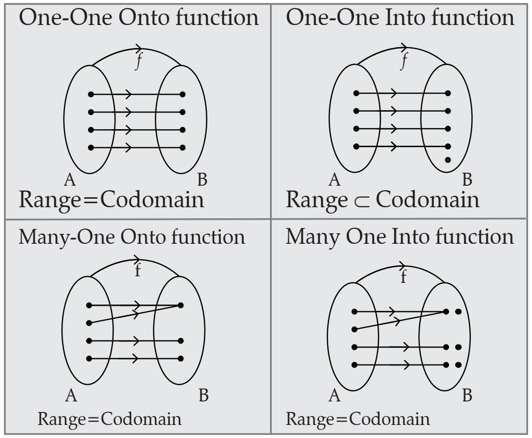
MIND MAP : LEARNING MADE SIMPLE CHAPTER - 2

Relations & Functions

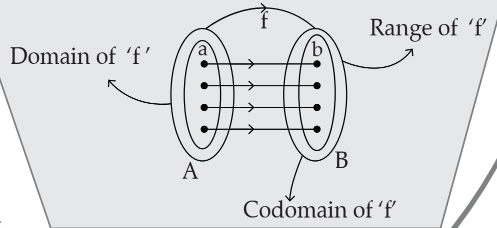
Kinds of Functions



Functions

Definition: A relation ' f ' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Notations: Domain(Input) $\xrightarrow{x}$ [f] ⟶ y=f(x) Range(Output)



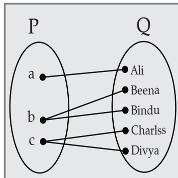
Cartesian product of sets

Given two non empty sets A&B. The cartesian product A×B is the set of all ordered pairs of elements from A&B i.e., A×B={(a,b) : a ∈ A ; b ∈ B}. If n(A)=p and n(B)=q , then n(A×B)=pq

Relation

Let A&B be two empty sets. Then any subset ' R ' of A x B is a relation from A to B.If (a,b) ∈R, then we write a R b , which is read as ' a is related to b ' by a relation R, 'b' is also called image of'a' under R.
The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A x B. If n(A)=p and n(B)=q, then n (A x B)=pq and total number relations is 2^{pq}.

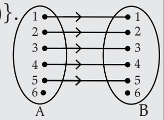
Pictorial representation of a relation



Given, R={(x,y): x is the first letter of the name y, x ∈P, y ∈Q}
Then, R={(a,Ali), (b, Beena), (c, charu)}This is a visual or pictorial representation of relation R (called an arrow diagram)is shown in figure.

Domain & range of a Relation

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. Symbolically,
Domain of R={x:(x,y) ∈ R}; Range of R={y:(x,y) ∈ R}
The set B is called co-domain of relation R.
Note: the range ⊆ Codomain.
Eg. Given, R={(1,2),(2,3),(3,4),(4,5),(5,6)}.
then Domain of R={1,2,3,4,5}
Range of R={2,3,4,5,6} and
codomain of R={1,2,3,4,5,6}



Inverse relation

Let A&B be two sets and R be a relation from set A to set B.Then inverse of R ,denoted by R^{-1}, is a relation from B to Aand is defined by
R^{-1}={(b,a) : (a,b) ∈ R}. Clearly, (a,b) ∈R ⇔ (b,a) ∈ R^{-1}
Also, Dom(R)=Range (R^{-1}) and Range(R)=Dom(R^{-1})

Even and odd function

Even function
$f(-x)=f(x)$, ∀x∈ Domain

Odd function
$f(-x)=-f(x)$, ∀x∈ Domain

Algebra of functions

Let $f: x \to R$ and $g \to R$ be any two real functions where X⊂R.
Addition: $(f+g)\,x = f(x)+g(x)$; ∀x∈ R
Subtraction: $(f-g)\,x = f(x)-g(x)$; ∀ x∈ R
Product: $(fg)\,x = f(x).g(x)$; ∀ x ∈ R
Quotient: $(f/g)\,(x) = f(x)/g(x)$; provided $g(x) \neq 0$, ∀ x ∈ R

Some standard real

Log function: $f(x)=\log_e x$, a>0 , a ≠ 1 Domain=x ∈ (0, ∞) Range=y ∈ R

Identity function: The function f:R → R defined by y=f(x)=x ∀ x ∈R is called identity function. Domain=R and Range=R

Constant function: The function f:R → R defined by y=f(x)=c, ∀ x ∈ R, where c is a constant is called constant function. Domain=R and Range ={c}

Modulus function: The function f:R → R defined by $f(x)=\begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$ is called modulus function.
It is denoted by y=f(x)=|x|. Domain=R and Range =(0, ∞)

Signum function: The function f:R → R defined by $f(x)=\begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is called signum function. It is usually denoted by y=f(x)=sgn(x) Domain=R and Range ={ 0, -1, 1}

Greatest integer function: The function f:R → R defined by as the greatest integer less than or equal to x .It is usually denoted by y=f(x)=[x]. Domain=R and Range=Z(All integers)

Exponential function: $f(x)=a^x$, a>0 , a ≠ 1,
Domain: x∈R: Range:f(x) ⊂ (0,∞)