APPLICATION OF DERIVATIVES

Marks	2	3	Total Marks
No. of Questions	1(A)	2(A+A)	8

2 MARK QUESTIONS

(Application)

1. A particle moves a distance $S = 6t^2 - t^3 + 5$ then find the velocity and acceleration after three seconds.
2. A particle moves a distance $S = 6t^2 - t^3 + 5$ then find the time at which it is at rest.
3. The distance s travelled by a particle in time t seconds is given by $S = t^3 - 6t^2 + 15t + 2$, find when the acceleration is zero.
4. According to law of motion of a particle if $s = t^3 - 6t^2 + 9t + 8$, find its initial velocity.
5. If the displacement 's' at any time 't' is given by $s = \sqrt{1-t}$. Show that the velocity is inversely proportional to the displacement.
6. The displacement 's' of a particle at time 't' is given by $s = 2t^3 - 5t^2 + 4t - 3$, find the time when acceleration is 14 ft/sec^2.
7. The displacement 's' of a particle at time 't' is given by $s = 2t^3 - 5t^2 + 4$. Find the velocity and displacement at that time.
8. When brakes are applied to a moving car, the car travels a distance 's' feet in 't' sec given by $s = 20t - 40t^2$ when and where does the car stop
9. A particle moves according to the laws $s = t^3 + at^2 + bt$. Find a and b if initial velocity is 5 unit and when t = 1 sec it is moving with a velocity which is 4 times its initial velocity.
10. A particle shot vertically upward rises 's' feet in 't' sec where $s = 40t - 16t^2$. Find the greatest height attained by the particle.
11. The displacement 's' of a particle at time 't' is given by $s = 4t^3 - 6t^2 + t - 7$. Find the velocity and acceleration when t is 2 sec.
12. If $s = 5t^2 + 4t - 8$ find the initial velocity and acceleration.

13. A stone thrown vertically upward rises 's' feet in 't' sec where $s = 80t - 16t^2$. What will be the velocity and acceleration after 2 seconds.

14. A body is thrown vertically upwards, its distance s feet in t sec is given by $s = 5 + 12t - t^2$. Find the greatest height reached by the body.

15. If $s = at^3 + bt$, find a and b given that when $t = 3$ velocity is zero and acceleration is 14 units.

16. When the brakes are applied to a moving car, the car travels a distance 's' feet in 't' sec given by $s = 8t - 6t^2$ when does the car stop?

17. If the cost function of a firm is given by $C(x) = x^3 - 3x + 7$. Find the average cost and marginal cost when the output $x = 6$ units.

18. The total cost function is given by $q^3 - 3q^2 + 15q + 27$. Find the average cost and the marginal cost.

19. If the total cost function $C = 9q - 3q^2 + \frac{q^3}{3}$ find the level of output at which average cost is minimized.

20. The total cost of a commodity is given by $C = x^2 - 7x + 2$ where x is the number of units and price/unit is ₹ 5.00. Find the profit function.

21. Find the average and marginal cost if the total cost function of an article is given by $C = 5x^2 + 2x + 3$.

22. The total cost of a commodity is given by $C = -x^2 + 5x + 7$ and the price per unit is ₹ 12. Find the profit function.

23. The sum of 2 natural numbers is 48. Find them if their product is maximum.

24. Product of 2 natural numbers is 64. Find the numbers when their sum is minimum.

25. For the demand function $2x - 5y = 7$ (x = number of units, y is price/unit) find the total revenue, manginal revenue and aveage revenue.

26. The total cost C for output Q is given by $C = 300Q - 10Q^2 + \frac{Q^3}{3}$ find the output level at which manginal cost is minimum.

27. If $R = 250x + 45x^2 - x^3$ what will be the manginal revenue if $x = 25$ unit and what is average revenue when $x = 10$ unit.

3 MARKS QUESTIONS

(Application based)

1. The radius of a circular plate increases at the rate $\frac{8}{3\pi}$ cm/sec. Find the

   (a) rate of change in the area when diameter is 12 cm.

   (b) rate of increase of circumference of circle after 3 sec.

3. The volume of a sphere is increasing at the rate of 4π c.c./sec. Find the rate at which the area of its surface increases when its radius is 10 cm.

4. A spherical balloon is being inflated so that its volume is increasing at the rate of 30 c.c./min. How fast its surface area increasing when its volume is 36π c.c.

5. A square plate is expanding uniformly, the side is increasing at the rate of 5 cm/sec. What is the rate at which the area and its perimeter is increasing when the side is 20 cm long.

6. The sides of an equilateral triangle are increasing at the rate of 3 cm/sec. How fast is its area increasing when the side is 10 cm.

8. The edge of a variable cube is increasing at the rate of 6 cm/min. How fast is the volume and its surface area increasing when the edge is 10 cm long.

10. A laddir 17 feet long leaus against a smooth vertical wall. If the lower end is moving at the rate of 12 ft/min. Find the rate at which the upper end is moving when the lower end is 8ft from the wall.

12. The radius of a sphere is increasing at the rate of 0.5 mt/sec. Find the rate of increase of its surface area and volume after 3 sec.

13. The surface area of a spherical bubble is increasing at the rate of 0.8 cm^2/sec find at what rate is its volume increasing when $r = 2.5$cm.

14. A spherical balloon is being inflated at the rate of 35cc/sec. Find the rate at which the surface area of the balloon increases when its diameter is 14 cm.

15. The radius of circular plate is increasing at the rate of $\frac{2}{3\pi}$ cm/sec find the rate of change of its area when the radius is 6 cm.

16. A circular patch of oil spreads a water the area growing at the rate of 16m^2/min. How fast are radius and circumference increasing when the diameter is 12cm.

19. Water is being poured at the rate of 30mt^3/min into a cylindical vessel whose base is a circle of radius 3 mt. Find the rate at which the level of water is rising.

20. Sand is being dropperd at the rate of 10mt^3/sec into a conical pile. If the height of pile is twice the radius of the base, at what rate is the height of the pile increasing when the sand in the pile is 8 mt high.

23. A man 6 ft tall is moving directly away from a lamp post of height 10 ft above the ground. It he is moving at the rate of 3 ft/sec. Find the rate at which the length of his shadow is increasing and also tip of shadow is moving.
25. The volume of a spherical ball is increasing at the rate of 4π c.c/sec. find the rate of increase of the radius of the ball when the volume is 288π cc.
26. A drop of ink spreads over a blotting paper so that the circumference of the blot is 4 π cm and it changes at 3cm/sec. Find the rate of increase of its radius and also find rate of increase of its area.
27. A circular plate of metal is heated so that its radius increases at the rate of 0.1mm/min. At what rate is the plates area increasing when the radius is 25 cm (1cm=10m).
29. Find the minimum value of $x^2 + \frac{250}{x}$
30. Find maxmum and minimum value of $x^3 - 9x^2 + 15x - 1$
32. Find maximum and minimum value of the function $f(x) = 3x^3 - 9x^2 - 27x + 30$
34. Find the maximum and minimum value of $f(x) = x^5 - 5x^4 + 5x^3 - 1$
35. Show that $x^3 - 6x^2 + 12x - 3$ has neither a maximum nor a minimum at $x = 2$.
36. Divide the number 40 into two parts such that their product is maximum.
37. The product of two natural number is 144, find the numbers if their sum is minimum.
38. Show that $f(x) = 4x^5 - 25x^4 + 40x^3 - 3$ has point of inflexion at $x = 0$.
39. Divide 64 into two parts such that the sum of the cubes of two parts in minimum.
40. Find the maximum and minimum of the function $f(x) = x^3 - 12x^2 + 36x - 4$
41. Find the maximum and minimum value of the function $f(x) = 2x^3 - 3x$
42. Find the maximum and minimum value of the function $f(x) = x^3 - 6x^2 + 9x + 15 \ \ (0 \le x \le 6)$
43. Find the maximum and minimum value of the function $f(x) = x^4 - 62x^2 + 120x + 9$
44. Find the maximum and minimum value of the function $f(x) = 2x^3 - 3x^2 - 312x + 12$
45. Find the maximum and minimum value of the function $f(x) = 2x^3 - 3x^2 - 36x + 10$
46. Find the maximum and minimum value of the function $f(x) = 9x^3 + 12x + 2$

47. $f(x) = 2x^3 - 15x^2 + 36x + 10$

48. $f(x) = 2x^3 - 21x^2 + 36x - 20$

49. $f(x) = 12x^5 - 45x^2 + 40x^3 + 6$

50. Find two positive number whose sum is 14 and sum of the squares is minimum.

51. Find two positive numbers whose sum is 30 and sum of their cubes is minimum.

4 MARKS QUESTIONS

(Application)

1. The total revenue function is given by R = 400 x – 2x^2 and total cost function is given by C = 2x^2 + 40x + 4000 find

   (a) Manginal revenue and marginal cost function.

   (b) Average revenue a average cost.

   (c) Output at which marginal revenue = marginal cost.

2. If the demand function is given by p = 50 – 2 q (p = price, Q = output) find the value of p and q at which revenue is maximum and revenue corresponding to these values.

3. The cost C of manufacturing an article is given by $C = 5 + \frac{48}{q} + 3q^2$ find the number of articles produced at minimum cost and also find the minimum cost.

4. Let the demand function an article be p = 75 – 2x and the cost function be $c(x) = 350 + 12x + \frac{x^2}{4}$ find the number of units and the price at which the total profit is maximum.

5. A manufacture gives the total cost $C = \frac{q^3}{3} - 7q^2 + 11q + 50$ and q = 100 – p. Find the level of output at which profit is maximised.

6. The demand function of a firm is p = 500 – 0.2q and total cost c = 25q + 10000. Find the output at which profit is maximised. What is the price charged.

7. The total cost c of output Q is given by $c = 300\,Q - 10Q^2 + \frac{Q^3}{3}$ find the output level at which marginal cost and average cost attain their repective minimum.
8. The total cost of the production of a firm is given by $C = 0.7x + 18$ find
   (a) Total cost for output of 10 units.
   (b) Average cost for an output of 9 units.
   (c) Marginal cost for output of 6 units.
9. The total cost function of a manufacture is $c = 5x^2 + 500x + 50,000$ find the output when AC = MC.
10. If $R = x\left(15 - \frac{x}{30}\right)$ what is marginal revenue function and what will be the marginal revenue if 100 units are produced.
11. The total revenue and total cost function is given by $R(q) = 300q - q^2$, $c(q) = 20 + 4q$ find the output at which the profit is maximised.

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