Chapter -13

Direct and Inverse Proportion

J8C3G5

Look at the following examples –

(a) A pump can draw 100 litres of water in 10 minutes. Same pump can draw 200 litres of water in 20 minutes.

(b) Price of a book is Rs.150. Price of 7 similar books is Rs.1050.

(c) A labour can earn Rs.300 daily. He can earn Rs.900 in 3 days.

(d) Weight of one bag of rice is 10 kg. Weight of 4 similar bags of rice is 40 kg.

(e) Price of 40 oranges is Rs. 400. Price of 20 oranges is Rs. 200.

From the above examples, you have seen that, in examples (a), (b), (c) and (d) if quantity of one increases than the others also increase at same proportion. On the other hand, in example (e), when the number of oranges decreases, the amount of money required also decreases at the same proportion.

Activity

Fill in the blanks of the following table based on example (a).

Time (in minute)	1	2	3		10		20
Quantity of water (in litre)	10	20		50		180	

Fill in the blanks of the following table based on example (b).

Number of books	1	2		7	9	12		20
Price (in Rupees)	150		750	1050			2100	

Work out other examples also by putting numbers of your own choice and show it to your teacher. Such types of proportional increase or decrease is known as **Direct Variation** or **proportion**.

13.1 Direct Proportion

Consider one example. Suppose, the price of two lemons is Rs.10, price of three lemons is Rs.15, price of 7 lemons is Rs.35, price of 15 lemons is Rs.75 etc.

Let us prepare a table –

Number of lemons (x)	2 (x_1)	3 (x_2)	7 (x_3)	15 (x_4)
Price (y) (In Rupees)	10 (y_1)	15 (y_2)	35 (y_3)	75 (y_4)
$\frac{x}{y}$	$\frac{2}{10}=\frac{1}{5}$	$\frac{3}{15}=\frac{1}{5}$	$\frac{7}{35}=\frac{1}{5}$	$\frac{15}{75}=\frac{1}{5}$

In the above table, we denote number of lemons by x and its price by y. Note that if the number of lemons increases, the cost also increases proportionaly. On the other hand, cost of lemon decreases when the number of lemons decreases proportionally.

Again, $\frac{x}{y}=\frac{x_1}{y_1}=\frac{x_2}{y_2}=\frac{x_3}{y_3}=\frac{x_4}{y_4}=\frac{1}{5}$, a constant

If we denote $\frac{1}{5}$ by k, then

$$\frac{x}{y}=k \text{ or } x=ky$$

Such type of variation for which the increase (or decrease) of one variable x corressponds the increase (or decrease) of other variable y proportionally, is known as **direct variation** and we say that x is directly proportional to y and denote it by $x \propto y$ and in such cases, $\frac{x}{y}$ is always a constant.

In the above example, $k=\frac{1}{5}$, a constant (a fixed number). For other examples, value of k may be different.

From the above table, we can determine the price of lemon as the number of lemons changes.

Activity

A table showing the ages of Hima and Sima is as follows :

	Age 5 years ago	Present age	Age 5 years later
Age of Hima (x)	9	14	19
Age of Sima (y)	10	15	20
$\frac{x}{y}$	$\frac{9}{10}$	$\frac{14}{15}$	$\frac{19}{20}$

Note that in the above table, when x increases y also increases, but $\frac{x}{y}$ is not a constant i.e. **two variables may increase (or decrease) simultaneously but they may not be proportional.**

Example 1 : A car consumes 20 litres petrol to cover a distance of 240 km. Find the respective distance covered by the car when it consumes 5 litres, 8 litres, 12 litres and 25 litres of petrol.

Solution : Let, the amount of petrol consumed be x litre and the distance covered is y km. Now let us prepare the following table :

Petrol (x litre)	$20(x_1)$	$5(x_2)$	$8(x_3)$	$12(x_4)$	$25(x_5)$
Distance (y km)	$240(y_1)$	y_2	y_3	y_4	y_5

When the distance covered by the car increases, the amount of petrol consumed also increases. Therefore, petrol consumed is directly proportional to the distance covered.

Therefore, $\frac{x}{y} = k$ or $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \frac{x_4}{y_4} = \frac{x_5}{y_5}$

(i) $x_1 = 20,\ y_1 = 240,\ x_2 = 5,\ y_2 = ?$

Now $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

or, $\frac{20}{240} = \frac{5}{y_2}$

or, $\frac{1}{12} = \frac{5}{y_2}$

or, $y_2 = 12 \times 5 = 60$

(ii) $x_3 = 8, y_3 = ?$

Now $\frac{x_1}{y_1} = \frac{x_3}{y_3}$

or, $\frac{20}{240} = \frac{8}{y_3}$

or, $\frac{1}{12} = \frac{8}{y_3}$

or, $y_3 = 96$

(iii) $x_4 = 12,\ y_4 = ?$

Now $\frac{x_1}{y_1} = \frac{x_4}{y_4}$

or, $\frac{20}{240} = \frac{12}{y_4}$

or, $\frac{1}{12} = \frac{12}{y_4}$

or, $y_4 = 144$

(iv) $x_5 = 25,\ y_5 = ?$

Now $\frac{x_1}{y_1} = \frac{x_5}{y_5}$

or, $\frac{20}{240} = \frac{25}{y_5}$

or, $\frac{1}{12} = \frac{25}{y_5}$

or, $y_5 = 12 \times 25 = 300$

Now complete the table as given below :

Petrol (x litre)	20	5	8	12	25
Distance (y km)	240	60	96	144	300

Example 2 : Madhurjya can cover a distance of 1 km in 10 minutes on a bicycle. What will be the distance he can cover in 1 hour 20 minutes with the same speed ?

Solution : 1 hour 20 minutes = (60 + 20) minutes

= 80 minutes

First method : Let x and y represent time and distance covered respectively. Now, look at the following table. Here, more time is taken, more distance will be covered. Therefore it is direct proportion.

Time (x)	10(x_1)	80(x_2)
Distance (y)	1(y_1)	y_2

(or, $\frac{x}{y} = k$, a costant)

So $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

or, $\frac{10}{1} = \frac{80}{y_2}$

or, $10 \times y_2 = 80 \times 1$

or, $y_2 = \frac{80}{10} = 8$

$\therefore$ Madhurjya can cover 8 km distance in 1 hour 20 minutes.

Second Method (Unitary Method) :

In 10 minutes distance covered = 1 km

$\therefore$ In 1 minute distance covered $= \frac{1}{10}$ km

$\therefore$ In 80 minutes distance covered $= \frac{1}{10} \times 80$ km

$= 8$ km.

Third Method :

$\because$ More distance is covered in more time

$\therefore$ It is direct proportion

Since, we are to find distance and more distance covered in more time.

$\therefore$ Required distance = 1 km $\times \frac{80}{10} = 8$ km

(For more distance, we are to multiply by the greater ratio of time i.e $\frac{80}{10}$)

Example 3 : A train covers a distance 720 km in 18 hours. How much time will the train require to cover a distance of 200 km with same speed?

Solution : First Method :

Let x and y represent time and distance respectively.

Now, let us prepare the following table –

Time (x)	18(x_1)	x_2
Distance (y)	720(y_1)	200(y_2)

Here, more distance, more time. Therefore, it is direct proportion

$\therefore \quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$

or, $\quad \frac{18}{720} = \frac{x_2}{200}$

or, $\quad x_2 = \frac{18}{720} \times 200 = 5$

$\therefore$ The train will require 5 hours to cover 200 km.

Second Method (Unitary Method) :

To cover 720 km, time required 18 hours

$\therefore$ To cover 1 km, time required $\frac{18}{720}$ hours

$\therefore$ To cover 200 km, time required $\left(\frac{18}{720}\times 200\right)$ hours = 5 hours

Third Method :

Since less time will be required to cover less distance, therefore, it is direct proportion. Here, we are to find time and less time will be required.

$\therefore$ Time required $=18\times\frac{200}{720}=5$ hours

(For less time, we should multiply by lesser ratio i.e. $\frac{200}{720}$ which is less than 1).

Example 4 : The weight of 12 pages thick paper is 40 grams, how many such pages will weigh $2\frac{1}{2}$ kg?

Solution : $2\frac{1}{2}$ kg = (2000 + 500) gm

= 2500 gm

Let, the number of pages required be x. Now, let us prepare the table.

No of papers (in pages)	12	x
Weight of paper (in gm)	40	2500

When number of pages increases, the weight also increases. So, it is direct proportion.

$\therefore \frac{12}{40}=\frac{x}{2500}$

or, $40\times x=12\times 2500$

or, $x=\frac{12\times 2500}{40}$ = 750

$\therefore$ Required number of pages = 750

Briefly,

Number of pages $= 12 \times \frac{2500}{40}$ $\quad (\frac{2500}{40} > 1$, more weight more pages)

$= 750$

Example 5 :

The scale of a map in centimeter is 1 : 300000. In that map, distance between two cities is shown as 4 cm. Find their actual distance.

Solution :

Here, $300000 = 3 \times 10^5$

Let the distance in the map be x cm and actual distance be y cm

Given, $1 : 300000 = x : y$

or, $\frac{1}{3 \times 10^5} = \frac{x}{y}$

$\therefore$ For $x = 4$, we have,

$\frac{1}{3 \times 10^5} = \frac{4}{y}$

or, $y = 4 \times 3 \times 10^5$

$= 12 \times 10^5$

$\therefore$ Actual distance = 12 km (10^5 cm = 1 km)

Exercise 13.1

1. Observe the following tables and find whether x and y are in direct proportion or not.

i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

iii)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

2. If Principal is Rs. 1000, rate of interest is 8% per annum, then fill up the following table and find out in which case it is direct proportion.

Time	1 year	2 year	3 year
Simple interest (in Rupees)			
Compound interest (in Rupees)			

3. Karim covers a distance of 15 km per hour by cycling. How much distance will he cover in (i) 3 hours (ii) 5 hours (iii) 1 hour 20 minutes?
4. In 3 hours 15 minutes, Julumi reaches Kaziranga by a car travelling at an average speed of 50 km/hr. From what distance does she come to Kaziranga?
5. What distance will a plane covers in 2 hours 20 minutes flying at a speed of 510 km per hour?
6. Mary can run at a speed of 4.5 km per hour and Gurpreet can run 600 meters in 9 minutes. Who can run faster in an hour?
7. In a soft drink factory, 840 bottles are filled in 6 hours. How many bottles will be filled in 5 hours?
8. The height of the post of mast of a model ship is 9 cm. Height of the post of mast of the actual ship is 12 metres. If the length of the actual ship is 28 metres, then what will be the length of the model ship?
9. A vertical tower of height 5 m 60 cm casts a shadow of length 3 m 20 cm. At the same time
   (i) What will be the length of the shadow cast by a tower of length 10 m 50 cm?
   (ii) What will be the length of the tower which casts a shadow of length 5 m?
10. Limsing has a map of road. The ratio of the distance shown in the map and actual distance is 1 cm : 18 km. If he drives 72 km, what will be the distance in the map?

13.2 Inverse Proportion

Observe the following examples –

Example 1 : Distance between Nagaon and Guwahati is 120 km. Ramen reaches Guwahati in 3 hours starting from Nagaon by driving a car at a speed of 40 km/hr. Again, returning back to Nagaon, he requires 2 hours while driving at a speed 60 km/hr. Therefore, if speed is increased, less time will be required to cover a fixed distance.

Example 2 : 5 men can do a piece of work in 18 days. If 10 men are engaged to do the same work, then it will be completed in 9 days i.e. if number of men is increased, time required will decrease.

Example 3 : Rohdoi wants to buy a rectangular plot of land of area 144 square metres. If the length of one side of the plot is 12 metres (suppose x), then length of other side of the plot will be 12 metres (suppose y).

If she decreases the length of x to 9 metres, then she must increase the length of y to 16 metres. On the other hand, if she increases the length of x to 36 metres, then she must decrease the length of y to 4 metres. Because in each case the area of the plot remains same. i.e. for the fixed area of the rectangular plot, if the length of one side (x) increases, then the length of other side (y) decreases.

In Example 1, Distance = time × speed

If distance covered is fixed, time decreases when speed increases and vice versa.

In Example 2, for a fixed work, time decreases when number of men increases.

Now, complete the following tables according to the above examples –

Table 1

Speed of the car (x km/hr)	10		20	40	60	80
Time (y hour)	12	8		3	2	
xy	120	120	120		120	120

Table 2

Number of men (x)	5	10		18	30	
Time (y days)	18	9	6		3	2
xy	90	90	90	90		90

Table 3

Length of one side (x metre)	12	9	36		8	24	
Length of other side (y metre)	12	16	4	26		6	3
xy	144	144	144	144	144		144

In Table 1, if speed is represented by x and time taken is respresented by y, then their product xy is a constant. Because $x \times y = 120$ in each case.

Similarly in Table 2 and Table 3, the products of number of men and time and products of lengths are constant respectively.

From the discussion in the previous page, we see that when x increases (decreases) y decreases (increases) proportionaly. Their product is a constant. In this case, x is said to be inversely proportional to y.

It is written as $x \propto \frac{1}{y}$

or, $xy = k$ (a constant)

In Example 1, speed is x, time is y and k is120.

Activity

In the morning assembly of a school, the students are to be arranged in 10 rows and each row consists of 40 students. Now, with the same number of students, changing the number of rows, determine the number of students in each row and fill in the blanks of the following table.

Number of rows (x)	10 (x_1)	16 (x_2)	20 (x_3)	8 (x_4)
Number of students in each row (y)	40 (y_1)	..... (y_2)	..... (y_3)	..... (y_4)

(i) What have you seen in the table? Here, does y decrease when x increases or does y increase when x decreases?

(ii) Is $x \times y$ the same in each case?

(iii) Is $\frac{x_1}{x_2} = \frac{y_2}{y_1}$?

(iv) Is $x_2 : x_3 = y_3 : y_2$?

(v) Is $x_1 : x_2 = y_1 : y_2$?

Example 1 : Observe the following table and find those pairs of x and y that are inversely proportional.

i)

x	50	40	30	20
y	5	6	7	8

ii)

x	100	200	300	400
y	60	30	20	15

iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35

Solution :

(i)

x	50	40	30	20
y	5	6	7	8
$x \times y$	250	240	210	160

Since in each case $x \times y$ is not same, so, $x \times y$ is not a constant. Therefore, in this case x and y are not inversely proportional.

(ii)

x	100	200	300	400
y	60	30	20	15
$x \times y$	6000	6000	6000	6000

Since in each case $x \times y$ is same, so, $x \times y$ is a constant. Therefore, in this case x and y are inversely proportional.

(iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35
$x \times y$	900	900	900	750	600	175

Since, in each case, $x \times y$ is not same, so, $x \times y$ is not a constant. Therefore, in this case x and y are not inversely proportional.

Example 2 : If the pencils in a pencil box are distributed among 15 students, each student gets 8 pencils. If the number of students increased by 5, find the number of pencils each student will get.

Solution : **First Method :**

If the number of students increases, the number of pencils each student gets will decrease.

$\therefore$ Number of students and the number of pencils that each student gets are inversely proportional.

Let number of students be x and number of pencil that each student gets be y

Now, the following table is prepared.

Number of students (x)	15 (x_1)	20 (x_2)
Number of pencils that each students get (y)	8 (y_1)	? (y_2)

Since x and y are inversely proportional.

$\therefore \quad x_1y_1 = x_2y_2$

Or, $15 \times 8 = 20 \times y_2$

Or, $y_2 = \frac{15 \times 8}{20} = 6$

$\therefore$ Each student will get 6 pencils.

Example 3 : A bus covers a distance in $4\frac{1}{2}$ hours moving at a speed 40 km/hr. If the bus returns back at a speed 45 km/hr., how much time will be required to cover the same distance?

Solution : Notice that if the bus moves at a higher speed to cover the same distance, it will take less time. Therefore, it is in inverse proportion.

Let the speed of the bus be x km/hr. and the time taken be y hours

Now, let us prepare the following table.

Speed of the bus (x km/hr.)	$40(x_1)$	$45(x_2)$
Time taken (y hrs)	$4\frac{1}{2}(y_1)$	$?\ (y_2)$

Since, x and y are inversely proportional

$x_1y_1 = x_2y_2$

or, $40 \times \frac{9}{2} = 45 \times y_2$

or, $\frac{9}{2} \times \frac{40}{45} = y_2$

or, $4 = y_2$

$\therefore$ The bus will take 4 hours to cover the same distance.

Example 4 : One man can complete a thatched house in 15 days. In how many days 3 men can complete the house?

Solution : Let the number of men be x and time taken be y.

x	$1(x_1)$	$3(x_2)$
y	$15(y_1)$	? (y_2)

Since, when the number of men increases, time taken will decrease. So it is a case of inverse proportion.

$\therefore\ x_1y_1 = x_2y_2$

or, $y_2 = \frac{1}{3} \times 15 = 5$

$\therefore$ 3 men can complete the work in 5 days.

Example 5 : 10 men can dig a pond in 17 days. In how many days 34 men can dig that pond?

Solution :

Let the number of men be x and time taken be y days.

x	$10(x_1)$	$34(x_2)$
y	$17(y_1)$	y_2

When number of men increases, time taken will decrease.

$\therefore$ It is a case of inverse proportion.

$\therefore\ x_1y_1 = x_2y_2$

or, $10 \times 17 = 34 \times y_2$

or, $y_2 = \frac{10 \times 17}{34} = 5$

$\therefore$ 34 men can dig the pond in 5 days.

Exercise 13.2

1. Write whether the following quantities are inversely proportional or not.
   (i) Number of workers and time taken to complete the work.
   (ii) Speed of a vehicle and time taken to cover a distance.
   (iii) Area of a cultivable land and its production.
2. People of a village decides to dig a public pond. If 30 villagers can dig it in 35 days, in how many days 210 villagers can dig the pond?
3. 15 women of a Self Help Group can weave 500 Gamosa in 24 days. In how many days 500 such Gamosa can be weaved by 5 women, 8 women or 24 women?
4. An empty tank can be filled up in 1 hour 20 minutes by 6 pipes. If 5 similar pipes are used, how much time will be required to fill up the tank?
5. A dam of length 4 km can be constructed in 42 days by 120 workers. How many workers will be needed to construct the same dam in 30 days?
6. A family consisting of 6 members can manage to run the expenditure of the family for 42 days by a fixed income. If one member in the family is increased, then how many days the family can be managed to run with the same income?
7. 35 women can complete a work in 160 days. In how many days 28 women can complete that work?
8. Two carpenters can complete 4 new windows in 7 days.
   (i) A carpenter fell ill before starting the work. In how many days the remaining carpenter can complete the work?
   (ii) How many carpenters will be needed to complete the work a single day?

Multiple Choice Questions :

1. If x and y are directly proportional, then, which one of the following is correct.
   a) $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ b) $x_1 y_1 = x_2 y_2$ c) $x_1 x_2 = y_1 y_2$ d) $\frac{x_1}{x_2} = \frac{y_2}{y_1}$
2. If $x_1 = 4, y_1 = 10, x_2 = 2$ and x, y are directly proportional, then $y_2 = ?$
   a) 20 b) 5 c) 25 d) 10
3. If x and y are inversely proportional, then which of the following is correct?
   a) y increases when x increases b) y decreases when x decrease
   c) y decreases when x increases d) None of the above

4. If weight of 5 books is 4 kg. then, what is the weight of 8 such books?
   a) 5 kg b) 6 kg c) 6.4 kg d) 10 kg
5. 10 persons can dig a tank in 6 hours. How many people can dig the same tank in 12 hours?
   a) 20 b) 5 c) 7 d) 15
6. A man can complete a work in 6 days. What portion of the work can be completed in a single day by 2 men?
   a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{2}$ d) $\frac{1}{12}$
7. 36 persons can do a piece of work in 20 days. In how many days 12 persons can do the same work?
   a) $\frac{20}{3}$ days b) 40 days c) 60 days d) 8 days
8. A canteen can provide food to 300 persons for 20 days. If 50 persons decreases, then, for how many days the canteen will run with the same food?
   a) 120 days b) 17 days c) 25 days d) 24 days

What we have learnt

1. Two quantities x and y are said to be directly proportional, if their ratios of increase or decrease are constant i.e. $\frac{x}{y} = k$ (where k is a positive number). If the values of x_1 and x_2 and the corresponding values of y be y_1 and y_2 then, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.
2. Two quantities x and y are said to be inversely proportional, if y decreases when x increases proportionally. In this case, the product of their corresponding values become a costant i.e. if $xy = k$, then x and y are said to be inversely propotional. If, for two values x_1 and x_2 the corresponding values of them be of y_1 and y_2, then,

   $x_1 y_1 = x_2 y_2$ or, $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

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