KVPY EXAMINATION 2010

PART-1

One-Marks Question

MATHEMATICS

1. Let A denote the matrix $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, where $i^2 = -1$, and let I denote the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Then $I + A + A^2 + \ldots + A^{2010}$ is-

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ (D) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Ans. **(C)**

Sol. $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$; $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$; $A^3 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I + A + A^2 + A^3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; $A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$I + A + A^2 + A^3 + \ldots\ldots A^{2010}$

$(I + A + A^2 + A^3) + A^4(I + A + A^2 + A^3) + \ldots + A^{2008}(I + A + A^2)$

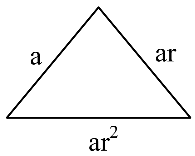
$= \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

2. Suppose the sides of a triangle form a geometric progression with common ratio r. Then r lies in the interval-

(A) $\left(0, \frac{-1+\sqrt{5}}{2}\right]$ (B) $\left(\frac{1+\sqrt{5}}{2}, \frac{2+\sqrt{5}}{2}\right]$ (C) $\left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ (D) $\left(\frac{2+\sqrt{5}}{2}, \infty\right)$

Ans. **(C)**

Sol.



$a + ar > ar^2$

$r^2 - r - 1 < 0$

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ …(1)

$ar^2 + ar > a$

$r^2 + r - 1 > 0$

$r > \frac{-1+\sqrt{5}}{2}$, $r < \frac{-1-\sqrt{5}}{2}$ ….(2)

$ar^2 + a > ar$; $r^2 - r + 1 > 0$ always true

Solving (1) & (2)

$r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$

3. The number of rectangles that can be obtained by joining four of the twelve vertices of a 12-sided regular polygon is-

(A) 66 (B) 30 (C) 24 (D) 15

Ans. **(D)**

Sol. Number of diagonals passing through centre = 6

number of rectangles = ${}^6C_2 = 15$

4. Let I, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial, with real coefficients, having $2\omega^2$, $3 + 4\omega$, $3 + 4\omega^2$ and $5 - \omega - \omega^2$ as roots is-

(A) 4 (B) 5 (C) 6 (D) 8

Ans. **(B)**

Sol. roots $\rightarrow$ $\underset{\alpha}{2\omega^2}$, $\underset{\beta}{3 + 4\omega}$, $\underset{\gamma}{3 + 4\omega^2}$, $\underset{\delta}{5 - \omega - \omega^2}$

$\delta = 5 - (\omega + \omega^2) = 5 - (-1) = 6$

If $\alpha = 2\omega^2$ is a root then 2ω has to be a root too.

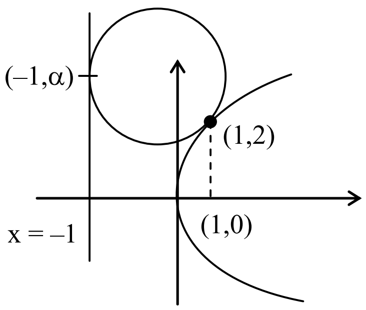
total $\rightarrow$ min. 5 roots, hence min. degree $\rightarrow$ 5

5. A circle touches the parabola $y^2 = 4x$ at (1, 2) and also touches its directrix. The y-coordinates of the point of contact of the circle and the directrix is-

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4

Ans. **(C)**

Sol.



$y^2 = 4x$

$2y \dfrac{dy}{dx} = 4$

$m_T = \dfrac{2}{y} = \dfrac{2}{2} = 1$

Circle $\rightarrow$ $S + \lambda L = 0$

$(x + 1)^2 + (y - \alpha)^2 + \lambda(x + 1) = 0$ ....(1)

differentiate

$2(x + 1) + 2(y - \alpha) \dfrac{dy}{dx} + \lambda = 0$

$x = 1, y = 2$

$4 + 2(2 - \alpha)\, m_T + \lambda = 0$

$\lambda = 2\alpha - 8$ ....(2)

(1, 2) satisfies eq.(1)

$2^2 + (2 - \alpha)^2 + 2\lambda = 0$

$\alpha^2 - 4\alpha + 8 + 2(2\alpha - 8) = 0$

$\alpha^2 = 8$

$\alpha = 2\sqrt{2}$

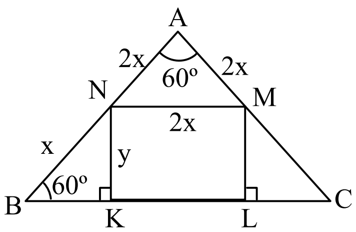
6. Let ABC be an equilateral triangle, let KLMN be a rectangle with K, L on BC, M on AC and N on AB. Suppose AN/NB = 2 and the area of triangle BKN is 6. The area of the triangle ABC is-

(A) 54 (B) 108

(C) 48 (D) not determinable with the above data

Ans. **(B)**

Sol.



$y = \frac{\sqrt{3}x}{2}$

$z = \frac{x}{2}$

$\frac{1}{2}yz = 6 \Rightarrow x^2 = \frac{48}{\sqrt{3}}$

Area of $\Delta ABC = 6 + 6 + 2xy + \frac{1}{2}(2x)(2x)\sin 60^\circ$

$= 12 + 2x\frac{\sqrt{3}x}{2} + 2x^2\frac{\sqrt{3}}{2}$

$12 + 2\sqrt{3} \times \frac{48}{\sqrt{3}} = \boxed{108}$

7. Let P be an arbitrary point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$, $a > b > 0$. Suppose F_1 and F_2 are the foci of the ellipse. The locus of the centroid of the triangle PF_1F_2 as P moves on the ellipse is-

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

Ans. **(C)**

Sol. $P \to a\cos\theta, b\sin\theta$

$G \to \left(\frac{\Sigma x_i}{3}, \frac{\Sigma y_i}{3}\right)$

$F_1 \to (ae, 0)$ $F_2 \to (-ae, 0)$

$h = \frac{a\cos\theta + ae - ae}{3}$; $\cos\theta = \frac{3h}{a}$

$k = \dfrac{b\sin\theta}{3}$; $\sin\theta = \dfrac{3k}{b}$

$\cos^2\theta + \sin^2\theta = 1$

$\dfrac{9h^2}{a^2} + \dfrac{9k^2}{b^2} = 1 \quad \Rightarrow \quad \boxed{\dfrac{x^2}{(a^2/9)} + \dfrac{y^2}{(b^2/9)} = 1}$

(Ellipse)

8. The number of roots of the equation $\cos^7\theta - \sin^4\theta = 1$ that lie in the interval $[0, 2\pi]$ is-

(A) 2 (B) 3 (C) 4 (D) 8

Ans. **(A)**

Sol. $\cos^7\theta = 1 + \sin^4\theta \geq 1$ but $\cos^7\theta \leq 1$

so $\cos\theta = 1$; $\sin\theta = 0$

$\boxed{\theta = 0, 2\pi}$

9. The product $(1 + \tan 1^\circ)(1 + \tan 2^\circ)(1 + \tan 3^\circ) \ldots (1 + \tan 45^\circ)$ equals-

(A) 2^{21} (B) 2^{22} (C) 2^{23} (D) 2^{25}

Ans. **(C)**

Sol. $(1 + \tan\theta)(1 + \tan(45 - \theta)) = (1 + \tan\theta)\left(1 + \dfrac{1-\tan\theta}{1+\tan\theta}\right) = 2$

$(1 + \tan 1^\circ)(1 + \tan 44^\circ) = 2$ etc

so product $= 2^{22}(1 + \tan 45^\circ) = 2^{23}$

10. Let $f : R \to R$ be a differentiable function such that $f(a) = 0 = f(b)$ and $f'(a)\, f'(b) > 0$ for some $a < b$. Then the minimum number of roots of $f'(x = 0$ in the interval (a, b) is-

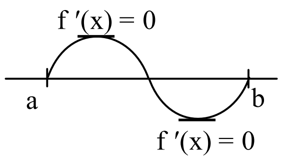
(A) 3 (B) 2 (C) 1 (D) 0

Ans. **(B)**

Sol. $f'(a) \cdot f'(b) > 0$

so either both are positive or both are negative

$f(a) = f(b) = 0$



$\boxed{\text{2 roots}}$

11. The roots of $(x - 41)^{49} + (x - 49)^{41} + (x - 2009)^{2009} = 0$ are -

(A) all necessarily real

(B) non-real except one positive real root

(C) non-real except three positive real roots

(D) non-real except for three real roots of which exactly one is positive

Ans. **(B)**

Sol. $(x - 41)^{49} + (x - 49)^{41} + (x - 2009)^{2009} = 0$

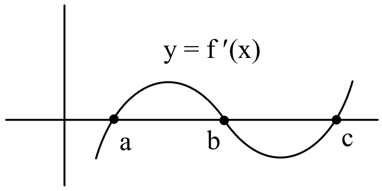
$f(x) = (x - 41)^{49} + (x - 49)^{41} + (x - 2009)^{2009}$

$f'(x) = 49(x - 41)^{48} + 41(x - 49)^{40} + 2009(x - 2009)^{48} > 0$

hence f(x) will cut x-axis only once.

$\boxed{\text{1 real root}}$

12. The figure shown below is the graph of the derivative of some function $y = f'(x)$.



Then-

(A) f has local minima at x = a, b and a local maximum at x = c

(B) f has local minima at x = b, c and a local maximum at x = a

(C) f has local minima at x = c, a and a local maximum at x = b

(D) the given figure is insufficient to conclude any thing about the local minima and local maxima of f

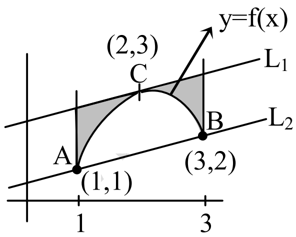
Ans. **(C)**

Sol. $f'(a) = f'(b) = f'(c) = 0$

$$\left.\begin{matrix} f'(a^-) < 0 & f'(a^+) > 0 \\ f'(c^-) < 0 & f'(c^+) > 0 \end{matrix}\right] \text{ minima at a \& c}$$

$f'(b^-) > 0 \;\; f'(b^+) < 0$] max. at b.

13. The following figure shows the graph of a continuous function y = f(x) on the interval [1, 3]. The points A, B, C have coordinates (1, 1), (3, 2), (2, 3) respectively, and the lines L_1 and L_2 are parallel, with L_1 being tangent to the curve at C. If the area under the graph of y = f(x) from x = 1 to x = 3 is 4 square units, then the area of the shaded region is-



(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(A)**

Sol. Equation of $\ell_2 \rightarrow y - 1 = \frac{2-1}{3-1}(x-1)$

$2y - 2 = x - 1$

$2y - x = 1$

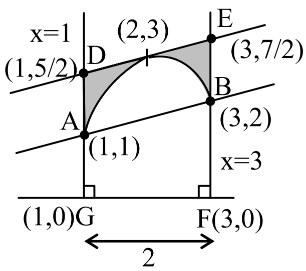
$\Rightarrow$ slope of $\ell_1 = 1/2$

Equation of $\ell_1 = y - 3 = \frac{1}{2}(x-2)$

$2y - 6 = x - 2$

$2y - x = 4$

$D \rightarrow \left(1, \frac{5}{2}\right)$, $E \rightarrow \left(3, \frac{7}{2}\right)$



area under f(x) = 4

shaded area = area of trapezium DEFG – area under f(x)

$$= \frac{1}{2}\left(\frac{5}{2}+\frac{7}{2}\right) \times 2 - 4$$

$$= 6 - 4 = 2$$

14. Let $I_n = \int_0^1 (\log x)^n \, dx$, where n is a non-negative integer. Then $I_{2001} - 2011\ I_{2010}$ is equal to-

(A) $I_{1000} + 999\ I_{998}$ (B) $I_{890} + 890\ I_{889}$

(C) $I_{100} + 100\ I_{99}$ (D) $I_{53} + 54\ I_{52}$

Ans. **(C)**

Sol. $I_n = \int_1^e \underset{II}{1}.\underset{I}{(\log x)^n}\ dx$

$I_n = (\log x)^n \ x\Big|_1^e - \int_1^e \frac{n(\log x)^{n-1}}{x}.x\,dx$

$I_n = e - 0 - n\ I_{n-1}$

$I_n + n\ I_{n-1} = e$

$I_{2001+2011}\ \ I_{2010} = e$

$\boxed{I_{100} + 100\ I_{99} = e}$

15. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \le 100\}$ and $= \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then the area of the region $A \cap B$ is-

(A) $10\ \pi$ (B) 100 (C) $100\ \pi$ (D) $50\ \pi$

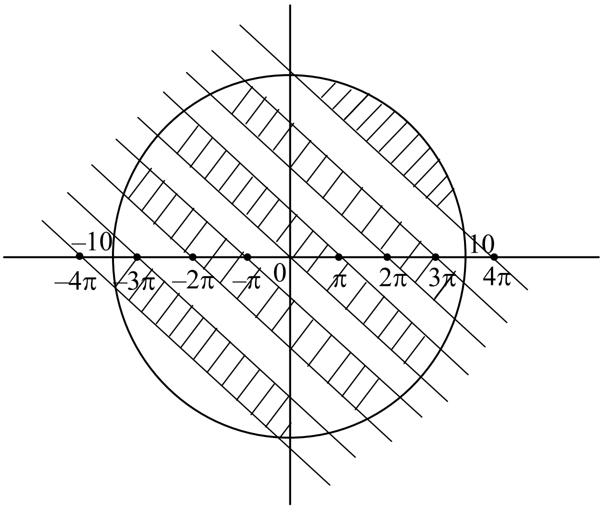
Ans. **(D)**

Sol. $x^2 + y^2 \le 100 \rightarrow$ inside of a circle

$\sin(x + y) > 0$

$x + y \in (0, \pi) \cup (2\pi, 3\pi)$ …..

$x + y = c \rightarrow$ equation of a line



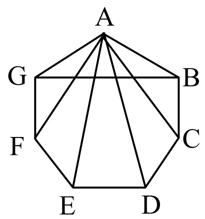
required area = shaded region = $\frac{1}{2}\pi(10)^2 = 50\pi$

16. Three vertices are chosen randomly from the seven vertices of a regular 7-sided polygon. The probability that they form the vertices of an isosceles triangle is-

(A) $\frac{1}{7}$ (B) $\frac{1}{3}$ (C) $\frac{3}{7}$ (D) $\frac{3}{5}$

Ans. **(D)**

Sol.



ΔAGB, ΔAFC & ΔAED are isosceles

$$P = \frac{{}^7C_1 \times 3}{{}^7C_3} = \frac{7\times 3}{\frac{7\times 6\times 5}{3\times 2}} = \frac{3}{5}$$

17. Let $\vec{u} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{v} = -3\hat{j} + 2\hat{k}$ be vectors in R^3 and $\vec{w}$ be a unit vector in the xy-plane. Then the maximum possible value of $|(\vec{u} \times \vec{v}) . \vec{w}|$ is-

(A) $\sqrt{5}$ (B) $\sqrt{12}$ (C) $\sqrt{13}$ (D) $\sqrt{17}$

Ans. **(D)**

Sol. $\vec{u}\times\vec{v} = (2\hat{i} - \hat{j} + \hat{k})\times(-3\hat{j} + 2\hat{k})$

$-6\hat{k} - 4\hat{j} - 2\hat{i} + 3\hat{i} = \hat{i} - 4\hat{j} - 6\hat{k}$

Let $\vec{w} = a\hat{i} + b\hat{j}$ $\quad a^2 + b^2 = 1$ $\quad a = \cos\theta$; $b = \sin\theta$

$\vec{u}\times\vec{v}.\vec{w} = a - 4b = \cos\theta - 4\sin\theta$

max. value = $\sqrt{1^2 + (-4)^2} = \sqrt{17}$

18. How many six-digit numbers are there in which no digit is repeated, even digits appear at even places, odd digits appear at odd places and the number is divisible by 4 ?

(A) 3600 (B) 2700 (C) 2160 (D) 1440

Ans. **(D)**

Sol.

3 ways	3 ways	4 ways	4 ways	5 ways	2 ways

(1,3,5,7,9) (2,6)

$3 \times 3 \times 4 \times 4 \times 5 \times 2 = 1440$

19. The number of natural numbers n in the interval [1005, 2010] for which the polynomial $1 + x + x^2 + x^3 + \ldots. x^{n-1}$ divides the polynomial $1 + x^2 + x^3 + x^4 + \ldots. + x^{2010}$ is-

(A) 0 (B) 100 (C) 503 (D) 1006

Ans. **(C)**

Sol. $1 + x^2 + x^4 + \ldots.. x^{2010} = \dfrac{1(1-x^{2012})}{1-x^2} = \dfrac{(1-x^{1006})(1+x^{1006})}{(1-x)(1+x)}$

$$= (1+x^{1006})\left(\frac{(1-x^{503})}{(1-x)}\right)\left(\frac{(1+x^{503})}{(1+x)}\right)$$

$= (1 + x^{1006})(1 + x + x^2 + \ldots. x^{502})(1 - x + x^2 - x^3 + \ldots. x^{502})$

this is divisible by $1 + x + x^2 + \ldots.. x^{n-1}$

if $n - 1 = 502$

$\boxed{n = 503}$

20. Let $a_0 = 0$ and $a_x = 3a_{n-1} + 1$ for $n \geq 1$. Then the remainder obtained dividing a_{2010} by 11 is-

(A) 0 (B) 7 (C) 3 (D) 4

Ans. **(A)**

Sol. $a_n = 3a_{n-1} + 1$

$a_{2010} = 3a_{2009} + 1$

$= 3(3a_{2008} + 1) + 1 = 3^2\, a_{2008} + 3 + 1$

$= 3^3\, a_{2007} + 3 + 3 + 1$

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$$3^{2010}\, a_0 + \underbrace{(3+3+\ldots 3)}_{2009\,\text{times}} + 1$$

$= 0 + 6027 + 1 = 6028$

Remainder $\left(\dfrac{6028}{11}\right) = 0$

PHYSICS

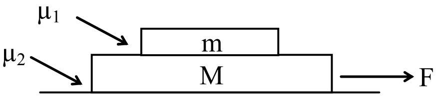
21. A pen of mass 'm' is lying on a piece of paper of mass M placed on a rough table. If the coefficient of friction between the pen and paper, and, the paper and table are μ_1 and μ_2, respectively, then the minimum horizontal force with which the paper has to be pulled for the pen to start slipping is given by-

(A) $(m + M)(\mu_1 + \mu_2)g$ (B) $(m\mu_1 + M\mu_2)g$

(C) $\{m\mu_1 + (m + M)\mu_2\}g$ (D) $m(\mu_1 + \mu_2)g$

Ans. **(A)**

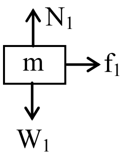
Sol.



For pen to start slipping maximum horizontal force on it is $f = \mu_1 mg$

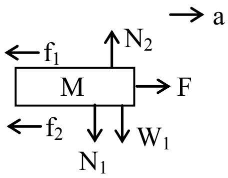
$\therefore$ $a = \mu_1 g$ is the maximum common acceleration for both pen and paper

F.B.D. for both pen and paper



$f_1 = \mu N_1$ & $N_1 = W_1$

$\therefore f_1 = \mu_1 mg$



$\therefore N_2 = N_1 + W_1$

$F - f_1 - f_2 = Ma$

also $f_2 = \mu_2 N_2 = \mu_2(m + M)g$

$\therefore F = f_1 + f_2 + Ma$

$F = \mu_1 mg + \mu_2(m + M)g + M(\mu_1 g)$

$\therefore \boxed{F = (m + M)(\mu_1 + \mu_2)g}$

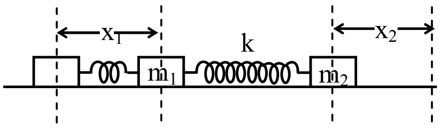
22. Two masses m_1 and m_2 connected by a spring of spring constant k rest on a frictionless surface. If the masses are pulled apart and let go, the time period of oscillation is-

(A) $T = 2\pi\sqrt{\frac{1}{k}\left(\frac{m_1 m_2}{m_1 + m_2}\right)}$ (B) $T = 2\pi\sqrt{k\left(\frac{m_1 + m_2}{m_1 m_2}\right)}$

(C) $T = 2\pi\sqrt{\left(\frac{m_1}{k}\right)}$ (D) $T = 2\pi\sqrt{\left(\frac{m_2}{k}\right)}$

Ans. **(A)**

Sol.



Let the masses be slightly displaced by x_1 and x_2 from this equilibrium position in opposite direction so net stretch in spring is $x = x_1 + x_2$. Because of this a restoring force kx will act on each mass and therefore equation for m_1 & m_2 will be

$$m_1 \frac{d^2x_1}{dt^2} = -kx \quad \text{and} \quad m_2 \frac{d^2x_2}{dt^2} = -kx$$

but as $x = x_1 + x_2$

$$\therefore \frac{d^2x}{dt^2} = \frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2}$$

replacing values of $\frac{d^2x_1}{dt^2}$ and $\frac{d^2x_2}{dt^2}$ from acceleration equations we get

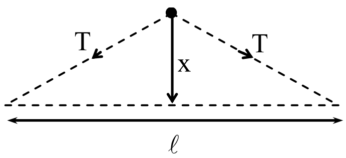
$$\frac{d^2x_2}{dt^2} = -\left(\frac{1}{m_1} + \frac{1}{m_2}\right) kx$$

also if $\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m}$ then m is the effective mass in this case therefore

$$\frac{d^2x}{dt^2} = -\omega^2 x = \frac{-kx}{m} \text{ or } \omega^2 = \frac{k}{m} \text{ and } T = \frac{2\pi}{\omega}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}}$$

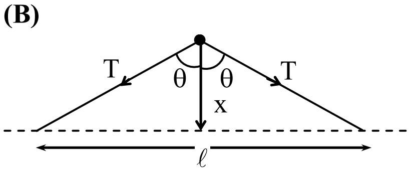
23. A bead of mass m is attached to the mid-point of a taut, weightless string of length ℓ and placed on a frictionless horizontal table.



Under a small transverse displacement x, as shown, if the tension in the string is T, then the frequency of oscillation is-

(A) $\frac{1}{2\pi}\sqrt{\frac{2T}{m\ell}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{4T}{m\ell}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{4T}{m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{2T}{m}}$

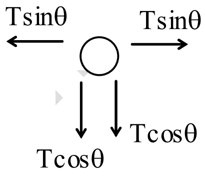
Ans. **(B)**



Sol.

Let the angle of T with the vertical be θ then

F.B.D.



$\therefore$ 2T cosθ = ma

also $\cos\theta = \frac{x}{\sqrt{x^2 + \left(\frac{\ell}{2}\right)^2}}$

given $\ell >> x \quad \therefore \quad \frac{\ell^2}{4} + x^2 \simeq \frac{\ell^2}{4}$

$\therefore\ a = \dfrac{-2Tx}{m\left(\dfrac{\ell}{2}\right)}$ (negative sign for restoring force)

or $a = -\left(\dfrac{4T}{m\ell}\right)x$ also this is similar to the equation of SHM i.e. $a = -\omega^2 x$

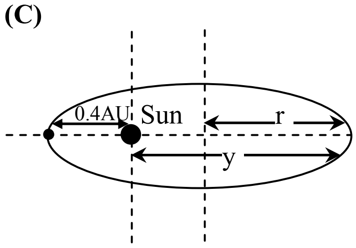
$\therefore\ \omega = \sqrt{\dfrac{4T}{m\ell}}$ & $f = \dfrac{\omega}{2\pi}$

$\therefore\ f = \dfrac{1}{2\pi}\sqrt{\dfrac{4T}{m\ell}}$

24. A comet (assumed to be in an elliptical orbit around the sun) is at a distance of 0.4 AU from the sun at the perihelion. If the time period of the comet is 125 years, what is the aphelion distance ? AU : Astronomical Unit.

(A) 50 AU (B) 25 AU (C) 49.6 AU (D) 24.6 AU

Ans. **(C)**



Sol.

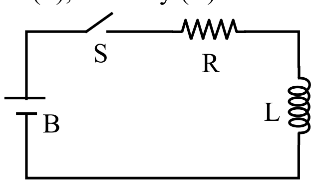
$\therefore r = \dfrac{0.4 + y}{2}$

also $\left(\dfrac{T_1}{T_2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^3$ by Kepler's law of time- periods (T_1, r_1 are taken for earth)

$\therefore\ \left(\dfrac{1\ y}{125\ y}\right)^2 = \left(\dfrac{1\ AU}{\left(\dfrac{0.4 + y}{2}\right)AU}\right)^3$

solving we get $\boxed{y = 49.6\ AU}$

25. The circuit shown consists of a switch (S), a battery (B) of emf E, a resistance R, and an inductor L.



The current in the circuit at the instant the switch is closed is-

(A) E/R (B) E/R(1 – e) (C) ∞ (D) 0

Ans. **(D)**

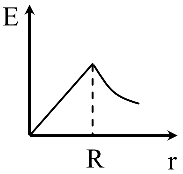
Sol. Using the equation $I = I_0\ (1 - e^{-\frac{t}{t_L}})$

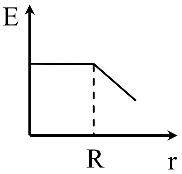
Put t = 0 we get $I = 0$

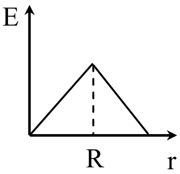
Just when the battery is closed inductor provides infinite resistance to the current flow

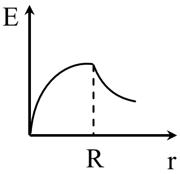
$\therefore$ current is zero initially.

26. Consider a uniform spherical volume charge distribution of radius R. Which of the following graphs correctly represents the magnitude of the electric field E as a distance r from the center of the sphere ?

(A) 

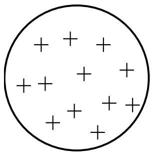
(B) 

(C) 

(D) 

Ans. **(A)**

Sol.

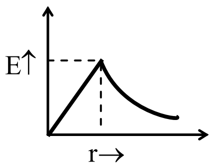


$E_r = \frac{\rho r}{3\varepsilon_0}$ for $0 \le r < R$

$E_r = \frac{kQ}{r^2}$ for $r \ge R$

$\therefore$ $E \propto r$ for $r < R$ and $E \propto \frac{1}{r^2}$ for $r \ge R$

$\therefore$ curve is

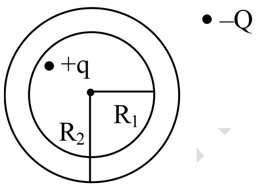


27. A charge +q is placed somewhere inside the cavity of a thick conducting spherical shell of inner radius R_1 and outer radius R_2. A charge – Q is placed at a distance $r > R_2$ from the centre of the shell. Then the electric field in the hollow cavity-

(A) depends on both +q and – Q
(B) is zero
(C) is only that due to – Q
(D) is only that due to +q

Ans. **(D)**

Sol.



For a conductor electric field inside its cavity is only due to inside charge and not due to outside charge.

28. The following travelling electromagnetic wave

$E_x = 0$, $E_y = E_0\sin(kx + \omega t)$, $E_z = -2E_0\sin(kx - \omega t)$ is-

(A) elliptically polarized
(B) circularly polarized
(C) linearly polarized
(D) unpolarized

Ans. **(B)**

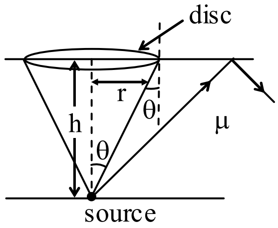
Sol. From the equation of E_y & E_z it is evident that wave is circularly polarized.

29. A point source of light is placed at the bottom of a vessel which is filled with water of refractive index μ to a height h. If a floating opaque disc has to be placed exactly above it so that the source is invisible from above, the radius of the disc should be-

(A) $\frac{h}{\sqrt{\mu - 1}}$ (B) $\frac{h}{\sqrt{\mu^2 - 1}}$ (C) $\frac{h}{\mu^2 - 1}$ (D) $\frac{\mu h}{\sqrt{\mu^2 - 1}}$

Ans. **(B)**

Sol.



r should be such that rays beyond it got totally internally reflected

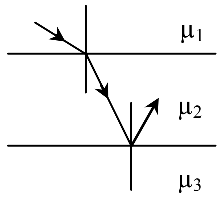
For this $\theta > C$ or $\sin\theta > \sin C$

also $\mu = \frac{1}{\sin C}$ $\therefore$ $\frac{r}{\sqrt{h^2 + r^2}} > \frac{1}{\mu}$

In limiting case $\frac{r}{\sqrt{h^2 + r^2}} = \frac{1}{\mu}$

solving we get $\boxed{r = \frac{h}{\sqrt{\mu^2 - 1}}}$

30. Three transparent media of refractive indices μ_1, μ_2, μ_3 respectively, are stacked as shown. A ray of light follows the path shown. No light enters the third medium.

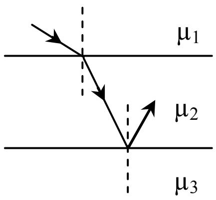


Then-

(A) $\mu_1 < \mu_2 < \mu_3$ (B) $\mu_2 < \mu_1 < \mu_3$ (C) $\mu_1 < \mu_3 < \mu_2$ (D) $\mu_3 < \mu_1 < \mu_2$

Ans. **(D)**

Sol. At first incidence light is deviated towards the normal therefore $\mu_2 > \mu_1$. Also at second incidence TIR takes place therefore $\mu_2 > \mu_3$, also $\mu_1 > \mu_3$ because for the same angle in medium μ_2, angle in μ_1 medium is less.



$\therefore$ $\mu_3 < \mu_1 < \mu_2$

31. A nucleus has a half-life of 30 minutes. At 3 PM its decay rate was measured as 120,000 counts/sec. What will be the decay rate at 5 PM ?

(A) 120,000 counts/sec (B) 60,000 counts/sec
(C) 30,000 counts/sec (D) 7,500 counts/sec

Ans. **(D)**

Sol. Given T = 30 minutes. $\frac{dN}{dt} = 120\text{ K}\ \frac{\text{counts}}{\text{sec}}$

After each half life, activity is reduced to half therefore after n half lives activity reduces to $\left(\frac{1}{2}\right)^n$.

Also $\frac{dN}{dt} \propto N$

$\frac{dN}{dt}$ at 5 P.M. will be equal to activity remaining after four half lives.

i.e. $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{16}\text{th}\right)$ of the initial activity

$\left(\frac{dN}{dt}\right)_{\text{at 5 P.M.}} = \left(\frac{1}{16}\text{th}\right)$ of the initial activity

$\left(\frac{dN}{dt}\right)_{\text{5 P.M.}} = \left(\frac{1}{16}\right)\left(\frac{dN}{dt}\right)_{\text{3 P.M.}}$

$\boxed{\left(\frac{dN}{dt}\right)_{\text{5 P.M.}} = 7500\text{ counts/sec}}$

32. A book is resting on shelf that is undergoing vertical simple harmonic oscillations with an amplitude of 2.5 cm. What is the minimum frequency of oscillation of the shell for which the book will lose contact with the shelf ? (Assume that $g = 10\text{ m/s}^2$)

(A) 20 Hz (B) 3.18 Hz (C) 125.6 Hz (D) 10 Hz

Ans. **(B)**

Sol. Book will loose contact with the shelf when a = g

Now $|a| = \omega^2 x \ \therefore\ g = \omega^2 A$ (A → Amplitude)

$\omega^2 = \frac{g}{A}$ also $f = \frac{\omega}{2\pi}$

$\therefore\ f = \frac{1}{2\pi}\sqrt{\frac{g}{A}}$

replacing $g = 10\text{ m/s}^2$ and $A = 2.5 \times 10^{-2}$ m

We get $\boxed{f = 3.18\text{ Hz}}$

33. A van der Waal's gas obeys the equation of state $\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$. Its internal energy is given by $U = CT - \frac{n^2 a}{V}$. The equation of a quasistatic adiabat for this gas is given by-

(A) $T^{C/nR}V$ = constant (B) $T^{(C+nR)/nR}V$ = constant
(C) $T^{C/nR}(V - nb)$ = constant (D) $P^{(C+nR)/nR}(V - nb)$ = constant

Ans. **(C)**

Sol. For adiabatic process

$dQ = 0$ and $-dU = dW \Rightarrow -nC_V\,\Delta T = P\Delta V$ or $-nC_V dT = PdV$

when change is very small

now given $U = CT - \frac{n^2 a}{V}$ $\therefore$ $dU = CdT + \frac{n^2 a}{V^2}dV$

put this value of dU in $-dU = dW$

$$\therefore -\left(CdT + \frac{n^2 a}{V^2}dV\right) = PdV \qquad ....(1)$$

also $P = \left(\frac{nRT}{V - nb}\right) - \frac{n^2 a}{V^2}$ replace it in (1)

$$-\left(CdT + \frac{n^2 a}{V^2}dV\right) = \left(\left(\frac{nRT}{V - nb}\right) - \frac{n^2 a}{V^2}\right)dV$$

$$\therefore -CdT = \left(\frac{nRT}{V - nb}\right)dV$$

$$\therefore -\frac{C}{nR}\frac{dT}{T} = \frac{dV}{V - nb}$$

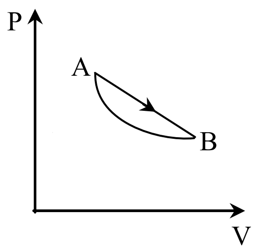
Integrating we get

$$-\ell n\, T^{C/nR} = \ell n\,(V - nb) + k \qquad (k \rightarrow \text{constant of integration})$$

$\therefore \ell n\,(T^{C/nR})(V - nb) = -k$

$\boxed{\text{or } (T^{C/nR})(V - nb) = \text{constant}}$

34. An ideal gas is made to undergo a cycle depicted by the PV diagram alongside. The curved line from A to B is an adiabat.



Then-

(A) The efficiency of this cycle is given by unity as no heat is released during the cycle

(B) Heat is absorbed in the upper part of the straight line path and released in the lower part

(C) If T_1 and T_2 are the maximum and minimum temperatures reached during the cycle, then the efficiency is given by $1 - \frac{T_2}{T_1}$

(D) The cycle can only be carried out in the reverse of the direction shown in figure

Ans. **(B)**

Sol. From the analysis of P-V diagram we can easily say that B is the correct option.

35. A bus driving along at 39.6 kmph is approaching a person who is standing at the bus stop, while honking repeatedly at an interval of 30 seconds. If the speed of the sound is 330 m/s, at what interval will the person hear the horn ?

(A) 31 seconds

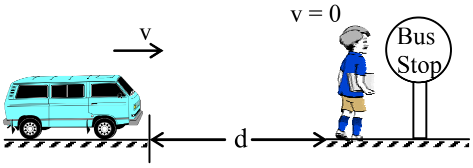
(B) 29 seconds

(C) 30 seconds

(D) the interval will depend on the distance of the bus from the passenger

Ans. **(B)**

Sol.



$v = 39.6$ km/hr $= 11$ m/s, $t_1 = \dfrac{d}{330}$ and $t_2 = \dfrac{d - 11\times 30}{330}$

Now $\Delta t = t_1 - t_2 = 1$

now $t_1 = 30$ sec $\therefore$ $t_2 = 29$ sec.

36. Velocity of sound measured at a given temperature in oxygen and hydrogen is in the ratio -

(A) 1 : 4 (B) 4 : 1 (C) 1 : 1 (D) 32 : 1

Ans. **(A)**

Sol. $v = \sqrt{\dfrac{\gamma RT}{M}}$ $\therefore v \propto \dfrac{1}{\sqrt{M}}$

$$\frac{V_0}{V_H} = \sqrt{\frac{M_H}{M_0}} = \sqrt{\frac{2}{32}} = \sqrt{\frac{1}{16}}$$

$$\frac{V_0}{V_H} = \frac{1}{4}$$

37. In Young's double slit experiment, the distance between the two slits is 0.1 mm, the distance between the slits and the screen is 1 m and the wavelength of the light used is 600 nm. The intensity at a point on the screen is 75% of the maximum intensity. What is the smallest distance of this point from the central fringe ?

(A) 1.0 mm (B) 2.0 mm (C) 0.5 mm (D) 1.5 mm

Ans. **(A)**

Sol. $d = 0.1$ mm, $D = 1$ m, $\lambda = 600$ nm

$I_P = 75$ % of maximum or $I_P = 3I_0$

Where I_0 is the intensity of a single wave

now $I_P = 3I_0 = (\sqrt{I_0})^2 + (\sqrt{I_0})^2 + 2\sqrt{I_0 \times I_0}\cos\phi$

$\therefore \cos\phi = \cos\dfrac{\pi}{3}$, also $\Delta x = \dfrac{yd}{D}$

now $\Delta x = \dfrac{\lambda}{2\pi}\times\dfrac{\pi}{3} = \dfrac{\lambda}{6}$ $\therefore$ $y = \dfrac{\lambda D}{6d} = \dfrac{600\times10^{-9}\times1}{6\times0.1\times10^{-3}}$ or $\boxed{y = 1m}$

38. Two masses m_1 and m_2 are connected by a massless spring of spring constant k and unstreched length ℓ. The masses are placed on a frictionless straight channel – which we consider our x-axis. They are initially at rest at x = 0 and x = ℓ, respectively. At t = 0, a velocity of v_0 is suddenly imparted to the first particle. At a later time t_0, the centre of mass of the two masses is at-

(A) $x = \dfrac{m_2\ell}{m_1+m_2}$

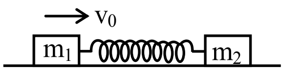
(B) $x = \dfrac{m_1\ell}{m_1+m_2} + \dfrac{m_2v_0t}{m_1+m_2}$

(C) $x = \dfrac{m_2\ell}{m_1+m_2} + \dfrac{m_2v_0t}{m_1+m_2}$

(D) $x = \dfrac{m_2\ell}{m_1+m_2} + \dfrac{m_1v_0t}{m_1+m_2}$

Ans. **(D)**

Sol.



$v_{COM} = \dfrac{m_1v_0 + 0}{m_1+m_2}$, $x_{iCOM} = \dfrac{m_1(0)+m_2(\ell)}{m_1+m_2}$

also $x_{COM} = x_{iCOM} + v_{COM}t$

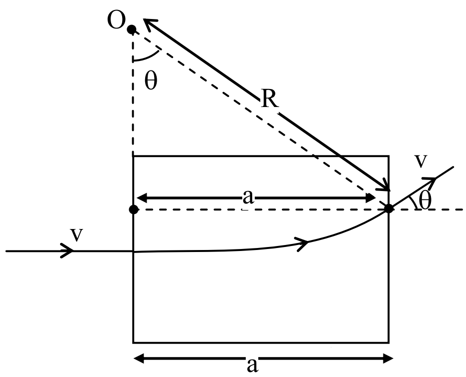
$\therefore\ x_{COM} = \left(\dfrac{m_2\ell}{m_1+m_2}\right) + \dfrac{m_1v_0t}{m_1+m_2}$

39. A charged particle of charge q and mass m, gets deflected through an angle θ upon passing through a square region of side 'a' which contains a uniform magnetic field B normal to its plane. Assuming that the particle entered the square at right angles to one side, what is the speed of the particle ?

(A) $\dfrac{qB}{m}a\cot\theta$ (B) $\dfrac{qB}{m}a\tan\theta$ (C) $\dfrac{qB}{m}a\cot^2\theta$ (D) $\dfrac{qB}{m}a\tan^2\theta$

Ans. **(A)**

Sol.



Now $\sin\theta = \dfrac{a}{R}$, $R = \dfrac{mv}{qB}$

$\therefore\ \boxed{v = \dfrac{qBa\cot\theta}{m}}$

40. A piece of hot copper at 100ºC is plunged into a pond at 30ºC. The copper cools down to 30ºC, while the pond, being huge, stays at its initial temperature. Then-

(A) copper loses some entropy, the pond stays at the same entropy

(B) copper loses some entropy, and the pond gains exactly the same amount of entropy

(C) copper loses entropy, and the pond gains more than this amount of entropy

(D) both copper and the pond gain in entropy

Ans. **(C)**

Sol. Using theory of entropy it is evident that answer is (C).

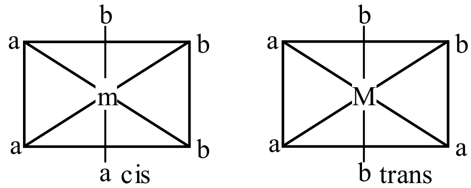
CHEMISTRY

41. The number of isomers of Co (diethylene triamine) Cl_3 is-

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(A)**

Sol. Isomers of [Co (dien) Cl_3] is ma_3b_3 type complex therefore it shows two cis & trans isomers



42. Among the following, the π-acid ligand is-

(A) F (B) NH_3 (C) CN^- (D) I^-

Ans. **(C)**

Sol. CN^- accept electrons from metal ion in its vacant π^* ABMO.

43. The bond order in O_2^{2-} is-

(A) 2 (B) 3 (C) 1.5 (D) 1

Ans. **(D)**

Sol. Bond order of O_2^{2-}

Total electron = 18

Configuration = KK $\sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \pi^*(2p_x)^2\ \pi^*(2p_y)^2$

Bond order = $\frac{N_b - N_a}{2} = \frac{8-6}{2}$ = 1.0

44. The energy of a photon of wavelength k = 1 meter is (Planck's constant = 6.625×10^{-34} Js, speed of light = 3×10^8 m/s)

(A) 1.988×10^{-23} J (B) 1.988×10^{-28} J (C) 1.988×10^{-30} J (D) 1.988×10^{-25} J

Ans. **(A)**

Sol. $E = \frac{hc}{\lambda} = \frac{6.02\times10^{-34}\times3\times10^{8}}{1}$

$E = 1.988 \times 10^{-23}$ J

45. The concentration of a substance undergoing a chemical reaction becomes one-half of its original value after time t regardless of the initial concentration. The reaction is an example of a-

(A) zero order reaction (B) first order reaction

(C) second order reaction (D) third order reaction

Ans. **(B)**

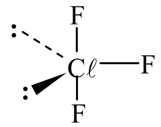
Sol. Informative

46. The shape of the molecule ClF_3 is-

(A) trigonal planar (B) pyramidal (C) T-shaped (D) Y-shaped

Ans. **(C)**

Sol. Three bond pair & two lone pair present in ClF_3 molecule.



T-shape

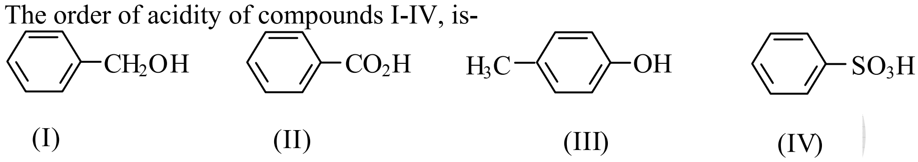
47. Friedel-Crafts acylation is-

(A) α-acylation of a carbonyl compound
(B) acylation of phenols to generate esters
(C) acylation of aliphatic olefins
(D) acylation of aromatic nucleus

Ans. **(D)**

Sol. Friedel craft reaction used for introducing an alkyl or acyl group in benzene nucleus by an alkylating or acylating agent in presence of a suitable catalyst.

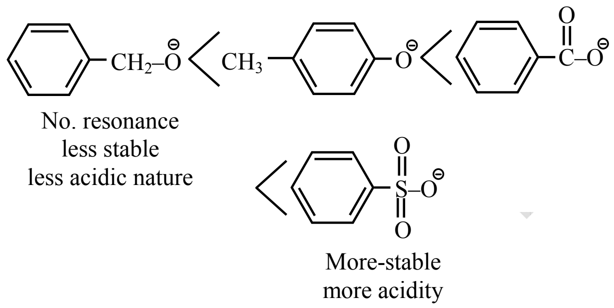
48. The order of acidity of compounds I-IV, is-



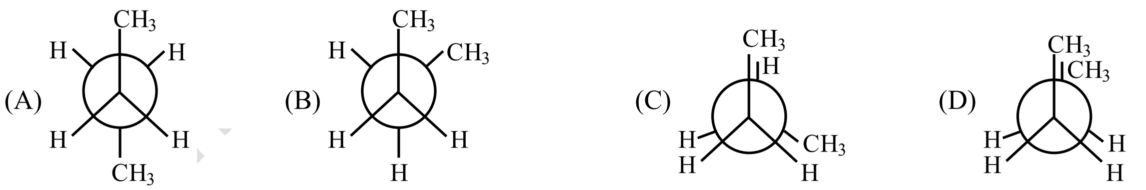
(A) I < III < II < IV
(B) IV < I < II < III
(C) III < I < II < IV
(D) II < IV < III < I

Ans. **(A)**

Sol.

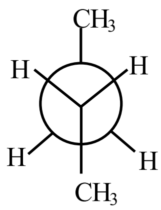


49. The most stable conformation for n-butane is-



Ans. **(A)**

Sol. Most stable conformer of n-butane is



because dihedral angle between CH_3 group is 180°

50. In the nuclear reaction ${}^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} + X$. X is-

(A) ${}^{0}_{-1}e$ (B) ${}^{0}_{1}e$ (C) H (D) ${}^{2}_{1}H$

Ans. **(A)**

Sol. ${}^{234}_{90}\text{Th} \longrightarrow {}^{234}_{91}\text{Pa} + {}_{-1}e^{0}$

51. A concentrated solution of copper sulphate, which is dark blue in colour, a mixed at room temperature with a dilute solution of copper sulphate, which is light blue. For this process-

(A) Entropy change is positive, but enthalpy change is negative

(B) Entropy and enthalpy changes are both positive

(C) Entropy change is positive and enthalpy does not change

(D) Entropy change is negative and enthalpy change is positive

Ans. **(C)**

Sol. Informative

52. Increasing the temperature increases the rate of reaction but does not increase the-

(A) number of collisions (B) activation energy

(C) average energy of collisions (D) average velocity of the reactant molecules

Ans. **(B)**

Sol. Informative

53. In metallic solids, the number of atoms for the face-centered and the body-centered cubic unit cells, are, respectively-

(A) 2, 4 (B) 2, 2 (C) 4, 2 (D) 4, 4

Ans. **(C)**

Sol. $\text{Fcc} = 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

$\text{Bcc} = 8 \times \frac{1}{8} + 1 \times 1 = 2$

54. From equations 1 and 2,

$CO_2 \rightleftharpoons CO + 1/2\, O_2$ [$K_{cl} = 9.1 \times 10^{-13}$ at 1000ºC] (eq. 1)

$H_2O \rightleftharpoons H_2 + 1/2\, O_2$ [$K_{cl} = 7.1 \times 10^{-12}$ at 1000ºC] (eq. 2)

the equilibrium constant for the reaction $CO_2 + H_2 \rightleftharpoons CO + H_2O$ at the same temperature, is-

(A) 0.78 (B) 2.0 (C) 16.2 (D) 1.28

Ans. **(D)**

Sol. (i) $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2 \quad K_1 = 9.1 \times 10^{-13}$

(ii) $H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2 \quad K_2 = 7.1 \times 10^{-12}$

Object $\quad CO_2 + H_2 \rightleftharpoons CO + H_2O$

equation (i) – (ii)

$\therefore\ K^1 = \dfrac{K_1}{K_2} = 1.28$

$C_t = C_0\, e^{-kt}$

$\therefore\ R = R_0\, e^{-kt}$

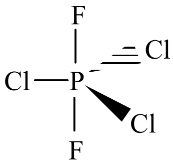
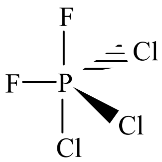
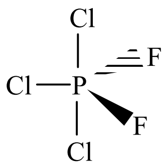
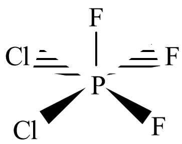
55. For a first order reaction $R \rightarrow P$, the rate constant is k. If the initial concentration of R is $[R_0]$, the concentration of R at any time 't' is given by the expression-

(A) $[R_0]\, e^{kt}$ (B) $[R_0](1 - e^{-kt}]$ (C) $[R_0]\, e^{-kt}$ (D) $[R_0]\, (1 - e^{kt})$

Ans. **(B)**

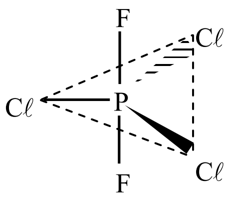
Sol.

56. The correct structure of PCl_3F_2 is-

(A)  (B)  (C)  (D) 

Ans. **(A)**

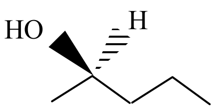
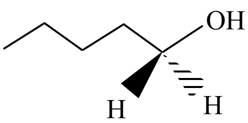
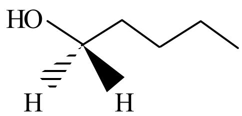
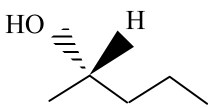
Sol. Correct structure of PCl_3F_2 is



Trigonal biprymidal

for minimum repulsion between atoms.

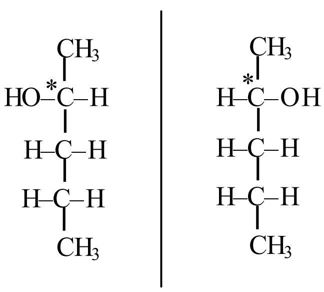
57. The enontiomeric pair among the following four structures-

I.  II.  III.  IV. 

(A) I & II (B) I & IV (C) II & III (D) II & IV

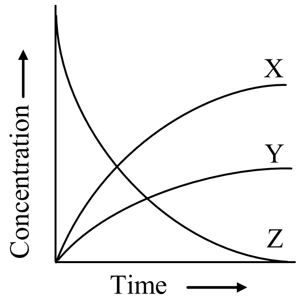
Ans. **(B)**

Sol.



Mirror image & not superimposable on each other. While chiral center absent in II and III.

58. Consider the reaction : $2\ NO_2(g) \rightarrow 2\ NO(g) + O_2\ (g)$. In the figure below, identify the curves X, Y and Z associated with the three species in the reaction-



(A) $X = NO, Y = O_2, Z = NO_2$ (B) $X = O_2, Y = NO, Z = NO_2$

(C) $X = NO_3, Y = NO, Z = O_2$ (D) $X = O_2, Y = NO_2, Z = NO$

Ans. **(A)**

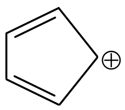
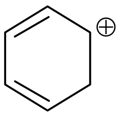
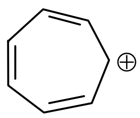
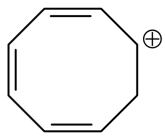
Sol. $$r = -\frac{1}{2}\frac{d[NO_2]}{dt} = +\frac{1}{2}\frac{d[NO]}{dt} = +\frac{d[O_2]}{dt}$$

$\therefore$ NO_2 is reactant so (Z)

rate of dis appearance of NO_2 = rate of formation of of NO

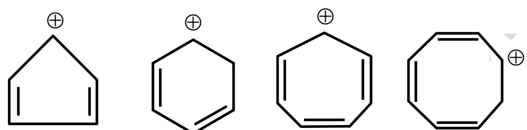
So NO is (X)

59. The aromatic carbocation among the following is-

(A)  (B)  (C)  (D) 

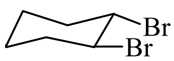
Ans. **(C)**

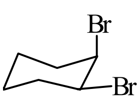
Sol.

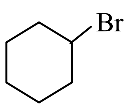


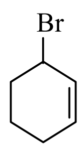
Not follow Huckel rule | sp^3 'C' present (nonplaner) | 6 π e^- & planer molecule (Aromatic) | sp^3 'C' present (non-planer)

60. Cyclohexene is reacted with bromine in CCl_4 in the dark. The product of the reaction is-

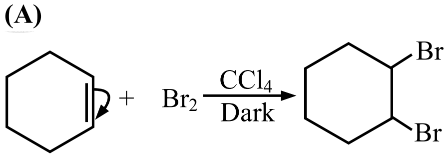
(A) 

(B) 

(C) 

(D) 

Ans. **(A)**

Sol. 

Br_2 molecule ionizes on interaction with π bond

$Br_2 \longrightarrow Br^+ + :Br^-$

Electrophile Nucleophile

BIOLOGY

61. Ribonucleic Acids (RNA) that catalyze enzymatic reactions are called ribozymes. Which one of the following acts as a ribozyme ?

(A) Ribosome (B) Amylase (C) tRNA (D) Riboflavin

Ans. **(A)**

62. In 1670, Robert Boyle conducted an experiment where in he placed a viper (a poisonous snake) in a chamber and rapidly reduced the pressure in that chamber. Which of the following would be true ?

(A) Gas bubbles developed in the tissues of the snake

(B) The basal metabolic rate of the snake increased tremendously

(C) The venom of the snake was found to decrease in potency

(D) The venom of the snake was found to increase in potency

Ans. **(A)**

63 Bacteria can survive by absorbing soluble nutrients via their outer body surface, but animals cannot, because

(A) Bacteria cannot ingest particles but animals can

(B) Bacteria have cell walls and animals do not

(C) Animals have too small a surface area per unit volume as compared to bacteria

(D) Animals cannot metabolize soluble nutrients

Ans. **(C)**

64. A horse has 64 chromosomes and a donkey has 62. Mules result from crossing a horse and a donkey. State which of the following is INCORRECT ?

(A) Mules can have either 64, 63 or 62 chromosomes

(B) Mules are infertile

(C) Mules have well defined gender (male/female)

(D) Mules have 63 chromosomes

Ans. **(A)**

65. If the total number of photons falling per unit area of a leaf per minute is kept constant, then which of the following will result in maximum photosynthesis ?

(A) Shining green light (B) Shining sunlight

(C) Shining blue light (D) Shining ultraviolet light

Ans. **(C)**

66. Path-finding by ants is by means of-

(A) Visually observing landmarks (B) Visually observing other ants

(C) Chemical signals between ants (D) Using the earth's magnetic field

Ans. **(C)**

67. Sometimes urea is fed to ruminants to improve their health. It works by-

(A) Helping growth of gut microbes that break down cellulose

(B) Killing harmful microorganisms in their gut

(C) Increasing salt content in the gut

(D) Directly stimulating blood cell proliferation

Ans. **(A)**

68. If you compare adults of two herbivore species of different sizes, but from the same geographical area, the amount of faeces produced per kg body weight would be-
(A) More in the smaller one than the larger one
(B) More in the larger one than the smaller one
(C) Roughly the same amount in both
(D) Not possible to predict which would be more

Ans. (A)

69. Fruit wrapped in paper ripens faster than when kept in open air because-
(A) Heat of respiration is retained better
(B) A chemical in the paper helps fruit ripening
(C) A volatile substance produced by the fruit is retained better and helps in ripening
(D) The fruit is cut off from the ambient oxygen which is an inhibitor to fruit repening

Ans. (C)

70. When a person is suffering from high fever, it is sometimes observed that the skin has a reddish tinge. Why does this happen ?
(A) Red colour of the skin radiates more heat
(B) Fever causes the release of a red pigment in the skin
(C) There is more blood circulation to the skin to keep the body warm
(D) There is more blood circulation to the skin to release heat from the body

Ans. (D)

71. Bacteriochlorophylls are photosynthetic pigments found in phototrophic bacteria. Their function is distinct from the plant chlorophylls in that they-
(A) do not produce oxygen
(B) do not conduct photosynthesis
(C) absorb only blue light
(D) function without a light source

Ans. (A)

72. Athletes often experience muscle cramps. Which of the following statements is true muscle cramps ?
(A) Muscle cramp is caused due to conversion of pyruvic acid into lactic acid in the cytoplasm
(B) Muscle cramp is caused due to conversion of pyruvic acid into lactic acid in the mitochondria
(C) Muscle cramp is caused due to nonconversion of glucose to pyruvate in the cytoplasm
(D) Muscle cramp is caused due to conversion of pyruvic acid into ethanol in the cytoplasm

Ans. (A)

73. A couple went to a doctor and reported that both of them are "carriers" for a particular disorder, their first child is suffering from that disorder and that they are expecting their second child. What is the probability that the new child would be affected by the same disorder ?
(A) 100 %
(B) 50 %
(C) 25 %
(D) 75 %

Ans. (C)

74. Of the following combinations of cell biological processes which one is associated with embryogenesis ?
(A) Mitosis and Meiosis
(B) Mitosis and Differentiation
(C) Meiosis and Differentiation
(D) Differentiation and Reprogramming

Ans. (B)

75. Conversion of the Bt protoxin produced by *Bacillus thuringiensis* to its active form in the gut of the insects is mediated by-

(A) acidic pH of the gut (B) alkaline pH of the gut

(C) lipid modification of the protein (D) cleavage by chymotrypsin

Ans. (B)

76. If you dip a sack full of paddy seeds in water overnight and then keep it out for a couple of days, it feels warm. What generates this heat ?

(A) Imbibation

(B) Exothermic reaction between water and seed coats

(C) Friction among seeds due to swelling

(D) Respiration

Ans. (D)

77. Restriction endonucleases are enzymes that cleave DNA molecules into smaller fragments. Which type of bond do they act on ?

(A) N-glycosidic Bond (B) Hydrogen bond

(C) Phosphodiester bond (D) Disulfide bond

Ans. (C)

78. The fluid part of blood flows in and out of capillaries in tissue to exchange nutrients and waste materials. Under which of the following conditions will fluid flow out from the capillaries into the surrounding tissue ?

(A) When arterial blood pressure exceeds blood osmotic pressure

(B) When arterial blood pressure is less than blood osmotic pressure

(C) When arterial blood pressure is equal to blood osmotic pressure

(D) Arterial blood pressure and blood osmotic pressure have nothing to do with the outflow of fluid from capilleries

Ans. (A)

79. The distance between two consecutive DNA base pairs is 0.34 nm. If the length of a chromosome is 1 mm, the number of base pairs in the chromosome is approximately-

(A) 3 million (B) 30 million (C) 1.5 million (D) 6 million

Ans. (A)

80. Estimate the order of the speed of propagation of an action potential or nerve impulse -

(A) nm/s (B) micron/s (C) cm/s (D) m/s

Ans. (D)

PART-2

Two-Marks Question

MATHEMATICS

81. Arrange the expansion of $\left(x^{1/2} + \frac{1}{2x^{1/4}}\right)^n$ in decreasing powers of x. Suppose the coefficient of the first three terms form an arithmetic progression. Then the number of terms in the expansion having integer powers of x is-

(A) 1 (B) 2 (C) 3 (D) more than 3

Ans. **(C)**

Sol. $T_{r+1} = {}^nC_n\,(x^{1/2})^{n-r}\,\frac{1}{(2x^{1/4})^r} = \frac{{}^nC_r}{2^r}\,x^{\frac{2n-3r}{4}}$

$T_1, T_2, T_3 \to AP$

$\frac{2\;{}^nC_1}{2} = {}^nC_0 + \frac{{}^nC_2}{2^2}$

$n - 1 = \frac{n(n-1)}{8} \Rightarrow n = 8$

$\frac{16-3r}{4}$ = Integers $\boxed{r = 0, 4, 8}$

82. Let r be a real number and $n \in N$ be such that the polynomial $2x^2 + 2x + 1$ divides the polynomial $(x + 1)^n - r$. Then (n, r) can be-

(A) $(4000, 4^{1000})$ (B $\left(4000, \frac{1}{4^{1000}}\right)$ (C) $\left(4^{1000}, \frac{1}{4^{1000}}\right)$ (D) $\left(4000, \frac{1}{4000}\right)$

Ans. **(B)**

Sol. $2x^2 + 2x + 1 = 0$

$x = \frac{-1+i}{2}, \frac{-1-i}{2}$

x satisfies $(x + 1)^n - r = 0$

$\left(\frac{-1 \pm i}{2} + 1\right)^n - r = 0$

$\left(\frac{1 \pm i}{2}\right)^n - r = 0$

$\left(\frac{1}{\sqrt{2}}\right)^n \left(\frac{1+i}{\sqrt{2}}\right)^n = r$

$\left(\frac{1}{\sqrt{2}}\right)^n \left(e^{\pm\frac{i\pi}{4}}\right)^n = r$

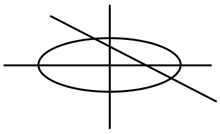
RHS = real

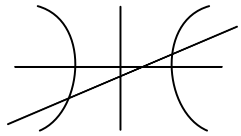
LHS = real only when n = multiply of 4

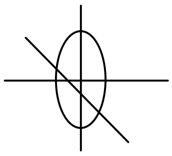
n = 4000

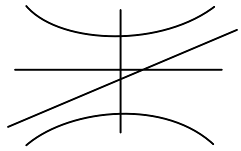
$r = \left(\frac{1}{\sqrt{2}}\right)^{4000} = \frac{1}{4^{1000}}$

83. Suppose a, b are real numbers such that $ab \neq 0$. Which of the following four figures represents the curve $(y - ax - b)(bx^2 + ay^2 - ab) = 0$?

(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol. $y = ax + b$ and $\frac{x^2}{a} + \frac{y^2}{b} = 1$

slope = a

for the line, y intercept = b

Fig.1 for line $\rightarrow a < 0 , b > 0$ hence the other fig. cannot be an ellipse

Fig.2 $a > 0 , b < 0$ hence the fig. is a hyperbola

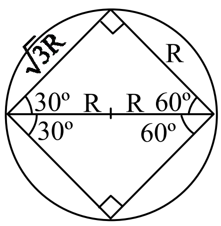
Similarly you can check rest 2 options

84. Among all cyclic quadrilaterals inscribed in a circle of radius R with one of its angles equal to 120º. Consider the one with maximum possible area. Its area is-

(A) $\sqrt{2}\ R^2$ (B) $\sqrt{3}\ R^2$ (C) $2\ R^2$ (D) $2\sqrt{3}\ R^2$

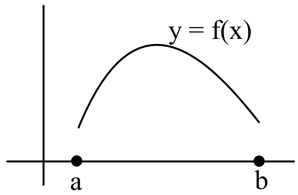
Ans. **(B)**

Sol.



$$A = 2 \times \frac{1}{2} \times \sqrt{3}\,R \times R = \sqrt{3}\,R^2$$

85. The following figure shows the graph of a differentiable function y = f(x) on the interval [a, b] (not containing 0).



Let g(x) = f(x) / x which of the following is a possible graph of y = g(x) ?

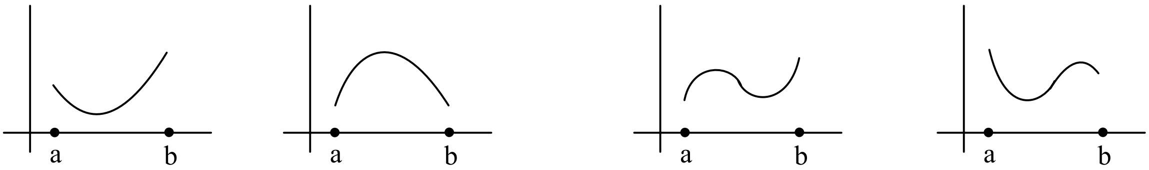
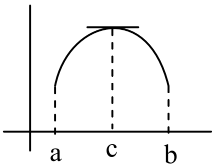


Fig.1 Fig.2 Fig.3 Fig.4

(A) Fig. 1 (B) Fig. 2 (C) Fig. 3 (D) Fig. 4

Ans. **(B)**

Sol.



$f'(c) = 0 \quad f'(c^-) > 0 \quad f'(c^+) < 0$

$g(x) = \dfrac{f(x)}{x} \quad g'(x) = \dfrac{xf'(x) - 1}{x^2}$

$g'(c^+) = \lim\limits_{h\to 0} \dfrac{(c+h)f'(c+h) - 1}{(c+h)^2} < 0 \qquad (f'(c+h) < 0)$

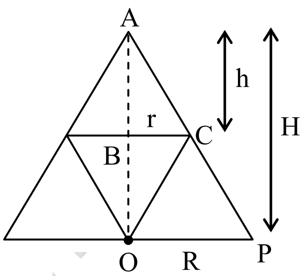
hence fig.(2)

86. Let V_1 be the volume of a given right circular cone with O as the centre of the base and A as its apex. Let V_2 be the maximum volume of the right circular cone inscribed in the given cone whose apex is O and whose base is parallel to the base of the given cone. Then the ratio V_2/V_1 is-

(A) $\dfrac{3}{25}$ (B) $\dfrac{4}{9}$ (C) $\dfrac{4}{27}$ (D) $\dfrac{8}{27}$

Ans. **(C)**

Sol.



ΔABC and ΔAOP are similar

$$\frac{h}{r} = \frac{H}{R} \quad \Rightarrow h = \frac{rH}{R}$$

$V_2 = \dfrac{1}{3}\pi r^2(H-h) = \dfrac{\pi}{3} r^2 H\left(1 - \dfrac{r}{R}\right) = \dfrac{\pi H}{3}\left(r^3 - \dfrac{r^3}{R}\right)$

$\dfrac{dV_2}{dr} = 2r - \dfrac{3r^2}{R} = 0 \qquad r = \dfrac{2R}{3}$

$V_{2max} = \dfrac{4\pi R^2 H}{81}$

$V_1 = \dfrac{\pi R^2 H}{3}$

$\therefore \dfrac{V_2}{V_1} = \dfrac{4}{27}$

87. Let $f : R \to R$ be a continuous function satisfying $f(x) = x + \int_0^x f(t)\,dt$, for all $x \in R$. Then the number of elements in the set $S = \{x \in R ; f(x) = 0\}$ is-

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(A)**

Sol. $f'(x) = 1 + f(x) \Rightarrow f(x) = e^x - 1$

$f(x) = 0 \Rightarrow e^x = 1$ $\boxed{x = 0}$ One solution

88. The value of $\int_0^{2\pi} \min\{|x - \pi|, \cos^{-1}(\cos x)\}\,dx$ is-

(A) $\dfrac{\pi^2}{4}$ (B) $\dfrac{\pi^2}{2}$ (C) $\dfrac{\pi^2}{8}$ (D) π^2

Ans. **(B)**

Sol. $I = \int_0^{\pi/2} x\,dx + \int_{\pi/2}^{\pi} (\pi - x)\,dx + \int_{\pi}^{3\pi/2} (x - \pi)\,dx + \int_{3\pi/2}^{2\pi} 2\pi - x\,dx$

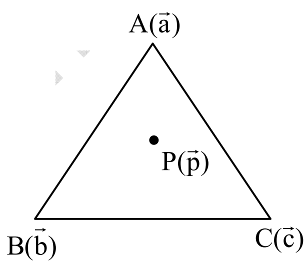
$\dfrac{\pi^2}{8} + \dfrac{\pi^2}{8} + \dfrac{\pi^2}{8} + \dfrac{\pi^2}{8} = \dfrac{\pi^2}{2}$

89. Let ABC be a triangle and P be a point inside ABC such that $\overrightarrow{PA} + 2\overrightarrow{PB} + 3\overrightarrow{PC} = \vec{0}$. The ratio of the area of triangle ABC to that of APC is-

(A) 2 (B) $\dfrac{3}{2}$ (C) $\dfrac{5}{3}$ (D) 3

Ans. **(D)**

Sol.



$\overrightarrow{PA} + 2\overrightarrow{PB} + 3\overrightarrow{PC} = 0$

$(\vec{a} - \vec{p}) + 2(\vec{b} - \vec{p}) + 3(\vec{c} - \vec{p}) = 0$

$\vec{p} = \dfrac{\vec{a} + 2\vec{b} + 3\vec{c}}{6}$

$$\frac{\text{Area } \Delta ABC}{\text{Area } \Delta APC} = \frac{\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{\frac{1}{2}|\vec{a}\times\vec{p}+\vec{p}\times\vec{c}+\vec{c}\times\vec{a}|}$$

put $\vec{p} = \dfrac{\vec{a}+2\vec{b}+3\vec{c}}{6}$

ratio = 3

90. Suppose m, n are positive integers such that $6^m + 2^{m+n}\ 3^w + 2^n = 332$. The value of the expression $m^2 + mn + n^2$ is-

(A) 7 (B) 13 (C) 19 (D) 21

Ans. **(C)**

Sol. $6^m + 2^{m+n}\quad 3^w + 2^n = 332$

maximum possible value of m is 3

checking for m = 3, 2 and 1

we get m = 2, n = 3, w = 2

$m^2 + mn + n^2 = 4 + 6 + 9 = 19$

PHYSICS

91. A ball is dropped vertically from a height of *h* onto a hard surface. If the ball rebounds from the surface with a fraction *r* of the speed with which it strikes the latter on each impact, what is the net distance traveled by the ball up to the 10^{th} impact ?

(A) $2h\dfrac{1-r^{10}}{1-r}$ (B) $h\dfrac{1-r^{20}}{1-r^2}$ (C) $2h\dfrac{1-r^{22}}{1-r^2}-h$ (D) $2h\dfrac{1-r^{20}}{1-r^2}-h$

Ans. **(D)**

Sol. Total distance = $\left(\dfrac{v_0^2}{g}+r^2\dfrac{v_0^2}{g}+r^4\dfrac{v_0^2}{g}+\text{...upto } 10^{th} \text{ terms}\right)-h = \dfrac{v_0^2}{g}(1+r^2+r^4+....+10^{th}\text{ term})-h$

also $v_0 = \sqrt{2gh}$

$\therefore$ Total distance = $2h\left(\dfrac{1-(r^2)^{10}}{1-r^2}\right)-h$

or $\boxed{\text{total distance} = \dfrac{2h(1-r^{20})}{(1-r^2)}-h}$

92. A certain planet completes one rotation about its axis in time T. The weight of an object placed at the equator on the planet's surface is a fraction f (f is close to unity) of its weight recorded at a latitude of 60º. The density of the planet (assumed to be a uniform perfect sphere is given by-

(A) $\dfrac{4-f}{1-f}\dfrac{3\pi}{4GT^2}$ (B) $\dfrac{4-f}{1+f}\dfrac{3\pi}{4GT^2}$ (C) $\dfrac{4-3f}{1-f}\dfrac{3\pi}{4GT^2}$ (D) $\dfrac{4-2f}{1-f}\dfrac{3\pi}{4GT^2}$

Ans. **(A)**

Sol. $v = \sqrt{\frac{GM}{r}}$ also $T = \frac{2\pi r}{v}$ or $T = 2\pi\sqrt{\frac{r^3}{GM}}$ when v is replaced by $\sqrt{\frac{GM}{r}}$

now $g_{eff} = g - \omega^2 R_e \cos^2\phi$

now $f = \frac{g - \omega^2 R_e \cos 0^o}{g - \omega^2 R_e \cos 60^o}$

solving we get $R_e = \frac{4g(f-1)}{\omega^2(f-4)}$ or $\frac{GM}{R_e^2}(f-1) = \frac{\omega^2 R_e}{4}(f-4)$

$R_e^3 = \frac{4GM}{\omega^2}\frac{(f-1)}{(f-4)}$

now $\rho = \frac{M}{\frac{4}{3}\pi R_e^3} = \frac{3}{16}\frac{\omega^2}{\pi G}\frac{(f-4)}{(f-1)}$

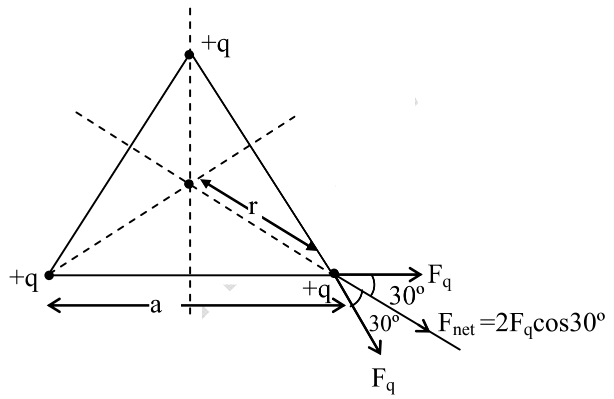
also $T = \frac{2\pi}{\omega}$ $\therefore$ $\boxed{\rho = \frac{3\pi(f-4)}{4T^2G(f-1)}}$

93. Three equal charges +q are placed at the three vertices of an equilateral triangle centered at the origin. They are held in equilibrium by a restoring force of magnitude F(r) = kr directed towards the origin, where k is a constant. What is the distance of the three charges from the origin ?

(A) $\left[\frac{1}{6\pi\varepsilon_0}\frac{q^2}{k}\right]^{1/2}$ (B) $\left[\frac{\sqrt{3}}{12\pi\varepsilon_0}\frac{q^2}{k}\right]^{1/3}$ (C) $\left[\frac{1}{6\pi\varepsilon_0}\frac{q^2}{k}\right]^{2/3}$ (D) $\left[\frac{\sqrt{3}}{4\pi\varepsilon_0}\frac{q^2}{k}\right]^{2/3}$

Ans. **(B)**

Sol.



F(r) = kr

now F_{net} on a particle is $2F_q \cos 30^o$ due to the other two charges

$F_{net} = \frac{2kq^2}{a^2} \times \frac{\sqrt{3}}{2}$

also $r = \frac{2}{3}\left(\frac{\sqrt{3}}{2}a\right)$

$\therefore$ $a = \sqrt{3}\,r$ replacing it in F_{net} we get

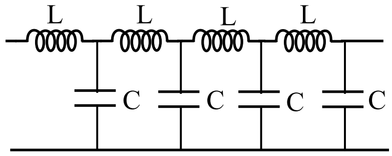
$$F_{net} = \frac{2kq^2}{(\sqrt{3}r)^2} \times \left(\frac{\sqrt{3}}{2}\right) = \frac{kq^2}{\sqrt{3}\,r^2}$$

this is balanced by F(r)

$$\therefore\ F(r) = F_{net} \Rightarrow kr = \frac{1 \times q^2}{4\pi\varepsilon_0 \times \sqrt{3}\,r^2}$$

$$\therefore\ r = \left(\frac{\sqrt{3}\ q^2}{12\pi\varepsilon_0 k}\right)^{1/3}$$

94. Consider the infinite ladder circuit shown below.



For which angular frequency ω will the circuit behave like a pure inductance ?

(A) $\frac{LC}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{LC}}$ (C) $\frac{2}{\sqrt{LC}}$ (D) $\frac{2L}{\sqrt{C}}$

Ans. **(C)**

Sol. Let the equivalent impedance of the circuit be Z

So $Z = \omega L + Z'$

now $Z' = \frac{ZX_C}{Z + X_C}$

$$\therefore\ Z = \omega L + \left(\frac{Z \times \frac{1}{\omega C}}{Z + \frac{1}{\omega C}}\right)$$

on solving we get $Z = \frac{\omega LC \pm \sqrt{(\omega LC)^2 - 4LC}}{2C}$

for Z to be purely inductive

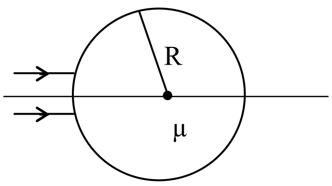
$\omega^2L^2C^2 - 4LC = 0$ or $\boxed{\omega = \frac{2}{\sqrt{LC}}}$

95. A narrow parallel beam of light falls on a glass sphere of radius R and refractive index μ at normal incidence. The distance of the image from the outer edge is given by-

(A) $\frac{R(2-\mu)}{2(\mu-1)}$ (B) $\frac{R(2+\mu)}{2(\mu-1)}$ (C) $\frac{R(2-\mu)}{2(\mu+1)}$ (D) $\frac{R(2+\mu)}{2(\mu+1)}$

Ans. **(A)**

Sol.



now $\frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R} \Rightarrow V_1 = \frac{\mu R}{\mu - 1}$

now $\frac{1}{V_f} - \frac{\mu}{-(2R - V_1)} = \frac{1-\mu}{-R}$

replace V_1 by $\frac{\mu R}{\mu - 1}$ and solving for V_f

we get $V_f = \frac{R(\mu - 2)}{2(\mu - 1)}$

First image is real and second is virtual.

96. A particle of mass m undergoes oscillations about x = 0 in a potential given by $V(x) = \frac{1}{2}kx^2 - V_0\cos\left(\frac{x}{a}\right)$, where V_0, k, a are constants. If the amplitude of oscillation is much smaller than a, the time period is given by-

(A) $2\pi\sqrt{\frac{ma^2}{ka^2 + V_0}}$ (B) $2\pi\sqrt{\frac{m}{k}}$ (C) $2\pi\sqrt{\frac{ma^2}{V_0}}$ (D) $2\pi\sqrt{\frac{ma^2}{ka^2 - V_0}}$

Ans. **(A)**

Sol. $V(x) = \frac{1}{2}kx^2 - V_0\cos\left(\frac{x}{a}\right)$

$E = -\frac{dV}{dx} = -(kx + V_0 \sin\left(\frac{x}{a}\right) \times \frac{1}{a})$

since x < < a

$\therefore \sin\left(\frac{x}{a}\right) \simeq \frac{x}{a}$ or $E = -\left(k + \frac{V_0}{a^2}\right)x$

This resembles $F = -kx$

$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{ma^2}{ka^2 + v_0}}$

$$\boxed{T = 2\pi\sqrt{\frac{m.a^2}{ka^2 + v_0}}}$$

97. An ideal gas with heat capacity at constant volume C_V undergoes a quasistatic process described by PV^α in a P-V diagram, where α is a constant. The heat capacity of the gas during this process is given by-

(A) C_V (B) $C_V + nR$

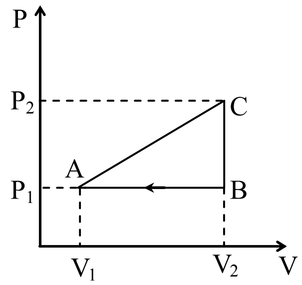
(C) $C_V + \frac{nR}{1-\alpha}$ (D) $C_V + \frac{nR}{1-\alpha^2}$

Ans. **(C)**

Sol. Direct formula is to be used

$$\boxed{C = C_V + \frac{nR}{1-\alpha}}$$

98. An ideal gas with constant heat capacity $C_V = \frac{3}{2}nR$ is made to carry out a cycle that is depicted by a triangle in the figure given below.



The following statement is true about the cycle-

(A) The efficiency is given $1 - \frac{P_1V_1}{P_2V_2}$

(B) The efficiency is given by $1 - \frac{1}{2}\frac{P_1V_1}{P_2V_2}$

(C) Net heat absorbed in the cycle is $(P_2 - P_1)(V_2 - V_1)$

(D) Heat absorbed in part AC is given by $2(P_2V_2 - P_1V_1) + \frac{1}{2}(P_1V_2 - P_2V_1)$

Ans. **(B)**

Sol. $C_V = \frac{3}{2}R,\ C_P = C_V + R = \frac{5R}{2}$

$f = 3$

$W = \frac{1}{2}(V_2 - V_1)(P_2 - P_1)$

For BA $\quad Q = nC_P\Delta T = n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}(P_1V_1 - P_2V_2)$

For AC $\quad Q_{AC} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + nC_V\Delta T$

now $nC_V\Delta T = \frac{3}{2}(P_2V_2 - P_1V_1)$

now $\eta = \frac{W}{Q} = \dfrac{\frac{1}{2}(V_2 - V_1)(P_2 - P_1)}{\frac{1}{2}\times(P_1 + P_2)(V_2 - V_1) + \frac{3}{2}(P_2V_2 - P_1V_1)}$

using formula for heat we can calculate heat absorbed in AC.

99. Two identical particles of mass 'm' and charge q are shot at each other from a very great distance with an initial speed v. The distance of closest approach of these charges is-

(A) $\frac{q^2}{8\pi\varepsilon_0 mv^2}$ (B) $\frac{q^2}{4\pi\varepsilon_0 mv^2}$ (C) $\frac{q^2}{2\pi\varepsilon_0 mv^2}$ (D) 0

Ans. **(B)**

Sol. Using law of conservation of mechanical energy

Initial K.E. = Final P.E.

$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{kq^2}{r} \quad \therefore\ r = \frac{q^2}{4\pi\varepsilon_0 mv^2}$

100. At time t = 0, a container has N_0 radioactive atoms with a decay constant λ. In addition, c numbers of atoms of the same type are being added to the container per unit time. How many atoms of this type are there at t = T ?

(A) $\frac{c}{\lambda}\exp(-\lambda T) - N_0 \exp(-\lambda T)$

(B) $\frac{c}{\lambda}\exp(-\lambda T) + N_0 \exp(-\lambda T)$

(C) $\frac{c}{\lambda}\{1 - \exp(-\lambda T)\} + N_0 \exp(-\lambda T)$

(D) $\frac{c}{\lambda}\{1 + \exp(-\lambda T)\} + N_0 \exp(-\lambda T)$

Ans. **(C)**

Sol. N_0 – initial nucleon

at t = 0, N_0

Addition is at a constant rate

$$(\lambda N - C) = -\frac{dN}{dt}$$

$$\int_0^k dt = \int_{N_0}^{N} \frac{dN}{\lambda N - C}$$

Integrating we get

$$N = \frac{C}{\lambda} + \frac{e^{-\lambda t}}{\lambda}(\lambda N_0 - C)$$

$$\therefore \boxed{N = \frac{C}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t}}$$

CHEMISTRY

101. 2.52 g of oxalic acid *dehydrate* was dissolved in 100 ml of water, 10 mL of this solution was diluted to 500 mL. The normality of the final solution and the amount of oxalic acid (mg/mL) in the solution are respectively-

(A) 0.16 N, 5.04 (B) 0.08 N, 3.60 (C) 0.04 N, 3.60 (D) 0.02 N, 10.08

Ans. **(C)**

Sol. Initial Normality $N = \frac{2.52 \times 1000}{63 \times 100} = 0.4$

$$\begin{matrix} COOH \\ | \\ COOH \end{matrix} \cdot 2\,H_2O$$

$\therefore N_1V_1 = N_2V_2$

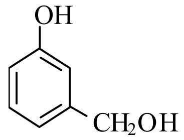
$0.4 \times 10 = N_2 \times 500$

$N_2 = \frac{0.4}{50} = 0.08$ N

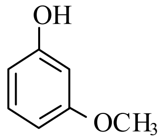
Then final weight $N = \frac{w \times 1000}{E \times V_{ml}}$

$$0.08 = \frac{w \times 1000}{63 \times 500}$$

102. Two isomeric compounds I and II are heated with HBr –

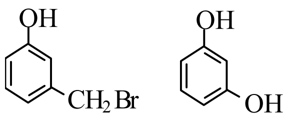


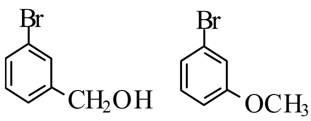
(I)

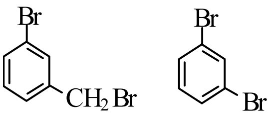


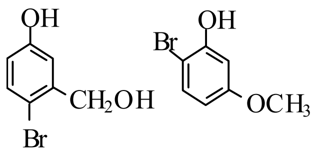
(II)

The products obtained are-

(A) 

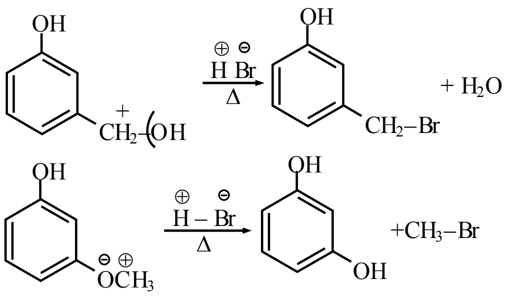
(B) 

(C) 

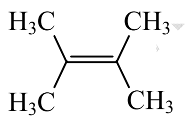
(D) 

Ans. **(A)**

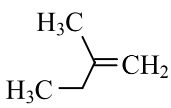
Sol.



103. The number of possible enatiomeric pair(s) produced from the bromination of I and II, respectively, are



I

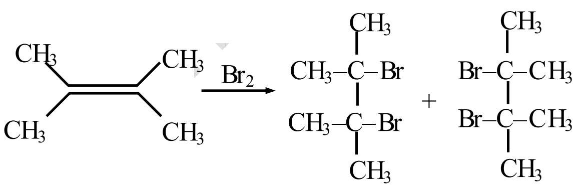


II

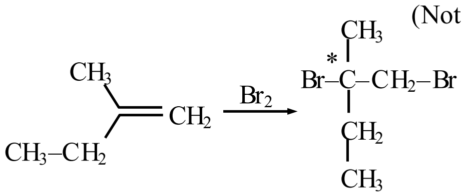
(A) 0, 1 (B) 1, 0 (C) 0, 2 (D) 1, 1

Ans. **(A)**

Sol.



Meso
(Not enantiomer's)



chiral center present

104. For the reaction A $\rightarrow$ B, ΔH^o = 7.5 mol^{-1} and ΔS^o = 2.5 J mol^{-1}. The value of ΔG^o and the temperature at which the reaction reaches equilibrium are, respectively,

(A) 0 kJ mol^{-1} and 400 K
(B) –2.5 kJ mol^{-1} and 400 K
(C) 2.5 kJ mol^{-1} and 200 K
(D) 0 kJ mol^{-1} and 300 K

Ans. **(D)**

Sol. At equation $\Delta G^\circ = 0$

$$\therefore T = \frac{\Delta H}{\Delta S} = \frac{7.5 \times 1000}{25} = 300 \text{ K}$$

105. The solubility product of $Mg(OH)_2$ is 1.0×10^{-12}. Concentrated aqueous NaOH solution is added to a 0.01 M aqueous solution of $MgCl_2$. The pH at which precipitation occur is-

(A) 7.2
(B) 7.8
(C) 8.0
(D) 9.0

Ans. **(D)**

Sol. $MgCl_2 \longrightarrow Mg^{+2} + 2Cl^-$

?

0.01 M

$K_{sp} = Q = [Mg^{+2}]\,[OH^-]^2$

$10^{-12} = [0.01]\,[OH^-]^2$

$[OH^-]^2 = 10^{-10}$

$[OH^-] = 10^{-5}$

pOH = 5 $\therefore$ pH = 9

106. A metal with an atomic radius of 141.4 pm crystallizes in the face centred cubic structure. The volume of the unit cell in pm is-

(A) 2.74×10^7
(B) 2.19×10^7
(C) 6.40×10^7
(D) 9.20×10^7

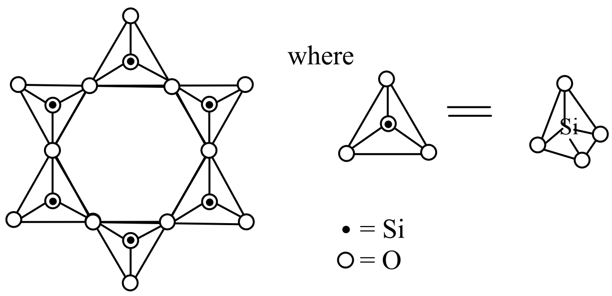
Ans. **(C)**

Sol. $r = \frac{a}{2\sqrt{2}} = 141.4$ pm

$a = 2 \times \sqrt{2} \times 141.4$

$\therefore V = a^3 = (2 \times \sqrt{2} \times 141.4)^3$

107. Identify the cyclic silicate ion given in the figure below



(A) $[Si_4O_{25}]^{24-}$
(B) $[Si_6O_{18}]^{18-}$
(C) $[Si_4O_{12}]^{12}$
(D) $[Si_6O_{24}]^{12-}$

Ans. **(B)**

Sol. Cyclic or ring silicates have general formula

$(Si{\cdot}O_3^{2-})_n$ or $(Si{\cdot}O_3)_n^{2n-}$

$\downarrow$ $\downarrow$

$Si_3O_9^{6-}$ $Si_6O_{18}^{12-}$

108. Diborane is formed the elements as shown in equation (1)

$2B(s) + 3H_2(g) \rightarrow B_2H_6\ (g)$ ....(1)

Given that

$H_2O(l) \rightarrow H_2O\ (g)$ $\Delta H_1^o = 44$ kJ

$2B(s) + 3/2O_2(g) \rightarrow B_2O_3(s)$ $\Delta H_2^o = -1273$ kJ

$B_2H_6(g) + 3O_2(g) \rightarrow B_2O_3(s) + 3H_2O\ (g)$ $\Delta H_3^o = -2035$ kJ

$H_2(g) + 1/2\ O_2(g) \rightarrow H_2O(l)$ $\Delta H_4^o = -286$ kJ

the ΔH^o for the reaction (1) is-

(A) 36 kJ (B) 509 kJ (C) 520 kJ (D) –3550 kJ

Ans. **(A)**

Sol. (1) $\underset{(\ell)}{H_2O} \longrightarrow \underset{(g)}{H_2O}$ $\Delta H = 44$ kJ

(2) $2B + \frac{1}{3}O_2 \rightarrow B_2O_3$ $\Delta H = -1273$ kJ

(3) $B_2H_6 + 3O_2 \rightarrow B_2O_3 + \underset{(g)}{3H_2O}$

$\Delta H = -2035$ kJ

(4) $H_2 + 1/2\ O_2 \rightarrow \underset{(\ell)}{H_2O}$ $\Delta H = -286$ kJ

equation (2) + (4) × 3 + (1) × 3 – (3)

$\Delta H = (-1273) + (-286) \times 3 + (44) \times 3 - (-2035) = 36$ kJ

109. The Crystal Field Stabilization Energy (CPSE) and the spin-only magnetic moment in Bohr Magneton (BM) for the complex $K_3[Fe(CN)_6]$ are, respectively-

(A) $0.0\ \Delta_s$ and $\sqrt{35}$ BM

(B) $-2.0\ \Delta_s$ and $\sqrt{3}$ BM

(C) $-0.4\ \Delta_s$ and $\sqrt{24}$ BM

(D) $-2.4\ \Delta_s$ and 0 BM

Ans. **(B)**

Sol. Informative

110. A solution containing 8.0 g of nicotine in 92 g of water freezes 0.925 degrees below the normal freezing point of water. If the molal freezing point depression constant $K_f = -1.85$°C mol^{-1} then the molar mass of nicotine is-

(A) 16 (B) 80 (C) 320 (D) 160

Ans. **(D)**

BIOLOGY

111. A bust cell has intracellular bacteria symbionts. If the growth rate of the bacterial symbiont is always 10% higher than that of the host cell, after 10 generations of the host cell the density of bacteria in host cells will increase -

(A) by 10 % (B) two-fold (C) ten-fold (D) hundred-fold

Ans. **(B)**

112. In a diploid organism, there are three different alleles for a particular gene. Of these three alleles one is recessive and the other two alleles exhibit co-dominance. How many phenotypes are possible with this set of alleles ?

(A) 3 (B) 6 (C) 4 (D) 2

Ans. **(C)**

113. Two students are given two different double stranded DNA molecules of equal length. They are asked to denature the DNA molecules by heating. The DNA given to student A has the following composition of bases (A:G:T:C:35:15:35:15) while that given to student B is (A:G:T:C::12:38:12:38). Which of the following statements is true ?

(A) Both the DNA molecules would denature at the same rate

(B) The information given is insufficient to draw any conclusion

(C) DNA molecule given to student B would denature faster than that of student A

(D) DNA molecule given to student A would denature faster than that given to student B

Ans. **(D)**

114. The amino acid sequences of a bacterial protein and a human protein carrying out similar function are found to be 60% identical. However the DNA sequences of the genes coding for these proteins are only 45% identical. This is possible because-

(A) Protein sequence does not depend on DNA sequence

(B) DNA codons having different nucleotides in the third position can code for the same amino acids

(C) DNA codons having different nucleotides in the second position can code for the same amino acids

(D) Same DNA codons can code for multiple amino acids

Ans. **(B)**

115. The following DNA sequence (5' $\rightarrow$ 3') specifies part of a protein coding sequence, starting from position I. Which of the following mutations will give rise to a protein that is shorter than the full-length protein ?

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	T	G	C	A	A	G	A	T	A	T	A	G	C	T

(A) Deletion of nucleotide 13

(B) Deletion of nucleotide 8

(C) Insertion of a single nucleotide between 3 and 4

(D) Insertion of a single nucleotide between 10 and 11

Ans. **(B)**

116. Which of the following correctly represents the results of an enzymatic reaction ? Enzyme is E, substrate is S and products are P1 & P2.

(A) P1 + S $\Leftrightarrow$ P2 + E (B) E + S $\Leftrightarrow$ P1 + P2

(C) P1 + P2 + E $\Leftrightarrow$ S (D) E + S $\Leftrightarrow$ P1 + P2 + E

Ans. **(D)**

117. Four species of birds have different egg colors : [1] white with no markings, [2] pale brown with no markings. [3] grey-brown with dark streaks and spots, [4] pale blue with dark blue-green spots. Based on egg color, which species is most likely to nest in a deep tree hole ?

(A) 1 (B) 2 (C) 3 (D) 4

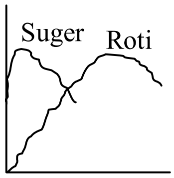
Ans. **(A)**

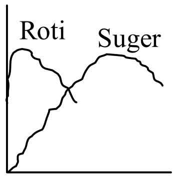
118. Consider a locus with two alleles, A and a. If the frequency of AA is 0.25, what is the frequency of A under Hardy-Weinberg equilibrium ?

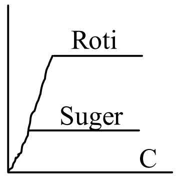
(A) 1 (B) 0.25 (C) 0.5 (D) 0

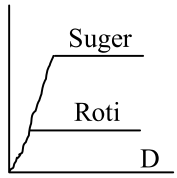
Ans. **(C)**

119. Which of the following graphs accurately represents the insulin levels (Y-axis) in the body as a function of time (X-axis) after eating sugar and bread/roti ?

(A) 

(B) 

(C) 

(D) 

Ans. **(A)**

120. You marked two ink-spots along the height at the base of a coconut tree and also at the top of the tree. When you examine the spots next year when the tree has grown taller, your will see-

(A) the two spots at the top have grown more apart than the two spots at the bottom

(B) the top two spots have grown less apart then the bottom two spots

(C) both sets of spots have grown apart to the same extent

(D) both sets of spots remain un-altered

Ans. **(A)**