2\,\hat{a}_z\,].$ Choice (D)

15. $\nabla . J = -\frac{\partial \rho \vartheta}{\partial t}$

$\Rightarrow\ 8\, z\sin(10^6 t)$ It $= -\partial\rho_\vee$

Integrating both sides

$\Rightarrow\ \rho_\vee = -[8z\cos(10^6 t)]\ \mu C/m^3.$

Choice (D)

16. $\nabla \times A = \frac{1}{\rho}\begin{bmatrix} \hat{a}_\rho & \rho\hat{a}_\varphi & \hat{a}_z \\ \frac{\partial}{\partial_\rho} & \frac{\partial}{\partial_\varphi} & \frac{\partial}{\partial z} \\ \rho z \sin\varphi & 0 & 3\rho z^2 \end{bmatrix}$

$= \frac{1}{\rho}\left[-\rho\hat{a}_\phi\left[3z^2 - \rho\sin\phi\right] + \hat{a}_z\left(\rho z\cos\phi\right)\right]$

$= [\hat{a}_\varphi\ [\rho\sin\phi - 3z^2] + \hat{a}_z\ z\cos\phi]_{(3, 90°, 2)}$

$= \hat{a}_\varphi\ [3 - 12] + \hat{a}_z\ (0) = -9\,\hat{a}_\varphi$ Choice (C)

17. Given that system in equilibrium

Force on Q is

$\Rightarrow \frac{qQ}{4\pi\varepsilon_0 x^2} = \frac{9qQ}{4\pi\varepsilon_0(d-x)^2}$

$\Rightarrow d = 4x = x = \frac{d}{4}$

Force on q is $\frac{9q^2}{4\pi\varepsilon_o d^2} = \frac{qQ}{4\pi\varepsilon_o X^2}$

$\Rightarrow Q = \frac{9}{16}q.$ Choice (B)

18. Displacement density $D = \frac{Q}{4\pi r^2}$

Displacement $\Psi = \int D.da = Q.$ Choice (C)

19. $E_t = -0.5\ \varepsilon_r$

$\Rightarrow \frac{2\eta_2}{\eta_2+\eta_1} = +\frac{1}{2}\left(\frac{\eta_1-\eta_2}{\eta_2+\eta_1}\right)$

$\Rightarrow 5\ \eta_2 = \eta_1$

$\Rightarrow 5\sqrt{\frac{4\mu_0}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{\varepsilon_1}}$

$\Rightarrow 10\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = 1$

$\frac{\varepsilon_1}{\varepsilon_2} = 10^{-2} = 0.01.$ Choice (D)

20. $(H_1 - H_2) \times a_{n12} = K$

$H_2 = \frac{B_2}{\mu_2} = \frac{1}{2\mu_0}\left(3\hat{a}_x + 7\hat{a}_z\right)$

$H_1 = \frac{B_1}{\mu_1} = \frac{1}{3\mu_0}\left(B_1\hat{a}_x + B_2\hat{a}_y + B_3\hat{a}_z\right)$

$\Rightarrow \frac{1}{6\mu_0}\left\{(2B_1-9)\hat{a}_x + 2B_2\hat{a}_y + (2B_3-21)\hat{a}_z\right\}\times\hat{a}_z = \frac{2\hat{a}_y}{\mu_0}$

$\left[\hat{a}_y \times \hat{a}_z = \hat{a}_x\right]$

$\left[\hat{a}_x \times \hat{a}_z = \hat{a}_y\right]$

$\Rightarrow \frac{1}{6\mu_o}\left(9 - 2B_1\right) = \frac{2}{\mu_o}$

$B_1 = 10.5,\ B_2 = 0$

$\Rightarrow H_1 = \frac{-1.5}{\mu_1} = \frac{-1.5}{\mu_o} = \frac{-0.5}{\mu_o}\hat{a}_x.$ Choice (D)

21. $\delta = \sqrt{\frac{2}{\omega\mu\varepsilon}}$

$\eta_S = R_S + j X_S = \sqrt{\frac{\omega\mu}{2\sigma}}\ (1+j) = \frac{0.08}{\sqrt{2}}\ (1+j)$

$\Rightarrow R_S.\ \delta = \frac{1}{\sigma}$

$\delta = \frac{\sqrt{2}}{1.8\times10^7\times0.08} = 0.98\mu m.$ Choice (C)

22. $\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$

$\Rightarrow 30\pi = \frac{120\pi}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r = 16$

$\Rightarrow \omega\sqrt{\mu\varepsilon} = \beta$

$\Rightarrow \omega = \frac{1\times3\times10^8}{4} = 75\text{MHz}$

$\Rightarrow \frac{E_x}{H_y} = \eta = \frac{-E_y}{H_x}$

$\Rightarrow E_x = 30\ \pi \times 0.1\ \sin(75 \times 10^6 t - z)\ \hat{a}_x$

$= 3\ \pi\ \sin(75 \times 10^6\ t - z)\ \hat{a}_x$

$\Rightarrow E_y = -\eta H_x = -0.5 \times 30\ \pi\ \cos(75 \times 10^6 t - z)\,\hat{a}_y$

$= -15\ \pi\ \cos(75 \times 10^6 t - z)\ \hat{a}_y$

$E = E_x + E_y = 3\ \pi\ \sin(75 \times 10^6 t - z)\ \hat{a}_x - 15\ \pi\ \cos(75 \times 10^6 t - z)\,\hat{a}_y.$ Choice (C)

23. $\frac{\sigma}{\omega\varepsilon} = \frac{5\times36\pi}{2\times10^8\times10^{-9}} = 900\pi \gg 1$

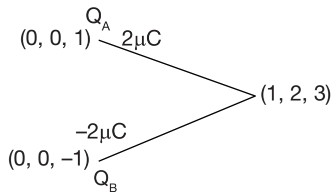
it represents a good conductor

$\infty = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$= \sqrt{\frac{2\times10^8\times4\pi\times10^{-7}\times16\times5}{2}}$

= 100.2 rad/m. Choice (D)

24.



$$\bar{E} = \frac{2\times10^{-6}}{4\pi\varepsilon_0}\left[\frac{(1-0)\hat{a}_x+(2-0)\hat{a}_y+(3-1)\hat{a}_z}{\sqrt{1^2+2^2+2^2}}\right]$$

$$\frac{-2\times10^{-6}}{4\pi\varepsilon_0}\left[\frac{(1-0)\hat{a}_x+(2-0)\hat{a}_y+(3+1)\hat{a}_z}{\sqrt{1^2+2^2+4^2}}\right]$$

$$= \frac{2\times10^{-6}}{4\pi\varepsilon_0}\left[\left(\frac{1}{3}-\frac{1}{\sqrt{21}}\right)\hat{a}_x+\left(\frac{2}{3}-\frac{2}{\sqrt{21}}\right)\hat{a}_y+\left(\frac{2}{3}-\frac{4}{\sqrt{21}}\right)\hat{a}_z\right]$$

$$\vec{E} = \frac{2\times10^{-6}}{4\pi\varepsilon_0}\left[0.115\hat{a}_x+0.23\hat{a}_y-0.206\hat{a}_z\right]$$

$$r = \sqrt{1+2^2+3^2} = \sqrt{14}$$

$$\phi = \tan^{-1}2 = 63.43^{o}$$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right) = 36.7^{o}$$

$$\overline{E_r} = \bar{E}\,\hat{a}_r = 2070\,\hat{a}_x\,.\hat{a}_r + 4140\,\hat{a}_y\,.\hat{a}_r +$$

$$4140\,\hat{a}_y\,.\hat{a}_r\ 3708\,\hat{a}_z\,.\hat{a}_r$$

$$= [2070 \sin\theta \cos\phi + 4140 \sin\theta \sin\phi - 3708 \cos\theta]\,\hat{a}_r$$

$$= [553.33 + 2212.86 - 2973]\,\hat{a}_r$$

$$= -206.81\,\hat{a}_r\ .$$

Choice (B)

25. $\bar{D} = \varepsilon.\bar{E} = \dfrac{\rho L}{2\pi d}\hat{a}_{12}$

z axis are $x = 0$ and $y = 0$

P_2 is (3, 4, –5) and P_1(0, 0, –5)

$\hat{a}_{12} = (3-0)\ \hat{a}_x + (4-0)\ \hat{a}_y = 3\,\hat{a}_x + 4\,\hat{a}_y$

$d = \left|\hat{a}_{12}\right| = 5$

$$\bar{D} = \frac{30\times10^{-6}}{2\pi[\times5}\left[\frac{3\hat{a}_x+4\hat{a}_y}{5}\right]$$

$$= 0.56\,\hat{a}_y + 0.76\,\hat{a}_y\ \frac{\mu C}{m^2}.$$

Choice (D)