CBSE Test Paper 03

Chapter 11 Three Dimensional Geometry

1. The distance d from a point P(x1, y1, z1) to the plane Ax + By + Cz + D = 0 is
   a. $d = \left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$
   b. $d = \left|\frac{Ax_1+By_1+2Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$
   c. $d = \left|\frac{Ax_1+2By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$
   d. $d = \left|\frac{Ax_1+By_1+Cz_1+2D}{\sqrt{A^2+B^2+C^2}}\right|$
2. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.
   a. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ +$\lambda \left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$,$\lambda \in R$
   b. $\vec{r} = \widehat{2i} + 2\hat{j} + 3\hat{k}$ +$\lambda \left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$
   c. $\vec{r} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ +$\lambda \left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$
   d. $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ +$\lambda \left(3\hat{i} + 2\hat{j} - 2\hat{k}.\right)$ $\lambda \in R$
3. Find the coordinates of the foot of the perpendicular drawn from the origin to 2x + 3y + 4z – 12 = 0.
   a. $\left(\frac{27}{29}, \frac{36}{29}, \frac{48}{29}\right)$
   b. $\left(\frac{24}{29}, \frac{39}{29}, \frac{48}{29}\right)$
   c. $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$
   d. $\left(\frac{24}{29}, \frac{36}{29}, \frac{49}{29}\right)$
4. Find the distance of the point (2, 3, – 5) from the plane x + 2y – 2z = 9.

a. 5
b. 4
c. 3
d. 2

5. What is the shortest distance between $\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_2}\ and\ \vec{r} = \overrightarrow{a_2} + \mu \overrightarrow{b_2}\,, \lambda, \mu \in R.$

a. $S.D = \left| \frac{\left(\overrightarrow{b_1} \times - \overrightarrow{b_2}\right) . \left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right|$

b. $S.D = \left| \frac{\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) . \left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right|$

c. $S.D = \left| \frac{\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) . \left(\overrightarrow{a_2} + \overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right|$

d. $S.D = \left| \frac{\left(-\overrightarrow{b_1} \times \overrightarrow{b_2}\right) . \left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right|$

6. The angle between skew lines is the angle between two intersecting lines drawn from any point ________ to each of the skew lines.
7. The vector equation of the line through the points (3, 4, -7) and (1, -1, 6) is ________.
8. Equation of a plane which is at a distance p from the origin with direction cosines of the normal to the plane as l, m, n, is ________.

9. Find the angle between the vector having direction ratios (3,4,5) and (4, -3, 5).

10. Equation of line is $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$. Find the direction cosines of a line parallel to above line.

11. Find the vector equation of the plane with intercepts 3, -4 and 2 on X,Y and Z-axes, respectively.

12. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ Write its vector form.

13. Prove that if a plane has the intercepts a,b,c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$ **(2)**

14. Show that the line through the points $(1, -1, 2), (3, 4, -2)$is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

15. Find the angles between the lines $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - 5\hat{k}\right) + \mu\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right)$.

16. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $r.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$.

17. Find the equation of the plane through the intersection of the planes 3x - y + 2z - 4 = 0 x + y + z - 2 = 0 and the point (2, 2, 1).

18. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

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Solution

1. a. $d=\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$, **Explanation:** The distance d from a point P(x1, y1, z1) to the plane Ax + By + Cz + D = 0 is given by : $d=\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$.

2. a. $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}$ +$\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}.\right)$,$\lambda\in R$, **Explanation:** The equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$, let vector $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}$ and vector $\overrightarrow{b}=3\hat{i}+2\hat{j}-2\hat{k}$, the equation of line is :
$\overrightarrow{a}+\lambda\overrightarrow{b}=(\hat{i}+\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$

3. c. $\left(\frac{24}{29},\frac{36}{29},\frac{48}{29}\right)$, **Explanation:** D.R.'s of the line are < 2 , 3 , 4 > .
Therefore, equation of the line is: $\frac{x-0}{2}=\frac{y-0}{3}=\frac{z-0}{4}=\lambda$
Thus , the coordinates of any point P on the above line are P(2λ , 3λ ,4λ).
But, this point P also lies on the given plane: 2(2λ) + 3(3λ) +4(4λ) - 12 =0.
$\Rightarrow 29\lambda=12\Rightarrow\lambda=\frac{12}{29}$
Therefore, the coordinates of the foot of perpendicular are given by :
$\left(2\times\frac{12}{29},3\times\frac{12}{29},4\times\frac{12}{29}\right)$

4. c. 3, **Explanation:** As we know that the length of the perpendicular from point p(x_1, y_1, z_1) from the plane $a_1x + b_1y + c_1z + d_1 = 0$ is given by:
$\frac{|a_1x+b_1y+c_1z+d_1|}{\sqrt{a^2{}_1+b^2{}_1+c^2{}_1}}$
Here, P(2, 3, -5) is the point and equation of plane is x + 2y - 2z = 9.
Therefore, the perpendicular distance is:
$\frac{|2+2(3)-2(-5)-9|}{\sqrt{1+4+4}}=\frac{|9|}{\sqrt{9}}=\frac{9}{3}=3$ units.

5. b. $S.D=\left|\frac{\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right).\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right|$, **Explanation:** The shortest distance between $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_2}\ and\ \vec{r}=\overrightarrow{a_2}+\mu\overrightarrow{b_2}$ is given by $S.D=\left|\frac{\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right).\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right|$

6. parallel
7. $(x-3)\hat{i}+(y-4)\hat{j}+(z+7)\hat{k}$ = $\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$
8. lx + my + nz = p

9. Let a_1 = 3, b_1 = 4, c_1 = 5 and a_2 = 4, b_2 = -3, c_2 = 5

$$\cos\theta=\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$
$$=\left(\frac{(3)(4)+(4)(-3)+(5)(5)}{\sqrt{3^2+4^2+5^2}\sqrt{4^2+(-3)^2+5^2}}\right)$$
$$=\frac{25}{\sqrt{50}\sqrt{50}}=\frac{25}{50}=\frac{1}{2}$$
$$\theta=60^0$$

10. According to the question, equation of line can be written as

$$\frac{x-4}{-2}=\frac{y+3}{2}=\frac{z+2}{1}$$

Here, Direction ratios of a line are (-2,2,1).

Direction cosines of a line parallel to above line are given by

$$\left(\frac{-2}{\sqrt{(-2)^2+(2)^2+(1)^2}},\frac{2}{\sqrt{(-2)^2+(2)^2+(1)^2}},\frac{1}{\sqrt{(-2)^2+(2)^2+(1)^2}}\right)$$
$$=\left(\frac{-2}{\sqrt{4+4+1}},\frac{2}{\sqrt{4+4+1}},\frac{1}{\sqrt{4+4+1}}\right)$$
$$=\left(\frac{-2}{\sqrt{9}},\frac{2}{\sqrt{9}},\frac{1}{\sqrt{9}}\right)=\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$$

Required Direction cosines of a line parallel to the given line are $\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$.

11. Here, a = 3, b = -4 ,c = 2.We know that the equation of plane having intercepts 3, - 4 and 2 is

$$\frac{x}{3}+\frac{y}{-4}+\frac{z}{2}=1$$

4x - 3y + 6z =12, which can be written as $(x\hat{i}+\hat{y}\hat{j}+z\hat{k})\cdot(4\hat{i}-3\hat{j}+6\hat{k})=12$

$\Rightarrow\quad \vec{r}.(4\hat{i}-3\hat{j}+6\hat{k})=12$

12. Given: The Cartesian equation of the line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}=\lambda$ (say)

$\Rightarrow x-5=3\lambda, y+4=7\lambda, z-6=2\lambda$

$\Rightarrow x=5+3\lambda, y=-4+7\lambda, z=6+2\lambda$

General equation for the required line is $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Putting the values of x, y, z in this equation,

$\vec{r}=(5+3\lambda)\hat{i}+(-4+7\lambda)\hat{j}+(6+2\lambda)\hat{k}=5\hat{i}+3\lambda\hat{j}-4\hat{j}+7\lambda\hat{j}+6\hat{k}+2\lambda\hat{k}$

$\Rightarrow \vec{r}=\left(5\hat{i}-4\hat{j}+6\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)\left[since\,\vec{r}=\vec{a}+\lambda\vec{b}\right]$

13. We know that equation of plane making intercepts a,b,c (on the axes) is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0$

Given: Perpendicular distance of the origin (0, 0, 0) from plane = p

$\therefore \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=p$

$=\frac{|-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=p$

Squaring both sides, $=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}=p^2$

$\Rightarrow p^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=1$

$\Rightarrow \left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=\frac{1}{p^2}$

14. We know that direction ratios of the line joining the points $A(1,-1,2)$ and $B(3,4,-2)$ are $x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow 3-1, 4-(-1), -2-2$

$\Rightarrow 2,5,-4=a_1,b_1,c_1$

Again, direction ratios of the line joining the points C (0, 3, 2) and D (3, 5, 6) are

$x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow 3-0, 5-3, 6-2$

$\Rightarrow 3,2,4=a_2,b_2,c_2$ (say)

For lines AB and CD, $a_1a_2+b_1b_2+c_1c_2=2\times 3+5\times 2(-4)\times 4$ = 6 + 10 - 16 = 0

Since, it is 0, therefore, line AB is perpendicular to line CD.

15. We have, $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

And $\vec{r}=\left(2\hat{i}-5\hat{k}\right)+\mu\left(6\hat{i}+3\hat{j}+2\hat{k}\right)$

Where, $\vec{a}_1 = 3\hat{i} - 2\hat{j} + 6\hat{k}, \vec{b}_1 = 2\hat{i} + \hat{j} + 2\hat{k}$

And $\vec{a}_2 = 2\vec{j} - 5\vec{k}, \vec{b}_2 = 6\hat{i} + 3\hat{j} + 2\hat{k}$

If θ is angle between the lines, then

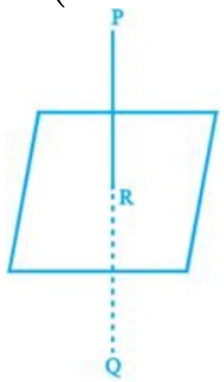
$$\cos\theta = \frac{\left|\vec{b}_1.\vec{b}_2\right|}{\left|\vec{b}_1\right|.\left|\vec{b}_1\right|}$$

$$= \frac{\left|\left(2\hat{i}+\hat{j}+2\hat{k}\right).\left(6\hat{i}+3\hat{j}+2\hat{k}\right)\right|}{\left|2\hat{i}+\hat{j}+2\hat{k}\right|\left|6\hat{i}+3\hat{j}+2\hat{k}\right|}$$

$$= \frac{|12+3+4|}{\sqrt{9}\sqrt{49}} = \frac{19}{21}$$

$$\theta = \cos^{-1}\frac{19}{21}$$

16. Let the given point be $P\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and Q be the image of P in the plane $\hat{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$ as shown in the Fig.



Then PQ is the normal to the plane. Since PQ passes through P and is normal to the given plane, so the equation of PQ is given by

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$$

Since Q lies on the line PQ, the position vector of Q can be expressed as

$\left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ i.e., $(1 + 2\lambda)\, i + (3 - \lambda)\,\hat{j} + 4\,(4 + \lambda)\,\hat{k}$

Since R is the mid point of PQ, the position vector of R is

$$\frac{\left[(1+2\lambda)\,\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}\right]+\left[\hat{i}+3\hat{j}+4\hat{k}\right]}{2}$$

i.e., $(\lambda + 1)\,\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}$

Again, since R lies on the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$, we have

$$\left\{(\lambda - 1)\,\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}\right\}.\left(2\hat{i} - \hat{j} - \hat{k}\right) + 3 = 0$$

$$\Rightarrow \lambda = -2$$

Hence, the position vector of Q is $\left(\hat{i}+3\hat{j}+4\hat{k}\right)-2\left(2\hat{i}-\hat{j}+\hat{k}\right)$, i.e., $-3\hat{i}+5\hat{j}+2\hat{k}$.

17. Equation of any plane through the intersection of given planes can be taken as
$(3x-y+2z-4)+\lambda(x+y+z-2)=0$ ...(1)
Since the point (2, 2, 1) lies in this plane,therefore,we get,
(3(2) - 2 + 2(1) - 4) + $\lambda[2+2+1-2]=0$
$\Rightarrow$2 + λ(3) = 0
$\Rightarrow \lambda=-\frac{2}{3}$. Put value of λ in eq (i), we get,
$(3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$
7x - 5y + 4z - 9 = 0

18. $\overrightarrow{a}_1=-\hat{i}-\hat{j}-\hat{k}$
$\overrightarrow{a}_2=3\hat{i}+5\hat{j}+7\hat{k}$
$\overrightarrow{b}_1=7\hat{i}-6\hat{j}+1\hat{k}$
$\overrightarrow{b}_2=\hat{i}-2\hat{j}+\hat{k}$
$\overrightarrow{a}_2-\overrightarrow{a}_1=4\hat{i}+6\hat{j}+8\hat{k}$
$\overrightarrow{b}_1\times\overrightarrow{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$
$=-4\hat{i}-6\hat{j}-8\hat{k}$
$(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=(4\hat{i}+6\hat{j}+8\hat{k}).(-4\hat{i}-6\hat{j}-8\hat{k})$ = -16 - 36 - 64 = -116
$\left|\overrightarrow{b}_1\times\overrightarrow{b}_2\right|=\sqrt{(-4)^2+(-6)^2+(-8)^2}$
$=\sqrt{116}=2\sqrt{29}$
$d=\left|\frac{(\overrightarrow{a}_2-\overrightarrow{a}_1).(b_1\times b_2)}{\left|\overrightarrow{b}_1\times\overrightarrow{b}_2\right|}\right|$
$=\left|\frac{-116}{2\sqrt{29}}\right|=\frac{4\times 29}{2\sqrt{29}}=2\sqrt{29}$