Short Answer Questions – II (PYQ)

Q. 1. Answer the following questions:

(i) Derive an expression for drift velocity of free electrons.

(ii) How does drift velocity of electrons in a metallic conductor vary with increase in temperature? Explain. [CBSE (Central) 2016]

Ans. (i) When a potential difference is applied across a conductor, an electric field is produced and free electrons are acted upon by an electric force (Fe). Due to this, electrons accelerate and keep colliding with each other and acquire a constant (average) velocity vd called drift velocity.

Electric force on electron $F_e = -eE$

If m is the mass of electron, then its acceleration

$$a = \frac{F}{m} = \frac{-e\vec{E}}{m}$$

Now, $v = u + at$

Here, $u = 0, t = \tau$ (relaxation time), $\vec{v} = \vec{v}_d$

$$\vec{v_d} = 0 - \frac{e\vec{E}}{m}\tau$$

$$\Rightarrow \quad \vec{v_d} = -\frac{e\tau}{m}\vec{E}$$

(ii) With rise of temperature, the rate of collision of electrons with ions of lattice increases, so relaxation time decreases. As a result the drift velocity of electrons decreases with the rise of temperature.

Q. 2. (a) State Kirchhoff's rules and explain on what basis they are justified.

(b) Two cells of emfs E_1 and E_2 and internal resistances r_1 and r_2 are connected in parallel. Derive the expression for the (i) emf and (ii) internal resistance of a single equivalent cell which can replace this combination. [CBSE Patna 2015]

Ans. (a) Kirchhoff's Laws

(i) First law (or junction law): The algebraic sum of currents meeting at any junction is zero, i.e.,

$\sum I = 0$

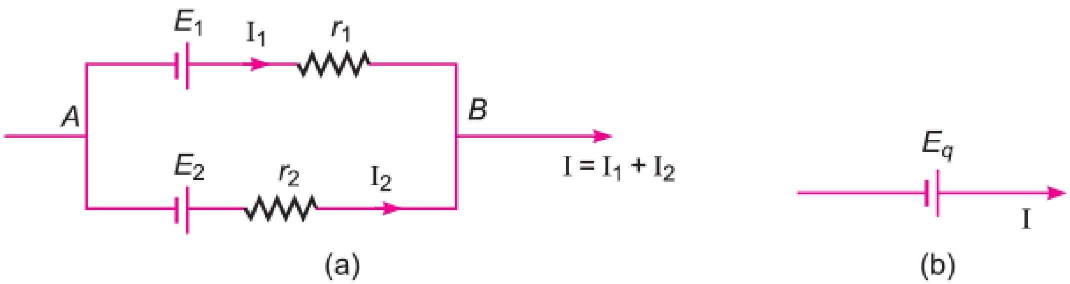
This law is based on conservation of charge.

(ii) Second law (or loop law): The algebraic sum of potential differences of different circuit elements of a closed circuit (or mesh) is zero, i.e.,

$\sum V = 0$

This law is based on conservation of energy.

(b)



Let I1 and I2 be the currents leaving the positive, terminals of the cells, and at the point B

$I = I_1 + I_2$ …(i)

Let V be the potential difference between points A and B of the combination of the cells, so

$V = E_1 - I_1r_1$ …(ii) (across the cells)

And $V = E_2 - I_2r_2$ …(iii)

From equation (i), (ii) and (iii), we get

$$I = \frac{(E_1 - V)}{r_1} + \frac{(E_2 - V)}{r_2}$$

$$= \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad ...(iv)$$

Fig. (b) Shows the equivalent cell, so for the same potential difference

$$V = E_{eq} - I_{rq}$$

or $$I = \frac{E_{eq}}{r_q} - \frac{V}{r_q} \quad \ldots(v)$$

On comparing Eq. (iv) and (v), we get

$$\frac{E_{eq}}{r_q} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

and $$\frac{1}{r_q} = \frac{1}{r_1} + \frac{1}{r_2} \quad \Rightarrow \quad r_q = \frac{r_1 r_2}{r_1 + r_2}$$

On further solving, we have

$$E_{eq}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

$$\Rightarrow \quad E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Q. 3. Plot a graph showing the variation of current density (J) versus the electric field (E) for two conductors of different materials. What information from this plot regarding the properties of the conducting material, can be obtained which can be used to select suitable materials for use in making (i) standard resistance and (ii) connecting wires in electric circuits?

Electron drift speed is estimated to be of the order of mms^{-1}. Yet large current of the order of few amperes can be set up in the wire. Explain briefly.
[CBSE Panchkula 2015]

Ans. As we know that the drift velocity of the free electrons in a conducting material is given by

$$v_d = \frac{-\,\mathrm{eE}}{m}\tau \quad \ldots(1)$$

And the flow of the current due to drifting electrons

$$I = neAv_d \quad \ldots(2)$$

From relation (1) and (2), we get

$$I = neA\left(\frac{eE}{m}\tau\right) = \frac{ne^2\ A}{m}\tau\ .\ E$$

Current density, $|J| = \frac{I}{A}$

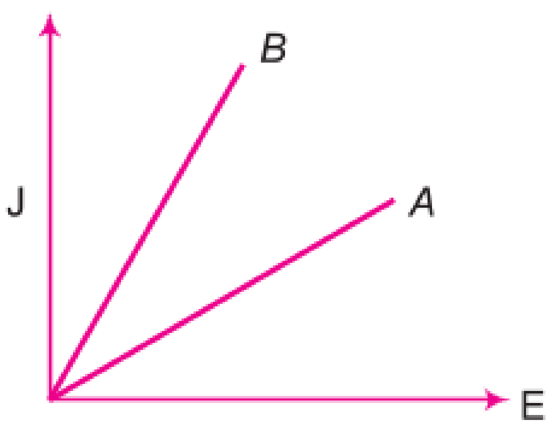
$$|J| = \frac{n\ e^2\ A}{m}\ E$$

$\Rightarrow$ $$|J| = \frac{1}{\rho}\ E$$

$$\left[\text{where}\ ,\ \rho\ =\ \frac{m}{ne^2\ P}\right]$$

$\Rightarrow$ Current density, $J \propto E$

Where represents the slope of J-E graph.



Let ρA the resistivity of the standard resistance A and ρ_B is the resistivity of the connective wire, i.e., $\rho_B < \rho_A$. So the graph of J vs E can be given as shown alongside.

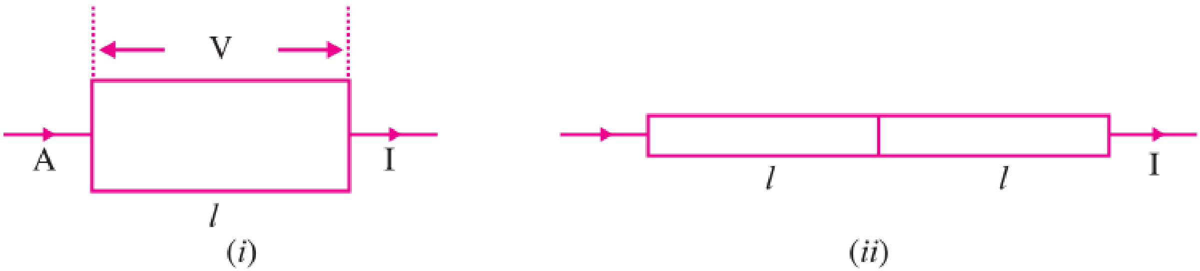
Information regarding current flow:

(i) When the circuit is closed, an electric field gets established through the conductor with the speed of light.

(ii) Localised electrons at all places inside the conductor begins to drift instantly and current flow occurs immediately.

(iii) In the relation i = neAvd, the free electron number density is of order of 10^{29} m^{-3}, even drift velocity is of order of few mm/sec or 10^{-3} m/sec, the current flow becomes of order of few ampere. I = neAvd, Here, drift velocity V_d is very small (in the order of mm s^{-1}), but electron number density (n) is very large. Hence, large current can be set up in the wire.

Q. 4. A metal rod of square cross-sectional area A having length l has current I flowing through it when a potential difference of V volt is applied across its ends (figure (i)). Now the rod is cut parallel to its length into two identical pieces and joined as shown in figure (ii). What potential difference must be maintained across the length 2l so that the current in the rod is still I? [CBSE (F) 2016]



Ans. Let resistance of metal rod having cross sectional area A and length l be R_1

$$\Rightarrow \quad R_1 = \rho \frac{l}{A}$$

Also, resistance of metal rod having cross sectional area A/2 and length 2l

$$R_2 = \rho \frac{2l}{\frac{A}{2}} \qquad \left[\because R = \rho \frac{l}{A}\right]$$

$$= 4\ R_1$$

Let V′ be potential difference maintained across rod. When the rod is cut parallel and rejoined by length, the length of the conductor becomes 2l and area decreases by A/2.

For maintaining same current,

$$I = \frac{V}{R_1} = \frac{V'}{R_2}$$

$$I = \frac{V}{R_1} = \frac{V'}{4R_1} \quad \Rightarrow \quad V' = 4V$$

The new potential applied across the metal rod will be four times the original potential (V).

Q. 5. Two metallic wires, P_1 and P_2 of the same material and same length but different cross-sectional areas, A_1 and A_2 are joined together and connected to a source of emf. Find the ratio of the drift velocities of free electrons in the two wires when they are connected (i) in series, and (ii) in parallel. [CBSE (F) 2017]

Ans. We know that,

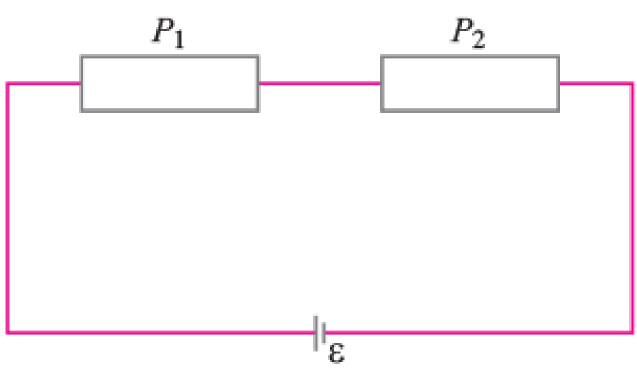
$$I = \text{neA}\ v_d \qquad \Rightarrow \qquad v_d = \frac{I}{\text{neA}}$$

Let R_1 and R_2 be resistances of P_1 & P_2 and A_1 & A_2 are their cross sectional areas respectively.

$$\therefore \qquad R_1 = \rho\frac{l}{A_1} \quad \text{and} \quad R_2 = \rho\frac{l}{A_2}$$

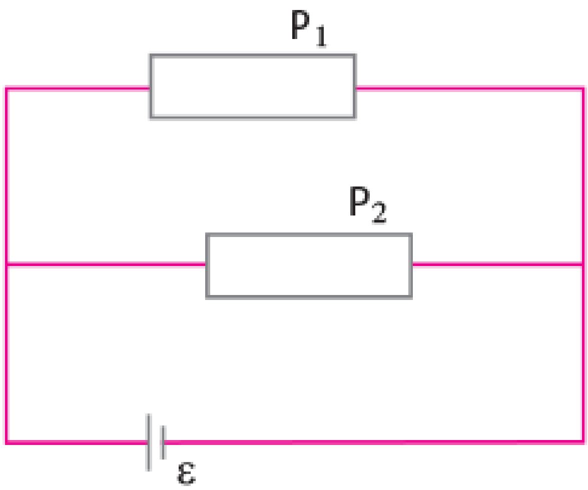
When connected in series,

$$\therefore \qquad \frac{v_{d_1}}{v_{d_2}} = \frac{\dfrac{\varepsilon}{\left(\dfrac{\rho l}{A_1}+\dfrac{\rho l}{A_2}\right)\text{neA}_1}}{\dfrac{\varepsilon}{\left(\dfrac{\rho l}{A_1}+\dfrac{\rho l}{A_2}\right)\text{neA}_2}} = \frac{A_2}{A_1}$$



When, connected in parallel,

$$\frac{v_{d_1}}{v_{d_2}} = \frac{\frac{\frac{\varepsilon}{\rho l} \cdot \frac{1}{neA_1}}{A_1}}{\frac{\frac{\varepsilon}{\rho l}}{A_2} \cdot \frac{1}{neA_2}} = 1$$



Q. 6. Two heating elements of resistance R_1 and R_2 when operated at a constant supply of voltage, V, consume powers P_1 and P_2 respectively. Deduce the expressions for the power of their combination when they are, in turn, connected in

(i) In series combination
(ii) In parallel combination

Ans. In series combination

Net resistance, $R = R_1 + R_2$ ...(i)

As heating elements are operated at same voltage V, we have

$$R = \frac{V^2}{P}, \quad R_1 = \frac{V^2}{P_1} \text{ and } R_2 = \frac{V^2}{P_2}$$

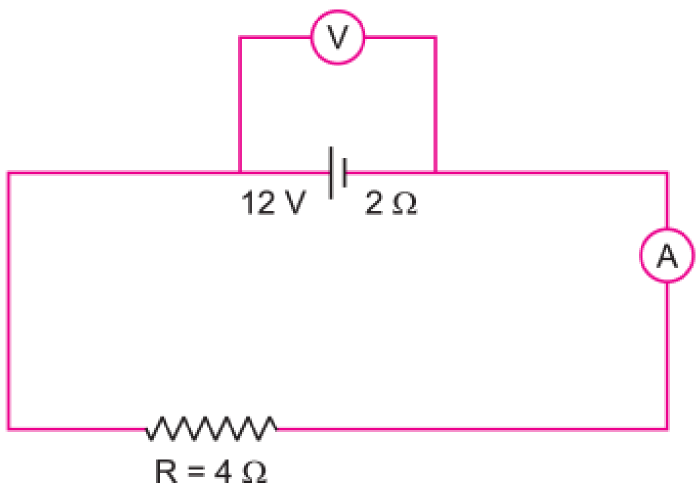
$\therefore$ From equation (i)

$$\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} \quad \Rightarrow \quad \frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

Q. 7. Answer the following questions:

(i) The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change?

(ii) In the figure shown, an ammeter A and a resistor of 4 Ω are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of 2 Ω. Calculate the voltmeter and ammeter readings. [CBSE AI 2017]



Ans. (i)

Heat generated $H = I_2 Rt = \frac{V^2 t}{R}$

Given, $H' = 9H$

$\therefore \quad \frac{V'^2 t}{R} = 9 \times \frac{V^2 t}{R}$

$\Rightarrow \quad V^2 = 9 \times V^2 \qquad \Rightarrow \quad V' = \sqrt{9} \times V$

$\therefore \quad V = 3V$

$\therefore$ Potential difference increases by a factor of $\sqrt{9}$ *i.e.*, 3.

(ii) Given: emf E = 12 V

Internal resistance r = 2 Ω

External resistance r = 4 Ω

Ammeter Reading,

$$I = \frac{E}{R+r} = \frac{12}{4+2} = \frac{12}{6}$$

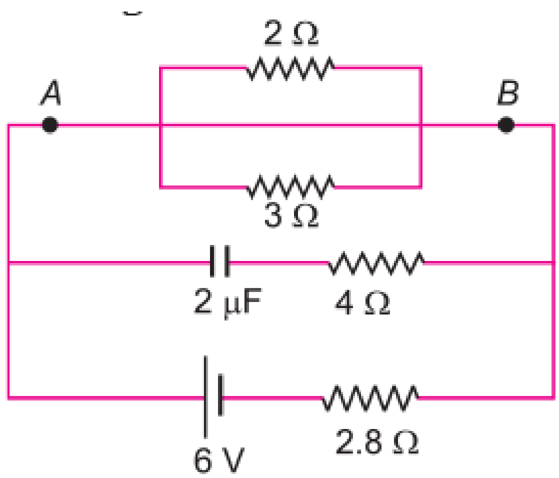
∴ I = 2 A

Voltmeter Reading,

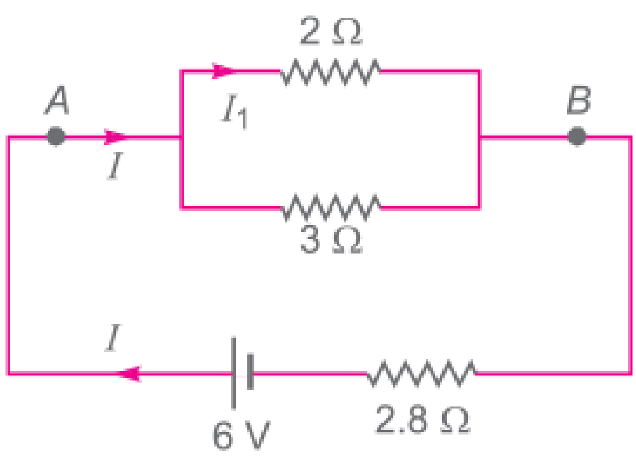
V = E – Ir = 12 – (2 × 2)

∴ V = 8 V

Q. 8. Calculate the steady current through the 2 Ω resistor in the circuit shown below.
[CBSE (F) 2010]



Ans. In steady state there is no current in capacitor branch, so equivalent circuit is shown in fig.



Net resistance of circuit,

$$R_{eq} = \frac{2\times3}{2+3} + 2.8 = 1.2 + 2.8 = 4\ \Omega$$

Net emf, $E = 6$ V

Current in circuit, $I = \frac{E}{R_{eq}} = \frac{6}{4} = 1.5\ A$

Potential difference across parallel combination of 2 Ω and 3 Ω resistances.

$$V' = IR' = 1.5 \times 1.2 = 1.8 \text{ V}$$

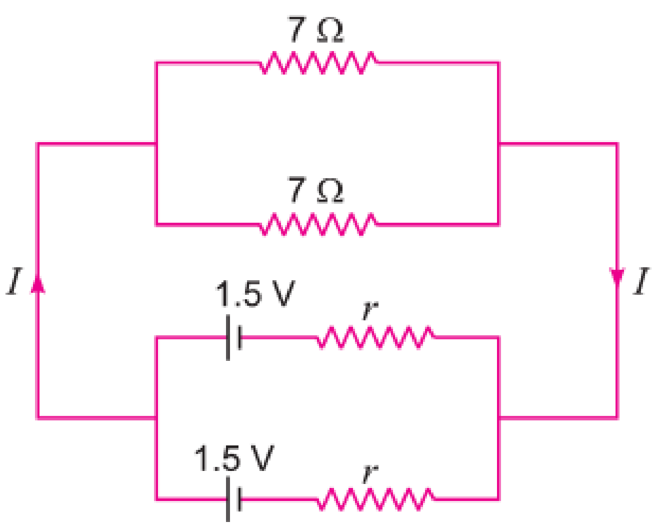
Current in 2 Ω resistance

$$I_1 = \frac{V'}{R_1} = \frac{1.8}{2} = 0.9A$$

Q. 9. Two identical cells of emf 1.5 V each joined in parallel supply energy to an external circuit consisting of two resistances of 7Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell. [CBSE (North) 2016]

Ans. Here, E = 1.5 V, V = 1.4 V

Resistance of external circuit = Equivalent resistance of two resistances of 7Ω connected in parallel



or $$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{7 \times 7}{7+7}\Omega = 3.5\Omega$$

Let r′ be the total internal resistance of the two cells, then

$$r' = \left(\frac{E-V}{V}\right) R = \left(\frac{1.5-1.4}{1.4}\right) 3.5 = 0.25\,\Omega$$

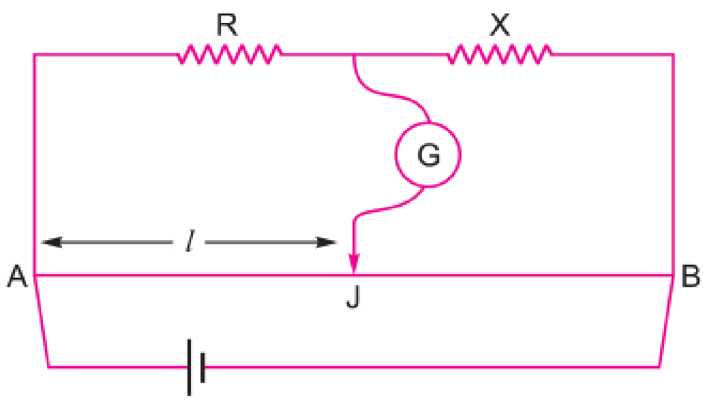
As the two cells of internal resistance r each have been connected in parallel, so

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} \quad \Rightarrow \quad \frac{1}{0.25} = \frac{2}{r} \quad \Rightarrow \quad r = 0.25 \times 2 = 0.5\Omega$$

Q. 10. In the meter bridge experiment, balance point was observed at J with AJ = l.

(i) The values of R and X were doubled and then interchanged. What would be the new position of balance point?

(ii) If the galvanometer and battery are interchanged at the balance position, how will the balance point get affected? [CBSE (AI) 2011]



Ans.

(i) $\frac{R}{X} = \frac{rl}{r(100-l)}$

$$\Rightarrow \quad \frac{R}{X} = \frac{l}{100-l} \qquad ...(i)$$

When both R and X are doubled and then interchanged, the new balance length becomes l' given by

$$\frac{2X}{2R} = \frac{l'}{(100-l')} \qquad ...(ii)$$

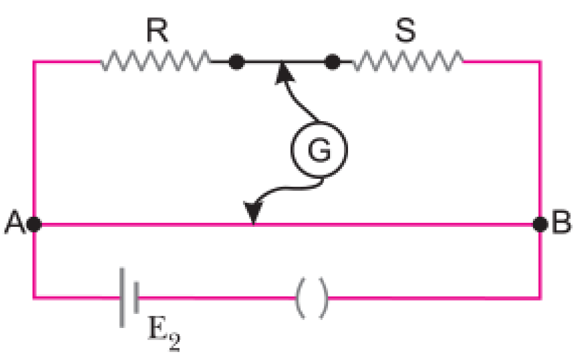
$$\Rightarrow \quad \frac{X}{R} = \frac{l'}{100-l'}$$

From (i) and (ii),

$$\frac{100-l}{l} = \frac{l'}{100-l'} \qquad \Rightarrow l' = (100-l)$$

(ii) If galvanometer and battery are interchanged, there is no effect on the balance point.

Q. 11. In a meter bridge shown in the figure, the balance point is found to be 40 cm from end A. If a resistance of 10 Ω is connected in series with R, balance point is obtained 60 cm from A. Calculate the value of R and S. [CBSE Patna 2015]



Ans.

$$\frac{R}{S} = \frac{40}{60} \quad \Rightarrow \quad 3R = 2S \quad \Rightarrow \quad R = \frac{2S}{3} \quad ...(i)$$

$$\frac{R+10}{S} = \frac{60}{40} \quad \Rightarrow \quad 2R + 20 = 3S \quad ...(ii)$$

From equation (i) and (ii), we get

$$2 \times \frac{2S}{3} + 20 = 3S$$

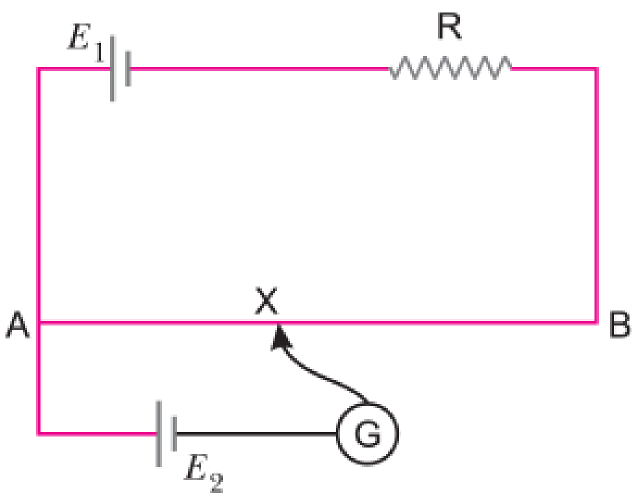
$\Rightarrow$ S = 12 ohm

From equation (i), we get

$$R = \frac{2 \times 12}{3}$$

$\Rightarrow$ R = 8 ohm

Q. 12. In the circuit diagram shown, AB is a uniform wire of resistance 15 Ω and length 1 m. It is connected to a cell E1 of emf 2 V and negligible internal resistance and a resistance R. The balance point with another cell E2 of emf 75 mV is found at 30 cm from end A. Calculate the value of the resistance R. [CBSE Chennai 2015]



Ans. Current drawn from the cell, E_1 = 2 V

$$I = \frac{E_1}{15 + R} = \frac{2}{15+R}$$

Potential drop across the wire AB

$$V_{AB} = I \times 15 = \frac{2\times15}{15+R} = \frac{30}{15+R}$$

Since wire length is 1 m or 100 cm.

So, potential gradient along the wire,

$$K = \frac{V_{AB}}{100\ \text{cm}} = \frac{30}{100(15+R)}$$

At the balance point

$$E_2 = kl_2$$

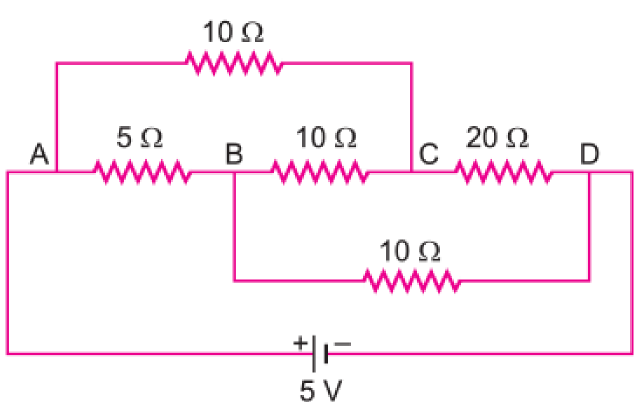
$$75\ \text{mV} = \frac{30}{100(15+R)} \times 30\ \text{cm}$$

$$75 \times 10-3 \times 100\ (15 + R) = 900$$

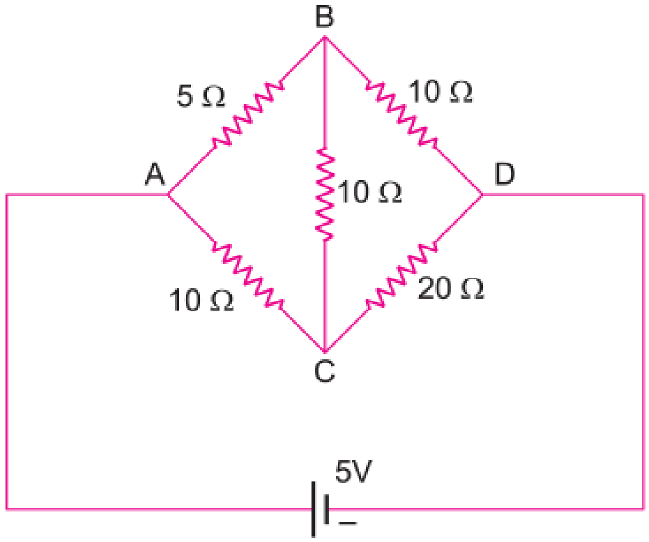
$$15 + R = \frac{9000}{75}$$

$$\therefore \quad R = 120 - 15 = 105\ \text{ohm}$$

Q. 13. Calculate the value of the current drawn from a 5 V battery in the circuit as shown. [CBSE (F) 2013]



Ans. The equivalent wheat stone bridge for the given combination is shown in figure alongside.



The resistance of arm ACD, R_{S_1} =10 + 20 = 30Ω

Also, the resistance of arm ABD, R_{S2} = 5+ 10= 15Ω

Since the condition $\frac{R}{Q} = \frac{R}{S}$ is satisfied, it is a balanced bridge.

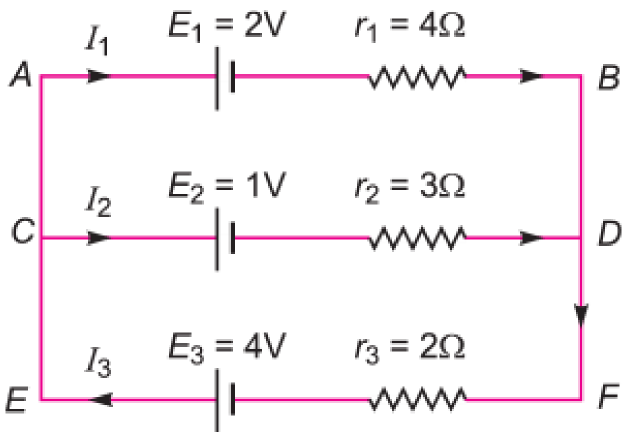
No current flows along arm BC.

$\therefore$ Equivalent resistance $\frac{R_{S_1} \times R_{S_2}}{R_s + R_{s_2}}$

$$= \frac{30 \times 15}{30 + 15} = \frac{30 \times 15}{45}$$

Current drawn from the source, I = $I = \frac{V}{R} = \frac{5}{10} = \frac{1}{2}A = 0.5\ A$

Q. 14. State Kirchhoff's rules. Use these rules to write the expressions for the currents I1, I2 and I3 in the circuit diagram shown. [CBSE (AI) 2010]



Ans. Kirchhoff's Rules:

(i) The algebraic sum of currents meeting at any junction is zero, i.e.,

$$\sum I = 0$$

(ii) The algebraic sum of potential differences across circuit elements of a closed circuit is zero, i.e., $\sum V = 0$

From Kirchhoff's first law

$$I_3 = I_1 + I_2 \quad \ldots(i)$$

For applying Kirchhoff's second law to mesh ABDC

$$-2 - 4I_1 + 3I_2 + 1 = 0$$

$$\Rightarrow \quad 4I_1 - 3I_2 = -1 \quad \ldots(ii)$$

Applying Kirchoff's II law to mesh ABCEA

$$-2 - 4I_1 - 2I_3 + 4 = 0$$

$$\Rightarrow \quad 4I_1 + 2I_3 = 2 \text{ or } 2I_1 + I_3 = 1$$

Using (i) we get

$$\Rightarrow \quad 2I_1 + (I_1 + I_2) = 1$$

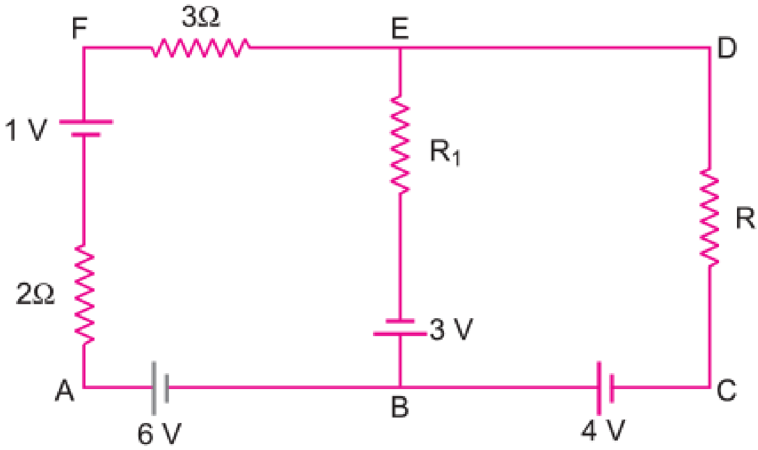
Or $\quad 3I_1 + I_2 = 1 \quad \ldots(iii)$

Solving (ii) and (iii), we get

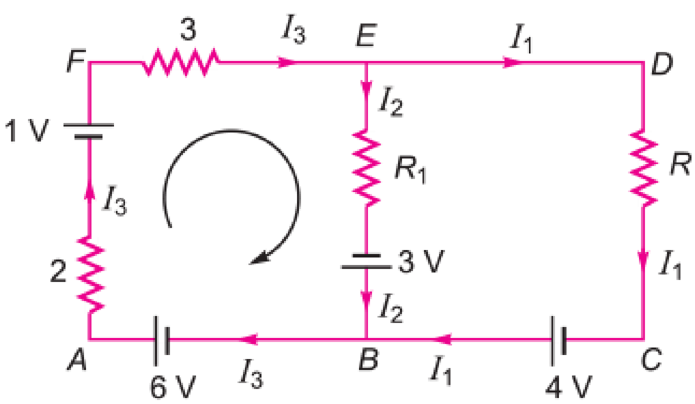
$$I_1 = \frac{2}{13}\ A,\ I_2 = 1 - 3I_1 = \frac{7}{13}\ A$$

so, $\quad I_3 = I_1 + I_2 = \frac{9}{13} A$

Q. 15. Use Kirchhoff's rules to determine the potential difference between the points A and D when no current flows in the arm BE of the electric network shown in the figure.
[CBSE Allahabad 2015]



Ans.



According to Kirchhoff's junction rule at E or B

$I_3 = I_2 + I1$

Since I2 = 0 in the arm BE as given in the question

⇒ $I_3 = I_1$

Using loop rule in the loop AFEBA

$+6V – 2I_3 + 1V – 3I_3 – I_2 R_1 + 3V = 0$

⇒ $2I_3 + 3I_3 + I_2R_1 = 10V$

Since $I_2=0$, so

$5I_3 = 10V$

⇒ $I_3 = 2$ A

∴ $I_3 = I_2 = 2$ A

The potential difference between A and D, along the branch AFED of the closed circuit.

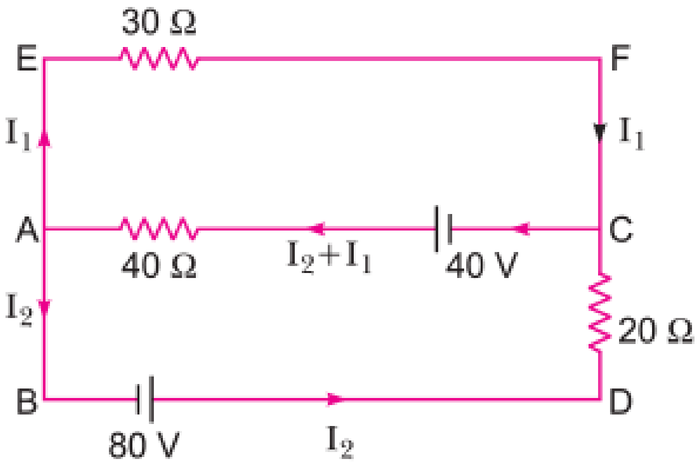
$$VA - 2I_3 + 1V - 3I_3 - V_D = 0$$

⇒ $V_A - V_D = 2I_3 - 1 + 3I_3$

$= 2 \times 2 - 1 + 3 \times 2 = 9$ V

Q. 16. Answer the following questions:

(i) Using Kirchhoff's rules, calculate the current in the arm AC of the given circuit.



(ii) On what principle does the meter bridge work? Why are the metal strips used in the bridge? [CBSE South 2016]

Ans. (i) For the mesh EFCAE

$$-30I_1 + 40 - 40(I_1+I_2) = 0$$

Or $7I_1 - 4I_2 = -4$

Or $7I_1 + 4I_2 = 4$ ...(i)

For the mesh ACDBA

$$40(I_1 + I_2) - 40 + 20I_2 - 80) = 0$$

Or $40(I_1 + 60I_2 - 120 = 0$

Or $2I_1 + 3I_2 = 6$ ...(ii)

Solving (i) and (ii), we get

$$I_1 = \frac{-12}{13} A$$

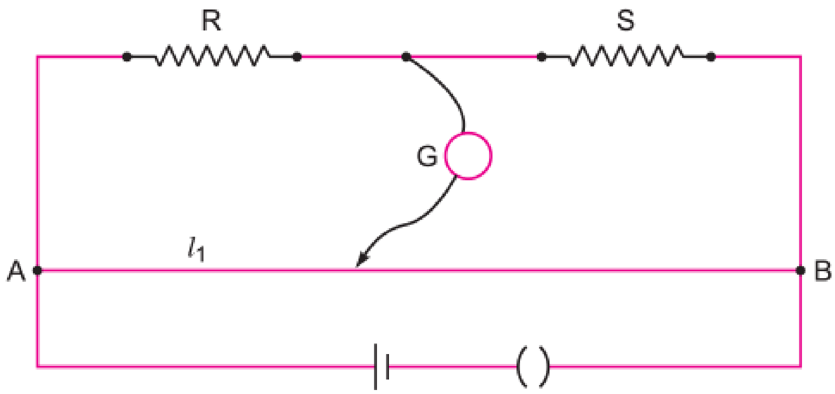
$$I_2 = \frac{34}{13} A$$

$\therefore$ Current through arm $\quad AC = I_1 + I_2 = \frac{22}{13} A$

Q. 17. Answer the following questions:

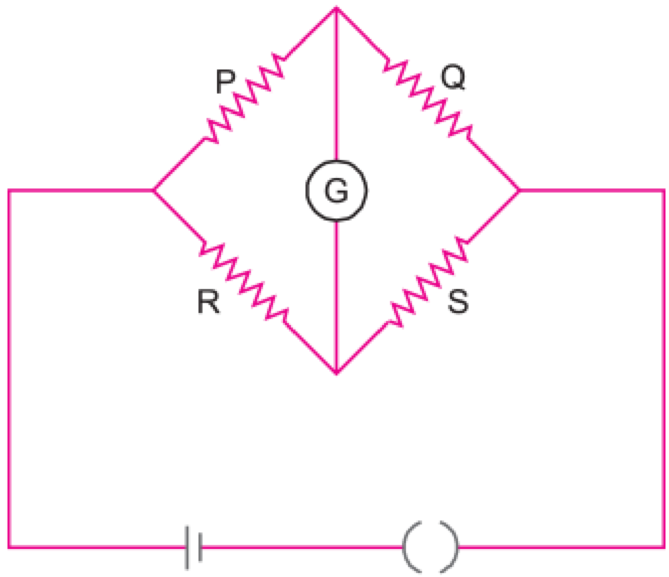
(i) Write the principle of working of a metre bridge.

(ii) In a metre bridge, the balance point is found at a distance l1 with resistances R and S as shown in the figure.



An unknown resistance X is now connected in parallel to the resistance S and the balance point is found at a distance l_2. Obtain a formula for X in terms of l_1, l_2 and S.
[CBSE (AI) 2017]

Ans. (i)



Working of a meter bridge is based on the principle of balanced Wheatstone bridge.

According to the principle, the balancing condition is

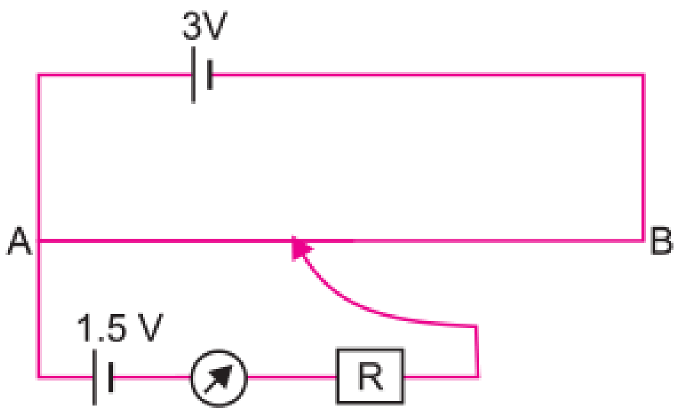
$$\frac{P}{Q} = \frac{R}{S} \text{ (When } I_g = 0)$$

For balancing lengths in a meter bridge,

$$\frac{P}{Q} = \frac{R}{S} \quad \Rightarrow \quad \frac{l}{100 - l} = \frac{R}{S}$$

$$\therefore \quad S = \frac{100 - l}{l} R$$

Q. 18. A potentiometer wire of length 1 m is connected to a driver cell of emf 3 V as shown in the figure. When a cell of 1.5 V emf is used in the secondary circuit, the balance point is found to be 60 cm. On replacing this cell and using a cell of unknown emf, the balance point shifts to 80 cm. [CBSE Delhi 2008]



(i) Calculate unknown emf of the cell.

(ii) Explain with reason, whether the circuit works, if the driver cell is replaced with a cell of emf 1 V.

(iii) Does the high resistance R, used in the secondary circuit affect the balance point? Justify our answer.

Ans. (i)

Unknown emf ε_2 is given by

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{l_2}{l_1} \quad \Rightarrow \quad \varepsilon_2 = \frac{l_2}{l_1}\varepsilon_1$$

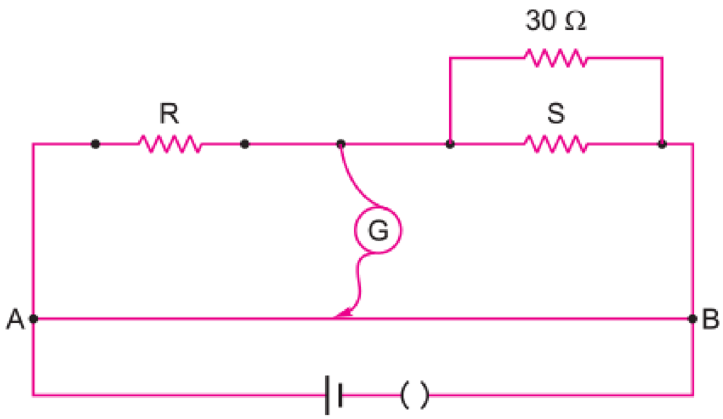
Given $\varepsilon_1 = 1.5$ V, $l_1 = 60$ cm, $l_2 = 80$ cm

$$\therefore \quad \varepsilon_2 = \frac{80}{60} \times 1.5V = 2.0V$$

(ii) The circuit will not work if emf of driver cell is 1 V (less than that of cell in secondary circuit), because total voltage across wire AB is 1 V which cannot balance the voltage V.

(iii) No, since at balance point no current flows through galvanometer G i.e., cell remains in open circuit.

Q. 19. In a meter bridge with R and S in the gaps, the null point is found at 40 cm from A. If a resistance of 30Ω is connected in parallel with S, the null point occurs at 50 cm from A. Determine the values of R and S. [CBSE East 2016]



Ans.

In first case l_1 =40cm

$$\frac{R}{S} = \frac{l_1}{100-l_1} \Rightarrow \frac{R}{S} = \frac{40}{60} = \frac{2}{3} \quad \ldots(i)$$

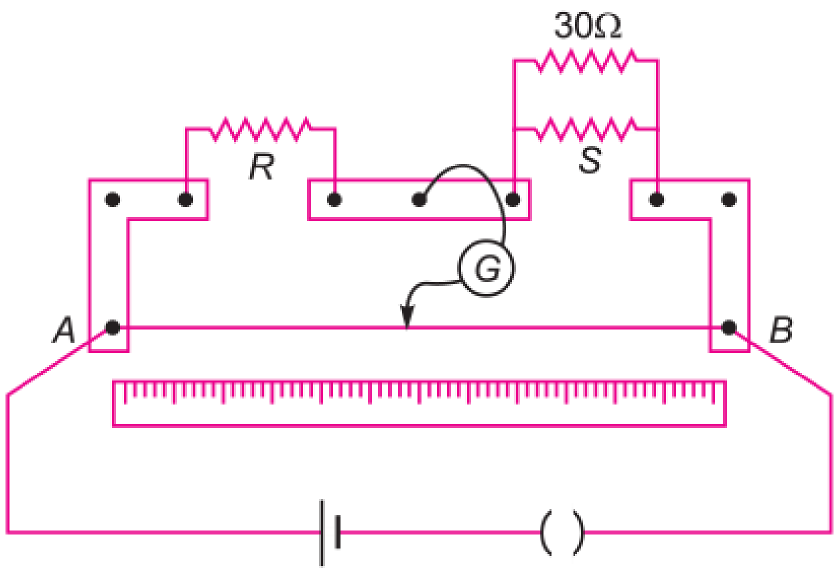
In second case when S and 30Ω are in parallel balancing length l_2=50 cm, so

$$S' = \frac{30S}{30+S} \quad \ldots(ii)$$

$$\frac{R}{S'} = \frac{50}{100-50} = 1 \Rightarrow S' = R \quad \ldots(iii)$$

From (i) $\quad S = \frac{3}{2}R$

Substituting this value in (ii), we get



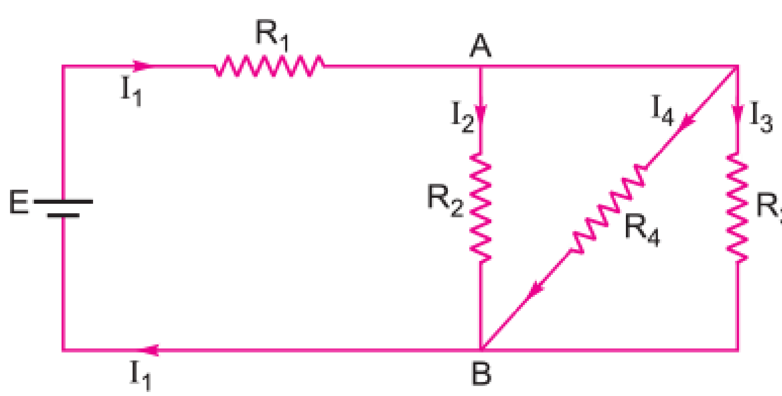
$$S' = \frac{30 \times \left(\frac{3}{2}R\right)}{30 + \left(\frac{3}{2}R\right)} = \frac{45R}{30 + \frac{3}{2}R}$$

Also from equation (*iii*), S′ = R

$$\therefore \quad \frac{45R}{30 + \frac{3}{2}R} = R \quad \Rightarrow \quad 45 = 30 + \frac{3}{2}R$$

$$\Rightarrow \quad \frac{3}{2}R = 15 \quad \text{or} \quad R = 10\,\Omega \quad \Rightarrow \quad S = \frac{3}{2} \times R = \frac{3}{2} \times 10 = 15\,\Omega$$

Q. 20. In the circuit shown, R1 = 4Ω , R2 = R3 = 15Ω, R4 = 30Ω and E = 10 V. Calculate the equivalent resistance of the circuit and the current in each resistor. [CBSE Delhi 2011] [HOTS]



Ans. Given $R_1 = 4\Omega$, $R_2 = R_3 = 15\Omega$, $R_4 = 30\Omega$, E = 10 V.

Equivalent Resistance:

R_2, R_3 and R_4 are in parallel, so their effective resistance (R) is given by

$$\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{15} + \frac{1}{15} + \frac{1}{30}$$

⇒ R = 6Ω

R1 is in series with R, so equivalent resistance

$R_{eq} = R + R_1 = 6 + 4 = 10\ \Omega$.

Currents:

$$I_1 = \frac{E}{R_{eq}} = \frac{10}{10} = 1\ \text{A} \qquad ...(i)$$

This current is divided at A into three parts I2, I3 and I4.

∴ $I_2 + I_3 + I_4 = 1$ A ....(ii)

Also, $I_2 R_2 = I_3 R3 = I4R4$

⇒ $I_2 \times 1_5 = I_3 \times 1_5 = I_4 \times 30$

⇒ $I_2 = I_3 = 2I_4$ ...(iii)

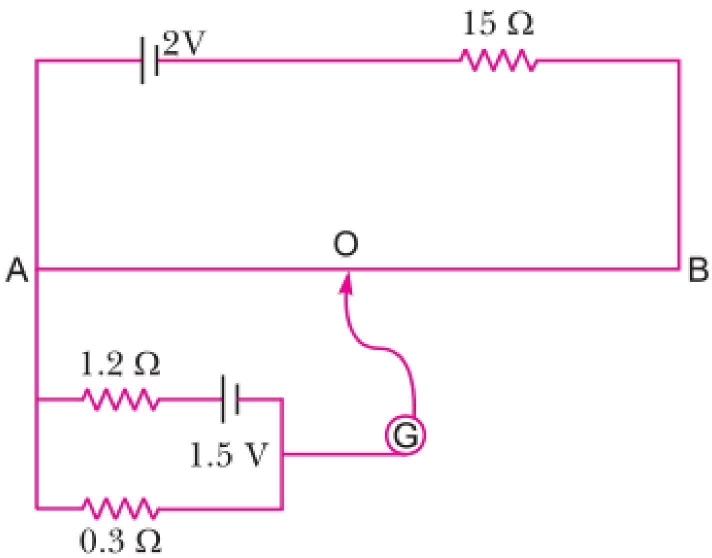
Substituting values of I2, I3 in (ii), we get

$2I_4 + 2I_4 + I_4 = 1$ A ⇒ I4 = 0.2 A

∴ $I_2 = I_3 = 2 \times 0.2 = 0.4$ A

Thus, I_1 = 1A, $I_2 = I_3$ = 0.4A and I_4 = 0.2A

Q. 21. In the following potentiometer circuit AB is a uniform wire of length 1 m and resistance 10Ω. Calculate the potential gradient along the wire and balance length AO.
[CBSE Delhi 2016] [HOTS]



Ans. Current flowing in the potentiometer wire

$$I = \frac{E}{R_{\text{total}}} = \frac{2.0}{15+10} = \frac{2}{25} A$$

Potential difference across the wire $\frac{2}{25} \times 10 = \frac{20}{25} = 0.8V$

Potential gradient $k = \frac{V_{AB}}{l_{AB}} = \frac{0.8}{1.0} = 0.8V/m$

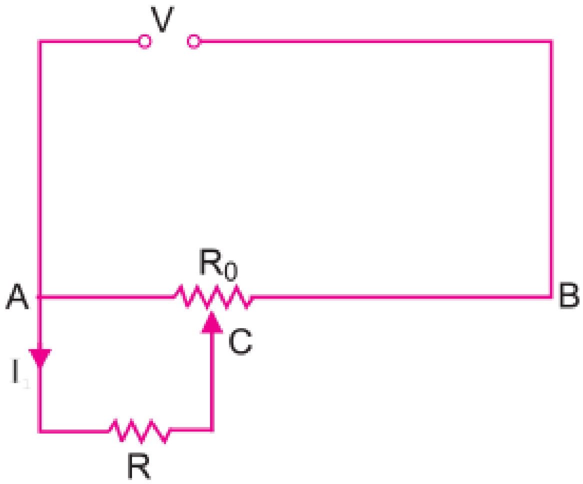
Now, current flowing in the circuit containing experimental cell,

$$\frac{1.5}{1.2+0.3} = 1 \ A$$

Potential difference across length $AO = 0.3 \times 1 = 0.3$ V

Length AO $= \frac{0.3}{0.8} m = \frac{0.3}{0.8} \times 100 \text{ cm} = 37.5 \text{ cm}$

Q. 22. A resistance of R draws current from a potentiometer. The potentiometer wire, AB, has a total resistance of R0. A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of potentiometer wire. [CBSE Delhi 2017]



Ans. Effective resistance between points A and C is given by

$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{\frac{R_0}{2}} = \frac{\frac{R_0}{2} + R}{R.\frac{R_0}{2}}$$

$$R_1 = \frac{R\frac{R_0}{2}}{\frac{R_0}{2} + R}$$

Effective resistance between points A and B is given by

$$R_2 = \left(\frac{R\frac{R_0}{2}}{\frac{R_0}{2}+R}\right) + \frac{R_0}{2}$$

Current drawn from the voltage source, $I = \frac{V}{R_2}$

$$I = \frac{V}{\left(\frac{R\frac{R_0}{2}}{R+\frac{R_0}{2}}\right)+\frac{R_0}{2}} = \frac{V}{R\frac{R_0}{2R+R_0}+\frac{R_0}{2}}$$

Let current through R be I_1

$$I_1 = \frac{I\left(\frac{R_0}{2}\right)}{R+\frac{R_0}{2}}$$

Voltage across R is given by $V_1 = I_1 R$

$$= \frac{IR_0}{2\left(R+\frac{R_0}{2}\right)} . R$$

$$= \frac{R\ R_0}{2\left(R+\frac{R_0}{2}\right)} \cdot \frac{V}{\left(\frac{R\ R_0}{2R\ +\ R_0}\right)+\frac{R_0}{2}}$$

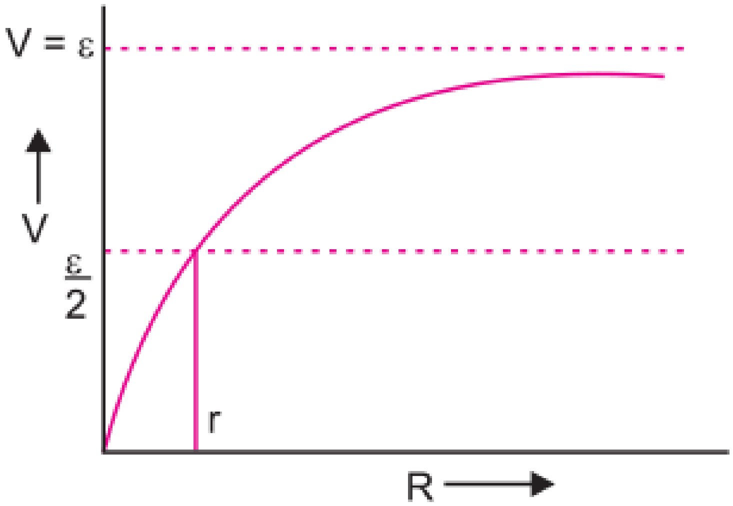
$$= \frac{R\ R_0}{2\left(\frac{2R\ +R_0}{2}\right)} \times \frac{V}{\frac{2R\ R_0+R_0(2R\ +\ R_0)}{2(2R\ +R_0)}}$$

$$= \frac{R\ R_0}{2R\ +\ R_0} \times \frac{2V(2R+R_0)}{R_0(2R\ +\ 2R\ +R_0)} = \frac{2\,RV}{R_0+4R}$$

Q. 23. A cell of emf 'E' and internal resistance 'r' is connected across a variable load resistor R. Draw the plots of the terminal voltage V versus (i) R and (ii) the

current I.
[CBSE Delhi 2015] [HOTS]

Ans.



(i) Current in circuit $I = \frac{\varepsilon}{R+r}$

Terminal potential difference

$$V = IR = \left(\frac{\varepsilon}{R+r}\right)R$$

$$= \frac{\varepsilon}{(R+r)/R} = \frac{\varepsilon}{1+\frac{r}{R}}$$

When R increases r/R decreases, so terminal potential difference increases with the increase of R.

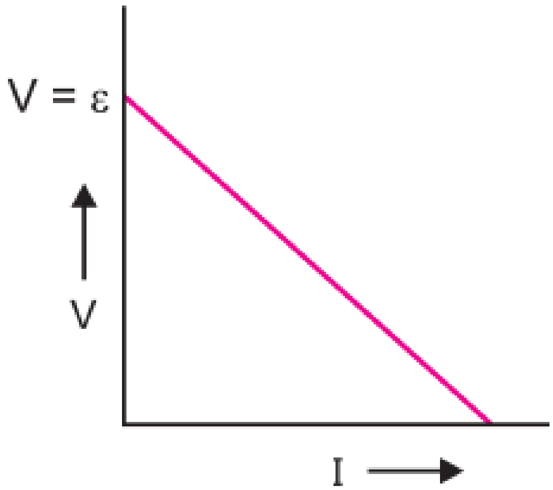
When $R = 0,\ V \rightarrow 0$

When $R = r,\ V = \frac{\varepsilon}{2}$

When $R \rightarrow \infty$ (open circuit), $V = \varepsilon$

The graph of terminal potential difference V versus R is shown in figure.

(ii) $V = E - Ir \quad \Rightarrow \quad I = \frac{E - V}{r}$



Short Answer Questions – II (OIQ)

Q. 1. Define resistivity of a conductor. Plot a graph showing the variation of resistivity with temperature for a metallic conductor. How does one explain such a behaviour, using the mathematical expression of the resistivity of a material?

Ans.

We know that, $R = \rho\frac{l}{A}$

If $l = 1, A = 1 \Rightarrow \rho = R$

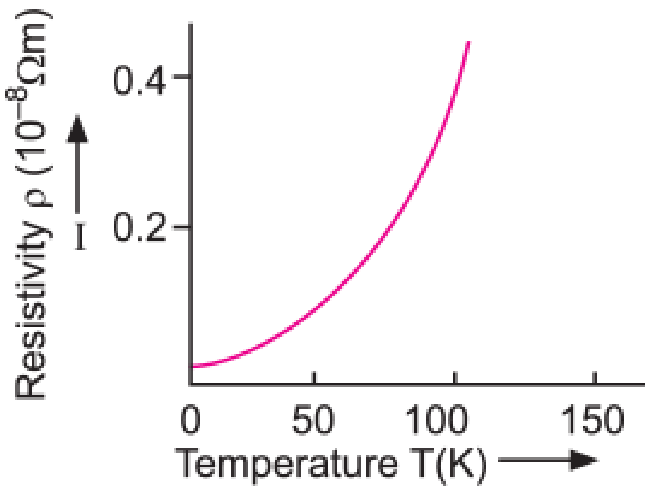
Thus, resistivity of a material is numerically equal to the resistance of the conductor having unit length and unit cross-sectional area.

The resistivity of a material is found to be dependent on the temperature. Different materials do not exhibit the same dependence on temperature. Over a limited range of temperatures, that is not too large, the resistivity of a metallic conductor is approximately given by,

$$\rho_T = \rho_0[1 + \alpha(T - T_0)] \qquad ...(i)$$

Where ρ_T is the resistivity at a temperature T and ρ_0 is the same at a reference temperature T0, α is called the temperature co-efficient of resistivity.

Relation (i) implies that a graph of ρ_T plotted against T would be a straight line. At temperatures much lower than 0°C, the graph, however, deviates considerably from a straight line (Fig.).



Resistivity ρ_T of metallic conductor as a function of temperature T

Q. 2. Write the mathematical relation for the resistivity of a material in terms of relaxation time, number density, mass and charge of charge carriers in it. Explain using this relation, why the resistivity of a metal increases and that of a semiconductor decreases with rise in temperature.

Ans. Resistivity of a material, $\rho = \frac{m}{ne^2 \tau}$

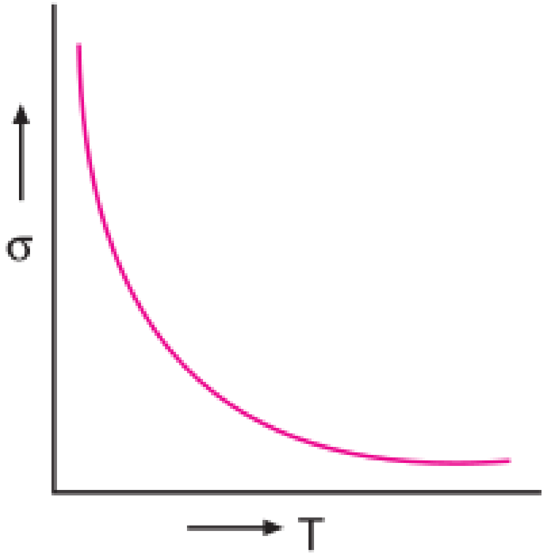
Where m is mass, e is charge on charge carrier, n is number density and τ is relaxation time.

For a metallic conductor: When temperature of a metal increases, the number of collisions of electrons with ion-lattice increases, so relaxation time decreases, as resistivity $\rho \propto \frac{1}{\tau}$, so resistivity of material increases with rise of temperature.

For a semiconductor: When temperature increases, the covalent bonds between valence electrons of atoms of semiconductor break, so more charge carriers (electrons and holes) becomes free. In other words the number density of charge carriers increases $\rho \propto \frac{1}{n}$, so resistivity of semiconductor decreases with the rise of temperature.

Q. 3. Explain the variation of conductivity with temperature for (i) a metallic conductor, (ii) ionic conductors and (iii) semiconductors.

Ans. Conductivity of a metallic conductor σ = $\frac{1}{\rho} = \frac{ne^2 \tau}{m}$.



Where m = mass of charge carrier, e = charge on each carrier

τ = relaxation time, n = number density of charge carriers

(i) With rise of temperature, the collision of electrons with fixed lattice ions/atoms increases so that relaxation time (τ) decreases. Consequently, the conductivity of metals decreases with rise of temperature.

(ii) Conductivity of ionic conductor increases with increase of temperature because with increase of temperature, the ionic bonds break releasing positive and negative ions which are charge carriers in ionic conductors.

(iii) In the case of a semiconductors, when temperature increases, covalent bonds break and charge carriers (electrons and holes) become free i.e., n increases, so conductivity increases with rise of temperature.

Q. 4. Two heater coils made of same material are connected in parallel across the mains. The length and diameter of the wire of one of coils are double that of the other. Which one of them will produce more heat?

Ans. We have resistance of one wire

$$R_1 = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2} = \frac{4\ \rho l}{\pi D^2}$$

Where l is length and D is diameter of the wire. The resistance of second wire of double the length and double the diameter is,

$$R_2 = \frac{4\rho\,(2l)}{\pi\,(2D)^2} = \frac{4\rho\, l}{\pi D^2}.\frac{1}{2},\ i.e.,\ R_2 = \frac{R_1}{2}$$

Heat produced per second

$$H = \frac{V^2}{R} \propto \frac{1}{R}$$

As second coil has resistance equal to half of first coil, therefore **heat produced in second coil is double than that in first coil.**

Q. 5. Two heaters are marked 200 V, 300 W and 200 V, 600 W. If the heaters are connected in series and the combination connected to a 200 V dc supply, which heater will produce more heat?

Ans.

Resistance of heaters $R_1 = \frac{V^2}{P_1} = \frac{(200)^2}{300} = \frac{400}{3}\Omega$

$$R_2 = \frac{V^2}{P_2} = \frac{(200)^2}{600} = \frac{400}{6}\Omega$$

When heaters are connected in series, current in circuit,

$$I = \frac{V}{R_1+R_2} = \frac{200}{\frac{400}{3}+\frac{400}{6}} = 1\ A$$

Heat produced in 200 V, 300 W heater per second

$$Q_1 = I^2R_1 = (1)^2 \times \frac{400}{3} = 133.33\ \text{Js}^{-1}$$

Heat produced in 200 V, 600 W heater per second

$$Q_2 = I^2R_2 = (1)^2 \times \frac{400}{6} = 66.66\ \text{Js}^{-1}$$

Clearly heat produced in 300 W heater is more that produced in 600 W heater.

Q. 6. Two cells of emf 1 V, 2 V and internal resistances 2Ω and 1Ω respectively are connected in (i) series. (ii) Parallel. What should be the external resistance in the circuit so that the current through the resistance be the same in the two cases? In which case is more heat generated in the cells?

Ans. For parallel combination,

$$\text{Net emf, } \varepsilon = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_2}{r_1 + r_2}$$

$$\text{Net internal resistance, } r_{\text{int}} = \frac{r_1 r_2}{r_1 + r_2}$$

For series combination,

Net emf, $\varepsilon = \varepsilon_1 + \varepsilon_2$

Net internal resistance $r_{int} = r_1 + r_2$

Given, $\varepsilon_1 = 1$ V, $\varepsilon_2 = 2$ V, and $r_1 = 2\ \Omega$, $r_2 = 1\ \Omega$, $R_{ext} = R$

$$\therefore \quad \text{Current, } I_1 = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R} = \frac{1+2}{2+1+R} = \frac{3}{3+R}\ A \quad \ldots(i)$$

$$\text{Current, } \quad I_2 = \frac{(\varepsilon_1 r_2 + \varepsilon_2 r_1)/(r_1 + r_2)}{R + \{(r_1 r_2/(r_1 + r_2)\}} \ldots(ii)$$

$$= \frac{(1\times1+2\times2)/(2+1)}{R+(2\times1)/(2+1)} = \frac{5/3}{R+\frac{2}{3}} = \frac{5}{3R+2}$$

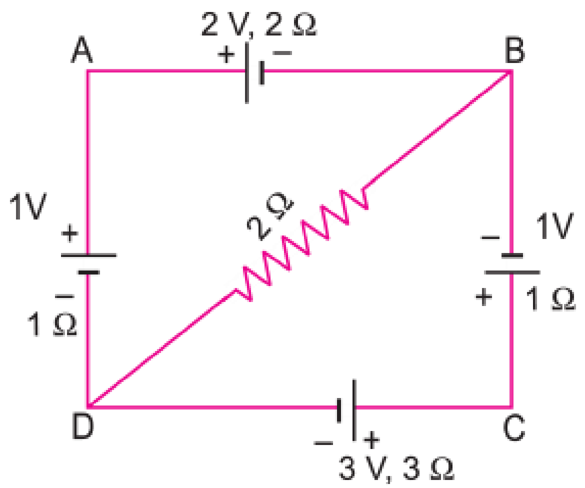
$$\text{Given } I_1 = I_2$$

$$\therefore \quad \frac{3}{3+R} = \frac{5}{3R+2} \quad \text{or} \quad 9R + 6 = 15 + 5\ R$$

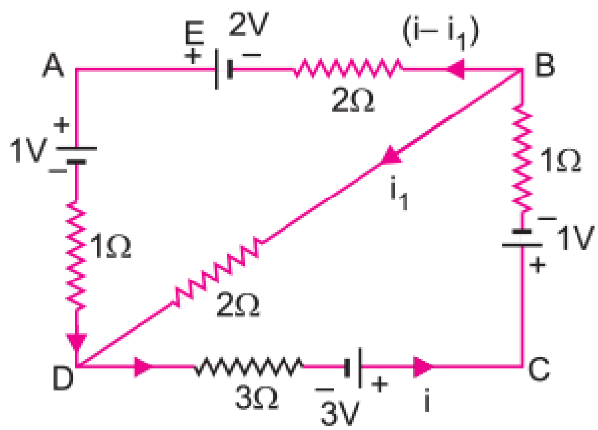
$$4R = 9 \qquad \Rightarrow R = \frac{9}{4} = 2.25\ \Omega$$

Heat generated in external resistance (I^2R) is same in both cases but heat generated in cells ($I^2\ r_{int}$) is more in series than that in parallel combination of cells.

Q. 7. For the circuit shown here, calculate the potential difference between points B and D.



Ans. According to Kirchhoff's first law the distribution of currents is shown in fig.



Applying Kirchhoff's second law to mesh BADB,

$$-2(i - i_1) + 2 - 1 - 1.\ (i - i_1) + 2i_1 = 0$$

$\Rightarrow$ $3i - 5i1 = 1$ …(i)

Applying Kirchhoff's law to mesh DCBD,

$$-3i + 3 - 1 - 1\times i - 2i_1 = 0$$

$\Rightarrow$ $4i + 2i_1 = 2$

Or $2i + i_1 = 1$ …(ii)

Multiplying equation (ii) with 5, we get

$10i + 5i1 = 5$ …(iii)

Adding (i) and (iii), we get

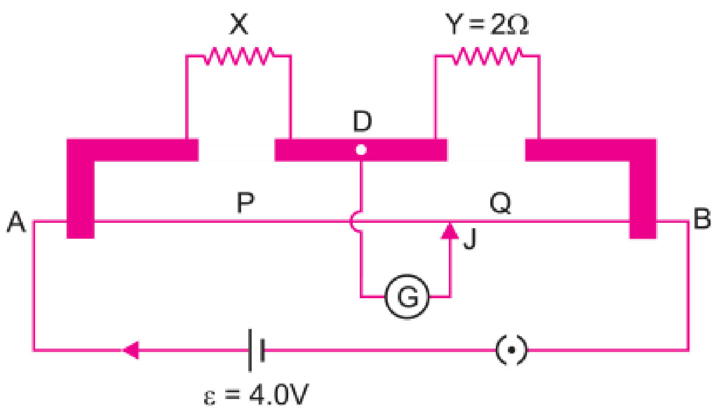
$$13i = 6 \Rightarrow i = \frac{6}{13} A$$

From (ii), $i_1 = 1 - 2i = 1 - \frac{12}{13} = \frac{1}{13} A$

Potential difference between B and D is

$$V_B - V_D = i_1 \times 2 = \frac{2}{13} V$$

Q. 8. In a practical Wheatstone bridge circuit, wire AB is 2 m long. When resistance Y = 2.0W and jockey is in position J such that AJ = 1.20 m, there is no current in galvanometer, find the value of unknown resistance X. The resistance per unit length of wire AB = 0.01 W/cm. Also calculate the current drawn by the cell of emf 4.0 V and negligible internal resistance.



Ans. P = Resistance of wire AJ

= (1.20 × 100 cm) × (0.01Ω/cm) = 1.20 Ω

Q = Resistance of wire BJ

= [(2–1.20) m×100] ×resistance per cm

= 0.80 × 100 cm × 0.01 Ω = 0.80Ω

Y = 2.0 Ω, X=?

When no current flows through the galvanometer, the bridge is balanced so

$$\frac{P}{Q} = \frac{X}{Y} \Rightarrow X = \frac{P}{Q}Y \quad \text{or} \quad X = \frac{1.20}{0.80} \times 2.0 = 3.0 \ \Omega$$

Total resistance of X and Y connected in series

$R_1 = X + Y = 3.0 + 2.0 = 5.0\ \Omega$

Total resistance of P and Q connected in series (or wire AB)

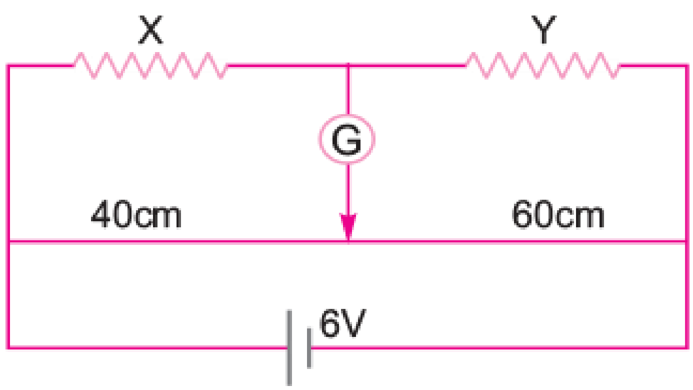
$R_2 = 2 \times 100 \times 0.01 = 2.0\Omega$

The resistance R_1 and R_2 are in parallel, so effective resistance between terminals A and B of bridge is

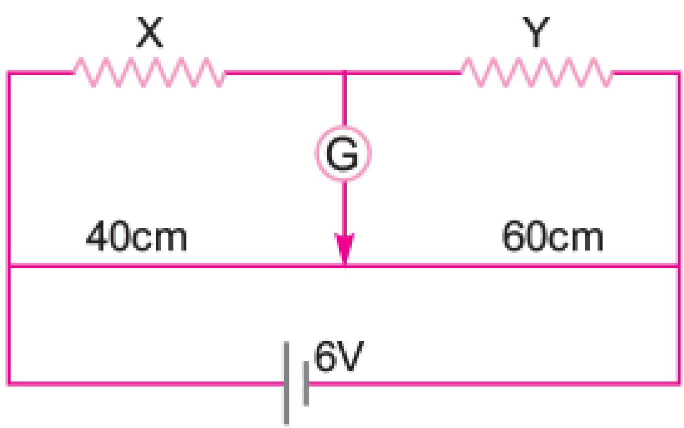
$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2} = \frac{5.0 \times 2.0}{5.0 + 2.0} = \frac{10}{7}\ \Omega$$

$$\text{Current drawn from battery } I = \frac{\varepsilon}{R_{AB}} = \frac{4.0}{10/7} = 2.8\text{A}$$

Q. 9. In the given circuit, a metre bridge is shown in the balanced state. The metre bridge wire has a resistance of 1 Ω cm^{-1}. Calculate the unknown resistance X and the current drawn from the battery of a negligible internal resistance if the magnitude of Y is 6Ω. If at the balancing point, we interchange the position of galvanometer and the cell, how it will affect the position of the galvanometer?



Ans. At balanced state



$$\frac{X}{Y} = \frac{40}{60} = \frac{2}{3} \Rightarrow \quad X = \frac{2}{3}Y = \frac{2}{3}\,?6$$

∴ $X = 4\Omega$

4Ω and 6Ω are in series, the equivalent resistance is given by

R_{eq} = 4Ω + 6Ω = 10Ω

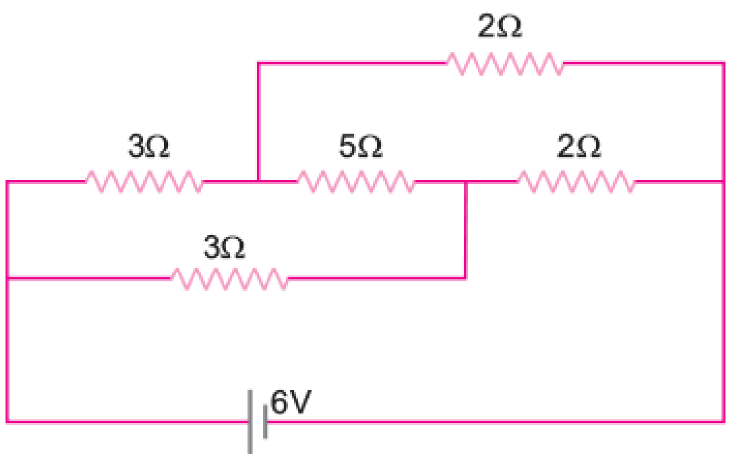
4Ω and 6Ω are in series R_{eq} = 10Ω

Resistance of bridge wire = 1 ohm/cm × 100 cm = 100Ω

10Ω and 100Ω are in parallel, = 1000/110Ω = 9.09Ω, Current = $\frac{6}{9.09}$

There will be no change in the balancing length if we interchange position of galvanometer and cell.

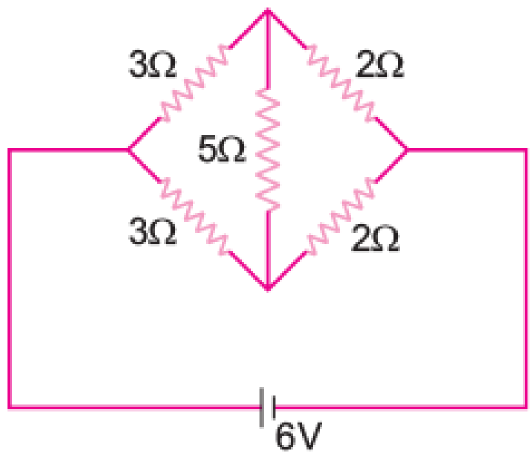
Q. 10. Calculate the current drawn from the battery in the given network shown here. State Kirchhoff's loop law and name the law on which it is based.



Ans. The equivalent circuit is as shown in figure alongside.

[∴ Bridge is in balanced condition, no current flows through 5Ω resistance]

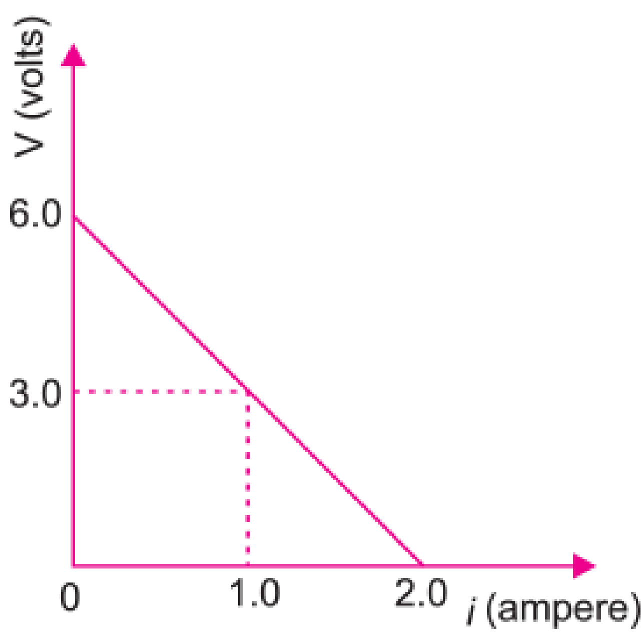
$\frac{1}{R} = \frac{1}{5} + \frac{1}{5}$ R = 5Ω



$$\text{Current in the circuit} = \frac{6}{2.5}A = 2.4A$$

Kirchhoff's Loop Law: The algebraic sum of potential differences of different circuit elements of a closed circuit (or mesh) is zero. This law is based on law of conservation of energy.

Q. 11. The graph shown here shows the variation of terminal potential difference V, across a combination of three cells in series to a resistor, versus current i: [HOTS]



(i) Calculate the emf of each cell.

(ii) For what current i, will the power dissipation of the circuit be maximum?

Ans. (i) Let ε be emf and r the internal resistance of each cell. The equation of terminal potential difference

$V = \varepsilon_{eff} - i\, r_{int}$ becomes

$V = 3\varepsilon - i\, r_{int}$ …(i)

Where r_{int} is effective (total) internal resistance.

From fig., when i = 0, V = 6.0 V

$$\therefore \text{ From } (i),\ 6 = 3\varepsilon - 0 \quad \Rightarrow \quad \varepsilon = \frac{6}{3} = 2\ \text{V}$$

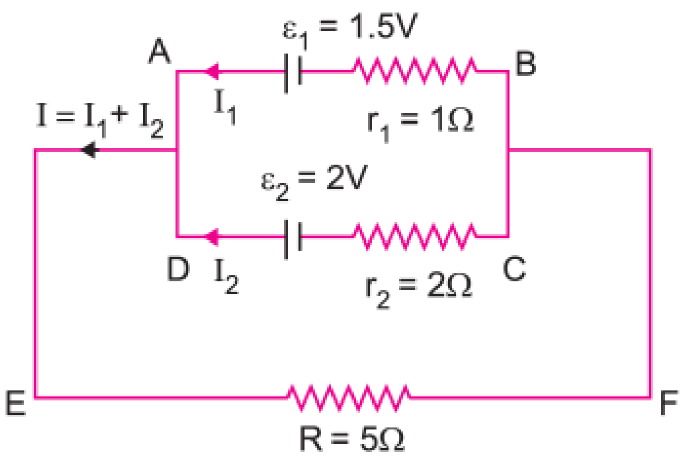
i.e., emf of each cell, ε = 2 V

Thus, emf of each cell, ε = 2V

Q. 12. Two cells of emf 1.5 V and 2 V and internal resistance 1Ω and 2Ω respectively are connected in parallel to pass a current in the same direction through an external resistance of 5Ω.

(i) Draw the circuit diagram.

(ii) Using Kirchhoff's laws, calculate the current through each branch of the circuit and potential difference across 5Ω resistor.



Ans. (i) The circuit is shown in figure.

(ii) Suppose I_1 are I_2 current drawn from cells ε1 and ε2 respectively, then according to Kirchhoff's junction law, current in R = 5Ω $I = I_1 + I_2$.

Applying Kirchhoff's second law to mesh ABFEA

$$1 \times I_1 + 1.5 - 5(I_1 + I_2) = 0$$

$\Rightarrow \quad 6I_1 + 5I_2 = 1.5$ …(i)

Applying Kirchhoff's second law to mesh CDEFC

$$-2I_2 + 2 - 5(I_1 + I_2) = 0$$

$\Rightarrow \quad 5I_1 + 7I_2 = 2 \quad \ldots(ii)$

Solving equation (i) and (ii), we get

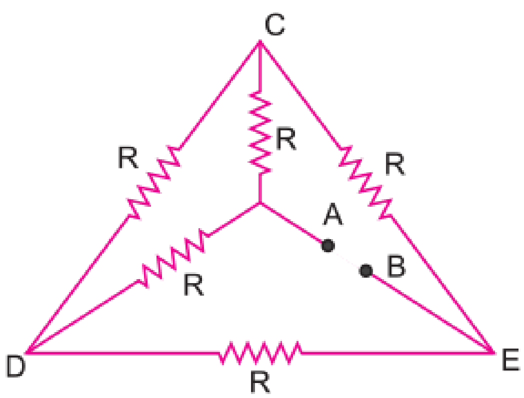
$$I_1 = \frac{1}{34}A,\ I_2 = \frac{9}{34}A$$

$$I = I_1 + I_2 = \frac{1}{34} + \frac{9}{34} = \frac{10}{34}A$$

Potential difference across $R=5\Omega$ resistor

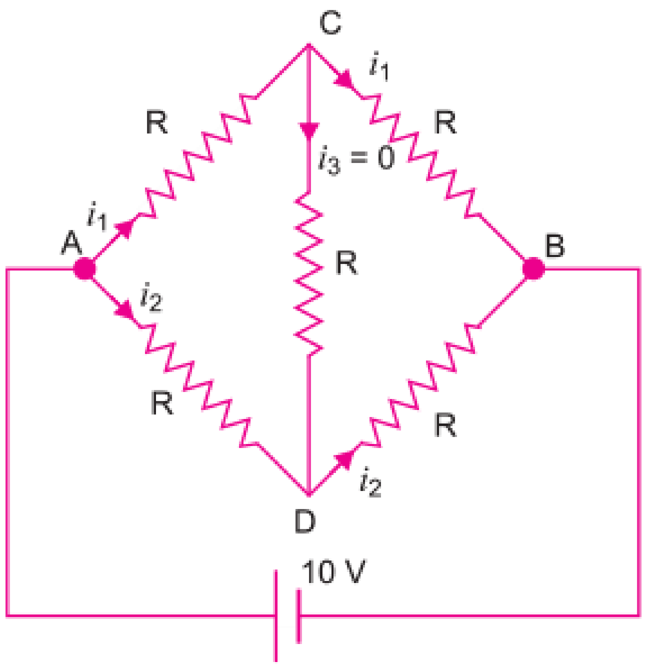
$$(I_1 + I_2)\,R = \frac{10}{34} \times 5 = \frac{25}{17} \text{ volt}$$

Q. 13. (i) Calculate the equivalent resistance of the given electrical network between points A and B.



(ii) Also calculate the current through CD and ACB if a 10 V dc source is connected between points A and B and the value of R = 2Ω.

Ans.



(i) The equivalent circuit is shown in fig. It is a balanced Wheatstone bridge. So, the resistance connected between C and D is ineffective.

Resistance of arm ACB, $R_1 = R + R = 2R$

Resistance of arm ADB, $R_2 = R + R = 2R$

Equivalent resistance between A and B, R_{AB} is given by

$$\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R} = \frac{2}{2R}$$

⇒ $R_{AB} = R = 2\Omega$

(ii) In arm CD, there is no current, $I_{CD} = 0$,

Current through arm ACB

$$i_1 = \frac{V}{R_1}$$

$$= \frac{10}{2R} = \frac{10}{2\times2} = \frac{10}{4} = 2.5A$$