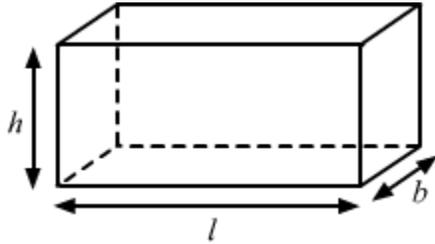# Surface Areas and Volumes

- **Surface areas of cuboid:**

Lateral surface area of the cuboid = $2h\ (l + b)$
Total surface area of the cuboid = $2\ (lb + bh + hl)$

Note: Length of the diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Example: **Find the edge of a cube whose surface area is 294 m².**
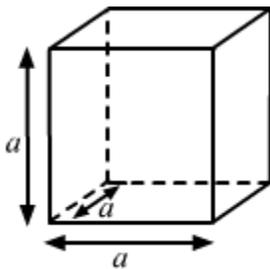
Solution: Let the edge of the given cube be a.
∴ Surface area of the cube = $6a^2$
Given, $6a^2 = 294$
$\Rightarrow a^2 = 49\ \text{m}^2$
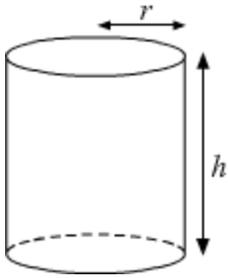$\therefore a = \sqrt{49}\ \text{m} = 7\ \text{m}$

- **Surface areas of cube:**

Lateral surface area of the cube = $4a^2$
Total surface area of the cube = $6a^2$

Note: Length of the diagonal of a cube = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

- **Surface areas of solid cylinder**

  o Curved surface area = $2\pi rh$, where r and h are the radius and height
  o Total surface area = $2\pi r\ (r + h)$, where r and h are the radius and height
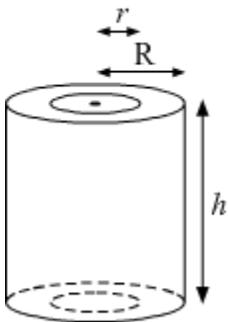
Example : What is the curved surface area of a cylinder of radius 2 cm and height 14 cm?

Solution: Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 2 \times 14 \text{ cm}^2$

$= 176 \text{ cm}^2$

- **Surface areas of hollow cylinder**

  o Curved surface area = $2\pi h \,(r + R)$, where r, R and h are the inner radius, outer radius and height

  o Total surface area = CSA of outer cylinder + CSA of inner cylinder + 2 × Area of base

    = $2\pi \,(r + R)\,(h + R - r)$, where r, R and h are the inner radius, outer radius and height

- **Surface areas of cone**

  o Curved surface area = πrl, where r and l are the radius and slant height
  o Total surface area = $\pi r \,(l + r)$, where r and l are the radius and slant height

Here, $l = \sqrt{h^2 + r^2}$, where h is the height.
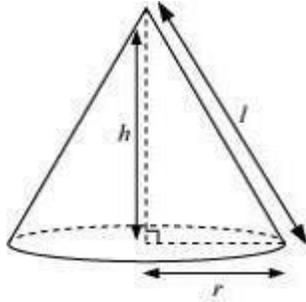
Example: Calculate the curved surface area of a cone of base radius 3 cm and height 4 cm.
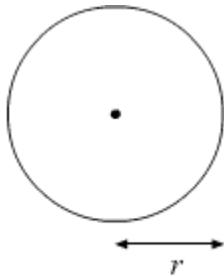
Solution: Here, $r = 3$ cm and $h = 4$ cm

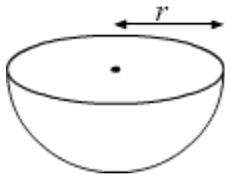$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{4^2 + 3^2}\ m = 5\ cm$$

Curved surface area = $\pi r l = \pi \times 3 \times 5$ cm² = 15π cm²

- **Surface areas of sphere and hemisphere**

o Surface area of sphere = $4\pi r^2$, where r is the radius

o Curved surface area of hemisphere = $2\pi r^2$, where r is the radius
o Total surface area of hemisphere = $3\pi r^2$, where r is the radius

Example: What is the radius of a balloon whose surface area is 5544 cm²?

Solution: Let radius of the balloon be r.
Surface area of the balloon = $4\pi r^2$ = 5544 cm²

$$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 5544\ cm^2$$
$$\Rightarrow r^2 = \frac{5544 \times 7}{88}\ cm^2$$
$$\Rightarrow r^2 = 441\ cm^2$$
$$\Rightarrow r = \sqrt{441} = 21\ cm$$

Thus, the radius of the balloon is 21 cm.

- **Volume of cube and cuboid**
  - Volume of cube = a^3, where a is the side of the cube
  - Volume of cuboid = $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.

    Example: What is the side of a cube of volume 512 cm³?
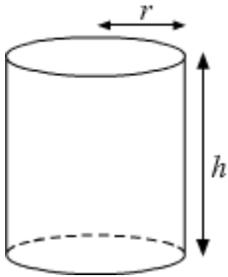   **Solution:** Volume of cube = 512 cm³

    $$\Rightarrow a^3 = 512\,\text{cm}^3$$
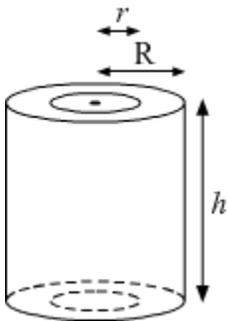 $$\Rightarrow a = \sqrt[3]{512}\,\text{cm}^3$$
    $$\Rightarrow a = 8\,\text{cm}$$

- **Volume of the solid cylinder and hollow cylinder**

  - Volume of solid cylinder = $\pi r^2 h$, where r and h are the radius and height of the solid cylinder

  - Volume of the hollow cylinder = $\pi (R^2 - r^2) h$, where r, R and h are the inner radius, outer radius and height of hollow cylinder

    Example: Find the volume of the pillar of radius 70 cm and height 10 m.

    Solution: Radius of the pillar (r) = 70 cm = $\frac{70\,\text{m}}{100}$ = 0.7 m

Height of the pillar (h) = 10 m

Volume of the pillar $= \pi r^2 h$

$$= \frac{22}{7} \times (0.7)^2 \times 10 \text{ m}^3$$

$$= 15.4 \text{ m}^3$$

- **Volume of a cone** $= \frac{1}{3}\pi r^2 h$, where r and h are the radius of base and height of the cone.

  Example: Calculate the volume of a cone of base radius 3 cm and height 4 cm.

  Solution: Here, r = 3 cm and h = 4 cm

  $$\text{Volume} = \frac{1}{3}\pi r^2 h$$

  $$= \frac{1}{3} \times \pi \times 3 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm}$$

  $$= 12\pi \text{ cm}^3$$

- **Volume of sphere and hemisphere**

  Volume of sphere $= \frac{4}{3}\pi r^3$

  Volume of hemisphere $= \frac{2}{3}\pi r^3$

  Example 1: Calculate the volume of a sphere whose surface area is 9π cm².

  Solution: Surface area = 9π cm²

  $$\Rightarrow 4\pi r^2 = 9\pi$$

  $$\Rightarrow r^2 = \frac{9}{4}$$

  $$\Rightarrow r = \frac{3}{2} \text{ cm}$$

  Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{3}{2} \text{ cm}\right)^3 = \frac{4}{3}\pi \left(\frac{27}{8}\right) \text{ cm}^3 = 4.5\pi \text{ cm}^3$

  Example 2: The inner radius of a hemispherical bowl is 4.2 cm. What is the capacity of the bowl?

  Solution: Here, r = 4.2 cm

  Volume of the bowl $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (4.2 \text{ cm})^3 = 155.232 \text{ cm}^3$

  1 ml = 1 cm³

  Thus, the capacity of the bowl is 155.232 ml.