Equation of Planes

Q.1. Find the coordinates of the point where the line joining the points (1 ,– 2 , 3) and (2 , – 1 , 5) cuts the plane $x - 2y + 3z = 19$. Hence, find the distance of this point from the point (5, 4, 1).

Solution : 1

Equation of line passing through the points (1 , – 2 , 3) and (2 , – 1 , 5) is

$(x - 1)/1 = (y + 2)/1 = (z - 3)/2 = k$.

Any point P on this line will be $(k + 1, k - 2 , 2k + 3)$

P lies on the plane , $x - 2y + 3z = 19$

Then $k + 1 - 2k + 4 + 6k + 9 = 19 \Rightarrow k = 1$.

The point P is (2 , – 1 , 5) and given point Q is (5 , 4, 1)

$PQ = \sqrt{(5 - 2)^2 + (4 + 1)^2 + (1 - 5)^2}$

$= \sqrt{9 + 25 + 16} = \sqrt{50} = 5\sqrt{2}$.

Q.2. Find the equation of the plane through the point (1, 2, 3) and perpendicular to the planes $x + y + 2z = 3$ and $3x + 2y + z = 4$.

Solution : 2

The equation of plane passing through (1, 2, 3) is

$A(x - 1) + B(y - 2) + C(z - 3) = 0$ ------------- (1)

The given equation are :

$x + y + 2z = 3$ ------------- (2)

and $3x + 2y + z = 4$ ------------- (3)

(1) is perpendicular to (2) and (3)

Therefore , $A + B + 2C = 0$ -------------- (4)

And $3A + 2B + C = 0$ -------------- (5)

Eliminating A, B and C from (1), (4) and (5) we get

$$\begin{vmatrix} x - 1 & y - 2 & z - 3 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

Or, $(x - 1)(1 - 4) - (y - 2)(1 - 6) + (z - 3)(2 - 3) = 0$

Or, $- 3x + 3 + 5y - 10 - z + 3 = 0$

Or, $3x - 5y + z + 4 = 0$.

Q.3. Find the equations to the two planes passing through the points (0, 4 , – 3) and (6, – 4 , 3) , if the sum of their intercepts on the axes is zero.

Solution : 3

Let the equation of plane be :

$x/a + y/b + z/c = 1$.

As (0, 4 , – 3) and (6, – 4 , 3) lie on it , then

$4/b - 3/c = 1$ ------------------ (1)

and $6/a - 4/b + 3/c = 1$

Adding we get , $6/a = 2 \Rightarrow a = 3$.

Also, $a + b + c = 0 \Rightarrow b + c = - a = - 3$

Putting in (1) we get

$4/b + 3/(3 + b) = 1$

Or, $b2 - 4b - 12 = 0$

Or, $(b - 6)(b + 2) = 0$

Therefore, $b = 6$ or $b = - 2$ and hence $c = - 9$ or $c = - 1$.

Therefore, equation of planes are : $x/3 + y/6 - z/9 = 1$ or $x/3 - y/2 - z/1 = 1$.

Q.4. A plane passes through the point (4, 2, 4) and is perpendicular to the planes $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$. Find the equation of the plane.

Solution : 4

Let the equation of the required plane be

$ax + by + c z + d = 0$ ---------------- (1)

As the plane passes through (4, 2, 4),

$4a + 2b + 4c + d = 0$ ---------------- (2)

As the plane is perpendicular to $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$,

Hence, $2a + 5b + 4c = 0$ ------------------------ (3)

and $4a + 7b + 6c = 0$ ------------------------- (4)

From (3) and (4) we get ,

$a/2 = b/4 = c/-6 = k$

Or, $a = 2k$, $b = 4k$, $c = -6k$.

Putting the values of a, b and c in (2) , we get

$8k + 8k - 24 k + d = 0 \Rightarrow d = 8k$.

Putting the values of a, b, c and d in (1), we get

$2kx + 4ky - 6kz + 8k = 0$

Or, $2x + 4y - 6z + 8 = 0$.

Q.5. Find the angle between the line $(x - 6)/3 = (y - 7)/2 = (z - 7)/-2$ and the plane $x + y + 2z = 0$.

Solution : 5

We have line as $(x - 6)/3 = (y - 7)/2 = (z - 7)/-2$ and plane as $x + y + 2z = 0$

Let l_1, m1 and n1 be direction cosines of normal to the plane , then

$l_1 = 1/\sqrt{6}$, $m_1 = 1/\sqrt{6}$, $n_1 = 2/\sqrt{6}$.

Let l_2, m_2 and n_2 be the direction cosines of the given line, then

$l2 = 3/\sqrt{17}$, $m_2 = 2/\sqrt{17}$, $n_2 = -2/\sqrt{17}$.

Let 'θ' be the angle between normal of the plane and the given line , then

$\cos\theta = l1l_2 + m_1m_2 + n_1n_2$

Or, $\cos\theta = 1/\sqrt{6} \times 3/\sqrt{17} + 1/\sqrt{6} \times 2/\sqrt{17} + 2/\sqrt{6} \times (-2/\sqrt{17})$

$= (3 + 2 - 4)/(\sqrt{6} \times \sqrt{17})$

$= 1/(\sqrt{6} \times \sqrt{17})$.

Therefore, angle between line and plane $= 90^o - \cos^{-1}[1/(\sqrt{6} \times \sqrt{17})]$

Q.6. Find the cosine of the angle between the planes :

$x + 2y - 2z + 6 = 0$
$2x + 2y + z + 8 = 0$

Solution : 6

Let angle between the given planes be θ,

$a_1 = 1/\sqrt{(1 + 4 + 4)} = 1/3$, $b_1 = 2/3$, $c_1 = -2/3$,

$a_2 = 2/\sqrt{(4 + 4 + 1)} = 2/3$, $b_2 = 2/3$ and $c_2 = 1/3$

Therefore, $\cos\theta = 1/3.2/3 + 2/3.2/3 + (-2)/3.1/3 = (2 + 4 - 2)/9 = 4/9$.

Q.7. Find the equation of the plane passing through (1, 2, 3) and perpendicular to the straight line $x/-2 = y/-4 = z/3$.

Solution : 7

Let equation of the plane be $Ax + By + Cz + D = 0$, ---------------- (1)

where A, B and C are direction ratios of the normal to the plane.

The straight line $x/-2 = y/-4 = z/3$

is perpendicular to (1) i.e. is normal to (1)

Equation of straight line passing through (α, β, γ) is

$(x - \alpha)/a = (y - \beta)/b = (z - \gamma)/c$

where a, b and c are direction ratio of the line.

By comparing, we get $a = -2$, $b = -4$ and $c = 3$.

Therefore, direction ratio of the normal to the plane are – 2, – 4 , 3.

Hence, A = – 2 B = – 4 C = 3.

Putting the value of A, B and C in (1), we get

$-2x - 4y + 3z + D = 0$

This passes through (1, 2, 3)

Therefore, $-2(1) - 4(2) + 3(3) + D = 0 \Rightarrow D = 1$

Therefore, equation of the plane is $-2x - 4y + 3z + 1 = 0$

Or, $2x + 4y - 3z - 1 = 0$

Q.8. A plane is passing through the point (2, –3, 1) and perpendicular to the straight line joining the points (3, 4, –1) and (2, –1, 5). Find the equation of the plane.

Solution : 8

The given line joining (3, 4, –1) and (2, –1 , 5) is perpendicular to the required plane. Hence direction ratio [(3 – 2), {4 – (–1)}, –1– 5}] = (1, 5, – 6) of the line is direction ratio of the normal to the plane. The plane passes through (2, – 3 , 1).

Therefore, equation of the plane is

$1(x - 2) + 5\{y - (-3)\} - 6(z - 1) = 0$

Or, $x - 2 + 5y + 15 - 6z + 6 = 0$

Or, $x + 5y - 6z + 19 = 0$.

Q.9. Find the equation of the plane which contains the line $(x - 1)/2 = (y + 1)/-1 = (z - 3)/4$ and is perpendicular to the plane $x + 2y + z = 12$.

Solution : 9

Equation of the plane through $(1, -1, 3)$ is given by

$A(x - 1) + B(y + 1) + C(z - 3) = 0$

This is perpendicular to the given line and plane

Therefore, $2A - B + 4C = 0$

and $A + 2B + C = 0$

Eliminating A, B and C from these two equation we get,

$$\begin{vmatrix} x - 1 & y + 1 & z - 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Or, $(x - 1)(-9) - (y + 1)(-2) + (z - 3)5 = 0$

Or, $-9x + 2y + 5z - 4 = 0$