7. Trigonometry

Exercise 7.1

1 A. Question

Determine whether each of the following is an identity or not.

$\cos^2\theta + \sec^2\theta = 2 + \sin\theta$

Answer

$\cos^2\theta + \sec^2\theta = 2 + \sin\theta$

Let $\theta = 0°$

LHS $= \cos^2\theta + \sec^2\theta = \cos^2\theta° + \sec^2 0° = 1 + 1 = 2$ … (1)

RHS $= 2 + \sin\theta = 2 + \sin 0° = 2 + 0 = 2$ … (2)

From (1) and (2), LHS = RHS.

∴ The given equation is an identity.

1 B. Question

Determine whether each of the following is an identity or not.

$\cot^2\theta + \cos\theta = \sin^2\theta$

Answer

$\cot^2\theta + \cos\theta = \sin^2\theta$

Let $\theta = 45°$.

LHS $= \cot^2\theta + \cos\theta = \cot^2 45° + \cos 45° = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}}$ … (1)

RHS $= \sin^2\theta = \sin^2(45°) = (\frac{1}{\sqrt{2}})^2 = 1/2$ … (2)

From (1) and (2), LHS ≠ RHS.

∴ The given equation is not an identity.

2 A. Question

Prove the following identities

$\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\ \text{cosec}^2\theta$

Answer

Consider LHS,

LHS $= \sec^2\theta + \text{cosec}^2\theta$

We know that $\sec\theta = \frac{1}{\cos\theta}$ and $\text{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \sec^2\theta + \text{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$

We know that $\sin^2\theta + \cos^2\theta = 1$.

$\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta\sin^2\theta}$

$= \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$

$= \sec^2\theta \ \text{cosec}^2\theta$ = RHS

Hence proved.

2 B. Question

Prove the following identities

$$\frac{\sin\theta}{1-\cos\theta} = \text{cosec}\theta + \cot\theta$$

Answer

Consider LHS,

LHS $= \frac{\sin\theta}{1-\cos\theta}$

Multiplying numerator and denominator by (1 + cosθ),

$\Rightarrow \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos\theta} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$

We know that (a + b) (a – b) = $a^2 - b^2$

$\Rightarrow \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

We know that 1 – $\cos^2\theta = \sin^2\theta$.

$\Rightarrow \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta+\sin\theta\cos\theta}{\sin^2\theta}$

$= \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta\cos\theta}{\sin^2\theta}$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

We know that $\frac{1}{\sin\theta}$ = cosecθ and $\frac{\cos\theta}{\sin\theta}$ = cotθ

$\therefore \frac{\sin\theta}{1-\cos\theta}$ = cosecθ + cotθ = RHS

Hence proved.

2 C. Question

Prove the following identities

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

Answer

Consider LHS,

LHS $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

Multiplying and dividing with (1 – sinθ),

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$

We know that (a + b) (a – b) = $a^2 - b^2$

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

We know that 1 – $\sin^2\theta = \cos^2\theta$.

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

We know that $\frac{1}{\cos\theta}$ = secθ and $\frac{\sin\theta}{\cos\theta}$ = tanθ.

$\therefore \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = secθ + tanθ = RHS

Hence proved.

2 D. Question

Prove the following identities

$$\frac{\cos\theta}{\sec\theta - \tan\theta} = 1 + \sin\theta$$

Answer

Consider LHS,

LHS = $\frac{\cos\theta}{\sec\theta-\tan\theta}$

Multiplying and dividing with (secθ + tanθ),

$\Rightarrow \frac{\cos\theta}{\sec\theta-\tan\theta} = \frac{\cos\theta}{\sec\theta-\tan\theta} \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

We know that (a + b) (a – b) = $a^2 - b^2$.

$\Rightarrow \frac{\cos\theta}{\sec\theta-\tan\theta} = \frac{\cos\theta(\sec\theta+\tan\theta)}{\sec^2\theta-\tan^2\theta}$

We know that $\sec^2\theta - \tan^2\theta = 1$.

$\Rightarrow \frac{\cos\theta}{\sec\theta-\tan\theta} = \frac{\cos\theta\sec\theta + \cos\theta\tan\theta}{1}$

We know that $\frac{1}{\cos\theta}$ = secθ and $\frac{\sin\theta}{\cos\theta}$ = tanθ.

$\Rightarrow \frac{\cos\theta}{\sec\theta-\tan\theta} = \cos\theta(\frac{1}{\cos\theta}) + \cos\theta\,(\frac{\sin\theta}{\cos\theta})$

= 1 + sinθ = RHS

Hence proved.

2 E. Question

Prove the following identities

$$\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$$

Answer

Consider LHS,

LHS = $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

We know that $\sec^2\theta = 1 + \tan^2\theta$ and $\text{coesc}^2\theta = 1 + \cot^2\theta$.

$\Rightarrow \sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2\,(1)}$

We know that $\tan\theta\cot\theta = 1$.

$\Rightarrow \sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{\tan^2\theta + \cot^2\theta + 2\,(\tan\theta\cot\theta)}$

We know that $a^2 + b^2 + 2ab = (a + b)^2$.

$\Rightarrow \sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{(\tan\theta + \cot\theta)^2}$

$= (\tan\theta + \cot\theta) = \text{RHS}$

Hence proved.

2 F. Question

Prove the following identities

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$$

Answer

Consider LHS,

$\text{LHS} = \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

We know that $\sin^2\theta = 1 - \cos^2\theta$.

$\Rightarrow \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{1+\cos\theta-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{1+\cos\theta-1+\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

We know that $\frac{\cos\theta}{\sin\theta} = \cot\theta$.

$\therefore \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta = \text{RHS}$

Hence proved.

2 G. Question

Prove the following identities

$\sec\theta\,(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

Answer

Consider LHS,

$\text{LHS} = \sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta)$

$\Rightarrow \sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta) = (\sec\theta - \frac{\sin\theta}{\cos\theta})\,(\sec\theta + \tan\theta)$

We know that $\frac{\sin\theta}{\cos\theta}$ = tanθ.

⇒ secθ (1 – sinθ) (secθ + tanθ) = (secθ – tanθ) (secθ + tanθ)

We know that (a + b) (a – b) = $a^2 - b^2$.

⇒ secθ (1 – sinθ) (secθ + tanθ) = $\sec^2\theta - \tan^2\theta$

We know that $\sec^2\theta - \tan^2\theta = 1$.

∴ secθ (1 – sinθ) (secθ + tanθ) = 1 = RHS

Hence proved.

2 H. Question

Prove the following identities

$$\frac{\sin\theta}{\operatorname{cosec}\theta + \cot\theta} = 1 - \cos\theta$$

Answer

Consider LHS,

LHS = $\frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta}$

Multiplying and dividing with (cosecθ - cotθ),

⇒ $\frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta} = \frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta} \times \frac{\operatorname{cosec}\theta-\cot\theta}{\operatorname{cosec}\theta-\cot\theta}$

We know that (a + b) (a – b) = $a^2 - b^2$.

⇒ $\frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta} = \frac{\sin\theta(\operatorname{cosec}\theta-\cot\theta)}{\operatorname{cosec}^2\theta-\cot^2\theta}$

We know that $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

⇒ $\frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta} = \frac{\sin\theta\operatorname{cosec}\theta-\sin\theta\cot\theta}{1}$

We know that $\frac{1}{\sin\theta}$ = cosecθ and $\frac{\cos\theta}{\sin\theta}$ = cotθ.

⇒ $\frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta}$ = sinθ ($\frac{1}{\sin\theta}$) - sinθ ($\frac{\cos\theta}{\sin\theta}$)

= 1 – cosθ = RHS

Hence proved.

3 A. Question

Prove the following identities.

$$\frac{\sin(90^\circ-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90^\circ-\theta)} = 2\sec\theta$$

Answer

Consider LHS,

LHS = $\frac{\sin(90^\circ-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90^\circ-\theta)}$

We know that sin (90° - θ) = cosθ and cos (90° - θ) = sinθ.

⇒ $\frac{\sin(90^\circ-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90^\circ-\theta)} = \frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta}$

Taking LCM,

$\Rightarrow \frac{\sin(90°-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90°-\theta)} = \frac{\cos\theta(1-\sin\theta)+\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

We know that (a + b) (a – b) = $a^2 - b^2$.

$\Rightarrow \frac{\sin(90°-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90°-\theta)} = \frac{\cos\theta-\cos\theta\sin\theta+\cos\theta+\cos\theta\sin\theta}{1-\sin^2\theta}$

We know that 1 $-\sin^2\theta = \cos^2\theta$.

$\Rightarrow \frac{\sin(90°-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90°-\theta)} = \frac{2\cos\theta}{\cos^2\theta}$

$= 2\left(\frac{1}{\cos\theta}\right)$

We know that $\frac{1}{\cos\theta}$ = secθ.

$\therefore \frac{\sin(90°-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90°-\theta)}$ = 2secθ = RHS

Hence proved.

3 B. Question

Prove the following identities.

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$$

Answer

Consider LHS,

LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

We know that $\frac{\cos\theta}{\sin\theta}$ = cotθ and $\frac{\sin\theta}{\cos\theta}$ = tanθ.

$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}\right]$

We know that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\cos\theta\sin\theta}\right]$

We know that $\sin^2\theta + \cos^2\theta = 1$.

$\Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\sin\theta\cos\theta+1}{\cos\theta\sin\theta}$

$= \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta} + \frac{1}{\cos\theta\sin\theta}$

We know that $\frac{1}{\sin\theta}$ = cosecθ and $\frac{1}{\cos\theta}$ = secθ.

$\therefore \Rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta = RHS$

Hence proved.

3 C. Question

Prove the following identities.

$$\frac{\sin(90^\circ - \theta)}{1-\tan\theta} + \frac{\cos\theta(90^\circ - \theta)}{1-\cot\theta} = \cos\theta + \sin\theta$$

Answer

Consider LHS,

LHS $= \frac{\sin(90^\circ-\theta)}{1-\tan\theta} + \frac{\cos(90^\circ-\theta)}{1-\cot\theta}$

We know that sin (90° - θ) = cosθ and cos (90° - θ) = sinθ.

$\Rightarrow \frac{\sin(90^\circ-\theta)}{1-\tan\theta} + \frac{\cos(90^\circ-\theta)}{1-\cot\theta} = \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$

We know that $\frac{\cos\theta}{\sin\theta}$ = cotθ and $\frac{\sin\theta}{\cos\theta}$ = tanθ.

$\Rightarrow \frac{\sin(90^\circ-\theta)}{1-\tan\theta} + \frac{\cos(90^\circ-\theta)}{1-\cot\theta} = \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$

$= \frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} + \frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}$

$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$

We know that (a + b) (a – b) = a^2 – b^2.

$\Rightarrow \frac{\sin(90^\circ-\theta)}{1-\tan\theta} + \frac{\cos(90^\circ-\theta)}{1-\cot\theta} = \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta-\sin\theta}$

= cosθ + sinθ = RHS

Hence proved.

3 D. Question

Prove the following identities.

$$\frac{\tan(90^\circ - \theta)}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta} = 2\sec\theta$$

Answer

Consider LHS,

LHS $= \frac{\tan(90^\circ-\theta)}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta}$

We know that tan (90° - θ) = cotθ.

$\Rightarrow \frac{\tan(90^\circ-\theta)}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta} = \frac{\cot\theta}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta}$

$= \frac{\cot^2\theta+(\operatorname{cosec}\theta+1)^2}{(\operatorname{cosec}\theta+1)\cot\theta}$

We know that $(a + b)^2 = a^2 + 2ab + b^2$.

$\Rightarrow \frac{\tan(90°-\theta)}{\mathrm{cosec}\theta+1} + \frac{\mathrm{cosec}\theta+1}{\cot\theta} = \frac{\cot^2\theta+\mathrm{cosec}^2\theta+2\mathrm{cosec}\theta+1}{(\mathrm{cosec}\theta+1)\cot\theta}$

We know that $\cot^2\theta + 1 = \mathrm{cosec}^2\theta$.

$\Rightarrow \frac{\tan(90°-\theta)}{\mathrm{cosec}\theta+1} + \frac{\mathrm{cosec}\theta+1}{\cot\theta} = \frac{\mathrm{cosec}^2\theta+\mathrm{cosec}^2\theta+2\mathrm{cosec}\theta}{(\mathrm{cosec}\theta+1)\cot\theta}$

$= \frac{2\mathrm{cosec}^2\theta+2\mathrm{cosec}\theta}{(\mathrm{cosec}\theta+1)\cot\theta}$

$= \frac{2\mathrm{cosec}\theta(\mathrm{cosec}\theta+1)}{(\mathrm{cosec}\theta+1)\cot\theta}$

$= \frac{2\mathrm{cosec}\theta}{\cot\theta}$

We know that $\frac{1}{\sin\theta} = \mathrm{cosec}\theta$ and $\frac{\cos\theta}{\sin\theta} = \cot\theta$.

$\Rightarrow \frac{\tan(90°-\theta)}{\mathrm{cosec}\theta+1} + \frac{\mathrm{cosec}\theta+1}{\cot\theta} = \frac{2(\frac{1}{\sin\theta})}{\frac{\cos\theta}{\sin\theta}}$

$= \frac{2(1)}{\cos\theta}$

We know that $\frac{1}{\cos\theta} = \sec\theta$.

$\Rightarrow \frac{\tan(90°-\theta)}{\mathrm{cosec}\theta+1} + \frac{\mathrm{cosec}\theta+1}{\cot\theta} = 2\sec\theta$ = RHS

Hence proved.

3 E. Question

Prove the following identities.

$$\frac{\cot\theta + \mathrm{cosec}\theta - 1}{\cot\theta - \mathrm{cosec}\theta + 1} = \mathrm{cosec}\theta + \cot\theta$$

Answer

Consider LHS,

LHS = $\frac{\cot\theta+\mathrm{cosec}\theta-1}{\cot\theta-\mathrm{cosec}\theta+1}$

We know that $\mathrm{cosec}^2\theta - \cot^2\theta = 1$.

$\Rightarrow \frac{\cot\theta+\mathrm{cosec}\theta-1}{\cot\theta-\mathrm{cosec}\theta+1} = \frac{\cot\theta+\mathrm{cosec}\theta-(\mathrm{cosec}^2\theta-\cot^2\theta)}{\cot\theta-\mathrm{cosec}\theta+1}$

We know that $(a + b)(a - b) = a^2 - b^2$.

$\Rightarrow \frac{\cot\theta+\mathrm{cosec}\theta-1}{\cot\theta-\mathrm{cosec}\theta+1} = \frac{\cot\theta+\mathrm{cosec}\theta-(\mathrm{cosec}\theta+\cot\theta)(\mathrm{cosec}\theta-\cot\theta)}{\cot\theta-\mathrm{cosec}\theta+1}$

$= \frac{(\mathrm{cosec}\theta+\cot\theta)(1-(\mathrm{cosec}\theta-\cot\theta))}{\cot\theta-\mathrm{cosec}\theta+1}$

$= \frac{(\mathrm{cosec}\theta+\cot\theta)(\cot\theta-\mathrm{cosec}\theta+1)}{\cot\theta-\mathrm{cosec}\theta+1}$

$= \mathrm{cosec}\theta + \cot\theta$ = RHS

Hence proved.

3 F. Question

Prove the following identities.

$(1 + \cot\theta - \mathrm{cosec}\,\theta)(1 + \tan\theta + \sec\theta) = 2$

Answer

Consider LHS,

LHS = (1 + cotθ – cosecθ) (1 + tanθ + secθ)

Expanding the above,

⇒ (1 + cotθ – cosecθ) (1 + tanθ + secθ)

= 1 + tanθ + secθ + cotθ + cotθtanθ + cotθsecθ – cosecθ –cosecθtanθ –cosecθsecθ

We know that $\frac{1}{\cos\theta}$ = secθ, $\frac{1}{\sin\theta}$ = cosecθ, $\frac{\sin\theta}{\cos\theta}$ = tanθ and $\frac{\cos\theta}{\sin\theta}$ = cotθ.

= 1 + tanθ + secθ + cotθ + 1 + cosecθ – cosecθ – secθ – cosecθsecθ

We know that tanθ + cotθ = cosecθsecθ.

= 1 + cosecθsecθ – cosecθsecθ + 1

∴ (1 + cotθ – cosecθ) (1 + tanθ + secθ) = 2 = RHS

Hence proved.

3 G. Question

Prove the following identities.

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$$

Answer

Consider LHS,

LHS = $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

Multiplying and dividing by sinθ + cosθ + 1,

⇒ $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} \times \frac{\sin\theta + \cos\theta + 1}{\sin\theta + \cos\theta + 1}$

We know that (a + b) (a – b) = a^2 – b^2.

⇒ $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{(1+\sin\theta)^2 - \cos^2\theta}{(\sin\theta + \cos\theta)^2 - 1}$

$= \frac{1+\sin^2\theta + 2\sin\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$

We know that 1 – $\cos^2\theta$ = $\sin^2\theta$ and $\sin^2\theta$ + $\cos^2\theta$ = 1.

$= \frac{\sin^2\theta + \sin^2\theta + 2\sin\theta}{1 + 2\sin\theta\cos\theta - 1}$

$= \frac{2\sin^2\theta + 2\sin\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin\theta(\sin\theta + 1)}{2\sin\theta\cos\theta}$

$= \frac{(\sin\theta + 1)}{\cos\theta}$

We know that $\frac{1}{\cos\theta}$ = secθ and $\frac{\sin\theta}{\cos\theta}$ = tanθ.

⇒ $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$ = secθ + tanθ

Multiplying and dividing by secθ – tanθ,

$\Rightarrow \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$ = secθ + tanθ × $\frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$

$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$

We know that $\sec^2 - \tan^2\theta = 1$.

$\therefore \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ = RHS

Hence proved.

3 H. Question

Prove the following identities.

$$\frac{\tan\theta}{1 - \tan^2\theta} = \frac{\sin\theta \sin(90^\circ - \theta)}{2\sin^2(90^\circ - \theta) - 1}$$

Answer

Consider RHS,

RHS $= \frac{\sin\theta \sin(90^\circ - \theta)}{2\sin^2(90^\circ - \theta) - 1}$

We know that sin (90° - θ) = cosθ.

$\Rightarrow \frac{\sin\theta \sin(90^\circ - \theta)}{2\sin^2(90^\circ - \theta) - 1} = \frac{\sin\theta\cos\theta}{2\cos^2\theta - 1}$

We know that $2\cos^2\theta - 1 = \cos^2\theta - \sin^2\theta$

$\Rightarrow \frac{\sin\theta \sin(90^\circ - \theta)}{2\sin^2(90^\circ - \theta) - 1} = \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Multiplying and dividing by $\cos^2\theta$,

$\Rightarrow \frac{\sin\theta \sin(90^\circ - \theta)}{2\sin^2(90^\circ - \theta) - 1} = \frac{\frac{\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$

$= \frac{\tan\theta}{1 - \tan^2\theta}$ = LHS

Hence proved.

3 I. Question

Prove the following identities.

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

Answer

Consider LHS,

LHS $= \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$

We know that $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ and $\frac{1}{\sin\theta}$ = cosecθ.

$\Rightarrow \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\operatorname{cosec}\theta - \cot\theta}$ – cosecθ

We know that $(a + b)(a - b) = a^2 - b^2$.

$\Rightarrow \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta)}{\operatorname{cosec}\theta - \cot\theta}$ – cosecθ

$= \text{cosec}\theta + \cot\theta - \text{cosec}\theta$

$= \cot\theta$ … (1)

Now consider RHS,

RHS $= \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\theta+\cot\theta}$

We know that $\text{cosec}^2\theta - \cot^2\theta = 1$ and $\frac{1}{\sin\theta} = \text{cosec}\theta$.

$\Rightarrow \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\theta+\cot\theta} = \text{cosec}\theta - \frac{\text{cosec}^2\theta-\cot^2\theta}{\text{cosec}\theta+\cot\theta}$

We know that $(a + b)(a - b) = a^2 - b^2$.

$\Rightarrow \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\theta+\cot\theta} = \text{cosec}\theta - \frac{(\text{cosec}\theta-\cot\theta)(\text{cosec}\theta+\cot\theta)}{\text{cosec}\theta+\cot\theta}$

$= \text{cosec}\theta - (\text{cosec}\theta - \cot\theta)$

$= \cot\theta$ … (2)

From (1) and (2), LHS = RHS

Hence proved.

3 J. Question

Prove the following identities.

$$\frac{\cot^2\theta + \sec^2\theta}{\tan^2\theta + \text{cosec}^2\theta} = (\sin\theta\cos\theta)(\tan\theta + \cot\theta)$$

Answer

Consider LHS,

LHS $= \frac{\cot^2\theta+\sec^2\theta}{\tan^2\theta+\text{cosec}^2\theta}$

We know that $\cot^2\theta = \text{cosec}^2 - 1$ and $\tan^2\theta = \sec^2\theta - 1$.

$\Rightarrow \frac{\cot^2\theta+\sec^2\theta}{\tan^2\theta+\text{cosec}^2\theta} = \frac{\text{cosec}^2\theta-1+\sec^2\theta}{\sec^2\theta-1+\text{cosec}^2\theta}$

$= \frac{\text{cosec}^2\theta+\sec^2\theta-1}{\text{cosec}^2\theta+\sec^2\theta-1}$

$= 1$ … (1)

Consider RHS,

RHS $= (\sin\theta\cos\theta)(\tan\theta + \cot\theta)$

Expanding,

$\Rightarrow (\sin\theta\cos\theta)(\tan\theta + \cot\theta) = \sin\theta\cos\theta\tan\theta + \sin\theta\cos\theta\cot\theta$

We know that $\frac{\sin\theta}{\cos\theta} = \tan\theta$ and $\frac{\cos\theta}{\sin\theta} = \cot\theta$.

$\Rightarrow (\sin\theta\cos\theta)(\tan\theta + \cot\theta) = \sin\theta\cos\theta(\frac{\sin\theta}{\cos\theta}) + \sin\theta\cos\theta(\frac{\cos\theta}{\sin\theta})$

$= \sin^2\theta + \cos^2\theta$

We know that $\sin^2\theta + \cos^2\theta = 1$.

$\therefore (\sin\theta\cos\theta)(\tan\theta + \cot\theta) = 1$ … (2)

From (1) and (2), LHS = RHS.

Hence proved.

4. Question

If x = a secθ + b tanθ and y = a tanθ + b secθ, then prove that $x^2 - y^2 = a^2 - b^2$.

Answer

$x^2 = (a \sec\theta + b \tan\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + ab \sec\theta \tan\theta$

$y^2 = (a \tan\theta + b \sec\theta)^2$

$= a^2\tan^2\theta + b^2\sec^2\theta + ab \sec\theta \tan\theta$

Now,

LHS = $x^2 - y^2$

$= (a^2\sec^2\theta + b^2\tan^2\theta + ab \sec\theta \tan\theta)$

$- (a^2\tan^2\theta + b^2\sec^2\theta + ab \sec\theta \tan\theta)$

$= a^2\sec^2\theta + b^2\tan^2\theta + ab \sec\theta \tan\theta$

$- a^2\tan^2\theta - b^2\sec^2\theta - ab \sec\theta \tan\theta$

$= a^2 (\sec^2\theta - \tan^2\theta) - b^2 (\sec^2\theta - \tan^2\theta)$

$= a^2 - b^2$ [$\because \sec^2\theta - \tan^2\theta = 1$]

=RHS

Hence proved.

5. Question

If tanθ = n tanα and sinθ = m sinα, then prove that $\cos^2\theta = \frac{m^2 - 1}{n^2 - 1}, n \neq \pm 1.$

Answer

We want to find value of $\cos^2\theta$ in terms of m and n.

So we first eliminate angle α,

tanθ =n tanα [∵ Given]

$\Rightarrow \frac{1}{\tan\alpha} = \frac{n}{\tan\theta}$

$\Rightarrow \cot\alpha = \frac{n}{\tan\theta}$ (1)

sinθ =m sinα [∵ Given]

$\Rightarrow \frac{1}{\sin\alpha} = \frac{m}{\sin\theta}$

$\Rightarrow \operatorname{cosec}\alpha = \frac{m}{\sin\alpha}$ (2)

We know that,

$\operatorname{cosec}^2\alpha - \cot^2\alpha = 1$

Substituting values from (1) and (2) gives,

$$\Rightarrow \frac{m^2}{\sin^2\theta} - \frac{n^2}{\tan^2\theta} = 1$$

$$\Rightarrow \frac{m^2}{\sin^2\theta} - \frac{n^2\cos^2\theta}{\sin^2\theta} = 1 \ [\because \tan\theta = \frac{\sin\theta}{\cos\theta}]$$

$$\Rightarrow m^2 - n^2\cos^2\theta = \sin^2\theta$$

$$\Rightarrow m^2 - n^2\cos^2\theta = 1 - \cos^2\theta \ [\because \sin^2\theta = 1 - \cos^2\theta]$$

$$\Rightarrow m^2 - 1 = \cos^2\theta\,(n^2 - 1)$$

$$\Rightarrow \frac{m^2 - 1}{n^2 - 1} = \cos^2\theta$$

Hence proved.

6. Question

If sinθ, cosθ and tanθ are in G.P., then prove that $\cot^6\theta - \cot^2\theta = 1$.

Answer

Given: sinθ, cosθ, tanθ are in G.P.

So,

$\cos^2\theta = \sin\theta \times \tan\theta$

$\cos^2\theta = \sin\theta \times \frac{\sin\theta}{\cos\theta}$

$\cos^2\theta = \frac{\sin^2\theta}{\cos\theta}$

Or, $\frac{\cos^2\theta}{\sin^2\theta} = \sec\theta$

$\Rightarrow \cot^2\theta = \sec\theta$ (1)

Taking LHS= $\cot^6\theta - \cot^2\theta$

$= (\cot^2\theta)^3 - \cot^2\theta$

$= \sec^3\theta - \sec\theta$ [Substituting from eqn. (1)]

$= \sec\theta\,(\sec^2\theta - 1)$

$= \sec\theta\,(\tan^2\theta)$

$= \cot^2\theta.\tan^2\theta$ [Substituting from eqn. (1)]

=1

=RHS

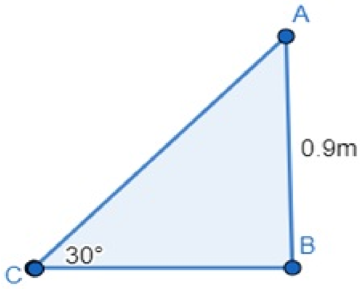
Hence proved.

Exercise 7.2

1. Question

A ramp for unloading a moving truck has an angle of elevation of 30°. If the top of the ramp is 0.9 m above the ground level, then find the length of the ramp.

Answer



Given AB = 0.9 m, AC = ?

In the above right triangle,

AC = Hypotenuse

AB = Perpendicular

BC = Base

We know,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\Rightarrow \sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{0.9}{AC}$$

⇒ AC = 0.9 x 2

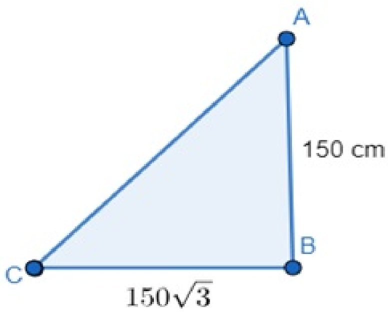
⇒ AC = 1.8 m

∴ length of ramp is 1.8 m

2. Question

A girl of height 150 cm stands in front of a lamp–post and casts a shadow of length 150√3 cm on the ground. Find the angle of elevation of the top of the lamp–post.

Answer



Given, AB = 150cm, BC = 150√3 and ∠C = ?

Here, AC = Hypotenuse

AB = Perpendicular

BC = Base

We know,

$$\Rightarrow \tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\Rightarrow \tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{150}{150\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

⇒ tan θ = tan 30°

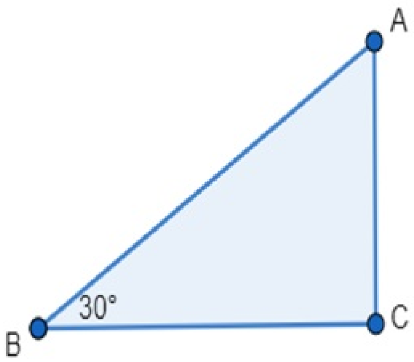
⇒ θ = 30°

∴ the angle of elevation of the top of the lamp–post is 30°

3. Question

Suppose two insects A and B can hear each other up to a range of 2 m. The insect A is on the ground 1 m away from a wall and sees her friend B on the wall, about to be eaten by a spider. If A sounds a warning to B and if the angle of elevation of B from A is 30°, will the spider have a meal or not? (Assume that B escapes if she hears A calling)

Answer



Given, BC = 1 m and A and B can hear each other up to a range of 2 m.

Here, AC = Hypotenuse

BC = Perpendicular

AC = Base

We know,

$$\text{Sin}\,\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\Rightarrow \text{Sin}\,\theta = \frac{BC}{AB}$$

$$\Rightarrow \sin 30^\circ = \frac{1}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{AC}$$

⇒ AC = 2 m

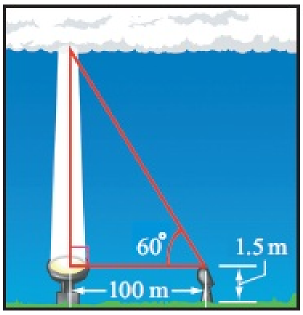
⇒ A and B can hear each other.

∴ Insect B escapes.

⇒ Spider will not have a meal.

4. Question

To find the cloud ceiling, one night an observer directed a spotlight vertically at the clouds. Using a theodolite placed 100 m from the spotlight and 1.5 m above the ground, he found the angle of elevation to be 60°. How high was the cloud ceiling? (Hint : See figure)



(**Note:** Cloud ceiling is the lowest altitude at which solid cloud is present. The cloud ceiling at airports must be sufficiently high for safe take offs and landings. At night the cloud ceiling can be determined by illuminating the base of the clouds by a spotlight pointing vertically upward.)

Answer

Given, Base = 100 m and height of cloud ceiling = ?

We know,

$\Rightarrow \tan\theta = \text{opposite side}/\text{adjacent side}$

$\Rightarrow \tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{AB}{100}$

⇒ $100\sqrt{3} = AB$

⇒ $AB = 100\sqrt{3}$

= 100 (1.732)

= 173.2

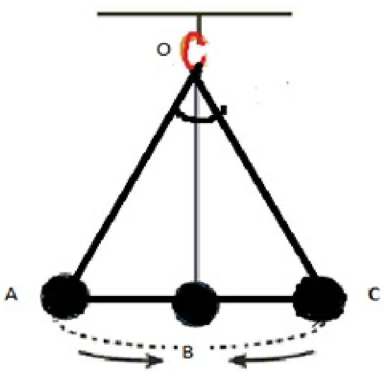
⇒ height of ceiling from ground = 173.2 + 1.5

= 174.7 m

5. Question

A simple pendulum of length 40 cm subtends 60° at the vertex in one full oscillation. What will be the shortest distance between the initial position and the final position of the bob? (between the extreme ends)

Answer



Given, OA = OC = length of pendulum = 40 cm, ∠AOC = 60°

In triangle OBC,

$\angle BOC = \frac{\angle AOC}{2}$

$= \frac{60°}{2}$

= 30°

Here, OC = hypotenuse

AB = perpendicular

BC = base

We know,

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

$$\Rightarrow \sin 30° = \frac{BC}{OC}$$

$$\Rightarrow \frac{1}{2} = \frac{BC}{40}$$

⇒ 40 = 2 BC

$$\Rightarrow BC = \frac{40}{2}$$

⇒ BC = 20 cm

∴ length of AC = 2(BC)

= 2(20)

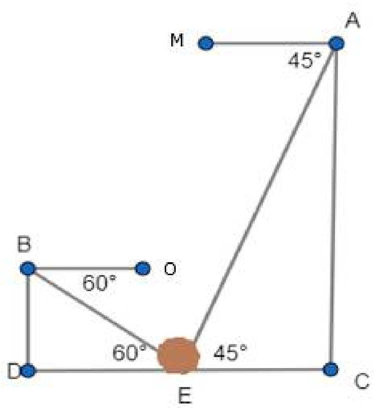
= 40 cm

∴ the shortest distance between the initial position and the final position of the bob = 40 cm

6. Question

Two crows A and B are sitting at a height of 15 m and 10 m in two different trees vertically opposite to each other . They view a vadai (an eatable) on the ground at an angle of depression 45° and 60° respectively. They start at the same time and fly at the same speed along the shortest path to pick up the vadai. Which bird will succeed in it? Hint : (foot of two trees and vadai (an eatable) are in a straight line)

Answer



Given, AC = 15 m, BD = 10 m, AE = ? and BE = ?

In triangle BED,

BE = hypotenuse

BD = perpendicular

ED = Base

∠ BED = ∠ OBE (adjacent angles are equal)

= 60°

We know,

$$\text{Sin}\ \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \text{Sin}\ 60° = \frac{BD}{BE}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{10}{BE}$$

⇒ BE√3 = 10 x 2

$$\Rightarrow BE = \frac{20}{\sqrt{3}}$$

= 11.55 m

And, in triangle AEC ,

AC = Perpendicular

AE = Hypotenuse

CE = Base

∠ AEC = ∠ MAE (adjacent angles are equal)

= 45°

We know,

$$\text{Sin}\ \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \text{Sin}\ 45° = \frac{AC}{AE}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{15}{AE}$$

⇒ AE = 15√2

= 21.21 m

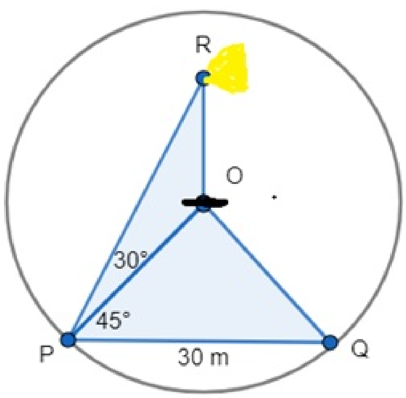
⇒ BD<AE

⇒ Crow B will succeed.

7. Question

A lamp–post stands at the centre of a circular park. Let P and Q be two points on the boundary such that PQ subtends an angle 90° at the foot of the lamp–post and the angle of elevation of the top of the lamp post from P is 30°. If PQ = 30 m, then find the height of the lamp post.

Answer



Given that PQ = 30m and ∠POQ = 90°

Let O be the centre of the park and OR be the lamp post and P and Q be two points on the boundary of the circular park.

In a right triangle OPQ,

OP = OQ = radius.

⇒ ∠OPQ = ∠OQP = 45° (∵ POQ = 90°)

We know,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\Rightarrow \cos 45° = \frac{\text{OP}}{\text{PQ}}$$

$$\Rightarrow \text{OP} = \frac{30}{\sqrt{2}}$$

Multiplying and dividing ihe fraction by √2,we get–

$$\text{OP} = \left(\frac{30}{\sqrt{2}}\right) \times \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

= 15 √2

And, in triangle RPO,

$$\tan 30° = \frac{\text{OR}}{\text{OP}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\text{OR}}{\left(\frac{30}{\sqrt{2}}\right)}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\text{OR}}{30} \times \sqrt{2}$$

$$\Rightarrow \text{OR} = \frac{30}{\sqrt{6}}$$

Multiplying and dividing the fraction by √6, we get–

$$\Rightarrow \text{OR} = \frac{30}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$\Rightarrow \text{OR} = \frac{30\sqrt{6}}{6}$$

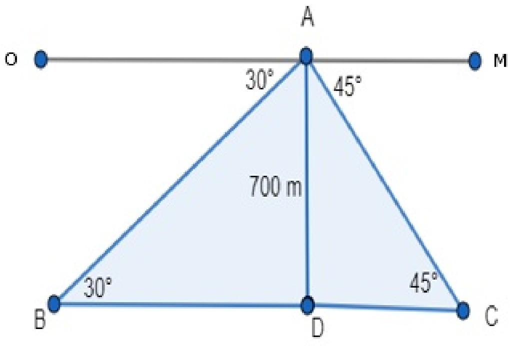
⇒ OR = 5√6

∴ the height of the lamp post is 5√6 m

8. Question

A person in an helicopter flying at a height of 700 m, observes two objects lying opposite to each other on either bank of a river. The angles of depression of the objects are 30° and 45°. Find the width of the river. (√3 = 1.732)

Answer



Given, AD = 700 m and BC = ?

In triangle ACD,

∠ACD = ∠ MAC (alternate angles are equal)

= 45°

We know,

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 45° = \frac{AD}{DC}$$

$$\Rightarrow 1 = \frac{700}{DC}$$

⇒ DC = 700 m ...... (1)

In triangle ABD,

∠ABD = ∠ OAB

= 30° (alternate angles are equal)

We know,

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{AD}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{700}{BD}$$

⇒ BD = 700√3 ...... (2)

Now, adding equation (1) and (2)–

Width of the river = BD + DC

= 700 + 700√3

= 700(1 + √3)

= 700 (1 + 1.732)

= 700 × 2.732

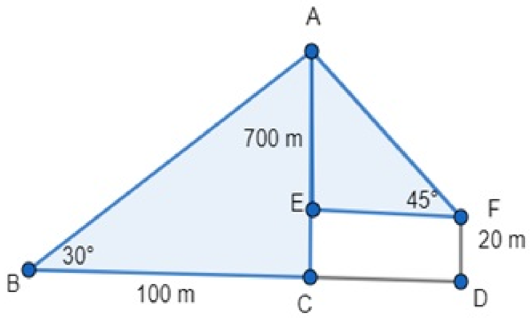
= 1912.40 m

∴ Width of the river is 1912.40 m

9. Question

A person X standing on a horizontal plane, observes a bird flying at a distance of 100 m from him at an angle of elevation of 30c. Another person Y standing on the roof of a 20 m high building, observes the bird at the same time at an angle of elevation of 45°. If X and Y are on the opposite sides of the bird, then find the distance of the bird from Y.

Answer



Given, AC = 700 m and EF = ?

Let position of person X be B, position of person Y be F and AE = x.

In triangle ABC

∠ABC = 30°

We know,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin 30° = \frac{AE + EC}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{x + 20}{100}$$

⇒ 100 = 2 (x + 20)

⇒ 100 = 2x + 40

⇒ 2x = 60

⇒ x = 30

And, in triangle AFE,

∠AFE = 45°

$$\Rightarrow \sin 45° = \frac{AE}{EF}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{x}{EF}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{30}{EF}$$

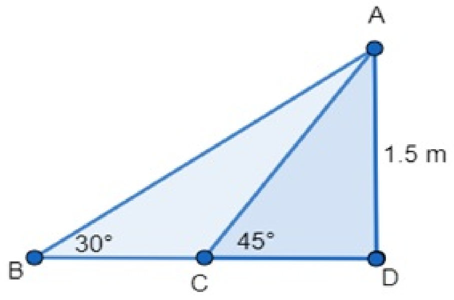
⇒ EF = 30√2

∴ Distance of the bird from Y is 30√2 m.

10. Question

A student sitting in a classroom sees a picture on the black board at a height of 1.5 m from the horizontal level of sight. The angle of elevation of the picture is 30°. As the picture is not clear to him, he moves straight towards the black board and sees the picture at an angle of elevation of 45°. Find the distance moved by the student.

Answer



Given, AD = 1.5 m and distance moved = BC = ?

In triangle ABD,

$\angle ABD = 30°$

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{AD}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1.5}{BD}$$

⇒ BD = 1.5 x √3

⇒ BD = 1.5√3

Now, in triangle ACD,

$\angle ACD = 45°$

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 45° = \frac{AD}{CD}$$

$$\Rightarrow 1 = \frac{1.5}{CD}$$

⇒ CD = 1.5

⇒ BC = BD – CD

⇒ BC = 1.5 √3 – 1.5

⇒ BC = 1.5 (√3 – 1)

⇒ BC = 1.5(1.732 – 1)

⇒ BC = 1.5 (0.732)

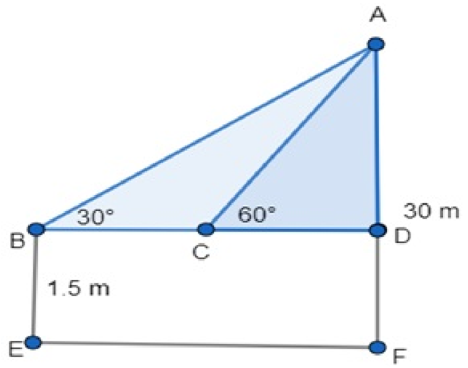
⇒ BC = 1.098 m

∴ the distance moved by the student is 1.098 m.

11. Question

A boy is standing at some distance from a 30 m tall building and his eye level from the ground is 1.5 m. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer



Given, AF = 30m, DF = BE = 1.5m,

AD = 28.5 m

In triangle ABD

∠ABD = 30°

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{AD}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{BD}$$

⇒ BD = 28.5 √3

Now, in triangle ACD,

∠ABD = 60°

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 60° = \frac{AD}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{CD}$$

$$\Rightarrow CD = \frac{28.5}{\sqrt{3}}$$

Multiplying and dividing the fraction by √3, we get

$$CD = \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

⇒ CD = 9.5√3

∴ distance he walked towards the building = BD – CD

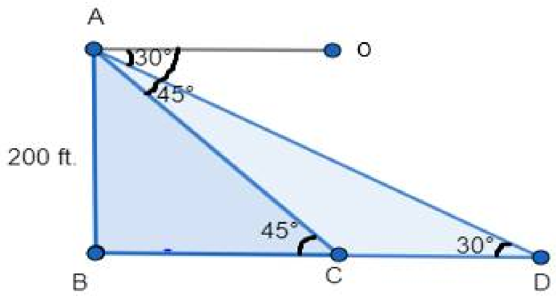
= 28.5√3 –9.5√3

= 19√3 m

12. Question

From the top of a lighthouse of height 200 feet, the lighthouse keeper observes a Yacht and a Barge along the same line of sight . The angles of depression for the Yacht and the Barge are 45° and 30° respectively. For safety purposes the two sea vessels should be atleast 300 feet apart. If they are less than 300 feet, the keeper has to sound the alarm. Does the keeper have to sound the alarm?

Answer



Given, AB = 200 ft. and CD = ?

In triangle ABC,

∠ACB = ∠ OAC (alternate angles are equal)

= 45°

We know,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin 45° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{200}{BC}$$

⇒ BC = 200 √2

And, in triangle ABD,

∠ADB = ∠OAD (alternate angles are equal)

= 30°

$$\Rightarrow \sin 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{2} = \frac{200}{BD}$$

⇒ BD = 200 x 2

⇒ BD = 400

Now, CD = BD – BC

⇒ CD = 400 – 200 √2

= 200(2 – √2)

= 200 (2 – 1.414)

= 200(0.586)

= 117.2 m

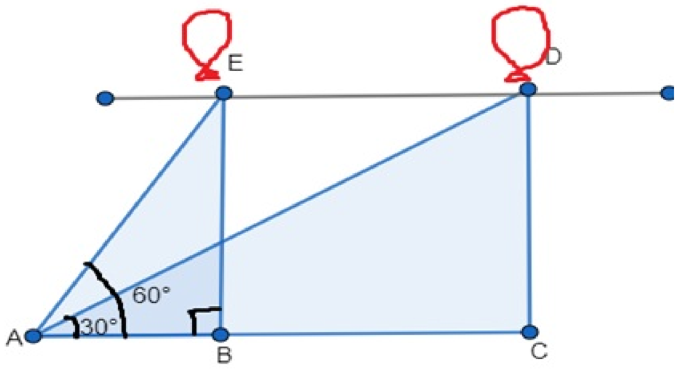
∴distance between Yacht and a Barge = 117.2<300 m.

⇒ the keeper has to sound the alarm.

13. Question

A boy standing on the ground, spots a balloon moving with the wind in a horizontal line at a constant height. The angle of elevation of the balloon from the boy at an instant is 60°. After 2 minutes, from the same point of observation, the angle of elevation reduces to 30°. If the speed of wind is 29√3 m/min. then, find the height of the balloon from the ground level.

Answer



Here, Distance covered by the balloon = BC

We know,

Distance = Time x Speed

⇒ BC = Time x Speed

= 2 x 29√3

= 58√3 m

Let AB = x

⇒ AC = x + 58√3

In triangle DAC,

∠DAC = 30°

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{DC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{DC}{x + 58\sqrt{3}}$$

$$\Rightarrow DC = \frac{x + 58\sqrt{3}}{\sqrt{3}}$$

Now, in triangle EAB,

∠EAB = 60°

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 60° = \frac{EB}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{EB}{x}$$

⇒ EB = √3x

∵ EB = DC

$$\Rightarrow \frac{x + 58\sqrt{3}}{\sqrt{3}} = \sqrt{3}x$$

⇒ $x + 58\sqrt{3} = 3x$

⇒ $2x = 58\sqrt{3}$

$$\Rightarrow x = \frac{(58\sqrt{3})}{2}$$

⇒ $x = 29\sqrt{3}$ m

And, Height of the balloon from ground level EB = $\sqrt{3}$ x

= 29 $\sqrt{3}$ ($\sqrt{3}$)

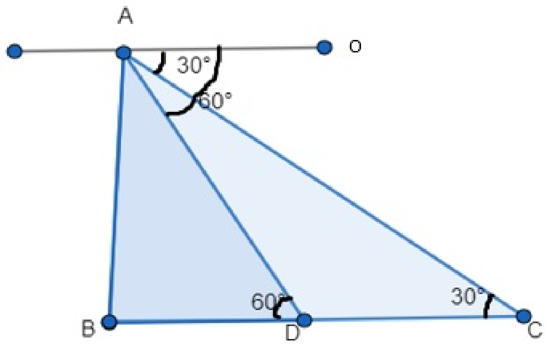
= 87 m

Hence height of the balloon from ground level is 87 m.

14. Question

A straight highway leads to the foot of a tower. A man standing on the top of the tower spots a van at an angle of depression of 30°. The van is approaching the tower with a uniform speed. After 6 minutes, the angle of depression of the van is found to be 60°. How many more minutes will it take for the van to reach the tower?

Answer



Given, time taken by van to reach D from C = 6 minutes.

And let the speed = x

We know,

Distance = speed × time

⇒ Distance between D and C = DC = 6x

In triangle ACB,

∠ACB = ∠ OAC (alternate angles are equal)

= 30°

Also,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD + DC}$$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD + 6x}$

$\Rightarrow AB = \frac{BD + 6x}{\sqrt{3}}$......... (1)

Now, in triangle ABD,

∠ABD = ∠ OAD (alternate angles are equal)

= 60°

We know,

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan 60° = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{AB}{BD}$

AB = BD√3 .... (2)

Now, equating (1) & (2), we get–

$\frac{BD + 6x}{\sqrt{3}} = BD\sqrt{3}$

⇒ BD + 6x = BD × 3

⇒ 2BD = 6x

$\Rightarrow BD = \frac{6x}{2}$

⇒ BD = 3x (where, x is speed)

Now, comparing it with Distance = speed × time, we have–

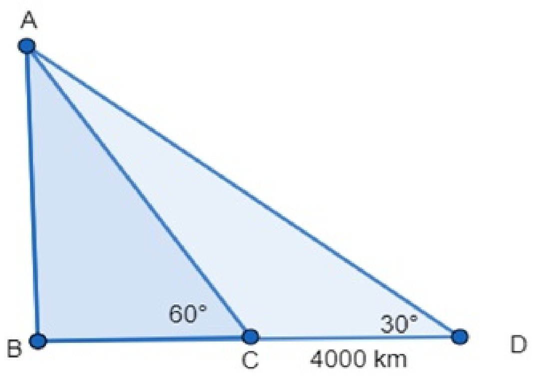
Time = 3 minutes.

Hence, it take 3 minutes more for the van to reach the tower.

15. Question

The angles of elevation of an artificial earth satellite is measured from two earth stations, situated on the same side of the satellite, are found to be 30° and 60°. The two earth stations and the satellite are in the same vertical plane. If the distance between the earth stations is 4000 km, find the distance between the satellite and earth. (√3 = 1.732)

Answer



Let C be position of station 1 and D of station 2 and BC = x.

Given, CD = 4000 km

Now, in triangle ABC,

$\angle ACB = 60°$

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BC}$$

$\Rightarrow AB = \sqrt{3}BC$

$\Rightarrow AB = x\sqrt{3}$ .... (1)

And, in triangle ABD,

$\angle ADB = 30°$

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{x + 4000}$$

$\Rightarrow AB = \frac{x+4000}{\sqrt{3}}$ .... (2)

Now, equating (1) & (2), we get–

$$x\sqrt{3} = (x + 4000)/\sqrt{3}$$

$\Rightarrow 3x = x + 4000$

$\Rightarrow 3x - x = 4000$

$\Rightarrow 2x = 4000$

$\Rightarrow x = 2000$

And distance between the satellite and earth(AB) = $x\sqrt{3}$

= 2000(1.732)

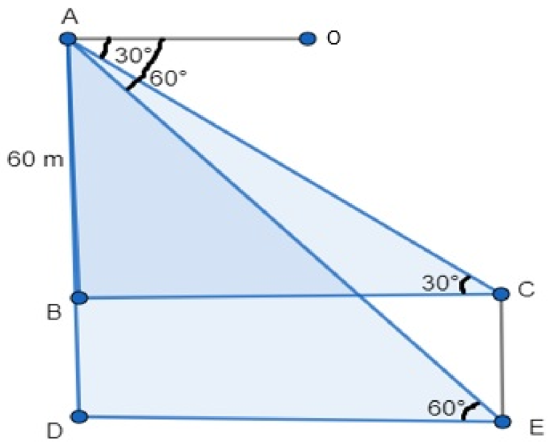
= 3464 km

∴ Distance between the satellite and earth = 3464 km

16. Question

From the top of a tower of height 60 m, the angles of depression of the top and the bottom of a building are observed to be 30° and 60° respectively. Find the height of the building.

Answer



Here, AB is tower and EC is building.

Given, AB = 60 m and EC = ?

In triangle ABC,

Let AB = x

⇒ BD = 60 – x

And, ∠ ACB = ∠OAC (alternate angles are equal)

∠ACB = 30°

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{BC}$$

⇒ BC = x √3 ……… (1)

In triangle ADE,

∠ AED = ∠OAE (alternate angles are equal)

∠AED = 60°

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 60° = \frac{AD}{DE}$$

$$\Rightarrow \sqrt{3} = \frac{60}{DE}$$

$\Rightarrow DE = \frac{60}{\sqrt{3}}$ ……… (2)

∵ BC = DE

⇒ equation (1) = equation (2)

$$\Rightarrow x\sqrt{3} = \frac{60}{\sqrt{3}}$$

⇒ 3x = 60

⇒ x = 20 m

∵ CE = BD and BD = 60–x.

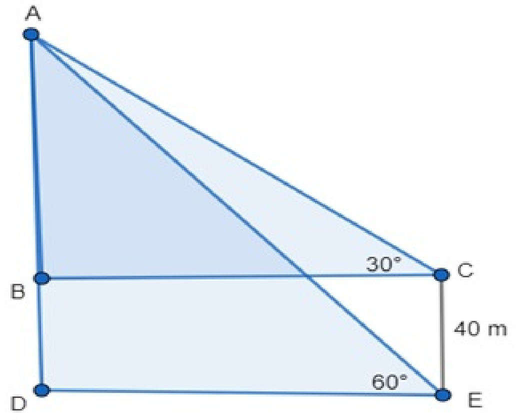
⇒ CE = 60 – 20 = 40 m

∴ Height of the building = 40 m.

17. Question

From the top and foot of a 40 m high tower, the angles of elevation of the top of a lighthouse are found to be 30° and 60° respectively. Find the height of the lighthouse. Also find the distance of the top of the lighthouse from the foot of the tower.

Answer



Given, CE = 40 m

Let AB = x and BD = CE = 40 m

In triangle ABC ,

∠ACB = 30°

We know,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\Rightarrow \tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{BC}$$

⇒ BC = x√3 ……… (1)

And, in triangle ADE,

∠AED = 60°

We know,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\Rightarrow \tan 60° = \frac{AD}{DE}$$

$$\Rightarrow \sqrt{3} = \frac{x + 40}{DE}$$

$\Rightarrow DE = \frac{x+40}{\sqrt{3}}$ ……… (2)

∵ BC = DE

⇒ equation (1) = equation (2)

$$\Rightarrow x\sqrt{3} = \frac{(x + 40)}{\sqrt{3}}$$

⇒ 3 x = x + 40

⇒ 2x = 40

⇒ x = 20

∴ Height of the tower = 40 + x

= 40 + 20

= 60 m

And, from (2)–

DE = BC

= x√3

= 20√3

And, in triangle ADE,

Also we know,

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\Rightarrow \cos 60° = \frac{DE}{AE}$$

$$\Rightarrow \frac{1}{2} = \frac{20\sqrt{3}}{AE}$$

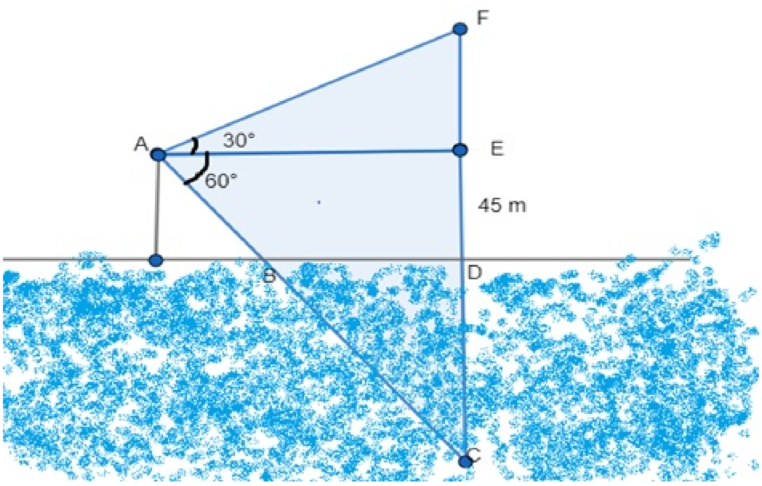
⇒ AE = 40√3 m

∴ the height of the lighthouse is 60 m and the distance of the top of the lighthouse from the foot of the tower is 40√3 m.

18. Question

The angle of elevation of a hovering helicopter as seen from a point 45 m above a lake is 30° and the angle of depression of its reflection in the lake, as seen from the same point and at the same time, is 60°. Find the distance of the helicopter from the surface of the lake.

Answer



Given, ED = 45 m and DF = ?

Let EF = x, DF = h

⇒ DC = h (∵ height of reflection = height of object)

In a right triangle FAE,

We know,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 30° = \frac{FE}{AE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-45}{AE}$$

AE = (h – 45) √3 .... (1)

And, in right triangle ACE,

$$\Rightarrow \tan 60° = \frac{EC}{AE}$$

$$\Rightarrow \sqrt{3} = \frac{ED + DC}{AE}$$

$$\Rightarrow \sqrt{3} = \frac{45 + h}{AE}$$

$\Rightarrow AE = \frac{45+h}{\sqrt{3}}$ ....(2)

Now, ∵BC = AD

⇒ equation (1) = equation (2)

$$\Rightarrow (h - 45)\sqrt{3} = \frac{(45 + h)}{\sqrt{3}}$$

⇒ 3(h – 45) = 45 + h

⇒ 3h – 135 = 45 + h

⇒ 2h = 45 + 135

⇒ 2h = 180

⇒ h = 90 m

∴ the distance of the helicopter from the surface of the lake is

90 m.

Exercise 7.3

1. Question

Choose the correct answer:$(1 – \sin^2\theta)\sec^2\theta$ =

A. 0

B. 1

C. $\tan^2\theta$

D. $\cos^2\theta$

Answer

Given trigonometric equation = $\left(1-\sin^2\theta\right)\sec^2\theta$

We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow (1\text{-}\sin^2\theta)\sec^2\theta = \cos^2\theta\sec^2\theta$

$\Rightarrow (1\text{-}\sin^2\theta)\sec^2\theta = \cos^2\theta \times \frac{1}{\cos^2\theta} (\because \cos\theta = \frac{1}{\sec\theta})$

$\Rightarrow (1\text{-}\sin^2\theta)\sec^2\theta = 1$

Hence, $(1\text{-}\sin^2\theta)\sec^2\theta = 1$.

2. Question

Choose the correct answer:$(1 + \tan^2\theta)\sin^2\theta =$

A. $\sin^2\theta$

B. $\cos^2\theta$

C. $\tan^2\theta$

D. $\cot^2\theta$

Answer

Given Trigonometric equation = $(1 + \tan^2\theta)\sin^2\theta$

We know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\therefore (1 + \tan^2\theta)\sin^2\theta = (1 + \frac{\sin^2\theta}{\cos^2\theta})\sin^2\theta$

$\Rightarrow (1 + \tan^2\theta)\sin^2\theta = (\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta})\sin^2\theta$

$\Rightarrow (1 + \tan^2\theta)\sin^2\theta = \frac{1}{\cos^2\theta}\sin^2\theta\ (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow (1 + \tan^2\theta)\sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow (1 + \tan^2\theta)\sin^2\theta = \tan^2\theta$

Hence, $(1 + \tan^2\theta)\sin^2\theta = \tan^2\theta$.

3. Question

Choose the correct answer:$(1 - \cos^2\theta)(1 + \cot^2\theta) =$

A. $\sin^2\theta$

B. 0

C. 1

D. $\tan^2\theta$

Answer

Given trigonometric equation is = $(1 - \cos^2\theta)(1 + \cot^2\theta)$

As we know that,

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

And $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\Rightarrow (1-\cos^2\theta)(1+\cot^2\theta) = \sin^2\theta(1+\frac{\cos^2\theta}{\sin^2\theta})$

$\Rightarrow (1-\cos^2\theta)(1+\cot^2\theta) = \sin^2\theta(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta})$

$\Rightarrow (1-\cos^2\theta)(1+\cot^2\theta) = \sin^2\theta \times \frac{1}{\sin^2\theta}$

$\Rightarrow (1-\cos^2\theta)(1+\cot^2\theta) = 1$

Hence, $\Rightarrow (1-\cos^2\theta)(1+\cot^2\theta) = 1.$

4. Question

Choose the correct answer:sin(90° – θ)cosθ + cos(90° – θ) sin θ =

A. 1

B. 0

C. 2

D. –1

Answer

From Trigonometric identities –

$\Rightarrow \sin(90° - \theta) = \cos\theta$

$\Rightarrow \cos(90° - \theta) = \sin\theta$

Using above formulas.

$\Rightarrow \sin(90° - \theta)\cos\theta + \cos(90° - \theta)\sin\theta = \cos\theta\cos\theta + \sin\theta\sin\theta$

$\Rightarrow \sin(90° - \theta)\cos\theta + \cos(90° - \theta)\sin\theta = \cos^2\theta + \sin^2\theta$

$\Rightarrow \sin(90° - \theta)\cos\theta + \cos(90° - \theta)\sin\theta = 1$ $(\because \sin^2\theta + \cos^2\theta = 1)$

5. Question

Choose the correct answer:$1 - \frac{\sin^2\theta}{1+\cos\theta} =$

A. cos θ

B. tan θ

C. cot θ

D. cosec θ

Answer

Given, $1 - \frac{\sin^2\theta}{1+\cos\theta}$

$\Rightarrow 1 - \frac{\sin^2\theta}{1+\cos\theta} = \frac{1+\cos\theta-\sin^2\theta}{1+\cos\theta}$ [taking (1+cosθ) as L.C.M.)

$\Rightarrow 1 - \frac{\sin^2\theta}{1+\cos\theta} = \frac{1+\cos\theta-(1-\cos^2\theta)}{1+\cos\theta}$ ($\because \sin^2\theta = 1 - \cos^2\theta$)

$\Rightarrow 1 - \frac{\sin^2\theta}{1+\cos\theta} = \frac{1+\cos\theta-1+\cos^2\theta}{1+\cos\theta}$

$\Rightarrow 1-\frac{\sin^2\theta}{1+\cos\theta}=\frac{\cos\theta+\cos^2\theta}{1+\cos\theta}$

$\Rightarrow 1-\frac{\sin^2\theta}{1+\cos\theta}=\frac{\cos\theta(1+\cos\theta)}{1+\cos\theta}$

$\Rightarrow 1-\frac{\sin^2\theta}{1+\cos\theta}=\cos\theta$

Hence, $1-\frac{\sin^2\theta}{1+\cos\theta}=\cos\theta$

6. Question

Choose the correct answer:$\cos^4 x - \sin^4 x =$

A. $2\sin^2 x - 1$

B. $2\cos^2 x - 1$

C. $1 + 2\sin^2 x$

D. $1 - 2\cos^2 x$

Answer

we can write –

$\cos^4 x=(\cos^2 x)^2$

$\sin^4 x=(\sin^2 x)^2$

$\therefore \cos^4 x-\sin^4 x=(\cos^2 x)^2-(\sin^2 x)^2$

Now applying formula,

$a^2-b^2=(a+b)(a-b)$

$\Rightarrow \cos^4 x-\sin^4 x=(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)$

We know that, $(\cos^2 x+\sin^2 x=1)$

$\Rightarrow \cos^4 x-\sin^4 x=(\cos^2 x-\sin^2 x)$

$\Rightarrow \cos^4 x-\sin^4 x=\cos^2 x-(1-\cos^2 x)$ $(\because \sin^2 x=1-\cos^2 x)$

$\Rightarrow \cos^4 x-\sin^4 x=\cos^2 x-1+\cos^2 x$

$\Rightarrow \cos^4 x-\sin^4 x=2\cos^2 x-1$

Hence, $\cos^4 x-\sin^4 x=2\cos^2 x-1$

7. Question

Choose the correct answer:If $\tan\theta=\frac{a}{x}$, then the value of $\frac{x}{\sqrt{a^2+x^2}}$ =

A. $\cos\theta$

B. $\sin\theta$

C. $\operatorname{cosec}\theta$

D. $\sec\theta$

Answer

Given, $\tan\theta=\frac{a}{x}$

∴ a=x tanθ

$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \frac{x}{\sqrt{(x\tan\theta)^2+x^2}}$

$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \frac{x}{\sqrt{x^2\tan^2\theta+x^2}}$

$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \frac{x}{x\sqrt{\tan^2\theta+1}}$

$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \frac{1}{\sqrt{\sec^2\theta}}$ (∵ $\sec^2\theta - \tan^2\theta = 1$)

$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \frac{1}{\sec\theta}$

$\Rightarrow \frac{x}{\sqrt{a^2+x^2}} = \cos\theta$ (∵ $\frac{1}{\sec\theta} = \cos\theta$)

8. Question

Choose the correct answer:If x = a secθ, y = b tan θ , then the value of $\frac{x^2}{a^2} - \frac{y^2}{b^2} =$

A. 1

B. –1

C. $\tan^2\theta$

D. $\text{cosec}^2\theta$

Answer

Given,

$x = a\sec\theta \text{ and } y = b\tan\theta$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{(a\ \sec\theta)^2}{a^2} - \frac{(b\ \tan\theta)^2}{b^2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{a^2\sec^2\theta}{a^2} - \frac{b^2\tan^2\theta}{b^2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (∵ $\sec^2\theta - \tan^2\theta = 1$)

9. Question

Choose the correct answer: $\frac{\sec\theta}{\cot\theta + \tan\theta} =$

A. cot θ

B. tan θ

C. sin θ

D. – cot θ

Answer

We know that,

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \frac{\sec\theta}{\cot\theta+\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{\sec\theta}{\cot\theta+\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}$

$\Rightarrow \frac{\sec\theta}{\cot\theta+\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta\cos\theta}}$ ($\because \cos^2\theta + \sin^2\theta = 1$)

$\Rightarrow \frac{\sec\theta}{\cot\theta+\tan\theta} = \frac{1}{\cos\theta} \times \sin\theta\cos\theta$

$\Rightarrow \frac{\sec\theta}{\cot\theta+\tan\theta} = \sin\theta$

Hence, proved $\frac{\sec\theta}{\cot\theta+\tan\theta} = \sin\theta.$

10. Question

Choose the correct answer: $\frac{\sin(90° - \theta)\sin\theta}{\tan\theta} + \frac{\cos(90° - \theta)\cos\theta}{\cot\theta} =$

A. tan θ

B. 1

C. -1

D. sin θ

Answer

We know that,

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\Rightarrow \frac{\sin(90°-\theta)\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta)\cos\theta}{\cot\theta} = \frac{\cos\theta\sin\theta}{\tan\theta} + \frac{\sin\theta\cos\theta}{\cot\theta}$

$\Rightarrow \frac{\sin(90°-\theta)\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta)\cos\theta}{\cot\theta} = \frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta\cos\theta}{\frac{\cos\theta}{\sin\theta}}$

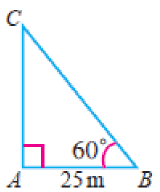
$\Rightarrow \frac{\sin(90°-\theta)\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta)\cos\theta}{\cot\theta} = \cos\theta\sin\theta \times \frac{\cos\theta}{\sin\theta} + \sin\theta\,\cos\theta \times \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \frac{\sin(90°-\theta)\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta)\cos\theta}{\cot\theta} = \cos^2\theta + \sin^2\theta$

$\Rightarrow \frac{\sin(90°-\theta)\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta)\cos\theta}{\cot\theta} = 1$ ($\because \cos^2\theta + \sin^2\theta = 1$)

11. Question

Choose the correct answer:In the adjoining figure, AC =



A. 25 m

B. 25√3 m

C. $\frac{25}{\sqrt{3}}$ m

D. 25√2 m

Answer

As we know that,

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AC}{AB}$

From given figure,

AB = 25 m.

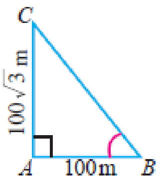
$\therefore AC = AB\ \tan\theta$

$\Rightarrow AC = 25\ \tan 60°$

$\Rightarrow AC = 25\sqrt{3}\text{ m } (\because \tan 60° = \sqrt{3})$

12. Question

Choose the correct answer:In the adjoining figure ∠ABC =



A. 45°

B. 30°

C. 60°

D. 50°

Answer

From given figure,

$AB = 100\text{ m}$

$AC = 100\sqrt{3}\text{ m}$

$\Rightarrow \tan\theta = \frac{\text{Perpendicular(AC)}}{\text{Base (AB)}}$

$\Rightarrow \tan\theta = \frac{100\sqrt{3}}{100}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\sqrt{3}$

$\Rightarrow \theta = 60°$

13. Question

Choose the correct answer:A man is 28.5 m away from a tower. His eye level above the ground is 1.5 m. The angle of elevation of the tower from his eyes is 45c. Then the height of the tower is

A. 30 m

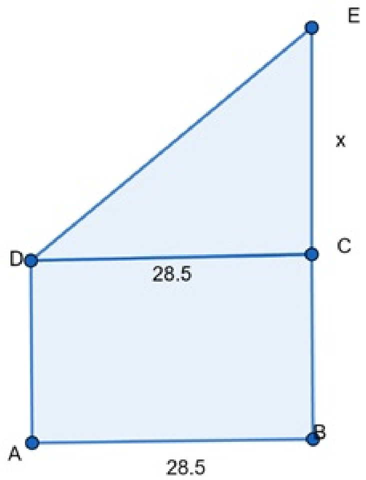
B. 27.5 m

C. 28.5 m

D. 27 m

Answer

figure can be drawn as –



∠CDE=45°

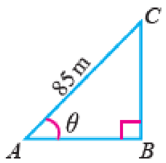
$\Rightarrow \tan 45° = \frac{x}{28.5}$

$\Rightarrow x = 28.5$

Height of the tower = 28.5+1.5= 30 m

14. Question

Choose the correct answer:In the adjoining figure, $\sin\theta = \frac{15}{17}$. Then BC =



A. 85 m

B. 65 m

C. 95 m

D. 75 m

Answer

We know that,

$\sin\theta = \frac{\text{Perpendicular(BC)}}{\text{Hypotaneous(AC)}}$

$\Rightarrow BC = AC \sin\theta$

Given that, $\sin\theta = \frac{15}{17}$

$\Rightarrow BC = 85 \times \frac{15}{17}$ (from figure AC = 85 m)

$\Rightarrow BC = 5 \times 15 = 75\ m$

15. Question

Choose the correct answer:$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) =$

A. $\cos^2\theta - \sin^2\theta$

B. $\sin^2\theta - \cos^2\theta$

C. $\sin^2\theta + \cos^2\theta$

D. 0

Answer

We know that,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = (1 + \frac{\sin^2\theta}{\cos^2\theta})(1 - \sin\theta)(1 + \sin\theta)$

Using formula, $(a + b)(a - b) = (a^2 - b^2)$

$\Rightarrow (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = (\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta})(1 - \sin^2\theta)$

$\Rightarrow (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} \times \cos^2\theta$

$(\because \cos^2\theta = 1 - \sin^2\theta)$

$\Rightarrow (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta + \sin^2\theta$

Hence, $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta + \sin^2\theta$

16. Question

Choose the correct answer:$(1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta) =$

A. $\tan^2\theta - \sec^2\theta$

B. $\sin^2\theta - \cos^2\theta$

C. $\sec^2\theta - \tan^2\theta$

D. $\cos^2\theta - \sin^2\theta$

Answer

We know that,

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

And $(a + b)(a - b) = a^2 - b^2$

$\Rightarrow (1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta) = \text{cosec}^2\theta\,(1 - \cos^2\theta)$

$[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$\Rightarrow (1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta) = \text{cosec}^2\theta \times \sin^2\theta$

$\because \sin\theta = \frac{1}{\text{cosec}\theta}$

$\therefore (1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta) = \text{cosec}^2\theta \times \frac{1}{\text{cosec}^2\theta}$

$\Rightarrow (1+\cot^2\theta)(1-\cos\theta)(1+\cos\theta)=1$

Also, we know that,

$1+\tan^2\theta = \sec^2\theta$

$\therefore \sec^2\theta - \tan^2\theta = 1$

17. Question

Choose the correct answer:(cos^{2}θ – 1)(cot^{2}θ + 1) + 1 =

A. 1

B. –1

C. 2

D. 0

Answer

We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow -\sin^2\theta = \cos^2\theta - 1$

Now,

$\Rightarrow (\cos^2\theta - 1)(\cot^2\theta + 1) + 1 = (-\sin^2\theta)(\mathrm{cosec}^2\theta) + 1$

$\because \mathrm{cosec}^2\theta - \cot^2\theta = 1$

And $\sin\theta = \frac{1}{\mathrm{cosec}\theta}$

$\Rightarrow (\cos^2\theta - 1)(\cot^2\theta + 1) + 1 = \left(-\sin^2\theta)(\frac{1}{\sin^2\theta}\right) + 1$

$\Rightarrow (\cos^2\theta - 1)(\cot^2\theta + 1) + 1 = (-1)(1) + 1$

$\Rightarrow (\cos^2\theta - 1)(\cot^2\theta + 1) + 1 = -1 + 1 = 0$

Hence, $(\cos^2\theta - 1)(\cot^2\theta + 1) + 1 = 0$

18. Question

Choose the correct answer: $\frac{1+\tan^2\theta}{1+\cot^2\theta} =$

A. cos^{2}θ

B. tan^{2}θ

C. sin^{2}θ

D. cot^{2}θ

Answer

We know that,

$\sec^2\theta - \tan^2\theta = 1$

$\therefore \sec^2\theta = 1 + \tan^2\theta$

$\text{cosec}^2\theta - \cot^2\theta = 1$

$\therefore \text{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{\sec^2\theta}{\text{cosec}^2\theta}$

Where, $\sec\theta = \frac{1}{\cos\theta}$ and $\text{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$

$\Rightarrow \dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{1}{\cos^2\theta} \times \dfrac{\sin^2\theta}{1}$

$\Rightarrow \dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow \dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \tan^2\theta$

Hence, $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \tan^2\theta$

19. Question

Choose the correct answer: $\sin^2\theta + \dfrac{1}{1+\tan^2\theta} =$

A. $\text{cosec}^2\theta + \cot^2\theta$

B. $\text{cosec}^2\theta - \cot^2\theta$

C. $\cot^2\theta - \text{cosec}^2\theta$

D. $\sin^2\theta - \cos^2\theta$

Answer

We know that,

$\sec^2\theta - \tan^2\theta = 1$

$\therefore \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$

$\Rightarrow \sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \cos^2\theta$ ($\because \cos\theta = \frac{1}{\sec\theta}$)

$\Rightarrow \sin^2\theta + \frac{1}{1+\tan^2\theta} = 1$ ($\because \sin^2\theta + \cos^2\theta = 1$)

Also,

$\therefore \text{cosec}^2\theta = 1 + \cot^2\theta$

$\text{cosec}^2\theta - \cot^2\theta = 1$

20. Question

Choose the correct answer: $9\tan^2\theta - 9\sec^2\theta =$

A. 1

B. 0

C. 9

D. –9

Answer

We know that,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow -(-\sec^2\theta + \tan^2\theta) = 1$

$\Rightarrow \tan^2\theta - \sec^2\theta = -1 - - - - - (\text{i})$

Now, from given questions –

$\Rightarrow 9\tan^2\theta - 9\sec^2\theta = 9(\tan^2\theta - \sec^2\theta)$

$\Rightarrow 9\tan^2\theta - \sec^2\theta = 9(-1)$ [from equation (i)]

$\Rightarrow 9\tan^2\theta - \sec^2\theta = -9$