

Topic : Definite Integration

Type of Questions

M.M., Min.

Single choice Objective (no negative marking) Q.1,2,3,4,5	(3 marks, 3 min.)	[15, 15]
Subjective Questions (no negative marking) Q.6,7	(4 marks, 5 min.)	[8, 10]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

1. $\int_2^4 \left(\log_x 2 - \frac{(\log_x 2)^2}{\ln 2} \right) dx$ equals to :

- (A) 0 (B) 1 (C) 2 (D) 4

2. The value of $\int_1^2 ([x^2] - [x]^2) dx$, where $[.]$ denotes the greatest integer function, is equal to :

- (A) $4 + \sqrt{2} - \sqrt{3}$ (B) $4 - \sqrt{2} + \sqrt{3}$ (C) $4 - \sqrt{3} - \sqrt{2}$ (D) none of these

3. The area of the closed figure bounded by $x = -1$, $x = 2$ and $y = \begin{cases} -x^2 + 2, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$ and the abscissa axis is

- (A) $16/3$ sq. units (B) $10/3$ sq. units (C) $13/3$ sq. units (D) $7/3$ sq. units

4. The area of the region for which $0 < y < 3 - 2x - x^2$ and $x > 0$ is

- (A) $\int_1^3 (3 - 2x - x^2) dx$ (B) $\int_0^3 (3 - 2x - x^2) dx$
(C) $\int_0^1 (3 - 2x - x^2) dx$ (D) $\int_{-1}^3 (3 - 2x - x^2) dx$

5. The area bounded by the curves $\sqrt{x} + \sqrt{y} = 1$ and $x + y = 1$ is

- (A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) none of these

6. Evaluate :
$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{\cos^{-1}\left(\frac{2x}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x + 1} dx$$

7. For $\theta \in (0, \pi) \cup (\pi, 2\pi)$ show that
$$\int_0^\infty \frac{dx}{x^2 + 2x\cos\theta + 1} = 2 \int_0^1 \frac{dx}{x^2 + 2x\cos\theta + 1}$$

8. **Column – I**

Column – II

(A) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function and $f(1) = 1$, $f'(1) = 3$. (P) 2

Then the value of $\lim_{x \rightarrow 1} \frac{\int_1^{x^2} (f(t) - t) dt}{(x-1)^2}$ is

(B) If $\int_0^3 (3ax^2 + 2bx + c) dx = \int_1^3 (3ax^2 + 2bx + c) dx$ where a, b, c are constants, then $a + b + c =$ (Q) 0

(C) Number of rational points $P \equiv (\alpha, \beta)$ lying on $(x - \sqrt{2})^2 + (y - \sqrt{3})^2 = 4$ (R) 1
is (rational point means x and y co-ordinate both are rational)

(D) Number of integral values of 'a' for which the function (S) 4
 $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$ is monotonic in \mathbb{R} is

Answers Key

1. (A) 2. (C) 3. (A) 4. (C)

5. (A) 6. $\frac{\pi}{2\sqrt{3}}$

8. $(A) \rightarrow (S), (B) \rightarrow (Q), (C) \rightarrow (Q), (D) \rightarrow (S)$