Chapter

2

Motion In One Dimension

Position

Any object is situated at point O and three observers from three different places are looking at same object, then all three observers will have different observations about the position of point O and no one will be wrong. Because they are observing the object from different positions.

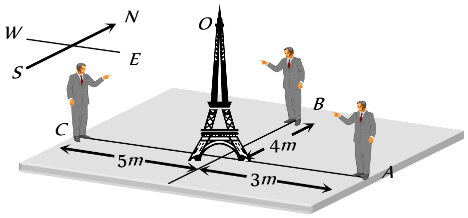


Fig. 2.1

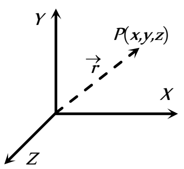
Observer 'A' says : Point O is 3 m away in west direction.

Observer 'B' says : Point O is 4 m away in south direction.

Observer 'C' says : Point O is 5 m away in east direction.

Therefore position of any point is completely expressed by two factors: Its distance from the observer and its direction with respect to observer.

That is why position is characterised by a vector known as position vector.



Consider a point P in xy plane and its coordinates are (x, y). Then position vector $(\vec{r})$ of point will be $x\hat{i} + y\hat{j}$ and if the point P is in space and its coordinates are (x, y, z) then position vector can be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Rest and Motion

If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference : It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that a tree on a platform is at rest. But the same passenger passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But observations are different because in first situation observer stands on a platform, which is reference frame at rest and in second situation observer moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references.

Table 2.1 : Types of motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
Ex. (i) Motion of car on a straight road. (ii) Motion of freely falling body.	*Ex.* (i) Motion of car on a circular turn. (ii) Motion of billiards ball.	*Ex.* (i) Motion of flying kite. (ii) Motion of flying insect.

Particle or Point Mass or Point object

The smallest part of matter with zero dimension which can be described by its mass and position is defined as a particle or point mass.

If the size of a body is negligible in comparison to its range of motion then that body is known as a particle.

A body (Group of particles) can be treated as a particle, depends upon types of motion. For example in a planetary motion around the sun the different planets can be presumed to be the particles.

In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

Distance and Displacement

(1) **Distance :** It is the actual length of the path covered by a moving particle in a given interval of time.

(i) If a particle starts from A and reach to C through point B as shown in the figure.

Then distance travelled by particle

$= AB + BC = 7\ m$

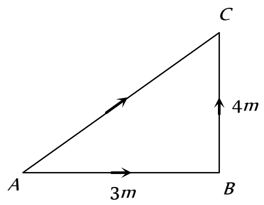


Fig. 2.2

(ii) Distance is a scalar quantity.

(iii) Dimension : $[M^0L^1T^0]$

(iv) Unit : metre (S.I.)

(2) **Displacement :** Displacement is the change in position vector i.e., A vector joining initial to final position.

(i) Displacement is a vector quantity

(ii) Dimension : $[M^0L^1T^0]$

(iii) Unit : metre (S.I.)

(iv) In the above figure the displacement of the particle $\vec{AC} = \vec{AB} + \vec{BC} \Rightarrow |\ AC|$

$= \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 90^o} = 5\ m$

(v) If $\vec{S}_1, \vec{S}_2, \vec{S}_3$ .... $\vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + ... + \vec{S}_n$

(3) **Comparison between distance and displacement :**

(i) The magnitude of displacement is equal to minimum possible distance between two positions.

So distance $\geq$ |Displacement|.

(ii) For a moving particle distance can never be negative or zero while displacement can be.

(zero displacement means that body after motion has came back to initial position)

i.e., Distance > 0 but Displacement > = or < 0

(iii) For motion between two points, displacement is single valued while distance depends on actual path and so can have many values.

(iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(v) In general, magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.

(vi) If $\vec{r}_A$ and $\vec{r}_B$ are the position vectors of particle initially and finally.

Then displacement of the particle $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

and s is the distance travelled if the particle has gone through the path *APB*.

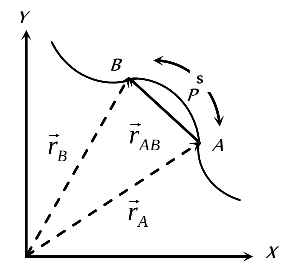


Fig. 2.3

Speed and Velocity

(1) **Speed :** The rate of distance covered with time is called speed.

(i) It is a scalar quantity having symbol υ .

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

(iv) Types of speed :

(a) **Uniform speed :** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5*m*) in each second. So we can say that particle is moving with uniform speed of 5 *m/s*.

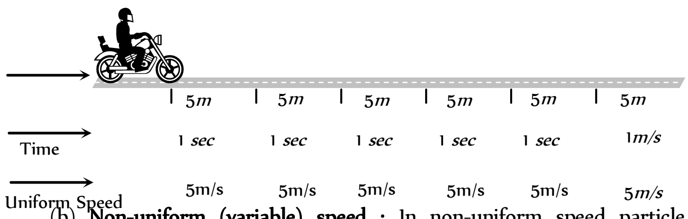


Fig. 2.4

(b) **Non-uniform (variable) speed :** In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5*m* in 1[st] second, 8*m* in 2[nd] second, 10*m* in 3[rd] second, 4*m* in 4[th] second *etc.*

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.

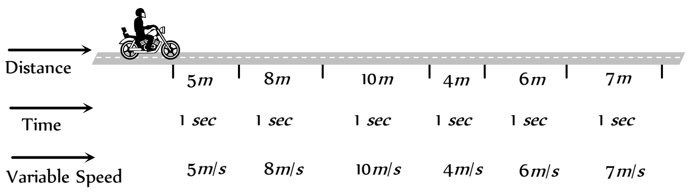


Fig. 2.5

(c) **Average speed :** The average speed of a particle for a given 'Interval of time' is defined as the ratio of total distance travelled to the time taken.

Average speed $= \dfrac{\text{Total distance travelled}}{\text{Time taken}}$; $v_{av} = \dfrac{\Delta s}{\Delta t}$

❑ Time average speed : When particle moves with different uniform speed υ_1, υ_2, υ_3 ... *etc* in different time intervals t_1, t_2, t_3, ... *etc* respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}}$$

$$= \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

❑ Distance averaged speed : When a particle describes different distances d_1, d_2, d_3, ... with different time intervals t_1, t_2, t_3, ... with speeds v_1, v_2, v_3...... respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...}$$

$$= \frac{d_1 + d_2 + d_3 + ...}{\dfrac{d_1}{\upsilon_1} + \dfrac{d_2}{\upsilon_2} + \dfrac{d_3}{\upsilon_3} + ...}$$

❑ If speed is continuously changing with time then

$$v_{av} = \frac{\int v dt}{\int dt}$$

(d) **Instantaneous speed :** It is the speed of a particle at a particular instant of time. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.*, $\Delta t \to 0$). Thus

Instantaneous speed $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

(2) **Velocity :** The rate of change of position *i.e.* rate of displacement with time is called velocity.

(i) It is a vector quantity having symbol $\vec{v}$.

(ii) Dimension : $[MLT^{-1}]$

(iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

(iv) Types of velocity :

(a) **Uniform velocity :** A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity :** A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes or both of them change.

(c) **Average velocity** : It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

(d) **Instantaneous velocity :** Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(v) **Comparison between instantaneous speed and instantaneous velocity**

(a) instantaneous velocity is always tangential to the path followed by the particle.

When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is $\vec{v}_1$, at point A the instantaneous velocity of stone is $\vec{v}_2$, similarly at point B and C are $\vec{v}_3$ and $\vec{v}_4$ respectively.

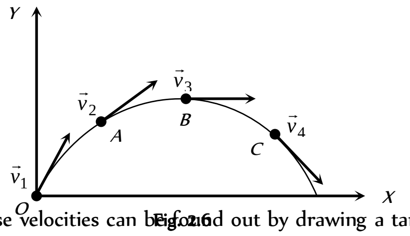


Fig. 2.6

Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

Instantaneous velocity $\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$

$\vec{v} = -A_1 + 2A_2 t$

For the given value of t, we can find out the instantaneous velocity.

e.g for $t = 0$,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

(vi) **Comparison between average speed and average velocity**

(a) Average speed is a scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = 0$ (as $\Delta \vec{r} = 0$) but $v_{av} > 0$ and finite as $(\Delta s > 0)$.

(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be *i.e.* $v_{av} > 0$ while $\vec{v}_{av}$ = or < 0.

(f) As we know for a given time interval

Distance ≥ |displacement|

∴ Average speed ≥ |Average velocity|

Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)

Table 2.2 : Possible ways of velocity change

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
Ex. Uniform circular motion	*Ex.* Motion under gravity	*Ex.* Projectile motion

(2) Dimension : $[MLT^{-2}]$

(3) Unit : *metre/second²* (S.I.); *cm/second²* (C.G.S.)

(4) Types of acceleration :

(i) **Uniform acceleration :** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

(ii) **Non-uniform acceleration :** A body is said to have non-uniform acceleration, if either magnitude or direction or both of them change during motion.

(iii) **Average acceleration :** $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

(iv) **Instantaneous acceleration** = $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.

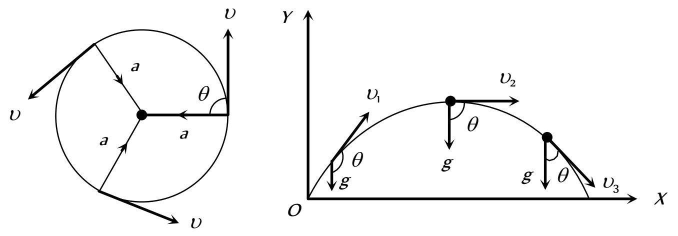


Fig. 2.7

Ex. (a) In uniform circular motion θ = 90° always

(b) In a projectile motion θ is variable for every point of trajectory.

(vi) If a force $\vec{F}$ acts on a particle of mass *m*, by Newton's 2nd law, acceleration $\vec{a} = \frac{\vec{F}}{m}$

(vii) By definition $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$ $\left[\text{As } \vec{v} = \frac{d\vec{x}}{dt}\right]$

i.e., if *x* is given as a function of time, second time derivative of displacement gives acceleration

(viii) If velocity is given as a function of position, then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v.\frac{d\upsilon}{dx}\left[\text{as } v = \frac{dx}{dt}\right]$$

(xi) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

(xii) For motion of a body under gravity, acceleration will be equal to "*g*", where g is the acceleration due to gravity. Its value is $9.8\ m/s^2$ or $980\ cm/s^2$ or $32\ feet/s^2$.

Position time Graph

During motion of the particle its parameters of kinematical analysis (*v*, *a*, *s*) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time *t* along *x*-axis and position of the particle on *y*-axis.

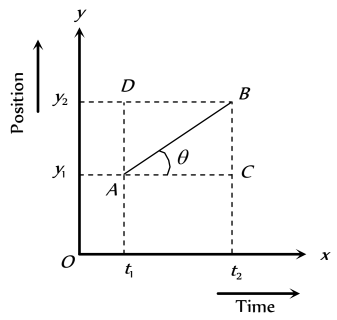


Fig. 2.8

Let *AB* is a position-time graph for any moving particle

As Velocity = $\frac{\text{Change in position}}{\text{Time taken}} = \frac{y_2 - y_1}{t_2 - t_1}$ ...(i)

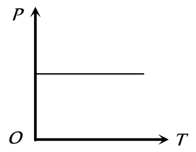
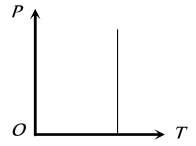
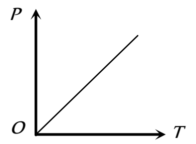
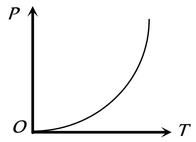
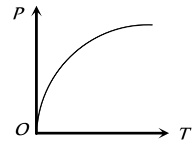
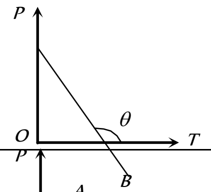
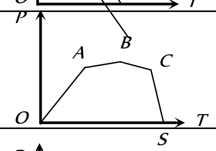
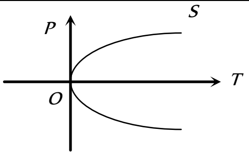
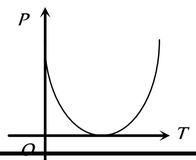
From triangle *ABC*, $\tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii) Velocity = tanθ

v = tanθ

It is clear that slope of tangent on position-time graph represents the velocity of the particle.

Table 2.3 : Various position -time graphs and their interpretation

	$\theta = 0°$ so $v = 0$ *i.e.*, line parallel to time axis represents that the particle is at rest.
	$\theta = 90°$ so $v = \infty$ i.e., line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity. Practically this is not possible.
	θ = constant so v = constant, $a = 0$ i.e., line with constant slope represents uniform velocity of the particle.
	θ is increasing so v is increasing, a is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so v will be constant but negative *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions, which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Note : ❑ If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A, it is not valid as shown in the figure.

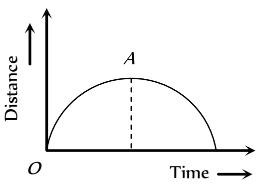


Fig. 2.9

Velocity-time Graph

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Calculation of Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

Total distance $= |A_1| + |A_2| + |A_3|$

= Addition of modulus of different area. *i.e.* $s = \int |v|\, dt$

Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign.

i.e. $r = \int v\, dt$

Area above time axis is taken as positive, while area below time axis is taken as negative

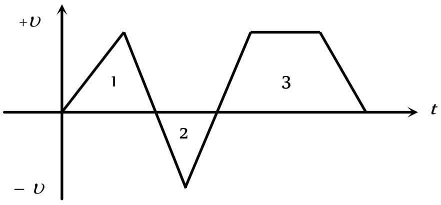


Fig. 2.10

here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Calculation of Acceleration : Let *AB* is a velocity-time graph for any moving particle

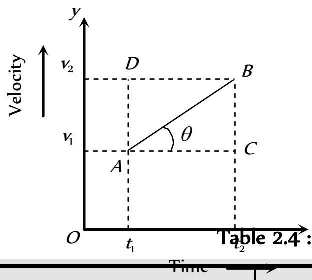


Fig. 2.11

As Acceleration = $\dfrac{\text{Change in velocity}}{\text{Time taken}}$

$$= \frac{v_2 - v_1}{t_2 - t_1} \qquad ...(i)$$

From triangle *ABC*, $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC}$

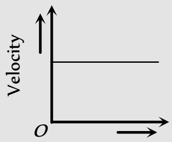
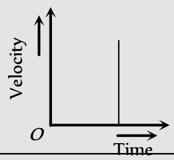
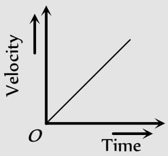
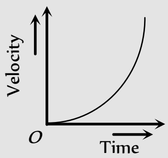
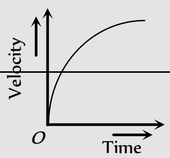
$$= \frac{v_2 - v_1}{t_2 - t_1} \qquad ....(ii)$$

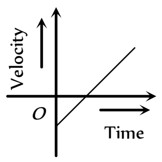
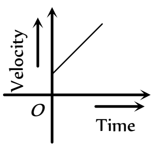
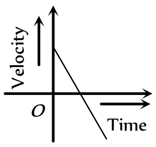
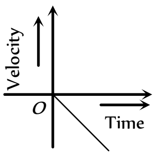
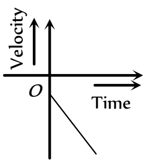
By comparing (i) and (ii)

Acceleration $(a) = \tan\theta$

It is clear that slope of tangent on velocity-time graph represents the acceleration of the particle.

Table 2.4 : Various velocity -time graphs and their interpretation

	$\theta = 0°$, $a = 0$, v = constant *i.e.*, line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90°$, $a = \infty$, v = increasing *i.e.*, line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	θ = constant, so a = constant and v is increasing uniformly with time *i.e.*, line with constant slope represents uniform acceleration of the particle.
	θ increasing so acceleration increasing *i.e.*, line bending towards velocity axis represent the increasing acceleration in the body.
	θ decreasing so acceleration decreasing

	i.e. line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and < 90° but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and < 90° but initial velocity of particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is negative.

Equation of Kinematics

These are the various relations between u, v, a, t and s for the particle moving with uniform acceleration where the notations are used as :

u = Initial velocity of the particle at time t = 0 *sec*

v = Final velocity at time t *sec*

a = Acceleration of the particle

s = Distance travelled in time t *sec*

s_n = Distance travelled by the body in n *sec*

(1) **When particle moves with zero acceleration**

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) $v = u$, $s = u\,t$ [As a = 0]

(2) **When particle moves with constant acceleration**

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion (in scalar from) / Equation of motion (in vector from)

Equations of motion (in scalar from)	Equation of motion (in vector from)
$\upsilon = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$\upsilon^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$

$$s_n = u + \frac{a}{2}(2n-1) \qquad \vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

Motion of Body Under Gravity (Free Fall)

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h << R$) is called free fall.

An ideal example of one-dimensional motion is motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) **If a body is dropped from some height (initial velocity zero)**

(i) Equations of motion : Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

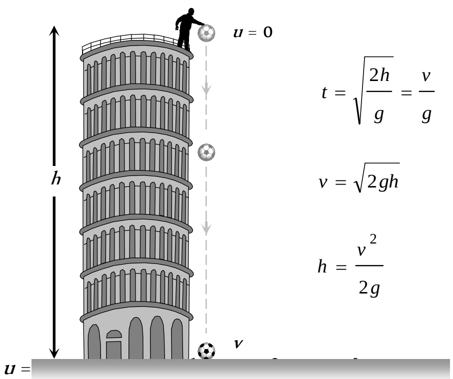


Fig. 2.12

$u = 0$

$a = +g$ [As acceleration is in the direction of motion]

$v = g\,t$ ...(i)

$h = \frac{1}{2}gt^2$ ...(ii)

$v^2 = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)

(ii) Graph of distance, velocity and acceleration with respect to time :

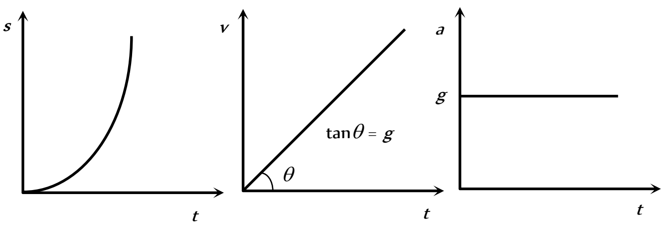


Fig. 2.13

(iii) As $h = (1/2)gt^2$, *i.e.*, $h \propto t^2$, distance covered in time t, $2t$, $3t$, *etc.*, will be in the ratio of $1^2 : 2^2 : 3^2$, *i.e.*, square of integers.

(iv) The distance covered in the *nth sec*, $h_n = \frac{1}{2}g(2n-1)$

So distance covered in 1st, 2nd, 3rd *sec*, *etc.*, will be in the ratio of 1 : 3 : 5, *i.e.*, odd integers only.

(2) **If a body is projected vertically downward with some initial velocity**

Equation of motion : $v = u + g\,t$

$h = ut + \frac{1}{2}g\,t^2$

$v^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n-1)$

(3) **If a body is projected vertically upward**

(i) Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$$v = u - g\,t\,; h = ut - \frac{1}{2}g\,t^2\,; v^2 = u^2 - 2gh; h_n = u - \frac{g}{2}(2n-1)$$

(ii) For maximum height $v = 0$

So from above equation $u = gt$,

$h = \frac{1}{2}gt^2$

and $u^2 = 2gh$

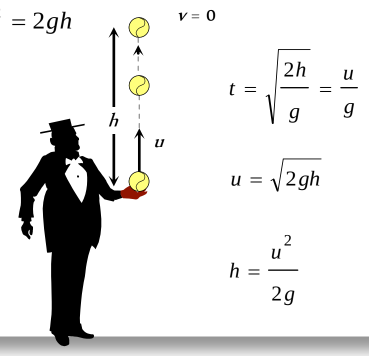


Fig. 2.14

(iii) Graph of displacement, velocity and acceleration with respect to time (for maximum height) :

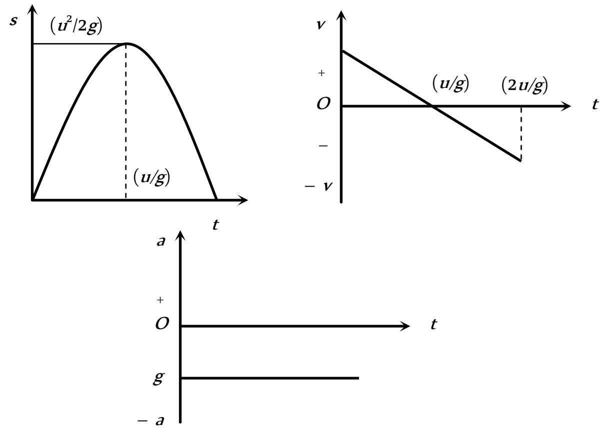


Fig. 2.15

It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.

(5) In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_2) = time of ascent (t_1) = u/g

$\therefore$ Total time of flight $T = t_1 + t_2 = \dfrac{2u}{g}$

(6) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(7) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \dfrac{u}{g+a}$

and $h = \dfrac{u^2}{2(g+a)}$

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

So $h = \dfrac{1}{2}(g-a)t_2^2$

$$\Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2$$

$$\Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing t_1 and t_2 we can say that $t_2 > t_1$

since $(g + a) > (g - a)$

Motion with Variable Acceleration

(i) If acceleration is a function of time

$a = f(t)$ then $v = u + \int_0^t f(t)\,dt$

and $s = ut + \int_0^t \left(\int f(t)\,dt \right) dt$

(ii) If acceleration is a function of distance

$a = f(x)$ then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)\,dx$

(iii) If acceleration is a function of velocity

$a = f(v)$ then $t = \int_u^v \dfrac{dv}{f(v)}$ and $x = x_0 + \int_u^v \dfrac{v\,dv}{f(v)}$

Tips & Tricks

✍ During translational motion of the body, there is change in the location of the body.

✍ During rotational motion of the body, there is change in the orientation of the body, while there is no change in the location of the body from the axis of rotation.

✍ A point object is just a mathematical point. This concept is introduced to study the motion of a body in a simple manner.

✍ The choice of the origin is purely arbitrary.

✍ For one dimensional motion the angle between acceleration and velocity is either 0° or 180° and it does not change with time.

✍ For two dimensional motion, the angle between acceleration and velocity is other than 0° or 180° and also it may change with time.

✍ If the angle between $\vec{a}$ and $\vec{v}$ is 90°, the path of the particle is a circle.

✍ The particle speed up, that is the speed of the particle increases when the angle between $\vec{a}$ and $\vec{v}$ lies between –90° and +90°.

✍ The particle speeds down, that is the speed of the particle decreases, when the angle between $\vec{a}$ and $\vec{v}$ lies between +90° and 270°.

✍ The speed of the particle remains constant when the angle between $\vec{a}$ and $\vec{v}$ is equal to 90°.

✍ The distance covered by a particle never decreases with time, it always increases.

✍ Displacement of a particle is the unique path between the initial and final positions of the particle. It may or may not be the actually travelled path of the particle.

✍ Displacement of a particle gives no information regarding the nature of the path followed by the particle.

✍ Magnitude of displacement ≤ Distance covered.

✍ Since distance ≥ |Displacement|, so average speed of a body is equal or greater than the magnitude of the average velocity of the body.

✍ The average speed of a body is equal to its instantaneous speed if the body moves with a constant speed

✍ No force is required to move the body or an object with uniform velocity.

✍ Velocity of the body is positive, if it moves to the right side of the origin. Velocity is negative if the body moves to the left side of the origin.

✍ When a particle returns to the starting point, its displacement is zero but the distance covered is not zero.

✍ When a body reverses its direction of motion while moving along a straight line, then the distance travelled by the body is greater than the magnitude of the displacement of the body. In this case, average speed of the body is greater than its average velocity.

- Speedometer measures the instantaneous speed of a vehicle.

- When particle moves with speed v_1 upto half time of its total motion and in rest time it is moving with speed v_2 then $v_{av} = \frac{v_1 + v_2}{2}$

- When particle moves the first half of a distance at a speed of v_1 and second half of the distance at speed v_2 then

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

- When particle covers one-third distance at speed v_1 next one third at speed v_2 and last one third at speed v_3 then

$$v_{av} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

- For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{\upsilon_1}{\upsilon_2} = \frac{\tan\theta_1}{\tan\theta_2}$

- Velocity of a particle having uniform motion = slope of displacement–time graph.

- Greater the slope of displacement-time graph, greater is the velocity and vice-versa.

- Area under $v - t$ graph = displacement of the particle.

- Slope of velocity-time graph = acceleration.

- If a particle is accelerated for a time t_1 with acceleration a_1 and for time t_2 with acceleration a_2 then average acceleration is

$$a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$$

- If same force is applied on two bodies of different masses m_1 and m_2 separately then it produces accelerations a_1 and a_2 respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that a be the acceleration of the combined system, then

$$a = \frac{a_1 a_2}{a_1 + a_2}$$

- If a body starts from rest and moves with uniform acceleration then distance covered by the body in t *sec* is proportional to t (*i.e.* $s \propto t^2$).

  So we can say that the ratio of distance covered in 1 *sec*, 2 *sec* and 3 *sec* is $1^2 : 2^2 : 3^2$ or 1 : 4 : 9.

- If a body starts from rest and moves with uniform acceleration then distance covered by the body in *n*th sec is proportional to $(2n - 1)$ (*i.e.* $s_n \propto (2n - 1)$)

So we can say that the ratio of distance covered in 1st, 2nd and 3rd is 1 : 3 : 5.

- A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of ns.

As $\upsilon^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a}$, $s \propto u^2$

[since a is constant]

So we can say that if u becomes n times then s becomes n times that of previous value.

- A particle moving with uniform acceleration from A to B along a straight line has velocities υ_1 and υ_2 at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

$$\upsilon = \sqrt{\frac{\upsilon_1^2 + \upsilon_2^2}{2}}$$

- The body returns to its point of projection with the same magnitude of the velocity with which it was thrown vertically upward, provided air resistance is neglected.

- All bodies fall freely with the same acceleration.

- The acceleration of the falling bodies does not depend on the mass of the body.

- If two bodies are dropped from the same height, they reach the ground in the same time and with the same velocity.

- If a body is thrown upwards with velocity u from the top of a tower and another body is thrown downwards from the same point and with the same velocity, then both reach the ground with the same speed.

- When a particle returns to the starting point, its average velocity is zero but the average speed is not zero.

- If both the objects A and B move along parallel lines in the same direction, then the relative velocity of A w.r.t. B is given by $v_{AB} = v_A - v_B$

and the relative velocity of B w.r.t. A is given by $v_{BA} = v_B - v_A$

- If both the objects A and B move along parallel lines in the opposite direction, then the relative velocity of A w.r.t. B is given by $v_{AB} = v_A - (-v_B) = v_A + v_B$

and the relative velocity of B w.r.t. A is given by $v_{BA} = -v_B - v_A$

- Suppose a body is projected upwards from the ground and with the velocity u. It is assumed that the friction of the air is negligible. The characteristics of motion of such a body are as follows.

(i) The maximum height attained = $H = u/2g$.

(ii) Time taken to go up (ascent) = Time taken to come down (descent) = $t = u/g$.

(iii) Time of flight $T = 2t = 2u/g$.

(iv) The speed of the body on return to the ground = speed with which it was thrown upwards.

(v) When the height attained is not large, that is u is not large, the mass, the weight as well as the acceleration remain constant with time. But its speed, velocity, momentum, potential energy and kinetic energy change with time.

(vi) Let m be the mass of the body. Then in going from the ground to the highest point, following changes take place.

(a) Change in speed = u

(b) Change in velocity = u

(c) Change in momentum = $m\ u$

(d) Change in kinetic energy = Change in potential energy = (1/2) mu.

(vii) On return to the ground the changes in these quantities are as follows

(a) Change in speed = 0

(b) Change in velocity = $2u$

(c) Change in momentum = $2mu$

(d) Change in kinetic energy = Change in potential energy = 0

(viii) If, the friction of air be taken into account, then the motion of the object thrown upwards will have the following properties

(a) Time taken to go up (ascent) < time taken to come down (descent)

(b) The speed of the object on return to the ground is less than the initial speed. Same is true for velocity (magnitude), momentum (magnitude) and kinetic energy.

(c) Maximum height attained is less than $u/2g$.

(d) A part of the kinetic energy is used up in overcoming the friction.

✍ A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two balls are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then

$$t = \sqrt{t_1 t_2}$$

✍ A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of $1m$ each will then be in the ratio of the difference in the square roots of the integers *i.e.*

$$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}) .... (\sqrt{4} - \sqrt{3}), ...$$