3. PRINCIPAL STRESS/PRINCIPAL STRAIN

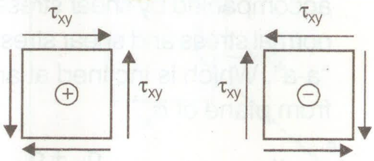
PRINCIPAL STRESS

Principal stress are maximum or minimum ***normal*** stress which may be developed on a loaded body.

> **Remember:** The plane of principal stress carry ***zero shear stress***.

SIGN CONVENTIONS

- ***Tensile*** stress is considered ***positive*** and ***compressive*** stress is ***negative***.
- Angle 'θ' is considered ***positive*** if it is in ***anti-clockwise direction***.



Positive shear stress　　Negative shear stress

- ***Shear*** stress acting on a positive face of an element is considered positive if it is acts in positive direction of one of the coordinate axes and negative if acts in the negative direction of the axes. Similarly on a negative face of an element is positive if it act in negative direction of the axes and negative if it act in the positive direction.

> **Remember:** Normally the reference plane taken are major principal plane or vertical plane.

ANALYTICAL METHOD OF ANALYSIS

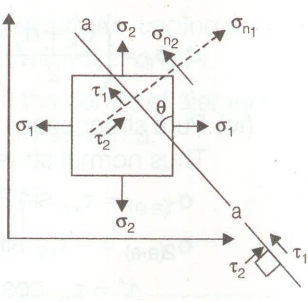
(i) If σ_1 and σ_2 are given ***principal*** stress as shown in figure, then normal and shear stress on plane a-a which is inclined at angle 'θ' from major principal plane ($\sigma_1 > \sigma_2$)

$$\therefore \quad \sigma_{n_1} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$\sigma_{n_1} = \frac{\sigma_1 + \sigma_2}{2} + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cdot \cos 2\theta$$

$$\sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2} - \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cdot \cos 2\theta$$

$$\tau_1 = \tau_2 = -\left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta$$



Remember

$\sigma_{n_1} + \sigma_{n_2} = \sigma_1 + \sigma_2 = \text{constant}$

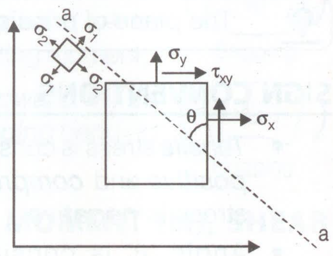
If $\theta = 45°$ or $135°$ then,

$$\tau_1 = \tau_2 = \tau_{max} = -\left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

On the plane of τ_{max}.

$$\sigma_{n_1} = \sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2}$$

(ii) If σ_x and 'σ_y' are normal stress on vertical and horizontal plane respectively and this plane is accompanied by shear stress "τ_{xy}" then normal stress and shear stress on plane "a-a". Which is inclined at an angle 'θ' from plane of σ_x.



$$\sigma'_{1(a-a)} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma'_{2(a-a)} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{(a-a)} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Remember

If θ occupies a position such that $\tau_{(a-a)}$ becomes zero, then such a plane is called principal plane and σ_1 and σ_2 become principal stress.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

θ_p = Angle of principal plane

(iii) If σ_x, σ_y and τ_{xy} are given and we have to find out principal stresses

$$\sigma_1/\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

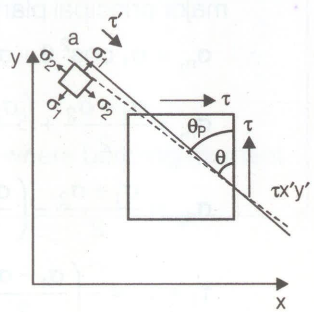
(iv) Pure shear case

Thus normal stress on plane a – a

$$\sigma_{1(a-a)} = \tau_{xy} \sin 2\theta$$

$$\sigma_{2(a-a)} = -\tau_{xy} \sin 2\theta$$

$$\tau' = \tau_{xy} \cos 2\theta$$



Remember

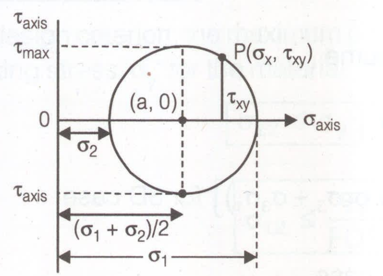
$\sigma_n + \sigma_n' = \sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_x' + \sigma_y' = \text{constant}$

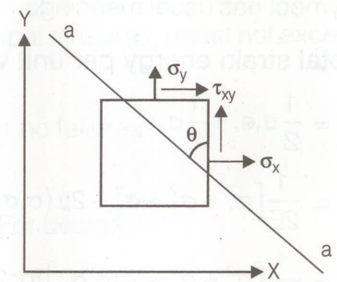
$\epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2 = \text{constant}$

$I_x + I_y = I_x' + I_y' = \text{constant}$

GRAPHICAL METHOD OF ANALYSIS/MOHR CIRCLE

Mohr circle is the locus of points representing magnitude of ***normal*** and ***shear stress*** at various plane in a given stress element.





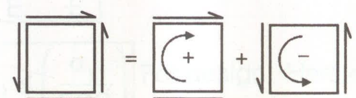
$$\sigma_1/\sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \qquad a = \frac{\sigma_1 + \sigma_2}{2}$$

- **Radius of Mohr's circle**

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

Remember

- Radius of Mohr circle represent the value of maximum shear stress.
- Normal stress on the plane of maximum shear stress is represented by coordinate of centre of Mohr circle.
- Mohr circle reduce to ***a point*** in case of ***hydrostatic*** loading and ***zero*** shear. In case of ***pure shear***, centre will fall at ***origin***.
- If shear stress causes clockwise couple at the centre of element then it will be plotted above σ_{axis} (+ve) and vise-versa.



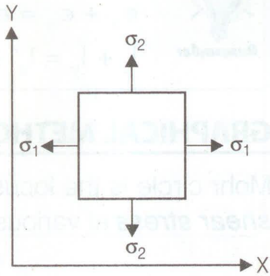
ANALYSIS OF STRAIN

ϵ_1 = Major principal strain = $\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$

ϵ_2 = Minor principal strain = $\frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$

$$\sigma_1 = \frac{E}{1-\mu^2}[\epsilon_1 + \mu\epsilon_2], \quad \sigma_2 = \frac{E}{1-\mu^2}[\epsilon_2 + \mu\epsilon_1]$$

Symbol has usual meanings.



- **Total strain energy per unit volume**

$$U = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^3 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right] \text{ for 3D case}$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \text{for 2D case}$$

Remember

- Plane stress does not lead to plain strains.
- For strain analysis formulas, put $\frac{\phi_{xy}}{2}$ is place of τ_{xy} every where in stress formulas.
- Max shear stress = $\frac{1}{2}$(difference of principal stress)
  Max shear strain = difference of principal strains
- **For shear:** Radius of Mohr circle = τ_{max}
  For strain: Radius of Mohr circle = $\frac{\phi_{max}}{2}$

■■■