

Topic : Rigid Body Dynamics, String Wave, Kinetic Theory of Gases, Thermal Expansion, Simple Harmonic Motion, Electrostatics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.2

(3 marks, 3 min.)

M.M., Min.

[6, 6]

Multiple choice objective ('-1' negative marking) Q.3 to Q.4

(4 marks, 4 min.)

[8, 8]

Subjective Questions ('-1' negative marking) Q.5 to Q.6

(4 marks, 5 min.)

[8, 10]

Comprehension ('-1' negative marking) Q.7 to Q.9

(3 marks, 3 min.)

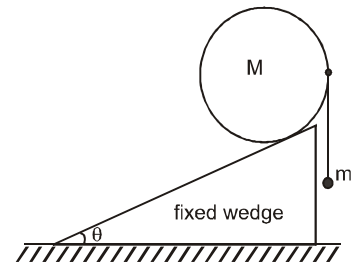
[9, 9]

Match the Following (no negative marking) (2 × 4)Q.10

(8 marks, 10 min.)

[8, 10]

1. A uniform cylinder of mass M lies on a fixed plane inclined at an angle θ with horizontal. A light string is tied to the cylinder's right most point, and a mass m hangs from the string, as shown. Assume that the coefficient of friction between the cylinder and the plane is sufficiently large to prevent slipping. For the cylinder to remain static, the value of mass m is-



(A) $\frac{M \cos \theta}{1 + \sin \theta}$

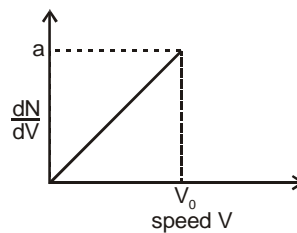
(B) $M \frac{\sin \theta}{1 + \sin \theta}$

(C) $M \frac{\cos \theta}{1 - \sin \theta}$

(D) $M \frac{\sin \theta}{1 - \sin \theta}$

2. At $t = 0$, a transverse wave pulse travelling in the positive x direction with a speed of 2 m/s in a long wire is described by the function $y = \frac{6}{x^2}$, given that $x \neq 0$. Transverse velocity of a particle at $x = 2$ m and $t = 2$ seconds is :
- (A) 3 m/s (B) - 3 m/s (C) 8 m/s (D) - 8 m/s

3. Graph shows a hypothetical speed distribution for a sample of N gas particle (for $V > V_0$; $\frac{dN}{dV} = 0$, $\frac{dN}{dV}$ is rate of change of number of particles with change velocity)

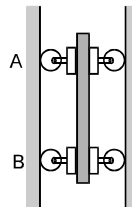


- (A) The value of aV_0 is $2N$.
 (B) The ratio V_{avg}/V_0 is equal to $2/3$.
 (C) The ratio V_{rms}/V_0 is equal to $1/\sqrt{2}$.
 (D) Three fourth of the total particle has a speed between $0.5 V_0$ and V_0 .
4. A vessel is partly filled with liquid. When the vessel is cooled to a lower temperature, the space in the vessel, unoccupied by the liquid remains constant. Then the volume of the liquid (V_L), volume of the vessel (V_v), the coefficients of cubical expansion of the material of the vessel (γ_v) and of the liquid (γ_L) are related as
- (A) $\gamma_L > \gamma_v$ (B) $\gamma_L < \gamma_v$ (C) $\gamma_v/\gamma_L = V_v/V_L$ (D) $\gamma_v/\gamma_L = V_L/V_v$

5. Two particles perform SHM with the same amplitude and same frequency about the same mean position and along the same line. If the maximum distance between them during the motion is A (A is the amplitude of either) then the phase difference between their SHM is _____.
6. Calculate the magnitude of electrostatic force on a charge placed at a vertex of a triangular pyramid (4 vertices, 4 faces), if 4 equal point charges are placed at all four vertices of pyramid of side ' a '.

COMPREHENSION

A bar of mass m is held as shown between 4 uniform discs each of mass m' and radius $r = 75$ mm. The bar has been released from rest, knowing that the normal forces exerted on the discs by vertical walls are sufficient to prevent any slipping. Answer the following questions assuming attachments are massless and fixed to the bar. (Acceleration due to gravity is g)



7. If $m = 5$ kg and $m' = 2$ kg, then acceleration of the bar is
 (A) g (B) $13g/7$ (C) $13g/17$ (D) $\frac{g}{17}$
8. If the mass m' of the discs is negligible, then acceleration of the bar is
 (A) g (B) $2g/3$ (C) $13g/17$ (D) $\frac{g}{17}$
9. If the mass m of the bar is negligible, then acceleration of the bar is
 (A) g (B) $2g/3$ (C) $13g/17$ (D) $\frac{g}{17}$
10. Assume only electrostatic interaction forces :

Column – I

- (A) Three charges are kept along a straight line
 (B) Three charges are kept at the vertices of an equilateral triangle
 (C) Three charges are kept at the three vertices of a square, and a fourth charge is kept at the point of intersection of the diagonals.
 (D) Three charges are kept at the vertices of an equilateral triangle with the fourth charge at the centroid.

Column – II

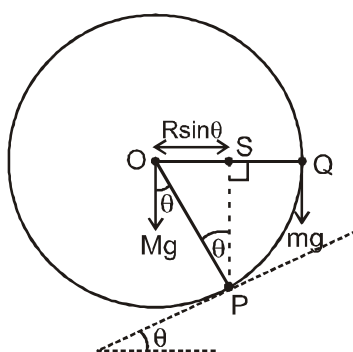
- (p) The system may be in equilibrium with proper choice of the value of charges.
 (q) The system will be in equilibrium for any value of the charges.
 (r) The system will not be in equilibrium for any choice of the value of charges.
 (s) The equilibrium is unstable.

Answers Key

1. (D) 2. (B) 3. (A), (B), (C), (D)
4. (A), (D) 5. $\phi = \pi/3$
6. $\frac{\sqrt{6} q^2}{4\pi \epsilon_0 a^2}$ 7. (C) 8. (A)
9. (B) 10. (A) p, s ; (B) r ; (C) r ; (D) p, s

Hints & Solutions

1.



Let the radius of cylinder be R

For the cylinder to remain static, net torque on cylinder about point P (point of contact with inclined surface) should be zero.

$$\therefore Mg (OS) = mg (SQ)$$

$$\text{or } Mg R \sin \theta = mg R (1 - \sin \theta)$$

$$\text{or } m = \frac{M \sin \theta}{1 - \sin \theta}$$

$$2. (B) y(x, t = 0) = \frac{6}{x^2} \quad \text{then} \quad y(x, t)$$

$$= \frac{6}{(x - 2t)^2}$$

$$\Rightarrow \frac{\partial y}{\partial t} = \frac{24}{(x - 2t)^3}$$

$$\text{at } x = 2, t = 2$$

$$V_y = \frac{24}{(-2)^3} = -3 \text{ m/s.}$$

3. Area under the curve is equal to number of molecules of the gas sample. Hence

$$N = \frac{1}{2} \cdot a \cdot V_0 \Rightarrow aV_0 = 2N$$

$$V_{\text{avg}} = \frac{1}{N} \int_0^{\infty} v N(v) dv$$

$$= \frac{1}{N} \int_0^{V_0} C \cdot \left(\frac{a}{V_0} \cdot v \right) dv = \frac{2}{3} V_0 \Rightarrow \frac{V_{\text{avg}}}{V_0} = \frac{2}{3}$$

$$v_{\text{rms}}^2 = \frac{1}{N} \int_0^{\infty} v^2 N(v) dv$$

$$= \frac{1}{N} \int_0^{V_0} v^2 \left(\frac{a}{V_0} \cdot v \right) dv = \frac{V_0^2}{2} \Rightarrow \frac{V_{\text{rms}}}{V_0} = \frac{1}{\sqrt{2}}$$

Area under the curve from $0.5 V_0$ to V_0 is $\frac{3}{4}$ of total area.

4. $\Delta V_L = \Delta V_V$
 $\Rightarrow Y_L V_L = Y_V V_V$

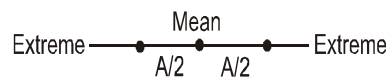
or $\frac{Y_L}{Y_V} = \frac{V_V}{V_L}$

but $V_V > V_L$
 $\Rightarrow Y_L > Y_V$

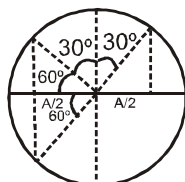
5. $Y_1 = A \sin \omega t$
 $Y_2 = A \sin(\omega t + \phi) \quad \phi = 2\pi/3$
 As, $Y_2 + Y_1 = A$

Now, solve and use the condition of maxima.

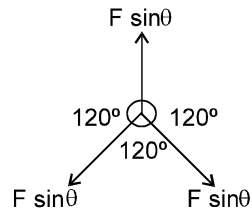
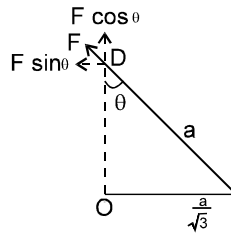
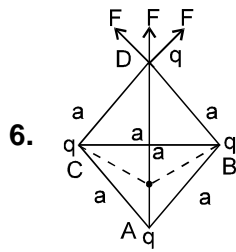
Alternate : by symmetry both should be on opposite side of mean position at equal distance from mean (for max. separation)



As projection of SHM on circular path



The phase difference from figure is $\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$



$$\cos \theta = \sqrt{\frac{2}{3}}$$

$$\begin{aligned} \therefore F_{\text{net}} &= 3F \cos \theta = 3 \frac{Kq^2}{a^2} \sqrt{\frac{2}{3}} \\ &= \frac{\sqrt{6} q^2}{4\pi \epsilon_0 a^2} \end{aligned}$$

$$\text{[Ans. } \frac{\sqrt{6} q^2}{4\pi \epsilon_0 a^2} \text{]}$$

Sol. 7 to 9

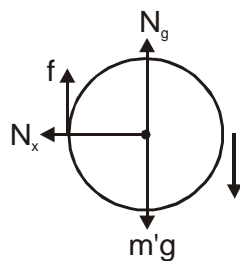
7. F.B.D. of disc

\Rightarrow In horizontal direction $N = N_x$ (1)

In vertical direction $m'g - f - N_y = m'a$ (2)

Torque due to friction ;

$$fR = \frac{m'R^2}{2} \propto$$



$$\text{As } a = R\alpha$$

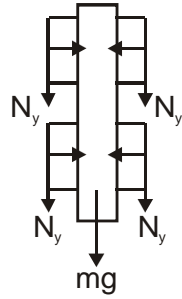
$$\therefore fR = \frac{m'R^2}{2} \frac{a}{R}$$

$$\Rightarrow f = \frac{m'a}{2} \text{ put in (2)}$$

$$\therefore m'g - N_y = \frac{3m'a}{2}$$

$$N_y = m'g - \frac{3m'a}{2} \quad \text{.....(3)}$$

Now, F.B.D. of bar ;



$$4N_y + mg = ma$$

Put N_y from (3)

$$4m'g - 6m'a + mg = ma \quad \dots (4)$$

As $m' = 2\text{ kg}$ & $m = 5\text{ kg}$

$$\therefore 8g - 12a + 5g = 5a$$

$$a = \frac{13g}{17} \downarrow$$

8. From equation (4) (neglecting m') $a = g \downarrow$

9. From equation (4) (neglecting m) $a = \frac{2g}{3} \downarrow$

10. (A) \rightarrow p, s ; (B) \rightarrow r ; (C) \rightarrow r ; (D) \rightarrow p, s