Elastic Waves. Acoustics (Part - 1)

Q.150. How long will it take sound waves to travel the distance l between the points A and B if the air temperature between them varies linearly from T_1 to T_2? The velocity of sound propagation in air is equal to $v = \alpha\sqrt{T}$, where a is a constant.

Ans. Since the temperature varies linearly we can write the temperature as a function of x, which is, the distance from the point A towards B.

i.e., $T = T_1 + \frac{T_2 - T_1}{l}x, \ [0 < x < l]$

Hence, $dT = \left(\frac{T_2 - T_1}{l}\right)dx$ (1)

In order to travel an elemental distance of dx which is at a distance of x from A it will take a time

$$dt = \frac{dx}{\alpha\sqrt{T}} \quad (2)$$

From Eqns (1) and (2), expressing dx in terms of dT, we get

$$dt = \frac{l}{\alpha\sqrt{T}}\left(\frac{l\,dT}{T_2 - T_1}\right)$$

Which on integration gives

$$\int_0^t dt = \frac{l}{\alpha(T_2 - T_1)}\int_{T_1}^{T_2}\frac{dT}{\sqrt{T}}$$

Or,

$$t = \frac{2l}{(T_2 - T_1)}\left(\sqrt{T_2} - \sqrt{T_1}\right)$$

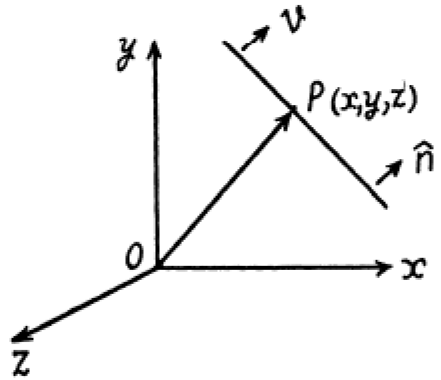
Hence the sought time $t = \frac{2l}{\alpha\left(\sqrt{T_1} + \sqrt{T_2}\right)}$

Q.151. A plane harmonic wave with frequency ω propagates at a velocity v in a direction forming angles α, β, γ with the x, y, z axes. Find the phase difference

between the oscillations at the points of medium with coordinates x_1, y_1, z_1 a nd x_2, y_2, z_2.

Ans. Equation of plane wave is given by

$\xi(r, t) = a\cos(\omega t - \vec{k}\cdot\vec{r})$, where $\vec{k} = \frac{\omega}{v}\hat{n}$ called the wave vector and $\hat{n}$ is the unit vector normal to the wave surface in the direction of the propagation of wave.



$$\text{or,} \quad \xi(x, y, z) = a\cos(\omega t - k_x x - k_y y - k_z z)$$
$$= a\cos(\omega t - kx\cos\alpha - ky\cos\beta - kz\cos\gamma)$$
$$\text{Thus} \quad \xi(x_1, y_1, z_1, t) = a\cos(\omega t - kx_1\cos\alpha - ky_1\cos\beta - kz_1\cos\gamma)$$
$$\text{and} \quad \xi(x_2, y_2, z_2, t) = a\cos(\omega t - kx_2\cos\alpha - ky_2\cos\beta - kz_2\cos\gamma)$$

Hence the sought wave phase difference

$$\varphi_2 - \varphi_1 = k\left[(x_1 - x_2)\cos\alpha + (y_1 - y_2)\cos\beta + (z_1 - z_2)\cos\gamma\right]$$
$$\text{or} \quad \Delta\varphi = |\varphi_2 - \varphi_1| = k\left|\left[(x_1 - x_2)\cos\alpha + (y_1 - y_2)\cos\beta + (z_1 - z_2)\cos\gamma\right]\right|$$
$$= \frac{\omega}{v}\left|\left[(x_1 - x_2)\cos\alpha + (y_1 - y_2)\cos\beta + (z_1 - z_2)\cos\gamma\right]\right|$$

Q.152. A plane wave of frequency ω propagates so that a certain phase of oscillation moves along the x, y, z axes with velocities v_1, v_2, v_3 respectively. Find the wave vector k, assuming the unit vectors e_x, e_y, e_z of the coordinate axes to be assigned.

Ans. The phase of the oscillation can be written as

$$\Phi = \omega t - \vec{k}\cdot\vec{r}$$

When the wave moves along the x-axis

$\Phi = \omega t - k_x x$ (On putting $k_y = k_z = 0$).

Since the velocity associated with this wave is v_1

We have $k_x = \frac{\omega}{v_1}$

Similarly $k_y = \frac{\omega}{v_2}$ and $k_z = \frac{\omega}{v_3}$

Thus $\vec{k} = \frac{\omega}{v_1}\hat{e}_x + \frac{\omega}{v_2}\hat{e}_y + \frac{\omega}{v_3}\hat{e}_3$.

Q.153. A plane elastic wave $\xi = a\cos(\omega t - kx)$ propagates in a medium K. Find the equation of this wave in a reference frame K' moving in the positive direction of the x axis with a constant velocity V relative to the medium K. Investigate the expression obtained.

Ans. The wave equation propagating in the direction of +ve x axis in medium K is give as

$$\xi = a\cos(\omega t - kx)$$

So, $\xi = a\cos k(vt - x)$, where $k = \frac{\omega}{v}$ and v is the wave velocity

In the reference frame K', the wave velocity will be (v - V) propagating in the

direction of +ve x axis and x will be x'. Thus the sought wave equation.

$$\xi = a\cos k[(v - V)t - x']$$

or, $$\xi = a\cos\left[\left(\omega - \frac{\omega}{v}V\right)t - kx'\right] = a\cos\left[\omega t\left(1 - \frac{V}{v}\right) - kx'\right]$$

Q.154. Demonstrate that any differentiable function f (t + ax), where a is a constant, provides a solution of wave equation. What is the physical meaning of the constant α?

Ans. This follows on actually putting

$$\xi = f(t + \alpha x)$$

In the wave equation

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}$$

(We have written the one dimensional form of the wave equation.) Then

$$\frac{1}{v^2} f''(t+\alpha x) = \alpha^2 f''(t+\alpha x)$$

So the wave equation is satisfied if

$$\alpha = \pm \frac{1}{v'}$$

That is the physical meaning of the constant α.

Q.155. The equation of a travelling plane sound wave has the form ξ = 60 cos (1800t — 5.3x), where ξ is expressed in micrometres, t in seconds, and x in metres. Find:
(a) the ratio of the displacement amplitude, with which the particles of medium oscillate, to the wavelength;
(b) the velocity oscillation amplitude of particles of the medium and its ratio to the wave propagation velocity;
(c) the oscillation amplitude of relative deformation of the medium and its relation to the velocity oscillation amplitude of particles of the medium.

Ans. The given wave equation

ξ **= 60 cos (1800t — 5.3x)**

Is of the type

$$\xi = a\cos(\omega t - kx), \quad \text{where } a = 60\times 10^{-6}\,\text{m}$$
$$\omega = 1800 \text{ per sec and } k = 5{\cdot}3 \text{ per metre}$$

As $k = \frac{2\pi}{\lambda}$, so $\lambda = \frac{2\pi}{k}$

And also $k = \frac{\omega}{v}$, so $v = \frac{\omega}{k} = 340$ m/s

(a) Sought ratio $= \frac{a}{\lambda} = \frac{ak}{2\pi} = 5{\cdot}1\times 10^{-5}$

(b) Since

$$\xi = a\cos(\omega t - kx)$$
$$\frac{\partial \xi}{\partial t} = -a\omega \sin(\omega t - kx)$$

So velocity oscillation amplitude

$$\left(\frac{\partial \xi}{\partial t}\right)_m \quad \text{or} \quad v_m = a\omega = 0{\cdot}11 \text{ m/s} \qquad (1)$$

And the sought ratio of velocity oscillation amplitude to the wave propagation velocity

$$= \frac{v_m}{v} = \frac{0{\cdot}11}{340} = 3{\cdot}2 \times 10^{-4}$$

(c) Relative deformation $= \frac{\partial \xi}{\partial x} = ak \sin(\omega t - kx)$

So, relative deformation amplitude

$$= \left(\frac{\partial \xi}{\partial x}\right)_m = ak = (60 \times 10^{-6} \times 5{\cdot}3)\,\text{m} = 3{\cdot}2 \times 10^{-4} \text{ m} \qquad (2)$$

From Eqns (1) and (2)

$$\left(\frac{\partial \xi}{\partial x}\right)_m = ak = \frac{a\omega}{v} = \frac{1}{v}\left(\frac{\partial \xi}{\partial t}\right)_m$$

Thus $\left(\frac{\partial \xi}{\partial x}\right)_m = \frac{1}{v}\left(\frac{\partial \xi}{\partial t}\right)_m,$ where v = 340 m/s is the wave velocity.

Q.156. A plane wave ξ = a cos (ωt — kx) propagates in a homogeneous elastic medium. For the moment t = 0 draw
(a) the plots of ξ, $\partial\xi/\partial t$, and $\partial\xi/\partial x$ vs x;
(b) the velocity direction of the particles of the medium at the points where ξ = 0, for the cases of longitudinal and transverse waves;
(c) the approximate plot of density distribution p (x) of the medium for the case of longitudinal waves.

Ans. (a) The given equation is,

$$\xi = a\cos(\omega t - kx)$$

So at $t = o,$

$$\xi = a\cos kx$$

Now, $$\frac{d\xi}{dt} = -a\omega\sin(\omega t - kx)$$

and $$\frac{d\xi}{dt} = a\omega\sin kx,\ \text{at}\ t = 0.$$

Also, $$\frac{d\xi}{dx} = +ak\sin(\omega t - kx)$$

and at $t = 0,$

$$\frac{d\xi}{dx} = -ak\sin kx.$$

Hence all the graphs are similar having different amplitudes, as shown in the answer-sheet of the problem book.

(b) At the points, where $\xi = 0$, the velocity direction is positive, i.e., along + ve x - axis in the case of longitudinal and + ve y- axis in the case of transverse waves, where $\frac{d\xi}{dt}$ is positive and vice versa.

For sought plots see the answer-sheet of the problem book.

Q.157. A plane elastic wave ξ = ae $^{-Yx}$ cos (ωt — kx), where a, γ, ω, and k are constants, propagates in a homogeneous medium. Find the phase difference between the oscillations at the points where the particles' displacement amplitudes differ by η = 1.0%, if γ = = 0.42 m $^{-1}$ and the wavelength is γ = 50 cm.

Ans. In the given wave equation the particle's displacement amplitude = ae $^{-yx}$
Let two points x_1 and x_2 , between which the displacement amplitude differ by η = 1 %

So, $$ae^{-\gamma x_1} - ae^{-\gamma x_2} = \eta a e^{-\gamma x_1}$$

or $$e^{-\gamma x_1}(1-\eta) = e^{-\gamma x_2}$$

or $$\ln(1-\eta) - \gamma x_1 = -\gamma x_2$$

Or, $$x_2 - x_1 = -\frac{\ln(1-\eta)}{\gamma}$$

So path difference $= -\frac{\ln(1-\eta)}{\gamma}$

And phase different $= \frac{2\pi}{\lambda} \times$ path difference

$$= -\frac{2\pi}{\lambda}\frac{\ln(1-\eta)}{\gamma} = \frac{2\pi\eta}{\lambda\cdot\gamma} = 0\cdot3 \text{ rad}$$

Q.158. Find the radius vector defining the position of a point source of spherical waves if that source is known to be located on the straight line between the points with radius vectors r_1 and r_2 at which the oscillation amplitudes of particles of the medium are equal to al and a_2. The damping of the wave is negligible, the medium is homogeneous.

Ans. Let S be the source whose position vector relative to the reference point O is $\vec{r}$. since intensities are inversely proportional to the square of distances,

$$\frac{\text{Intensity at } P\,(I_1)}{\text{Intensity at } Q\,(I_2)} = \frac{d_2^2}{d_1^2}$$

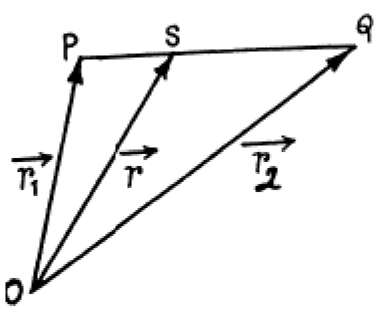
where $d_1 = PS$ and $d_2 = QS$.

But intensity is proportional to the square of amplitude.

So, $\frac{a_1^2}{a_2^2} = \frac{d_2^2}{d_1^2}$ or $a_1 d_1 = a_2 d_2 = k$ (say)

Thus $d_1 = \frac{k}{a_1}$ and $d_2 = \frac{k}{a_2}$

Let $\hat{n}$ be the unit vector along PQ directed from P to Q.



Then $\vec{PS} = d_1\hat{n} = \frac{k}{a_1}\hat{n}$

And $\vec{SQ} = d_2\hat{n} = \frac{k}{a_2}\hat{n}$

From the triangle law of vector addition.

$\vec{OP} + \vec{PS} = \vec{OS}$ or $\vec{r_1} + \frac{k}{a_1}\hat{n} = \vec{r}$

Or $a_1\vec{r_1} + k\hat{n} = a_1\vec{r}$ (1)

Similarly $\vec{r} + \frac{k}{a_2}\hat{n} = \vec{r_2}$ or $a_2\vec{r_2} - k\hat{n} = a_2\vec{r}$ (2)

Adding (1) and (2),

$$a_1\vec{r_1} + a_2\vec{r_2} = (a_1 - a_2)\vec{r}$$

Hence $\vec{r} = \dfrac{a_1\vec{r_1} + a_2\vec{r_2}}{a_1 + a_2}$

Q.159. A point isotropic source generates sound oscillations with frequency v = 1.45 kHz. At a distance r, = 5.0 m from the source the displacement amplitude of particles of the medium is equal to a_0 = 50 pm, and at the point A located at a distance r = 10.0 m from the source the displacement amplitude is η = 3.0 times less than a_0. Find:
(a) the damping coefficient γ of the wave;
(b) the velocity oscillation amplitude of particles of the medium at the point A.

Ans. (a) We know that the equation of a spherical wave in a homogeneous absorbing medium of wave damping coefficient γ is:

$$\xi = \frac{a'_0 e^{-\gamma r}}{r}\cos(\omega t - kr)$$

Thus particle’s displacement amplitude equals

$$\frac{a'_0 e^{-\gamma r}}{r}$$

According to the conditions of the problem,

at $r = r_0,\ a_0 = \dfrac{a'_0 e^{-\gamma r_0}}{r_0}$ (1)

And when $r = r, \quad \frac{a_0}{\eta} = \frac{a'_0{}^{-\gamma r}}{r}$ (2)

Thus from Eqns (1) and (2)

$$e^{\gamma(r-r_0)} = \eta \frac{r_0}{r}$$

or, $$\gamma(r-r_0) = \ln(\eta r_0) - \ln r$$

or, $$\gamma = \frac{\ln \eta + \ln r_0 - \ln r}{r - r_0} = \frac{\ln 3 + \ln 5 - \ln 10}{5} = 0{\cdot}08 \text{ m}^{-1}$$

(b)

As $\xi = \frac{a'_0 e^{-\gamma r}}{r} \cos(\omega t - k r)$

So, $$\frac{\partial \xi}{\partial t} = -\frac{a'_0 e^{-\gamma r}}{r} \omega \sin(\omega t - k r)$$

$$\left(\frac{\partial \xi}{\partial t}\right)_n = \frac{a'_0 e^{-\gamma r}}{r} \omega$$

But at point A, $\frac{a'_0 e^{-\gamma r}}{r} = \frac{a_0}{\eta}$

So, $\left(\frac{\partial \xi}{\partial t}\right)_m = \frac{a_0 \omega}{\eta} = \frac{a_0 2\pi}{\eta} = \frac{50 \times 10^{-6}}{3} \times 2 \times \frac{22}{7} \times 1{\cdot}45 \times 10^3 = 15 \text{ m/s}$

Q.160. Two plane waves propagate in a homogeneous elastic medium, one along the x axis and the other along the y axis: ξ_1 = = a cos (ωt - kx), ξ_2 = a cos (wt - ky). Find the wave motion pattern of particles in the plane xy if both waves
(a) are transverse and their oscillation directions coincide;
(b) are longitudinal.

Ans. (a) Equation of the resultant wave,

$$\xi = \xi_1 + \xi_2 = 2a \cos k\left(\frac{y-x}{2}\right) \cos\left\{\omega t - \frac{k(x+y)}{2}\right\},$$

$$= a' \cos\left\{\omega t - \frac{k(x+y)}{2}\right\}, \quad \text{where} \quad a' = 2a \cos k'\left(\frac{y-x}{2}\right)$$

Now, the equation of wave pattern is,
x + y = k , (a Const.)

For sought plots see the answer-sheet of the problem book.

For antinodes, i.e. maximum intensity

$$\cos\frac{k(y-x)}{2} = \pm 1 = \cos n\pi$$

or, $$\pm(x-y) = \frac{2n\pi}{k} = n\lambda$$

or, $$y = x \pm n\lambda,\ n = 0, 1, 2, \ldots$$

Hence, the particles of the medium at the points, lying on the solid straight lines $(y = x \pm n\lambda)$, oscillate with maximum amplitude.

For nodes, i.e. minimum intensity,

$$\cos\frac{k(y-x)}{2} = 0$$

Or,

$$\pm\frac{k(y-x)}{2} = (2n+1)\frac{\pi}{2}$$

Or, $y = x \pm (2n+1)\lambda/2,$

And hence the particles at the points, lying on dotted lines do not oscillate.

(b) When the waves are longitudinal,

For sought plots see the answer-sheet of the problem book.

$$k(y-x) = \cos^{-1}\frac{\xi_1}{a} - \cos^{-1}\frac{\xi_2}{a}$$

$$\frac{\xi_1}{a} = \cos\left\{k(y-x) + \cos^{-1}\frac{\xi_2}{a}\right\}$$

$$= \frac{\xi_2}{a}\cos k(y-x) - \sin k(y-x)\sin\left(\cos^{-1}\frac{\xi_2}{a}\right)$$

$$= \frac{\xi_2}{a}\cos k(y-x) - \sin k(y-x)\sqrt{1-\frac{\xi_2^2}{a^2}} \qquad (1)$$

From (1), if

$$\sin k(y-x) = 0 \ \sin(n\pi)$$

$$\xi_1 = \xi_2(-1)^n$$

thus, the particles of the medium at the points lying on the straight

lines, $y = x \pm \frac{n\lambda}{2}$ will oscillate along those lines (even n), or at right angles to them (odd n).

Also from (1),

If

$$\cos k(y-x) = 0 = \cos(2n+1)\frac{\pi}{2}$$

$$\frac{\xi_1^2}{a^2} = 1 - \xi_2^2/a^2, \text{ a circle.}$$

Thus the particles, at the points, where $y = x \pm (n \pm 1/4)\lambda$ will oscillate along circles. In general, all other particles will move along ellipses.

Q.161. A plane undamped harmonic wave propagates in a medium. Find the mean space density of the total oscillation energy (w), if at any point of the medium the space density of energy becomes equal to w_0 one-sixth of an oscillation period after passing the displacement maximum.

Ans. The displacement of oscillations is given by $\xi = a\cos(\omega t - kx)$
Without loss of generality, we confine ourselves to x = 0. Then the displacement maxima occur at ωt = nπ. Concentrate at ωt = 0. Now the energy density is given by

$$w = \rho a^2 \omega^2 \sin^2 \omega t \quad \text{at } x = 0$$

$T/6$ time later (where $T = \frac{2\pi}{\omega}$ is the time period) than $t = 0$.

$$w = \rho a^2 \omega^2 \sin^2 \frac{\pi}{3} = \frac{3}{4}\rho a^2 \omega^2 = w_0$$

Thus

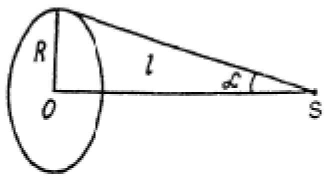
$$\langle w \rangle = \frac{1}{2}\rho a^2 \omega^2 = \frac{2w_0}{3}.$$

Q.162. A point isotropic sound source is located on the perpendicular to the plane of a ring drawn through the centre 0 of the ring. The distance between the point 0 and the source is l = 1.00 m, the radius of the ring is R = 0.50 m. Find the mean energy flow across the area enclosed by the ring if at the point 0 the intensity of sound is equal to I_0 = 30 μW/m^2. The damping of the waves is negligible.

Ans. The power output of the source much be

$$4\pi l^2 I_0 = Q \text{ Watt.}$$

The required flux of acoustic power is then: $Q = \frac{\Omega}{4\pi}$



Where Ω is the solid angle subtended by the disc enclosed by the ring at S. This solid angle is

$$\Omega = 2\pi\,(1 - \cos\alpha)$$

So flux $\Phi = I_0\, I_0 \left(1 - \frac{l}{\sqrt{r^2 + R^2}}\right) 2\pi l^2$

Substitution gives $\Phi = 2\Pi \times 30 \left(1 - \frac{l}{\sqrt{1 + \frac{1}{4}}}\right) \mu W = 1.99\ \mu W.$

Eqn. (1) is a well known result stich is derived as follows; Let SO be the polar axis.

Then the required solid angle is the area of that part of the surface a sphere of much radius whose colatitude is ≤ α.

Thus $\Omega = \int_0^{\alpha} 2\pi \sin\theta\, d\theta = 2\pi\,(1 - \cos\alpha).$

Q.163. A point isotropic source with sonic power P = 0.10 W is located at the centre of a round hollow cylinder with radius R = 1.0 m and height h = 2.0 m. Assuming the sound to be completely absorbed by the walls of the cylinder, find the mean energy flow reaching the lateral surface of the cylinder.

Ans. From the result of 4.162 power flowing out through anyone of the opening

$$= \frac{P}{2}\left(1 - \frac{h/2}{\sqrt{R^2 + (h/2)^2}}\right)$$

$$= \frac{P}{2}\left(1 - \frac{h}{\sqrt{4R^2 + h^2}}\right)$$

As total power output equals P, so the power reaching the lateral surface must be.

$$= P - 2\cdot\frac{P}{2}\left(1 - \frac{h}{\sqrt{4R^2 + h^2}}\right) = \frac{ph}{\sqrt{4R^2+h^2}} = 0{\cdot}07\,\text{W}$$

Q.164. The equation of a plane standing wave in a homogeneous elastic medium has the form ξ = a cos kx• cos wt. Plot:
(a) ξ and $\partial\xi/\partial x$ as functions of x at the moments t = 0 and t = T/2, where T is the oscillation period;
(b) the distribution of density p (x) of the medium at the moments t = 0 and t = T/2 in the case of longitudinal oscillations;
(c) the velocity distribution of particles of the medium at the moment t = T/4; indicate the directions of velocities at the antinodes, both for longitudinal and transverse oscillations.

Ans. We are given

$$\xi = a\cos kx\,\omega t$$

so $$\frac{\partial\xi}{\partial x} = -a k\sin kx\cos\omega t \quad \text{and} \quad \frac{\partial\xi}{\partial t} = -a\omega\cos kx\sin\omega t$$

Thus $$(\xi)_{t=0} = a\cos kx,\ (\xi)_{t=T/2} = -a\cos kx$$

$$\left(\frac{\partial\xi}{\partial x}\right)_{t=0} = a k\sin kx,\ \left(\frac{\partial\xi}{\partial x}\right)_{t=T/2} = -a k\sin kx$$

(a) The graphs of (ξ) and $\left(\frac{\partial\xi}{\partial x}\right)$ are as shown in Fig. (35) of the book (p.332).

(b) We can calculate the density as follows:

Take a parallelepiped of cross section unity and length dx with its edges at x and x + dx. After the oscillation the edge at x goes to x + ξ (x) and the edge at x + d x goes to

$$x + dx + \xi(x + dx)$$
$$= x + dx + \xi(x) + \frac{\partial\xi}{\partial x}dx.$$ Thus the volume of the element (originally dx) becomes

$$\left(1 + \frac{\partial\xi}{\partial x}\right)dx$$

And hence the density becomes $\rho = \frac{\rho_0}{1 + \frac{\partial \xi}{\partial x}}$.

On substituting we get for the density p (x) the curves shown in Fig.(35). Referred to above.

(c) The velocity v (x) at time t = T/4 is

$$\left(\frac{\partial \xi}{\partial t}\right)_{t = T/4} = -a\,\omega \cos kx$$

On plotting we get the figure (36).

Q.165. A longitudinal standing wave ξ = a cos kx- cos wt is maintained in a homogeneous medium of density ρ. Find the expressions for the space density of
(a) potential energy w_p (x, t);
(b) kinetic energy w_k (x, t).
Plot the space density distribution of the total energy w in the space between the displacement nodes at the moments t = 0 and t = T/4, where T is the oscillation period.

Ans. Given ξ = **a cos kx- cos wt**

(a) The potential eneigy density (per unit volume) is the eneigy of longitudinal strain. This is

$$w_p = \left(\frac{1}{2}\text{stress} \times \text{strain}\right) = \frac{1}{2}E\left(\frac{\partial \xi}{\partial x}\right)^2, \quad \left(\frac{\partial \xi}{\partial x} \text{ is the longitudinal strain}\right)$$

$$w_p = \frac{1}{2}E\,a^2 k^2 \sin^2 kx \cos^2 \omega t$$

But $\frac{\omega^2}{k^2} = \frac{E}{\rho}$ or $E k^2 = \rho\,\omega^2$

Thus $w_p = \frac{1}{2}\rho\,a^2 \omega^2 \sin^2 kx \cos^2 \omega t$

(b) The kinetic energy density is

$$= \frac{1}{2}\rho\left(\frac{\partial \xi}{\partial t}\right)^2 = \frac{1}{2}\rho\,a^2 \omega^2 \cos^2 kx \sin^2 \omega t.$$

On plotting we get Fig. 37 given in the book (p. 332). For example at t = 0

$$w = w_p + w_k = \frac{1}{2}\rho a^2 \omega^2 \sin^2 kx$$

And the displacement nodes are at $x = \pm\frac{\pi}{2k}$ so we do get the figure.

Q.166. A string 120 cm in length sustains a standing wave, with the points of the string at which the displacement amplitude is equal to 3.5 mm being separated by 15.0 cm. Find the maximum displacement amplitude. To which overtone do these oscillations correspond?

Ans. Let us denote the displacement of the elements of the string by

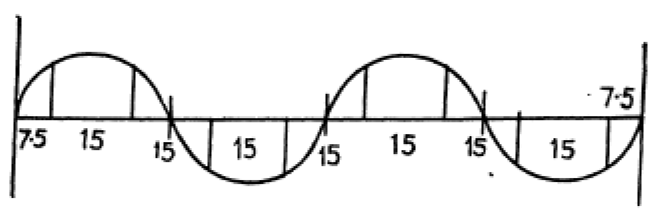
$$\xi = a \sin kx \cos \omega t$$

Since the string is 120 cm long we must have $k \cdot 120 = n\pi$

If X_1 is the distance at which the displacement amplitude first equals 3.5 mm then

Then $kx_1 + 15k = \pi - kx_1$ or $kx_1 = \frac{\pi - 15k}{2}$

One can convince our self that the string has the form shown below



It shows that $k \times 120 = 4\pi$, so $k = \frac{\pi}{30}\,\text{cm}^{-1}$

Thus we are dealing with the third overtone

Also $kx_1 = \frac{\pi}{4}$ so $a = 3{\cdot}5\sqrt{2}\text{ mm} = 4{\cdot}949\text{ mm}$.

Q.167. Find the ratio of the fundamental tone frequencies of two identical strings after one of them was stretched by η_1 = 2.0% and the other, by η_2 = 4.0%. The tension is assumed to 'be proportional to the elongation.

Ans. We have : $n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Tl}{M}}$ here M = total mass of the wire. When the write is stretched, total mass of the wire remains constant. For the first wire the new length = l + η_1 l and for the record wire, the length is l+ η_2l. Also $T_1 = \alpha(\eta_1 l)$ where α is a constant and $T_2 = \alpha(\eta_2 l)$. Substituting in the above formula.

$$\nu_1 = \frac{1}{2(l+\eta_1 l)}\sqrt{\frac{(\alpha\eta_1 l)(l+\eta_1 l)}{M}}$$

$$\nu_2 = \frac{1}{2(l+\eta_2 l)}\sqrt{\frac{(\alpha\eta_2 l)(l+\eta_2 l)}{M}}$$

$$\therefore \quad \frac{\nu_2}{\nu_1} = \frac{1+\eta_1}{1+\eta_2}\sqrt{\frac{\eta_2}{\eta_1}\cdot\frac{1+\eta_2}{1+\eta_1}}$$

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{\eta_2(1+\eta_1)}{\eta_2(1+\eta_2)}} = \sqrt{\frac{0\cdot 04(1+0\cdot 02)}{0\cdot 02(1+0\cdot 04)}} = 1\cdot 4$$

Q.168. Determine in what way and how many times will the fundamental tone frequency of a stretched wire change if its length is shortened by 35% and the tension increased by 70%.

Ans. Let initial length and tension be l and T respectively.

So, $\nu_1 = \frac{1}{2l}\sqrt{\frac{T}{\rho_1}}$

In accordance with the problem, the new length

$$l' = l - \frac{l\times 35}{100} = 0\cdot 65\, l$$

and new tension, $T' = T + \frac{T\times 70}{100} = 1\cdot 7\, T$

Thus the new frequency

$$\nu_2 = \frac{1}{2l'}\sqrt{\frac{T'}{\rho_1}} = \frac{1}{2\times 0\cdot 65\, l}\sqrt{\frac{1\cdot 7\, T}{\rho_1}}$$

Hence $\frac{\nu_2}{\nu_1} = \frac{\sqrt{1\cdot 7}}{0\cdot 65} = \frac{1\cdot 3}{0\cdot 65} = 2$

Q.169. To determine the sound propagation velocity in air by acoustic resonance technique one can use a pipe with a piston and a sonic membrane closing one of its ends. Find the velocity of sound if the distance between the adjacent positions of the piston at which resonance is observed at a frequency v = 2000 Hz is equal to l =

= 8.5 cm.

Ans. Obviously in this case the velocity of sound propagation
v - 2v (/$_2$ - l$_1$)

where l_2 and l_1 are consecutive lengths at which resonance occur
In our problem,

In our problem, $(l_2 - l_1) = l$

So $\quad v = 2\nu l = 2 \times 2000 \times 8{\cdot}5 \text{ cm/s} = 0{\cdot}34 \text{ km/s}.$

Q.170. Find the number of possible natural oscillations of air column in a pipe whose frequencies lie below v, = 1250 Hz. The length of the pipe is l = 85 cm. The velocity of sound is v = 340 m/s. Consider the two cases:
(a) the pipe is closed from one end;
(b) the pipe is opened from both ends. The open ends of the pipe are assumed to be the antinodes of displacement.

Ans. (a) When the tube is closed at one end

$$\nu = \frac{v}{41}(2n+1), \quad \text{where} \quad n = 0, 1, 2, \ldots$$

$$= \frac{340}{4 \times 0{\cdot}85}(2n+1) = 100(2n+1)$$

Thus for

$n = 0, 1, 2, 3, 4, 5, 6, \ldots$, we get

$n_1 = 100\,1H_z$, $n_2 = 300\,1H_z$, $n_3 = 500\,1H_z$, $n_4 = 700\,1H_z$,

$n_5 = 900\,1H_z$, $n_6 = 1100\,1H_z$, $n_7 = 1300\,1H_z$

Since v should be < v_0 = 1250 $1H_z$ we need not go beyond n_6.
Thus 6 natural oscillations are possible.

(b) Organ pipe opened from both ends vibrate with all harmonics of the fundamental frequency. Now, the fundamental mode frequency is given as
v = v/λ
or, v = v/2l Here, also, end correction has been neglected. So, the frequencies of higher modes of vibrations are given by
v = n (v/2l) (1)

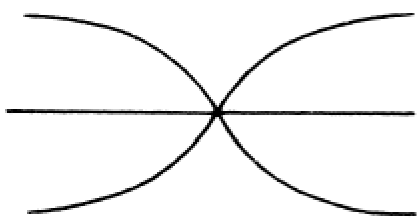
$\nu_1 = v/2l, \quad \nu_2 = 2(v/2l), \quad \nu_3 = 3(v/2l)$

It may be checked by putting the values of n in the equation (1) that below 1285 Hz, there are a total of six possible natural oscillation frequencies of air column in the open pipe.

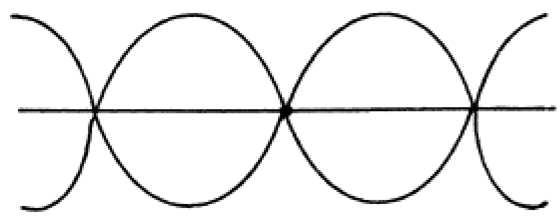
Elastic Waves. Acoustics (Part - 2)

Q.171. A copper rod of length l = 50 cm is clamped at its midpoint. Find the number of natural longitudinal oscillations of the rod in the frequency range from 20 to 50 kHz. What are those frequencies equal to?

Ans. Since the copper rod is clamped at mid-point, it becomes a mode and the two free ends will be antinodes. Thus the fundamental mode formed in the rod is as shown in the Fig. (a).



4.171 (a)



4.171 (b)

In this case $l = \frac{\lambda}{2}$

So,

$$\nu_0 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{E}{\rho}}\sqrt{\frac{E}{e}}$$

Where E = Young’s modules and p is the density of the copper
Similarly the second mode or the first overtone in the rod is as shown above in Fig. (b).

Here $l = \frac{3\lambda}{2}$

Hence

$$\nu_1 = \frac{3v}{2l} = \frac{3}{2l}\sqrt{\frac{E}{\rho}}$$

$$\nu = \frac{2n+1}{2l}\sqrt{\frac{E}{\rho}} \text{ where } n = 0, 1, 2 \ldots$$

Putting the given values of E and p in the general equation

$$\nu = 3{\cdot}8\,(2n+1)\,\text{k Hz}$$

Hence $\nu_0 = 3\cdot 8$ k.Hz, $\nu_1 = (3{\cdot}8\times 3)$ k Hz, $\nu_2 = (3{\cdot}8)\times 5 = 19$ k Hz,

$\nu_3 = (3{\cdot}8\times 7) = 26{\cdot}6$ k Hz, $\nu_4 = (3{\cdot}8\times 9) = 34{\cdot}2$ k Hz,

$\nu_5 = (3{\cdot}8\times 11) = 41{\cdot}8$ k Hz, $\nu_6 = (3{\cdot}8)\times 13$ k Hz $= 49{\cdot}4$ k Hz and

$\nu_7 = (3{\cdot}8)\times 14$ k Hz > 50 k Hz.

Hence the sought number of frequencies between 20 to 50 k Hz equals 4.

Q.172. A string of mass rn, is fixed at both ends. The fundamental tone oscillations are excited with circular frequency ω and maximum displacement amplitude Amax. Find:
(a) the maximum kinetic energy of the string;
(b) the mean kinetic energy of the string averaged over one oscillation period.

Ans. Let two waves $\xi_1 = a\cos(\omega t - kx)$ and $\xi_2 = a\cos(\omega t + kx)$, superpose and as a result, we have a standing wave (the resultant wave) in the string of the form $\xi = 2a\cos kx\,\cos\omega t$.

According to the problem 2 a = a_m.
Hence the standing wave excited in the string is

$$\xi = a_m \cos kx\,\cos\omega t \qquad (1)$$

Or, $$\frac{\partial\xi}{\partial t} = -\omega\, a_m \cos kx\,\sin\omega t \qquad (2)$$

So the kinetic energy confined in the string element of length dx, is given by

$$dT = \frac{1}{2}\left(\frac{m}{l}dx\right)\left(\frac{\partial\xi}{\partial t}\right)^2$$

Or, $$dT = \frac{1}{2}\left(\frac{m}{l}dx\right)a_m^2\,\omega^2\cos^2 kx\,\sin^2\omega t$$

Or, $$dT = \frac{m\,a_m^2\,\omega^2}{2l}\sin^2\omega t\,\cos^2\frac{2\pi}{\lambda}x\,dx$$

Hence the kinetic energy confined in the string corresponding to the fundamental tone

$$T = \int dT = \frac{m\,a_m^2\,\omega^2}{2l}\sin^2\omega t\int_0^{\lambda/2}\cos^2\frac{2\pi}{\lambda}x\,dx$$

Because, for the fundamental tone, length of the string $l = \frac{\lambda}{2}$

Integrating we get, $T = \frac{1}{4} m\, a_m^2\, \omega^2 \sin^2 \omega t$

Hence the sought maximum kinetic energy equals, $T_{max} = \frac{1}{4} m\, a_m^2\, \omega^2$,

Because for T_{max}, $\sin^2 \omega t = 1$

(ii) Mean kinetic energy averaged over one oscillation period

$$< T > = \frac{\int T\, dt}{\int dt} = \frac{1}{4} m\, a_m^2\, \omega^2 \quad \frac{\int_0^{2\pi/\omega} \sin^2 \omega t\, dt}{\int_0^{2\pi/\omega} dt}$$

Or, $< T > = \frac{1}{8} m\, a_m^2\, \omega^2$.

Q.173. A standing wave ξ = a sin kx•cos cot is maintained in a homogeneous rod with cross-sectional area S and density p. Find the total mechanical energy confined between the sections corresponding to the adjacent displacement nodes.

Ans. We have a standing wave given by the equation

$$\xi = a \sin k x \cos \omega t$$

So, $$\frac{\partial \xi}{\partial t} = - a\, \omega \sin k x \sin \omega t \qquad (1)$$

and $$\frac{\partial \xi}{\partial t} = a\, k \cos k x \cos \omega t \qquad (2)$$

The kinetic energy confined in an element of length dx of the rod

$$d T = \frac{1}{2} (\rho S\, d x) \left(\frac{\partial \xi}{\partial t} \right)^2 = \frac{1}{2} \rho S\, a^2 \omega^2 \sin^2 \omega t \sin^2 k x\, dx$$

So total kinetic energy confined into rod

$$T = \int dT = \frac{1}{2} \rho S\, a^2 \omega^2 \sin^2 \omega t \int_0^{\lambda/2} \sin^2 \frac{2\pi}{\lambda} x\, dx$$

Or, $T = \frac{\pi S a^2 \omega^2 \rho \sin^2 \omega t}{4k}$ (3)

The potential energy in the above rod element

$$dU = \int \partial U = -\int_0^{\xi} F_\xi \, d\xi, \text{ where } F_\xi = (\rho S \, dx) \frac{\partial^2 \xi}{\partial t^2}$$

Or, $F_\xi = -(\rho S \, dx)\, \omega^2 \xi$

So, $dU = \omega^2 \rho S \, dx \int_0^{\xi} \xi \, d\xi$

Or, $dU = \frac{\rho \omega^2 S \xi^2}{2} dx = \frac{\rho \omega^2 S a^2 \cos^2 \omega t \; \sin^2 kx \; dx}{2}$

Thus the total potential energy stored in the rod $U = \int dU$

Or, $U = \rho \omega^2 S a^2 \cos^2 \omega t \int_0^{\lambda/2} \sin^2 \frac{2\pi}{\lambda} x \, dx$

So, $U = \frac{\pi \rho S a^2 \omega^2 \cos^2 \omega t}{4k}$

To find the potential energy stored in the rod element we may adopt an easier way. We know that the potential energy density confined in a rod under elastic force equals:

$$U_D = \frac{1}{2}(\text{stress} \times \text{strain}) = \frac{1}{2}\sigma \varepsilon = \frac{1}{2} Y \varepsilon^2$$
$$= \frac{1}{2}\rho v^2 \varepsilon^2 = \frac{1}{2}\frac{\rho \omega^2}{k^2}\varepsilon^2$$
$$= \frac{1}{2}\frac{\rho \omega^2}{k^2}\left(\frac{\partial \xi}{\partial x}\right)^2 = \frac{1}{2}\rho a^2 \omega^2 \cos^2 \omega t \cos^2 kx$$

Hence the total potential energy stored in the rod

$$U = \int U_D \, dV = \int_0^{\lambda/2} \frac{1}{2} \rho a^2 \omega^2 \cos^2 \omega t \cos^2 kx \, S \, dx$$

$$= \frac{\pi \rho S a^2 \omega^2 \cos^2 \omega t}{4k}$$

Hence the sought mechanical energy confined in the rod between the two adjacent nodes

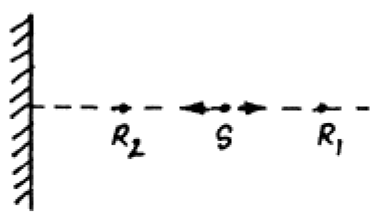
$$E = T + U = \frac{\pi \rho \omega^2 a^2 S}{4k}.$$

Q.174. A source of sonic oscillations with frequency ν_0 = 1000 Hz moves at right angles to the wall with a velocity u = 0.17 m/s. Two stationary receivers R_1 and R_2 are located on a straight line, coinciding with the trajectory of the source, in the following succession: R_1-source-R_2-wall. Which receiver registers the beatings and what is the beat frequency? The velocity of sound is equal to v = 340 m/s.

Ans. Receiver R_1 registers the beating, due to the sound waves reaching directly to it from source and the other due to the reflection from the wall.
Frequency of sound reaching directly from S to R_1

$$\nu_{S \to R_1} = \nu_0 \frac{v}{v - u} \quad \text{when } S \text{ moves towards } R_1$$

and $$\nu'_{S \to R_1} = \nu_0 \frac{v}{v + u} \quad \text{when } S \text{ moves towards the wall}$$



Now frequency reaching to R_1 after reflection from wall

$$\nu_{W \to R_1} = \nu_0 \frac{v}{v + u}, \text{ when } S \text{ moves towards } R_1$$

and $$\nu'_{W \to R_1} = \nu_0 \frac{v}{v - u}, \text{ when } S \text{ moves towards the wall}$$

Thus the sought beat frequency

$$\Delta \nu = \left(\nu_{S \to R_1} - \nu_{W \to R_1}\right) \text{ or } , \left(\nu'_{W \to R_1} - \nu_{S \to R_1}\right)$$

$$= \nu_0 \frac{v}{v - u} - \nu_0 \frac{v}{v + u} = \frac{2 \nu_0 v u}{v^2 - u^2} \approx \frac{2 u \nu_0}{v} = 1 \text{ Hz}$$

Q.175. A stationary observer receives sonic oscillations from two tuning forks one of which approaches, and the other recedes with the same velocity. As this takes place, the observer hears the beatings with frequency v = 2.0 Hz. Find the velocity of each tuning fork if -their oscillation frequency is v_o = 680 Hz and the velocity of sound in air is v = 340 m/s.

Ans. Let the velocity of tuning fork is u. Thus frequency reaching to the observer due to the tuning fork that approaches the observer

$$\nu' = \nu_0 \frac{v}{v-u} \quad [v = \text{velocity of sound}]$$

Frequency reaching the observer due to the tuning fork that recedes from the observer

$$\nu'' = \nu_0 \frac{v}{v+u}$$

So, Beat frequency $\nu - \nu'' = \nu = \nu_0 v \left(\frac{1}{v-u} - \frac{1}{v+u}\right)$

or, $$\nu = \frac{2\nu_0 v u}{v^2 - u^2}$$

So, $$\nu u^2 + (2 v \nu_0) u - v^2 \nu = 0$$

Hence $$u = \frac{-2 v \nu_0 \pm \sqrt{4\nu_0^2 v^2 + 4 \nu^2 v^2}}{2\nu}$$

Hence the sought value of u, on sim playing and noting that u > 0

$$u = \frac{v \nu_0}{\nu}\left(\sqrt{1 + \left(\frac{\nu}{\nu_0}\right)^2} - 1\right)$$

Q.176. A receiver and a source of sonic oscillations of frequency v_o = 2000 Hz are located on the x axis. The source swings harmonically along that axis with a circular frequency ω and an amplitude a = 50 cm. At what value of ω will the frequency bandwidth registered by the stationary receiver be equal to Av = 200 Hz? The velocity of sound is equal to v = 340 m/s.

Ans. Obviously the maximum, frequency will be heard when the source is moving with maximum velocity towards the receiver and minimum frequency will be heard when the source recedes with maximum velocity. As the source swing harmonically its maximum velocity equals ω to. Hence

$$\nu_{max} = \nu_0 \frac{v}{v - a\omega} \text{ and } \nu_{min} = \nu_0 \frac{v}{v + a\omega}$$

So the frequency band width $\Delta\nu = \nu_{max} - \nu_{min} = \nu_0 v \left(\frac{2a\omega}{v^2 - a^2\omega^2}\right)$

or, $$(\Delta\nu a^2)\omega^2 + (2\nu_0 v a)\omega - \Delta\nu v^2 = 0$$

So, $$\omega = \frac{-2\nu_0 v a \pm \sqrt{4\nu_0^2 v^2 a^2 + \Delta\nu^2 a^2 v^2}}{2\Delta\nu a^2}$$

On simplifying (and taking + sign as $\omega \to 0$ if $\Delta\nu \to 0$)

$$\omega = \frac{v\nu_0}{\Delta\nu a}\left(\sqrt{1 + \left(\frac{\Delta\nu}{\nu_0}\right)^2} - 1\right)$$

Q.177. A source of sonic oscillations with frequency ν_o = 1700 Hz and a receiver are located at the same point. At the moment t = 0 the source starts receding from the receiver with constant acceleration w = 10.0 m/s^2. Assuming the velocity of sound to be equal to v = 340 m/s, find the oscillation frequency registered by the stationary receiver t = 10.0 s after the start of motion.

Ans. It should be noted that the frequency emitted by the source at time t could not be received at the same moment by the receiver, because till that time the source will cover the distance $\frac{1}{2}wt^2$ and the sound wave will take the further time $\frac{1}{2}wt^2/v$ to reach the receiver. Therefore the frequency noted by the receiver at time t should be emitted by the source at the time $t_1 < t$. Therefore

$$t_1 + \left(\frac{1}{2}wt_1^2 / v\right) = t \quad (1)$$

And the frequency noted by the receiver

$$\nu = \nu_0 \frac{v}{v + wt_1} \quad (2)$$

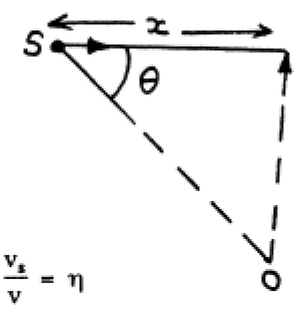
Solving Eqns (1) and (2), we get

$$\nu = \frac{\nu_0}{\sqrt{1 + \frac{2wt}{v}}} = 1 \cdot 35 \text{ kHz.}$$

Q.178. A source of sound with natural frequency ν_o = 1.8 kHz moves uniformly along a straight line separated from a stationary observer by a distance l = 250 m. The velocity of the source is equal to η = 0.80 fraction of the velocity of sound. Find:
(a) the frequency of sound received by the observer at the moment when the source gets closest to him;
(b) the distance between the source and the observer at the moment when the observer receives a frequency $\nu = \nu_o$.

Ans. (a) When the observer receives the sound, the source is closest to him. It means, that frequency is emitted by the source sometimes before (Fig.) Figure shows that the source approaches the stationary observer with

velocity $v_s \cos\theta$.
Hence the frequency noted by the observer



$$\nu = \nu_0 \left(\frac{v}{v - v_s \cos\theta} \right)$$

$$= \nu_0 \left(\frac{v}{v - \eta\, v \cos\theta} \right) = \frac{\nu_0}{1 - \eta \cos\theta} \quad (1)$$

But $\frac{x}{v_s} = \frac{\sqrt{l^2 + x^2}}{v}$, So, $\frac{x}{\sqrt{l^2 + x^2}} = \frac{v_s}{v} = \eta$

or, $\cos\theta = \eta$ (2)

Hence from Eqns. (1) and (2) the sought frequency

$$\nu = \frac{\nu_0}{1 - \eta^2} = 5 \text{ kHz}$$

(b) When the source is right in front of O, the sound emitted by it will not be Doppler shifted because 0 = 90°. This sound will be received at O at time $t = \frac{l}{v}$ after the source has v passed it. The source will by then have moved ahead by a distance $v_s t = l\eta$. The distance between the source and the observer at this time will be $l\sqrt{1 + \eta^2} = 0.32 \text{ km.}$

Q.179. A stationary source sends forth monochromatic sound. A wall approaches it with velocity u = 33 cm/s. The propagation velocity of sound in the medium is v = 330 m/s. In what way and how much, in per cent, does the wavelength of sound change on reflection from the wall?

Ans. Frequency of sound when it reaches the wall

$$\nu' = \nu \frac{v + u}{v}$$

Wall will reflect the sound with same frequency v'. Thus frequency noticed by a

stationary observer after reflection from wall

$\nu'' = \nu' \frac{v}{v - u}$, Since wall behaves as a source of frequency v'

Thus, $$\nu'' = \nu \frac{v + u}{v} \cdot \frac{v}{v - u} = \nu \frac{v + u}{v - u}$$

or, $$\lambda'' = \lambda \frac{v - u}{v + u} \quad \text{or} \quad \frac{\lambda''}{\lambda} = \frac{v - u}{v + u}$$

So, $$1 - \frac{\lambda''}{\lambda} = 1 - \frac{v - u}{v + u} = \frac{2u}{v + u}.$$

Hence the sought percentage change in wavelength

$$= \frac{\lambda - \lambda''}{\lambda} \times 100 = \frac{2u}{v + u} \times 100\ \% = 0.2\%\ \text{decrease.}$$

Q.180. A source of sonic oscillations with frequency ν_0 = 1700 Hz and a receiver are located on the same normal to a wall. Both the source and the receiver are stationary, and the wall recedes from the source with velocity u = 6.0 cm/s. Find the beat frequency registered by the receiver. The velocity of sound is equal to'. v = 340 m/s.

Ans. Frequency of sound reaching the wall.

$$\nu = \nu_0 \left(\frac{v - u}{v} \right) \quad (1)$$

Now for the observer the wall becomes a source of frequency v receding from it with velocity u Thus, the frequency reaching the observer

$$\nu' = \nu\left(\frac{v}{v+u}\right) = \nu_0\left(\frac{v-u}{v+u}\right)$$ [Using (1)]

Hence the beat frequency registered by the receiver (observer)

$$\Delta\nu = \nu_0 - \nu' = \frac{2u\nu_0}{v+u} = 0.6 \text{ Hz.}$$

Q.181. Find the damping coefficient y of a sound wave if at distances r_1 = 10 m and r_2 = 20 m from a point isotropic source of sound the sound wave intensity values differ by a factor η = 4.5.

Ans. Intensity of a spherical sound wave emitted from a point source in a homogeneous absorbing medium of wave damping coefficient y is given by

$$I = \frac{1}{2}\rho a^2 e^{-2\gamma r}\omega^2 v$$

So, Intensity of sound at a distance r_1 from the source

$$= \frac{I_1}{r_1^2} = \frac{1/2\,\rho a^2 e^{-2\gamma r_1}\omega^2 v}{r_1^2}$$

And intensity of sound at a distance r_2 from the source

$$= I_2/r_2^2 = \frac{1/2\,\rho a^2 e^{-2\gamma r_2}\omega^2 v}{r_2^2}$$

But according to the problem $\frac{1}{\eta}\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}$

So, $$\frac{\eta r_1^2}{r_2^2} = e^{2\gamma(r_2 - r_1)} \quad \text{or} \quad \ln\frac{\eta r_2^2}{r_1^2} = 2\gamma(r_2 - r_1)$$

or, $$\gamma = \frac{\ln(\eta r_2^2/r_1^2)}{2(r_2 - r_1)} = 6\times 10^{-3}\ \text{m}^{-1}$$

Q.182. A plane sound wave propagates along the x axis. The damping coefficient of the wave is $\gamma = 0.0230\ \text{m}^{-1}$. At the point x = 0 the loudness level is L = 60 dB. Find:
(a) the loudness level at a point with coordinate x = 50 m;
(b) the coordinate x of the point at which the sound is not heard any more.

Ans. (a)Loudness level in bells $= \log \frac{I}{I_0}$. (I_0 is the threshold of audibility.)

So, loudness level in decibels, $L = 10 \log \frac{I}{I_0}$

Thus loudness level at $x = x_1 = L_{x_1} = 10 \log \frac{I_{x_1}}{I_0}$

Similarly $L_{x_2} = 10 \log \frac{I_{x_2}}{I_0}$

Thus $L_{x_2} - L_{x_1} = 10 \log \frac{I_{x_2}}{I_{x_1}}$

$$\text{or,} \quad L_{x_2} = L_{x_1} + 10 \log \frac{1/2\, \rho\, a^2 \omega^2 v\, e^{-2\gamma x_2}}{1/2\, \rho\, a^2 \omega^2 v\, e^{-2\gamma x_1}} = L_{x_1} + 10 \log e^{-2\gamma(x_2 - x_1)}$$

$$L_{x_2} = L_{x_1} - 20\, \gamma\, (x_2 - x_1) \log e$$

$$\text{Hence} \quad L' = L - 20\, \gamma\, x \log e \; [\text{ since } (x_2 - x_1) = x\,]$$

$$= 20\, dB - 20 \times 0\cdot 23 \times 50 \times 0\cdot 4343 \; dB$$

$$= 60\, dB - 10\, dB = 50\, dB$$

(b) The point at which the sound is not heard any more, the loudness level should be zero.

Thus

$$0 = L - 20\, \gamma\, x \log e \quad \text{or} \quad x = \frac{L}{20\, \gamma \log e} = \frac{60}{20 \times 0\cdot 23 \times 0\cdot 4343} = 300 \text{ m}$$

Q.183. At a distance $r_o = 20.0$ m from a point isotropic source of sound the loudness level $L_o = 30.0$ dB. Neglecting the damping of the sound wave, find:
(a) the loudness level at a distance r = 10.0 m from the source;
(b) the distance from the source at which the sound is not heard.

Ans. (a) As there is no damping, so

$$L_{r_0} = 10 \log \frac{I}{I_0} = 10 \log \frac{1/2\, \rho\, a^2 \omega^2 v / r_0^2}{1/2\, \rho\, a^2 \omega^2 v} = -20 \log r_0$$

Similarly $L_r = -20 \log r$

So, $$L_r - L_{r_0} = 20 \log (r_0/r)$$

or, $L_r = L_{r_0} + 20 \log \left(\frac{r_0}{r} \right) = 30 + 20 \times \log \frac{20}{10} = 36\, dB$

(b) Let r be the sought distance at which the sound is not heard.

So, $L_r = L_{r_0} + 20 \log \frac{r_0}{r} = 0$ or, $L_{r_0} = 20 \log \frac{r}{r_0}$ or $30 = 20 \log \frac{r}{20}$

So, $$\log_{10} \frac{r}{20} = 3/2 \quad \text{or} \quad 10^{(3/2)} = r/20$$

Thus $$r = 200\sqrt{10} = 0{\cdot}63 \text{ Km.}$$

Thus for r > 0. 63 km no sound will be heard.

Q.184. An observer A located at a distance ζ_A = 5,0 m from a ringing tuning fork notes the sound to fade away ζ= 19 s later than an observer B who is located at a distance ζ_B = 50 m from the tuning fork. Find the damping coefficient β of `Oscillations of the tuning fork. The sound velocity v = 340 m/s.

Ans. We treat the fork as a point source. In the absence of damping the oscillation has the form

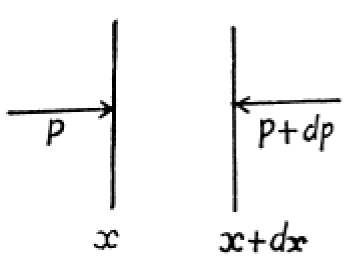
$$\frac{\text{Const.}}{r} \cos (\omega t - k r)$$

Because of the damping of the fork the amplitude of oscillation decreases exponentially with the retarded time (i.e. the time at which the wave started from the source.). Thus we write for the wave amplitude.

This means that

$$\xi = \frac{\text{Const.}}{r} e^{-\beta\left(t - \frac{r}{v}\right)}$$

$$\frac{e^{-\beta\left(t + \tau - \frac{r_A}{v}\right)}}{r_A} = \frac{e^{-\beta\left(t + \tau - \frac{r_B}{v}\right)}}{r_B}$$



Thus $e^{-\beta\left(\tau+\frac{r_B-r_A}{v}\right)}=\frac{r_A}{r_B}$ or $\beta=\frac{\ln\frac{r_B}{r_A}}{\tau+\frac{r_B-r_A}{v}}=0.12.\ s^{-1}$

Q.185. A plane longitudinal harmonic wave propagates in a medium with density ρ. The velocity of the wave propagation is v. Assuming that the density variations of the medium, induced by the propagating wave, Δρ ≪ ρ, demonstrate that
(a) the pressure increment in the medium $\Delta p=-\rho v^2(\partial\xi/\partial x)$, **where** $\partial\xi/\partial x$ **is the relative deformation;**
(b) the wave intensity is defined by Eq. (4.3i).

Ans. (a) Let us consider the motion of an element of the medium of thickness dx and unit area of cross-section. Let ξ displacement of the particles of the medium at location x. Then by the equation of motion

$$\rho\, dx\, \ddot{\xi}=-dp$$

Where dp is the pressure increment over the length dx
Recalling the wave equation

$$\ddot{\xi}=v^2\frac{\partial^2\xi}{\partial x^2}$$

We can write the foregoing equation as

$$\rho v^2\frac{\partial^2\xi}{\partial x^2}dx=-dp$$

Integrating this equation, we get

$$\Delta p=\text{surplus pressure}=-\rho v^2\frac{\partial\xi}{\partial x}+\text{Const.}$$

In the absence of a deformation (a wave), the surplus pressure is Δp = 0. So 'Const' = 0 and

$$\Delta p=-\rho v^2\frac{\partial\xi}{\partial x}.$$

(b) We have found earlier that

$$w = w_k + w_p = \text{total energy density}$$

$$w_k = \frac{1}{2}\rho\left(\frac{\partial \xi}{\partial t}\right)^2, \quad w_p = \frac{1}{2}E\left(\frac{\partial \xi}{\partial x}\right)^2 = \frac{1}{2}\rho v^2\left(\frac{\partial \xi}{\partial x}\right)^2$$

It is easy m see that the space-time average of both densities is the same and the space time average of total energy density is then

$$<w> = \left\langle \rho v^2\left(\frac{\partial \xi}{\partial x}\right)^2 \right\rangle$$

The intensity of the wave is

$$I = v <w> = \left\langle \frac{(\Delta p)^2}{\rho v} \right\rangle$$

Using

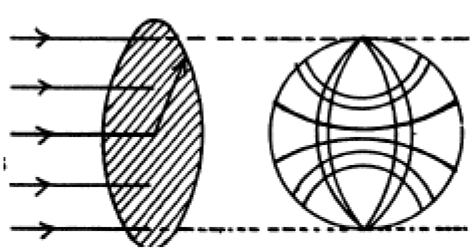
$$\left\langle (\Delta p)^2 \right\rangle = \frac{1}{2}(\Delta p)_m^2 \quad \text{we get} \quad I = \frac{(\Delta p)_m^2}{2\rho v}.$$

Q.186. A ball of radius R = 50 cm is located in the way of propagation of a plane sound wave. The sonic wavelength is λ = 20 cm, the frequency is ν = 1700 Hz, the pressure oscillation amplitude in air is $(\Delta p)_m$ = 3.5 Pa. Find the mean energy flow, averaged over an oscillation period, reaching the surface of the ball.

Ans. The intensity of the sound wave is

$$I = \frac{(\Delta p)_m^2}{2\rho v} = \frac{(\Delta p)_m^2}{2\rho \nu \lambda}$$

Using $v = \nu\lambda$, ρ is the density of air.



Thus the mean eneigy flow reaching the ball is

$$\pi R^2 I = \pi R^2 \frac{(\Delta p)_m^2}{2\rho v \lambda}$$

πR^2 Being the effective area (area of cross section) of the ball. Substitution gives 10.9 mW.

Q.187. A point A is located at a distance r = 1.5 m from a point isotropic source of sound of frequency v = 600 Hz. The sonic power of the source is P = 0.80 W. Neglecting the damping of the waves and assuming the velocity of sound in air to be equal to v = 340 m/s, find at the point A:
(a) the pressure oscillation amplitude $(\Delta p)_m$ and its ratio to the air pressure;
(b) the oscillation amplitude of particles of the medium; compare it with the wavelength of sound.

Ans.

We have $\frac{P}{4\pi r^2}$ = intensity $= \frac{(\Delta p)_m^2}{2\rho v}$

or $$(\Delta p)_m = \sqrt{\frac{\rho v P}{2\pi r^2}}$$

$$= \sqrt{\frac{1\cdot293\text{ kg/m}^3 \times 340\text{ m/s} \times 0\cdot80\text{ w}}{2\pi \times 1\cdot5 \times 1\cdot5\text{ m}^2}} = \sqrt{\frac{1\cdot293 \times 340 \times \cdot8}{2\pi \times 1\cdot5 \times 1\cdot5}} \left(\frac{\text{kg kg m}^2\text{s}^{-3}\text{ m s}^{-1}}{m^5}\right)^{\frac{1}{2}}$$

$$= 4\cdot9877 \left(\text{kg m}^{-1}\text{s}^{-2}\right) = 5\text{ Pa}.$$

$$\frac{(\Delta p)_m}{p} = 5 \times 10^{-5}$$

(b) We have

$$\Delta p = -\rho v^2 \frac{\partial \xi}{\partial x}$$

$$(\Delta p)_m = \rho v^2 k \xi_m = \rho v 2\pi v \xi_m$$

$$\xi_m = a = \frac{(\Delta p)_m}{2\pi \rho v v} = \frac{5}{2\pi \times 1\cdot293 \times 340 \times 600} = 3\,\mu\text{m}$$

$$\frac{\xi_m}{\lambda} = \frac{3 \times 10^{-6}}{340/600} = \frac{1800}{340} \times 10^{-6} = 5 \times 10^{-6}$$

Q.188. At a distance r = 100 m from a point isotropic source of sound of frequency 200 Hz the loudness level is equal to L = 50 dB. The audibility threshold at this frequency corresponds to the sound intensity I_0 = 0.10 nW/m^2. The damping coefficient of the sound wave is $\gamma = 5.0.10^{-4}$ m^{-1}. Find the sonic power of the source.

Ans. Express L in bels. (i.e. L = 5 bels).

Then the intensity at the relevant point (at a distance r from the source) is $: I_0 \cdot 10^L$

Had there been no damping the intensity would have been $: e^{2\gamma r} \, I_0 \cdot 10^L$

Now this must equal the quantity

$\frac{P}{4\pi r^2}$, where P = sonic power of the source.

Thus $\frac{P}{4\pi r^2} = e^{2\gamma r} \, I_0 \cdot 10^L$

Or $P = 4\pi r^2 e^{2\gamma r} I_0 \cdot 10^L = 1.39 \text{ W.}$