

15. STABILITY OF SLOPES

→ Forces which cause failure of Slopes:

- (i) Gravitational Force.
- (ii) Seepage force.
- (iii) Earthquake Force.
- (iv) Construction equipment loads.

→ Types of Slopes:

- (i) Infinite slope. Eg: mountain slope.
- (ii) Finite slope. Eg: embankment of roads, earthen dams, canals etc.

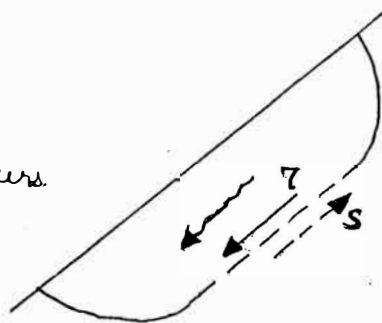
→ Types of Slope Failures:

- (i) Translational failure.
- (ii) Rotational Failure.
- (iii) Wedge Failure.
- (iv) Compound Failure.

* Translational Failure.

If $\tau > s$, translational failure occurs.

∴ to avoid failure, τ must be kept less than s .



$$\boxed{FOS = \frac{s}{\tau}}$$

if

$FOS > 1$; it is safe.

$FOS < 1$; unsafe.

$FOS = 1$; critical

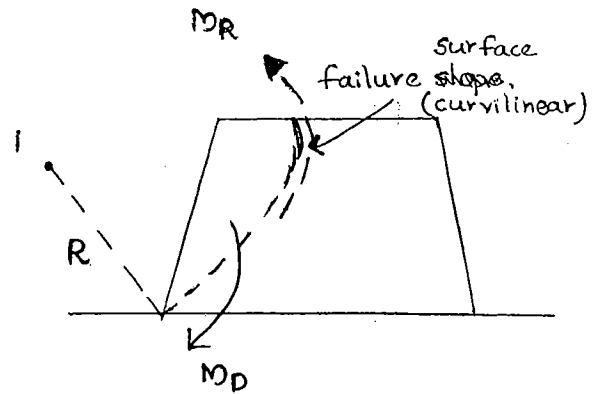
* Rotational Failure

M_D = driving moment.

M_R = resisting moment.

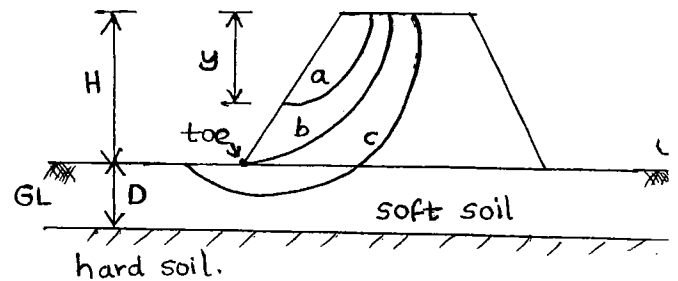
If $M_D > M_R$, failure occurs

$$\therefore FOS = \frac{M_R}{M_D}$$



— types of rotational failures:—

- Face failure
- Toe failure.
- Base failure.



Depth factor, $D_F = \frac{H+D}{H}$

For base failure, $D_F > 1$ (when there is soft soil)

For toe failure, $D_F = 1$ (when there is no soft soil).

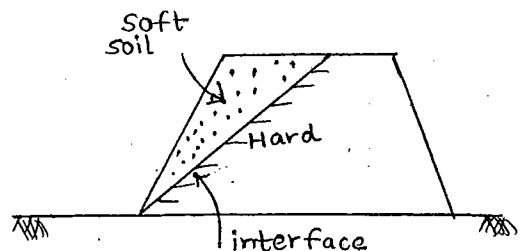
For face failure, $D_F < 1$

Depth factor, $D_F = \frac{y}{H}$

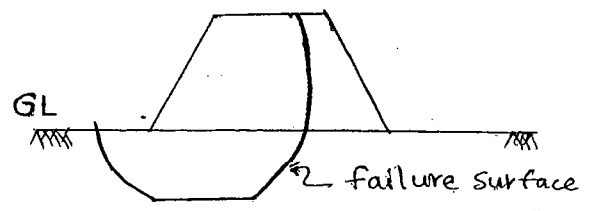
where $y \rightarrow$ vertical depth of point where failure surface passes as shown.

* Wedge Failure

The soft soil above the interface b/w soft soil and hard soil will fail as a wedge.



* Compound Failure.



→ Infinite Slope:

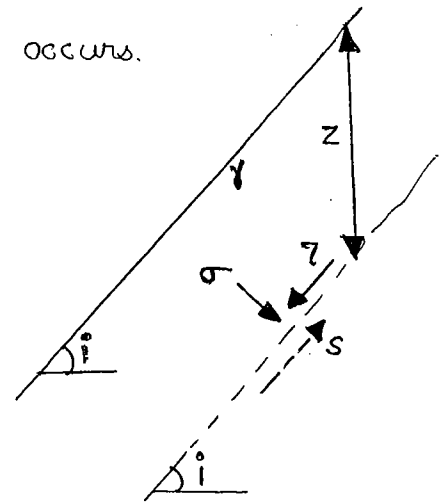
- generally, translational failure occurs.

$$FOS = \frac{S}{\tau}$$

$$S = c + \sigma \tan \phi$$

$$\sigma = \gamma z \cos^2 i$$

$$\tau = \gamma z \cos i \cdot \sin i$$



* Infinite Slope in c-φ soil:-

a) Dry or Partially Saturated Soil.

$$FOS = \frac{c + \sigma \tan \phi}{\tau} = \frac{c + \gamma z \cos^2 i \tan \phi}{\gamma z \cos i \cdot \sin i}; \text{ (at a depth } z \text{)}$$

b) Fully submerged soil. (mountain in ocean).

$$FOS = \frac{c' + \gamma' z \cos^2 i \cdot \tan \phi'}{\gamma' z \cos i \cdot \sin i}$$

c) If there is seepage parallel to slope. (rainwater seeping).

$$FOS = \frac{c' + \gamma' z \cos^2 i \cdot \tan \phi'}{\gamma_{sat} \cdot z \cdot \cos i \cdot \sin i}$$

* Infinite Slope in Cohesionless soils (c=0)

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a) Dry or Partially Saturated Soil.

$$FOS = \frac{\tan \phi}{\tan i} \text{ (if } i > \phi; \text{ it fails)}$$

b) For fully submerged slope.

$$FOS = \frac{\tan \phi'}{\tan i}$$

c) Seepage parallel to slope,

$$FOS = \left(\frac{\gamma'}{\gamma_{sat}} \right) \frac{\tan \phi'}{\tan i}$$

→ Finite Slope

- generally, rotational failure occurs.

- methods of analysis:

1. $\phi_u = 0$ Analysis.
2. Method of slices.
3. Bishop's method.
4. Friction circle method.
5. Stability number method.

* $\phi_u = 0$ Analysis:

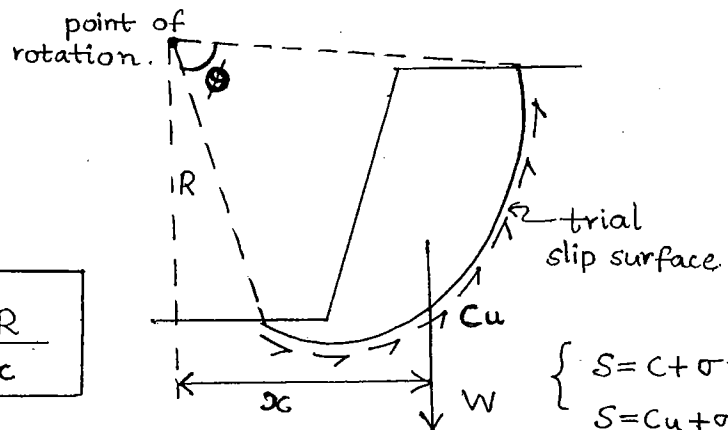
- suitable only for undrained saturated clays.

- graphical method based on trial & error.

$$\therefore M_D = W \cdot x$$

$$M_R = C_u \cdot \hat{L} \times 1 \times R$$

$$\boxed{FOS = \frac{M_R}{M_D} = \frac{C_u \hat{L} R}{W \cdot x}}$$



$$\begin{cases} S = C + \sigma \tan \phi \\ S = C_u + \sigma \tan \phi \\ \Rightarrow S = C_u \end{cases}$$

$\hat{L} \rightarrow$ arc length.

$$\hat{L} = R \theta \cdot \frac{\pi}{180}$$

$W \rightarrow$ weight of trial failure wedge.

$W = a \times 1 \times \gamma$; $a \rightarrow$ area of trial failure wedge.

$\hookrightarrow$ use γ' for submerged slope.
(canal running full)

use γ_{sat} for sudden drawdown condition

$$FOS \propto \frac{1}{W} ; \text{ but } W \propto \gamma$$

$$\Rightarrow FOS \propto \frac{1}{\gamma}$$

$\therefore$ for a canal slope, the critical condition is sudden drawdown condition.

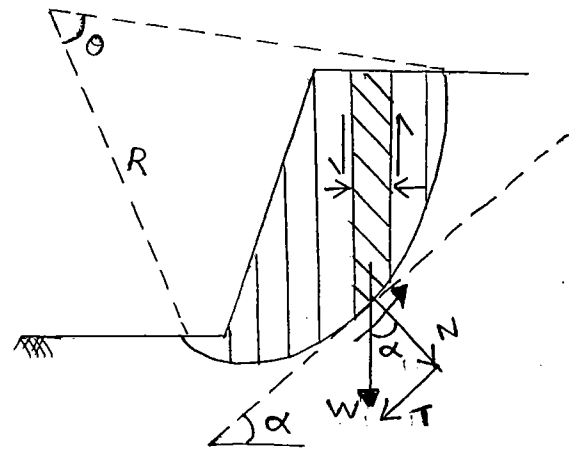
⑦ Canal will have more FOS when running full compared to sudden drawdown condition. 70

During sudden drawdown, γ increases to γ_{sat} and $\therefore$ FOS decreases.

⑧ Among the various trials, the trial slip surface which gives min. factor of safety is called 'Critical Slip Surface' and that min. FOS is taken as the FOS of the slope.

* Method of Slices:

- used for all soils.
- trial and error.
- forces acting on the sides of slices are neglected.



$$N = W \cos \alpha$$

$$T = W \sin \alpha$$

$$F = \frac{cL + \sum N \tan \phi}{\sum T}$$

⑨ When there is seepage,

$$F = \frac{c'L + \sum (N - u) \tan \phi'}{\sum T}$$

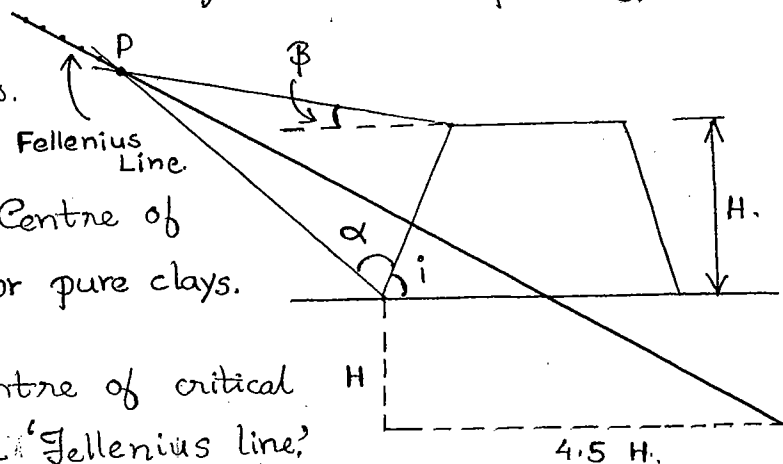
$\sum u \rightarrow$ sum of neutral forces.

⑩ Fellenius method to identify Critical Slip Circle:-

$\alpha, \beta \rightarrow$ direction angles.
(depend on i)

Point 'P' represents Centre of Critical Slip circle for pure clays.

For $c-\phi$ soils, the centre of critical slip circle lies on the 'Fellenius line'.



* Bishop's Method.

- forces acting on the sides of slices are also considered
- trial and error

* Friction Circle Method.

- trial and error

c = cohesion

c_m = mobilised cohesion.

ϕ = angle of internal friction.

ϕ_m = mobilised angle of internal friction.

S = shear strength.

S_m = mobilised shear strength.

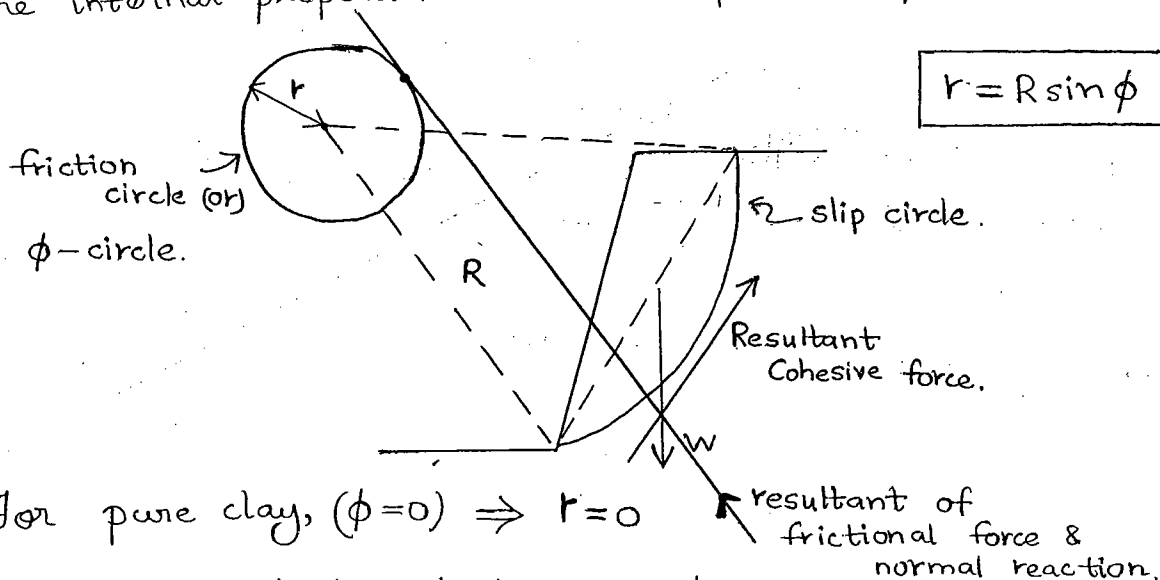
◉ 'Mobilised' means actually developed to keep system in eqbm. (wedge)

FOS wrt cohesion, $F_c = \frac{c}{c_m}$

FOS wrt friction, $F_\phi = \frac{\tan \phi}{\tan \phi_m}$

FOS wrt shear strength, $F = \frac{S}{S_m} = \frac{c + \sigma \tan \phi}{c_m + \sigma \tan \phi_m}$

σ remains the same as its related to weight, whereas c & ϕ are internal properties which develops as required.



◉ For pure clay, ($\phi = 0$) $\Rightarrow r = 0$

◉ Resultant of frictional force and normal reaction will be tangential to the friction circle.

* Taylor's Stability Number Method.

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72 ✓

- developed based on Friction Circle method.

$$\text{Stability number, } S_n = \frac{C}{F_c \cdot \gamma H}$$

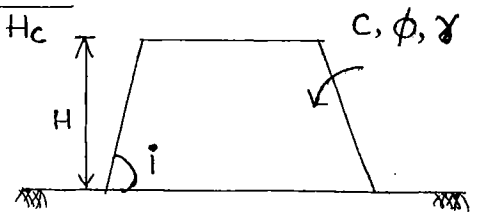
where $F_c = \frac{C}{C_m} = \frac{H_c}{H}$; $H_c \rightarrow$ critical height.

$H_c \rightarrow$ max. permitted height which can be provided for given soil against failure.

$H \rightarrow$ safe or actual height.

$$\Rightarrow S_n = \frac{C_m}{\gamma H} = \frac{C}{\gamma H_c}$$

$$\therefore F_c = \frac{C}{S_n \gamma H}$$



⊙ S_n depends on i & ϕ_m (or) i & depth factor

Knowing $F\phi$ & ϕ value, ϕ_m can be calculated by:-

$$F\phi = \frac{\tan \phi}{\tan \phi_m}$$

Max. value of $S_n = 0.261$ (for $i = 90^\circ$ & $\phi_m = 0$)

⊙ Except for cohesionless soil, this method is suitable for all soils. (for cohesionless, $c = 0 \Rightarrow F_c = 0$; meaningless)

⊙ For submerged slope, use γ'

For sudden draw down condition, use γ_{sat} & $\phi_m = \phi_w$

$$\phi_w = \text{weighted friction angle} = \frac{\gamma'}{\gamma_{sat}} \cdot \phi$$

25th Sept,
THURSDAY
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(71)
73

01. $F = \frac{c + \gamma z \cos^2 i \tan \phi}{\gamma z \cos i \sin i} \rightarrow \text{for } c-\phi \text{ soil.}$

i can be greater than ϕ .

03. $S_n = \frac{c}{F_c \gamma H} \Rightarrow F_c \propto \frac{1}{\gamma}$

04. $F = \frac{\gamma' \tan \phi'}{\gamma_{sat} \tan i}$

$$1.5 = \frac{19 - 9.81}{9.81} \frac{\tan 36}{\tan i}$$

$$\underline{i = 13.18^\circ}$$

If no seepage,

$$F = \frac{\tan \phi}{\tan i} \Rightarrow F = \underline{3.10^\circ}$$

05. Ignoring the crack,

a)

$$\theta = 109 + 12 = 121^\circ$$

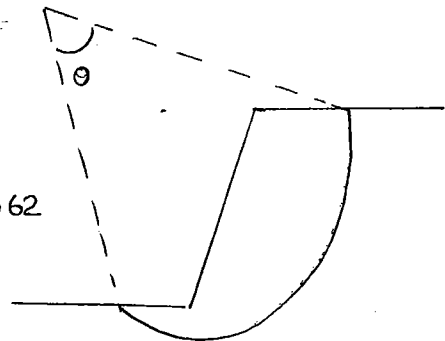
$$\hat{L} = R \theta \frac{\pi}{180} = 14.5 \times 121 \times \frac{\pi}{180} = 30.62$$

$$w = a \times 1 \times \gamma$$

$$= 110 \times 1 \times 18 = 1980$$

$$\bar{x} = 3.75 \text{ m.}$$

$$FOS = \frac{c \hat{L} R}{w \bar{x}} = \frac{27 \times 30.62 \times 14.5}{1980 \times 3.75} = \underline{1.61}$$



b). $\theta = 109^\circ$,

$$L = R \theta \times \frac{\pi}{180} = 27.585$$

$$w = a \times 1 \times \gamma \quad \bar{x} = 3.75 \text{ m.}$$

$$= (110 - 1.5) \times 18$$

$$= 1953$$

$$F = \frac{c_u \hat{L} R}{w \bar{x}}$$

$$= \frac{27 \times 27.585 \times 14.5}{1953 \times 3.75} = \underline{1.47}$$

6. $H = 25\text{m}$, $C = 35$, $\phi = 15^\circ$

a) $\gamma = 20$.

$$F_\phi = \frac{\tan \phi}{\tan \phi_m}$$

$$1.5 = \frac{\tan 15}{\tan \phi_m} \Rightarrow \phi_m \approx 10^\circ$$

For $\phi_m = 10^\circ$, $S_n = 0.06$.

$$S_n = \frac{C}{F_c \gamma H}$$

$$0.06 = \frac{35}{F_c \times 20 \times 25} \Rightarrow F_c = \underline{\underline{1.167}}$$

b) If $F_c = 1.5$, $F_\phi = ?$

$$S_n = \frac{C}{F_c \gamma H} = 0.05$$

$$\phi_m = 12.5^\circ \text{ (From table given).}$$

$$F_\phi = \frac{\tan \phi}{\tan \phi_m} = \frac{\tan 15^\circ}{\tan 12.5^\circ} = \underline{\underline{1.208}}$$

7. Canal running full, (submerged condition)

a) $S_n = \frac{C}{F_c \gamma' H}$

Since F_ϕ is not given, taken $\phi_m = \phi$.

$$\therefore \phi_m = 15^\circ \rightarrow S_n = 0.083$$

$$S_n = \frac{C}{F_c \gamma' H}$$

$$0.083 = \frac{1.4}{F_c \times 0.945 \times 5}$$

$$\Rightarrow F_c = \underline{\underline{3.56}}$$

b) For suddenly draw down,

(72)
74

$$S_n = \frac{C}{F_c \gamma_{sat} \cdot H}$$

$$\phi_m = \phi_w$$

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \phi \approx 7.5^\circ \Rightarrow S_n = 0.122$$

$$\Rightarrow 0.122 = \frac{1.4}{F_c \times 1.945 \times 5}$$

$$\therefore F_c = \underline{\underline{1.18}}$$

8. $F_c = \frac{C}{C_m} = \frac{30}{22} = \underline{\underline{1.36}}$

$$F_\phi = \frac{\tan \phi}{\tan \phi_m} = \frac{\tan 15}{\tan 12} = \underline{\underline{1.26}}$$

$$F = \frac{S}{S_m} = \frac{C' + \sigma \tan \phi'}{C_m + \sigma \tan \phi_m} = \frac{62.17}{47.5} = \underline{\underline{1.308}}$$

* To find F_ϕ when $F_c = 1$:

$$S_m = C_m + \sigma \tan \phi_m$$

$$47.5 = \frac{C}{F_c} + 120 \frac{\tan \phi'}{F_\phi}$$

$$= \frac{30}{1} + \frac{120 \tan 15}{F_\phi}$$

$$\Rightarrow F_\phi = \underline{\underline{1.829}}$$

Similarly, to find F_c when $F_\phi = 1$.

$$47.5 = \frac{C}{F_c} + \frac{120 \tan \phi'}{1}$$

$$F_c = \underline{\underline{1.95}}$$

- Q A granular soil possesses $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$. If $\phi' = 35^\circ$, and the designed FOS is 1.5, what is safe angle of slope for this soil when seepage occurs parallel to slope surface

$$F = \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi'}{\tan i}$$

$$1.5 = \frac{10}{20} \frac{\tan 35^\circ}{\tan i}$$

$$\Rightarrow i = \underline{\underline{13.14^\circ}}$$

Q

$$c = 15 \text{ kN/m}^2$$

$$\phi = 20^\circ$$

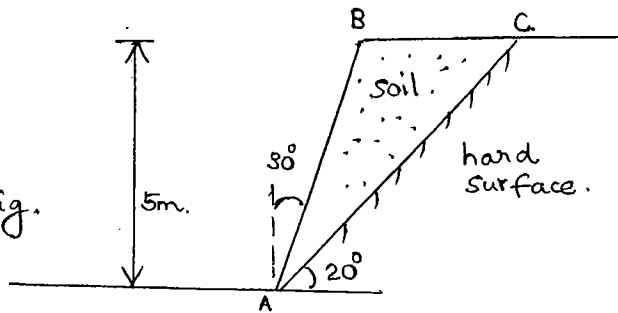
$$\gamma = 18 \text{ kN/m}^3$$

A slope is shown in the fig.

If the properties of soil

are as above, find the

FOS against possible wedge failure at the interface.



$$F = \frac{CL + N \tan \phi}{T}$$

L = length of AC

N = normal component of weight, w ; $N = w \cos 20^\circ$

$w \rightarrow$ weight of wedge, ABC.

$$T = w \sin 20^\circ$$

T = Tangential component of weight, w .

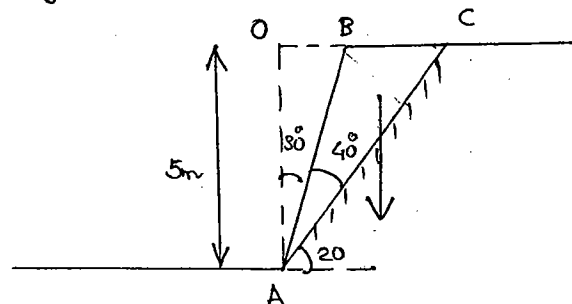
$$\tan 70^\circ = \frac{OC}{5}$$

$$OC = 13.73 \text{ m.}$$

$$\tan 30^\circ = \frac{OB}{5}$$

$$OB = 2.88 \text{ m.}$$

$$BC = OC - OB = 13.73 - 2.88 = 10.85 \text{ m.}$$



$$Ac = \sqrt{5^2 + 13.73^2} = 14.61 \text{ m}$$

(73)

$$W = \left(\frac{1}{2} \times 10.85 \times 5 \right) \gamma = 488.25 \text{ kN}$$

area of wedge.

74

$$N = 488.25 \cos 20 = 458.805$$

$$T = 488.25 \sin 20 = 166.99$$

$$F = \frac{CL + N \tan \phi}{T} = \frac{15 \times 14.61 + 458.805 \tan 20}{167}$$

$$= \underline{\underline{2.31}}$$