



Assignment

Fundamental Definite Integral

Basic Level

1. $\int_0^{-1} e^{2 \ln x} dx =$ [MP PET 1990]
 (a) 0 (b) 1/2 (c) 1/3 (d) 1/4
2. $\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx =$ [MNR 1990; AMU 1999; UPSEAT 2000]
 (a) $\frac{e^2}{2} + e$ (b) $e - \frac{e^2}{2}$ (c) $\frac{e^2}{2} - e$ (d) None of these
3. $\int_2^3 \frac{dx}{x^2 - x} =$ [EAMCET 2002]
 (a) $\log \frac{2}{3}$ (b) $\log \frac{1}{4}$ (c) $\log \frac{4}{3}$ (d) $\log \frac{8}{3}$
4. $\int_1^3 (x-1)(x-2)(x-3) dx =$ [Karnataka CET 2002]
 (a) 3 (b) 2 (c) 1 (d) 0
5. $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$ is equal to [DCE 2002]
 (a) $\pi/12$ (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/3$
6. $\int_1^e \frac{1}{x} dx$ is equal to [SCRA 1996]
 (a) ∞ (b) 0 (c) 1 (d) $\log(1+e)$
7. The value of $\int_0^{2/3} \frac{dx}{4+9x^2}$ is [Rajasthan PET 1992; MP PET 1997]
 (a) $\pi/12$ (b) $\pi/24$ (c) $\pi/4$ (d) 0
8. $\int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}} dx =$ [MNR 1987; UPSEAT 2000]
 (a) 0 (b) 2 (c) 8 (d) 4
9. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is
 (a) 3 (b) -1 (c) 2 (d) 0
10. $\int_0^{\pi/2} e^x \sin x dx$ is equal to [Roorkee 1978; EAMCET 1991]
 (a) $\frac{1}{2}(e^{\pi/2} - 1)$ (b) $\frac{1}{2}(e^{\pi/2} + 1)$ (c) $\frac{1}{2}(1 - e^{\pi/2})$ (d) $2(e^{\pi/2} + 1)$
11. The value of $\int_1^2 \log x dx$ is [Roorkee 1995]

- (a) $\log \frac{2}{e}$ (b) $\log 4$ (c) $\log \frac{4}{e}$ (d) $\log 2$
12. $\int_0^1 \frac{d}{dx} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right] dx$ is equal to [Kerala (Engg.) 2002]
 (a) 0 (b) π (c) $\pi/2$ (d) $\pi/4$
13. The value of $\int_{-2}^2 (ax^3 + bx + c) dx$ depends on [MNR 1988; Rajasthan PET 1990]
 (a) The value of a (b) The value of b (c) The value of c (d) The values of a and b
14. $\int_0^\pi \frac{dx}{1 + \sin x} =$ [CEE 1993]
 (a) 0 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$
15. $\int_0^1 \cos^{-1} x dx =$ [DSSE 1988]
 (a) 0 (b) 1 (c) 2 (d) None of these
16. If $I = \int_0^{\pi/4} \sin^2 x dx$ and $J = \int_0^{\pi/4} \cos^2 x dx$, then $I =$ [SCRA 1989]
 (a) $\frac{\pi}{4} - J$ (b) $2J$ (c) J (d) $\frac{J}{2}$
17. If $x(x^4 + 1)\phi(x) = 1$, then $\int_1^2 \phi(x) dx =$ [SCRA 1986]
 (a) $\frac{1}{4} \log \frac{32}{17}$ (b) $\frac{1}{2} \log \frac{32}{17}$ (c) $\frac{1}{4} \log \frac{16}{17}$ (d) None of these
18. $\int_{1/4}^{1/2} \frac{dx}{\sqrt{x-x^2}} =$ [SCRA 1986]
 (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$
19. $\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}} =$ [SCRA 1986]
 (a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{4\sqrt{2}}{3}$ (c) $\frac{8\sqrt{2}}{3}$ (d) None of these
20. $\int_0^{2\pi} (\sin x + \cos x) dx =$ [SCRA 1991]
 (a) 0 (b) 2 (c) -2 (d) 1
21. $\int_0^3 \frac{3x+1}{x^2+9} dx =$ [EAMCET 2003]
 (a) $\log(2\sqrt{2}) + \frac{\pi}{12}$ (b) $\log(2\sqrt{2}) + \frac{\pi}{2}$ (c) $\log(2\sqrt{2}) + \frac{\pi}{6}$ (d) $\log(2\sqrt{2}) + \frac{\pi}{3}$
22. $\int_0^{\pi/4} \tan^2 x dx =$ [Roorkee 1985]
 (a) $1 - \frac{\pi}{4}$ (b) $1 + \frac{\pi}{4}$ (c) $\frac{\pi}{4} - 1$ (d) $\frac{\pi}{4}$
23. $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx =$ [MP PET 1989]
 (a) $-\log 2$ (b) $\log 2$ (c) $\pi/2$ (d) 0
24. If $\int_0^1 f(x) dx = M$; $\int_0^1 g(x) dx = N$. Which of the following is correct

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- (a) $\int_0^1 (f(x) + g(x))dx = M + N$ (b) $\int_0^1 (f(x)g(x))dx = MN$ (c) $\int_0^1 \frac{1}{f(x)}dx = \frac{1}{M}$ (d) $\int_0^1 \frac{f(x)}{g(x)}dx = \frac{M}{N}$
25. $\int_{-\pi/4}^{\pi/2} e^{-x} \sin x dx =$ [CEE 1993]
 (a) $-\frac{1}{2}e^{-\pi/2}$ (b) $-\frac{\sqrt{2}}{2}e^{-\pi/4}$ (c) $-\sqrt{2}(e^{\pi/4} + e^{-\pi/4})$ (d) 0
26. $\int_1^e \frac{1 + \log x}{x} dx =$ [SCRA 1986]
 (a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{e}$ (d) None of these
27. If $\int_0^1 x \log\left(1 + \frac{x}{2}\right) dx = a + b \log \frac{2}{3}$, then [SCRA 1986]
 (a) $a = \frac{3}{2}, b = \frac{3}{2}$ (b) $a = \frac{3}{4}, b = -\frac{3}{4}$ (c) $a = \frac{3}{4}, b = \frac{3}{2}$ (d) $a = b$
28. The value of $\int_0^{\pi/4} \frac{1 + \tan x}{1 - \tan x} dx$ is [SCRA 1986]
 (a) $-\frac{1}{2} \log 2$ (b) $\frac{1}{4} \log 2$ (c) $\frac{1}{3} \log 2$ (d) None of these
29. $\int_{-1}^0 \frac{dx}{x^2 + 2x + 2}$ is equal to [MP PET 2000]
 (a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) $-\pi/4$
30. $\int_0^{\pi/6} (2 + 3x^2) \cos 3x dx =$ [DSSE 1985]
 (a) $\frac{1}{36}(\pi + 16)$ (b) $\frac{1}{36}(\pi - 16)$ (c) $\frac{1}{36}(\pi^2 - 16)$ (d) $\frac{1}{36}(\pi^2 + 16)$
31. The values of α which satisfy $\int_{\pi/2}^{\alpha} \sin x dx = \sin 2\alpha$ ($\alpha \in [0, 2\pi]$) are equal to [IIT Screening]
 (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{7\pi}{6}$ (d) All of the above
32. $\int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x)dx$ is equal to [Rajasthan PET 2000]
 (a) $\sqrt{2} - 2$ (b) $2\sqrt{2} - 2$ (c) $3\sqrt{2} - 2$ (d) $4\sqrt{2} - 2$
33. If $\int_0^a x dx \leq a + 4$, then [Rajasthan PET 2000]
 (a) $0 \leq a \leq 4$ (b) $-2 \leq a \leq 4$ (c) $-2 \leq a \leq 0$ (d) $a \leq -2$ or $a \geq 4$
34. $\int_0^{\pi/2} \{x - [\sin x]\} dx$ is equal to [AMU 1999]
 (a) $\pi^2/8$ (b) $\frac{\pi^2}{8} - 1$ (c) $\frac{\pi^2}{8} - 2$ (d) None of these
35. The value of $\int_0^1 \frac{x^4 + 1}{x^2 + 1} dx$ is [MP PET 1998]
 (a) $\frac{1}{6}(3\pi - 4)$ (b) $\frac{1}{6}(3 - 4\pi)$ (c) $\frac{1}{6}(3\pi + 4)$ (d) $\frac{1}{6}(3 + 4\pi)$
36. If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $I_m = k - l I_{m-1}$, then
 (a) $k = e$ (b) $l = m$ (c) $k = \frac{1}{e}$ (d) None of these

37. If $\int_{-1}^4 f(x)dx = 4$ and $\int_2^4 (3 - f(x))dx = 7$, then the value of $\int_2^{-1} f(x)dx$ is
 (a) 2 (b) -3 (c) -5 (d) None of these
38. The value of $\int_3^5 \frac{x^2}{x^2 - 4} dx$ is [Roorkee 1992]
 (a) $2 - \log_e \left(\frac{15}{7} \right)$ (b) $2 + \log_e \left(\frac{15}{7} \right)$
 (c) $2 + 4 \log_e 3 - 4 \log_e 7 + 4 \log_e 5$ (d) $2 - \tan^{-1} \left(\frac{15}{7} \right)$
39. Let $\int_0^1 f(x)dx = 1$, $\int_0^1 xf(x)dx = a$ and $\int_0^1 x^2 f(x)dx = a^2$, then the value of $\int_0^1 (x - a)^2 f(x)dx =$ [IIT 1990]
 (a) 0 (b) a^2 (c) $a^2 - 1$ (d) $a^2 - 2a + 2$
40. $\int_0^1 \frac{e^x(x-1)}{(x+1)^3} dx =$ [SCRA 1986]
 (a) $\frac{e}{4}$ (b) $\frac{e}{4} - 1$ (c) $\frac{e}{4} + 1$ (d) None of these
41. $\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx =$ [Roorkee 1989]
 (a) $\frac{\pi}{4} + \frac{1}{2} \log 2$ (b) $\frac{\pi}{4} + \log 2$ (c) $\frac{\pi}{4} - \frac{1}{2} \log 2$ (d) $\frac{\pi}{4} - \log 2$
42. $\int_{\pi/4}^{\pi/2} e^x [\log \sin x + \cot x] dx$ is equal to [AI CBSE 1991]
 (a) $e^{\pi/4} \log 2$ (b) $-e^{\pi/4} \log 2$ (c) $\frac{1}{2} e^{\pi/4} \log 2$ (d) $-\frac{1}{2} e^{\pi/4} \log 2$
43. $\int_0^{\infty} (a^{-x} - b^{-x}) dx =$ [EAMCET 1995]
 (a) $\frac{1}{\log a} - \frac{1}{\log b}$ (b) $\log a - \log b$ (c) $\log a + \log b$ (d) $\frac{1}{\log a} + \frac{1}{\log b}$
44. $\int_0^{1/2} |\sin \pi x| dx$ is equal to [DCE 1998]
 (a) 0 (b) π (c) $-\pi$ (d) $\frac{1}{\pi}$
45. $\int_0^1 \frac{x}{(1-x)^{3/4}} dx =$ [EAMCET 1992]
 (a) $\frac{12}{5}$ (b) $\frac{16}{5}$ (c) $\frac{-16}{5}$ (d) None of these
46. $\int_1^e \log x dx =$ [Karnataka CET 1994]
 (a) 1 (b) $e - 1$ (c) $e + 1$ (d) 0
47. Value of $\int_0^{1/2} \frac{dx}{\sqrt{x-x^2}}$ is [Assam JEE 1998]
 (a) $\frac{1}{2}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$
48. $\int_2^4 (3x-2)^2 dx$ equals [MP PET 1989]
 (a) 102 (b) 104 (c) 100 (d) 98

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49. $\int_1^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$ equals [MP PET 1989]
 (a) 20/3 (b) 19/3 (c) 13/2 (d) 6
50. $\int_0^\infty \sec hx dx$ equals [Karnataka CET 1996]
 (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{2} + 1$ (d) 1
51. If $\frac{d}{dx}[f(x)] = \phi(x)$, then the value of $\int_1^2 \phi(x) dx$ equals [Rajasthan PET 1995]
 (a) $f(1) - f(2)$ (b) $\phi(2) - \phi(1)$ (c) $f(2) - f(1)$ (d) $\phi(1) - \phi(2)$
52. $\int_0^a \sqrt{a^2 - x^2} dx$ equals [EAMCET 1996]
 (a) $\frac{\pi a}{4}$ (b) $\frac{\pi a^2}{4}$ (c) $\frac{\pi a^2}{2}$ (d) $\frac{\pi a}{2}$
53. $\int_0^{\pi/2} \sqrt{1 + \sin 2x} dx$ equals [Rajasthan PET 1990]
 (a) 1 (b) 1/2 (c) 2 (d) None of these
54. $\int_0^{\pi/4} \frac{dx}{1 + \cos 2x}$ equals [Rajasthan PET 1987]
 (a) 1 (b) -1 (c) 1/2 (d) -1/2
55. Let $I_1 = \int_1^2 \frac{1}{\sqrt{1+x^2}} dx$ and $I_2 = \int_1^2 \frac{1}{x} dx$. Then
 (a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_1 = I_2$ (d) $I_1 > 2I_2$
56. If for every integer n , $\int_n^{n+1} f(x) dx = n^2$, then the value of $\int_{-2}^4 f(x) dx$ is
 (a) 16 (b) 14 (c) 19 (d) None of these
57. If $I_n = \int_0^1 x^n e^{-x} dx$ for $n \in N$, then $I_n - nI_{n-1} =$
 (a) e (b) $1/e$ (c) $-1/e$ (d) None of these
58. $\int_0^3 x\sqrt{1+x} dx$ equals
 (a) 9/2 (b) 27/4 (c) 116/15 (d) None of these
59. $\int_1^{\frac{4\sqrt{3}-1}{5}} \frac{x+2}{\sqrt{x^2+2x-3}} dx =$
 (a) $\frac{2\sqrt{3}}{3} - \frac{1}{2} \log 3$ (b) $\frac{2\sqrt{3}}{3} + \frac{1}{2} \log 3$ (c) $\frac{2\sqrt{3}}{3} - \frac{1}{2} \log(\sqrt{3}+2)$ (d) $\frac{2\sqrt{3}}{3} + \frac{1}{2} \log(\sqrt{3}+2)$
60. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then
 (a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_3 > I_4$ (d) $I_4 > I_3$
61. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x) dx$ is
 (a) 3 (b) 2/3 (c) 1/3 (d) 0

62. If $\int_2^3 \frac{x^2 + 1}{(2x+1)(x^2-1)} dx = p \log \frac{7}{5} + q \log \frac{4}{3} + r \log 2$, then
- (a) $p = -\frac{5}{6}, q = 1, r = \frac{1}{3}$ (b) $p = \frac{5}{6}, q = 1, r = \frac{1}{3}$ (c) $p = -\frac{5}{6}, q = -1, r = -\frac{1}{3}$ (d) $p = \frac{5}{6}, q = 1, r = -\frac{1}{3}$
63. If $\frac{1}{\sqrt{a}} \int_1^a \left(\frac{3}{2} \sqrt{x} + 1 - \frac{1}{\sqrt{x}} \right) dx < 4$, then 'a' may take values
- (a) 0 (b) 4 (c) 9 (d) $\frac{13 + \sqrt{313}}{2}$
64. The value of $\int_0^{\frac{\pi}{2}} \frac{\cos 3x + 1}{\cos 2x - 1} dx$ is
- (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 0
65. $\int_0^2 (t - \log_2 a) dt$ equals
- (a) $\log_2(2/a)$ (b) $2 \log_2(2/a)$ (c) $2 \log_4(2/a)$ (d) None of these
66. $\int_0^{4/\pi} \left(3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x} \right) dx =$
- (a) $\frac{8\sqrt{2}}{\pi^3}$ (b) $\frac{32\sqrt{2}}{\pi^3}$ (c) $\frac{24\sqrt{2}}{\pi^3}$ (d) $\frac{\sqrt{2048}}{\pi^3}$
67. The value of $\int_a^b \frac{x}{|x|} dx, a < b < 0$ is [Orissa JEE 2003]
- (a) $-(|a| + |b|)$ (b) $|b| - |a|$ (c) $|a| - |b|$ (d) $|a| + |b|$
68. The value of $\alpha \in (-\pi, 0)$ satisfying $\sin \alpha + \int_{\alpha}^{2\alpha} \cos 2x dx = 0$ is
- (a) $-\pi/2$ (b) $-\pi$ (c) $-\pi/3$ (d) 0
69. If $\int_0^{36} \frac{1}{2x+9} dx = \log k$, then k is equal to
- (a) 3 (b) 9/2 (c) 9 (d) 81
70. The value of $\lim_{x \rightarrow \pi/2} \frac{\int_{\pi/2}^x t dt}{\sin(2x - \pi)}$ is
- (a) ∞ (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/8$
71. Suppose that $f''(x)$ is continuous for all x and $f(0) = f'(1) = 1$. If $\int_0^1 t f''(t) dt = 0$, then the value of $f(1)$ is
- (a) 2 (b) 3 (c) $4\frac{1}{2}$ (d) None of these
72. $I_{m,n} = \int_0^1 x^m (\ln x)^n dx =$ [EAMCET 1994]
- (a) $\frac{n}{n+1} I_{m,n-1}$ (b) $\frac{-m}{n+1} I_{m,n-1}$ (c) $\frac{-n}{n+1} I_{m,n-1}$ (d) $\frac{m}{n+1} I_{m,n-1}$

Advance Level

73. If $(n - m)$ is odd and $|m| \neq |n|$, then $\int_0^{\pi} \cos mx \sin nx dx$ is

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- (a) $\frac{2n}{n^2 - m^2}$ (b) 0 (c) $\frac{2n}{m^2 - n^2}$ (d) $\frac{2m}{n^2 - m^2}$
74. The value of the definite integral $\int_0^1 \frac{dx}{x^2 + 2x \cos \alpha + 1}$ for $0 < \alpha < \pi$ is equal to [Kurukshetra CEE 2002]
 (a) $\sin \alpha$ (b) $\tan^{-1}(\sin \alpha)$ (c) $\alpha \sin \alpha$ (d) $\frac{\alpha}{2}(\sin \alpha)^{-1}$
75. If $f(y) = e^y, g(y) = y; y > 0$ and $F(t) = \int_0^t f(t-y)g(y)dy$, then [AIEEE 2003]
 (a) $F(t) = 1 - e^{-t}(1+t)$ (b) $F(t) = e^t - (1+t)$ (c) $F(t) = te^t$ (d) $F(t) = te^{-t}$
76. If $l(m, n) = \int_0^1 t^m(1+t)^n dt$, then the expression for $l(m, n)$ in terms of $l(m+1, n-1)$ is [IIT Screening 2003]
 (a) $\frac{2^n}{m+1} - \frac{n}{m+1}l(m+1, n-1)$ (b) $\frac{n}{m+1}l(m+1, n-1)$
 (c) $\frac{2^n}{m+1} + \frac{n}{m+1}l(m+1, n-1)$ (d) $\frac{m}{n+1}l(m+1, n-1)$
77. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. The value of integral $\int_0^1 f(x)g(x)dx$ is equal to [AIEEE 2003]
 (a) $\frac{1}{4}(e-7)$ (b) $\frac{1}{4}(e-2)$ (c) $\frac{1}{2}(e-3)$ (d) None of these
78. $\int_0^{\pi/2} \left(\frac{\theta}{\sin \theta} \right)^2 d\theta =$
 (a) $\pi \log 2$ (b) $\frac{\pi}{\log 2}$ (c) π (d) None of these
79. The value of $\int_0^{\pi} |\sin^3 \theta| d\theta$ is [UPSEAT 2003]
 (a) 0 (b) $\frac{3}{8}$ (c) $\frac{4}{3}$ (d) π
80. $\int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin x} dx, (n \in N)$ is equal to [Kurukshetra CEE 1998]
 (a) $n\pi$ (b) $(2n+1)\frac{\pi}{2}$ (c) π (d) 0
81. If I is the greatest of the definite integrals $I_1 = \int_0^1 e^{-x} \cos^2 x dx$, $I_2 = \int_0^1 e^{-x^2} \cos^2 x dx$, $I_3 = \int_0^1 e^{-x^2} dx$, $I_4 = \int_0^1 e^{-x^2/2} dx$, then
 (a) $I = I_1$ (b) $I = I_2$ (c) $I = I_3$ (d) $I = I_4$
82. $\int_0^{\pi/2} x \cot x dx$ equals [Rajasthan PET 1997]
 (a) $-\frac{\pi}{2} \log 2$ (b) $\frac{\pi}{2} \log 2$ (c) $\pi \log 2$ (d) $-\pi \log 2$
83. The value of $\lim_{x \rightarrow \infty} \frac{\left(\int_0^x e^x dx \right)^2}{\int_0^x e^{2x^2} dx}$ is
 (a) 1 (b) 2 (c) 3 (d) 0

84. If $a < \int_0^{2\pi} \frac{1}{10 + 3 \cos x} dx < b$, then the ordered pair (a, b) is
- (a) $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$ (b) $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$ (c) $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$ (d) None of these
85. Let $a_n = \int_0^{\pi/2} \cos^n x \cos nx \, dx$, then $a_{n+1} : a_n =$
- (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these
86. If $I_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$, then
- (a) $I_n = \frac{n\pi}{2}$ (b) $I_1, I_2, I_3, I_4, \dots, I_n, \dots$ are in A.P.
(c) $\sin(I_{16}) = 0$ (d) All of these
87. If $n \in \mathbb{N}$ and $\int_0^1 e^x (x-1)^n dx = 2e - 5$, then $n =$
- (a) 1 (b) 2 (c) 3 (d) None of these
88. The value of $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) d(x - [x])$, (where $[.]$ denotes the greatest integer function) is
- (a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{2}{n-1}$ (d) None of these
89. The points of intersection of $f_1(x) = \int_2^x (2t-5) dt$ and $f_2(x) = \int_0^x 2t dt$, are [IIT Screening]
- (a) $\left(\frac{6}{5}, \frac{36}{25}\right)$ (b) $\left(\frac{2}{3}, \frac{4}{9}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{9}\right)$ (d) $\left(\frac{1}{5}, \frac{1}{25}\right)$
90. The value of integral $\int_0^1 e^{x^2} dx$ lies in interval [CEE 1993]
- (a) (0, 1) (b) (-1, 0) (c) (1, e) (d) None of these
91. The greatest value of the function $f(x) = \int_1^x |t| dt$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is given by [IIT Screening]
- (a) $\frac{3}{8}$ (b) $-\frac{1}{2}$ (c) $-\frac{3}{8}$ (d) $\frac{2}{5}$
92. The absolute value of $\int_{10}^{19} \frac{\cos x}{1+x^8} dx$ is
- (a) Less than 10^{-7} (b) More than 10^{-7} (c) Less than 10^{-6} (d) Both (a) and (c)
93. $\int_0^{\pi/4} \sin x (x - [x]) dx$ is equal to
- (a) $\frac{1}{2}$ (b) $1 - \frac{1}{\sqrt{2}}$ (c) 1 (d) None of these
94. $\int_{-1}^{10} \operatorname{sgn}(x - [x]) dx$ equals
- (a) 10 (b) 11 (c) 9 (d) $\frac{11}{2}$
95. If $I_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $a_n = \int_0^{\pi/2} \left(\frac{\sin n\theta}{\sin \theta}\right)^2 d\theta$, then $a_{n+1} - a_n =$
- (a) I_n (b) $2I_n$ (c) I_{n+1} (d) 0

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96. If $f'(x) = f(x) + \int_0^1 f(x) dx$ and given $f(0) = 1$ then $f(x) =$
- (a) $\frac{e^x}{2-e} + \left(\frac{1+e}{1-e}\right)$ (b) $\frac{2e^x}{3-e} + \left(\frac{1-e}{1+e}\right)$ (c) $\frac{e^x}{2-e}$ (d) $\frac{2e^x}{3-e}$
97. On the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, the greatest value of the function $f(x) = \int_{5\pi/3}^x (6 \cos t - 2 \sin t) dt =$
- (a) $3\sqrt{3} + 2\sqrt{2} + 1$ (b) $3\sqrt{3} - 2\sqrt{2} - 1$ (c) Does not exist (d) None of these
98. If $f'''(x) = k$ in $[0, a]$ then $\int_0^a f(x) dx - \left\{ xf(x) - \frac{x^2}{2!} f'(x) + \frac{x^3}{3!} f''(x) \right\}_0^a =$
- (a) $-\frac{ka^4}{12}$ (b) $\frac{ka^4}{24}$ (c) $-\frac{ka^4}{24}$ (d) None of these
99. The value of $\int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x} dx$ is
- (a) $\frac{3}{5} \left[\frac{2}{\log_e \left(\frac{2}{5}\right)} + \frac{1}{2 \log_e \left(\frac{5}{2}\right)} \right]$ (b) $-\frac{3}{5} \left[\frac{2}{\log_e \left(\frac{2}{5}\right)} + \frac{1}{2 \log_e \left(\frac{5}{2}\right)} \right]$
- (c) $\frac{3}{5} \left[\frac{2}{\log_e \left(\frac{2}{5}\right)} - \frac{1}{2 \log_e \left(\frac{5}{2}\right)} \right]$ (d) None of these
100. If $\int_0^1 e^{x^2} (x - \alpha) dx = 0$, then [MNR 1994]
- (a) $1 < \alpha < 2$ (b) $\alpha < 0$ (c) $0 < \alpha < 1$ (d) None of these
101. The value of $\lim_{x \rightarrow 0} \frac{\int_0^x \cos t^2}{x}$ dt is
- (a) 0 (b) 1 (c) -1 (d) None of these
102. If a be a positive integer, the number of value of a satisfying $\int_0^{\pi/2} \left\{ a^2 \left(\frac{\cos 3x}{4} + \frac{3}{4} \cos x \right) + a \sin x - 20 \cos x \right\} dx \leq -\frac{a^2}{3}$ is
- (a) Only one (b) Two (c) Three (d) Four
103. The expression $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$, where $[x]$ and $\{x\}$ are integral and fractional parts of x and $n \in N$ is equal to
- (a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) n (d) $n-1$
104. If $f(x) = x^3$ and $\int_a^b f(x) dx = \frac{b-a}{6} \left[f(a) + f(b) + kf\left(\frac{a+b}{2}\right) \right]$, then $k =$
- (a) 0 (b) 2 (c) 4 (d) None of these
105. If $I_n = \int_0^{\pi/2} x^n \sin x dx$ and $n > 1$ then $I_n + n(n-1)I_{n-2}$ is equal to
- (a) $n\left(\frac{\pi}{2}\right)^n$ (b) $(n-1)\left(\frac{\pi}{2}\right)^n$ (c) $n\left(\frac{\pi}{2}\right)^{n-1}$ (d) $(n-1)\left(\frac{\pi}{2}\right)^{n-1}$

106. The value of $\int_0^{\pi} e^{\sec x} \sec^3 x (\sin^2 x + \cos x + \sin x + \sin x \cos x) dx$ equals
- (a) 0 (b) $e + \left(\frac{1}{e}\right)$ (c) $-e - \left(\frac{1}{e}\right)$ (d) e
107. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x) dx = \frac{2A}{\pi}$, then the constants A and B are respectively [IIT 1995]
- (a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ (c) $\frac{4}{\pi}$ and 0 (d) 0 and $-\frac{4}{\pi}$
108. If for non-zero x , $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then $\int_1^2 f(x) dx =$ [IIT 1996]
- (a) $\frac{1}{(a^2 + b^2)} \left[a \log 2 - 5a + \frac{7}{2}b \right]$ (b) $\frac{1}{(a^2 - b^2)} \left[a \log 2 - 5a + \frac{7}{2}b \right]$
- (c) $\frac{1}{(a^2 - b^2)} \left[a \log 2 - 5a - \frac{7}{2}b \right]$ (d) $\frac{1}{(a^2 + b^2)} \left[a \log 2 - 5a - \frac{7}{2}b \right]$
109. If $u_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$, then $u_2 - u_1, u_3 - u_2, u_4 - u_3, \dots$ are in
- (a) A.P. (b) G.P. (c) H.P. (d) None
110. If $f(x) = \begin{cases} x, & \text{for } x < 1 \\ x - 1, & \text{for } x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x) dx$ is equal to
- (a) 1 (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{2}$
111. If $I = \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx$, then
- (a) $I \leq \frac{\pi}{6}$ (b) $I \geq \frac{\pi}{2}$ (c) $I \geq 0$ (d) All of these
112. If $f(x)$ and $g(x)$ are two integrable functions defined on $[a, b]$, then $\left| \int_a^b f(x)g(x) dx \right|$ is
- (a) Less than $\sqrt{\left(\int_a^b f(x) dx \right) \left(\int_a^b g(x) dx \right)}$ (b) Less than or equal to $\sqrt{\left(\int_a^b f^2(x) dx \right) \left(\int_a^b g^2(x) dx \right)}$
- (c) Less than or equal to $\sqrt{\left(\int_a^b f^2(x) dx \right) \left(\int_a^b g^2(x) dx \right)}$ (d) None of these
113. If $f(x) = a + bx + cx^2$ then $\int_0^4 f(x) dx$ has the value
- (a) $\frac{1}{6} \left\{ f(0) + 2f\left(\frac{1}{2}\right) + f(1) \right\}$ (b) $\frac{1}{6} \left\{ 3f(0) + 2f\left(\frac{1}{2}\right) + 3f(1) \right\}$ (c) $\frac{1}{6} \left\{ f(0) + 4f\left(\frac{1}{2}\right) + f(1) \right\}$ (d) $\frac{1}{6} \left\{ f(0) + f\left(\frac{1}{2}\right) + f(1) \right\}$

Definite integral by Substitution Method

Basic Level

114. The value of $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$ is [Rajasthan PET 1995]
- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

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115. $\int_0^a x^2 \sin x^3 dx$ equals [Rajasthan PET 1995]
- (a) $(1 - \cos a^3)$ (b) $3(1 - \cos a^3)$ (c) $-\frac{1}{3}(1 - \cos a^3)$ (d) $\frac{1}{3}(1 - \cos a^3)$
116. $\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx =$
- (a) $\pi \log \frac{1}{2}$ (b) $\pi \log 2$ (c) $2\pi \log \frac{1}{2}$ (d) $2\pi \log 2$
117. $\int_0^{\pi/4} \frac{dx}{\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x} =$
- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these
118. The value of the integral $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ equals [Kurukshetra CEE 1996]
- (a) 1 (b) 2 (c) 0 (d) 4
119. $\int_1^2 \frac{1}{x^2} e^{-1/x} dx =$ [DCE 2001]
- (a) $\sqrt{e} + 1$ (b) $\sqrt{e} - 1$ (c) $\frac{\sqrt{e} + 1}{e}$ (d) $\frac{\sqrt{e} - 1}{e}$
120. $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx =$ [SCRA 1987; MNR 1990; Rajasthan PET 2001]
- (a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{32}$
121. The value of $\int_{1/e}^{\tan x} \frac{t dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)} =$ [IIT Screening]
- (a) -1 (b) 1 (c) 0 (d) None of these
122. The value of the integral $\int_{1/\pi}^{2/\pi} \frac{\sin(1/x)}{x^2} dx =$ [IIT 1990]
- (a) 2 (b) -1 (c) 0 (d) 1
123. $\int_0^{\pi/3} \frac{\cos x}{3+4 \sin x} dx =$
- (a) $\frac{1}{4} \log \left(\frac{3+2\sqrt{3}}{2} \right)$ (b) $\frac{1}{2} \log \left(\frac{3+2\sqrt{3}}{2} \right)$ (c) $\frac{1}{3} \log \left(\frac{3+2\sqrt{3}}{2} \right)$ (d) None of these
124. $\int_0^1 \frac{e^{-x}}{1+e^x} dx =$ [Roorkee 1976]
- (a) $\log \left(\frac{1+e}{e} \right) - \frac{1}{e} + 1$ (b) $\log \left(\frac{1+e}{2e} \right) - \frac{1}{e} + 1$ (c) $\log \left(\frac{1+e}{2e} \right) + \frac{1}{e} - 1$ (d) None of these
125. The value of the integral $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx =$
- (a) $3 + 2\pi$ (b) $4 - \pi$ (c) $2 + \pi$ (d) None of these
126. If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then [Karnataka CET 2000]
- (a) $I_1 = I_2$ (b) $I_1 > I_2$ (c) $I_1 < I_2$ (d) None of these

127. $\int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx =$ [SCRA 1979]
 (a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) 2 (d) $\frac{1}{3}$
128. If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$, then for any positive integer n , the value of $n(I_{n-1} + I_{n+1})$ is [AIEEE 2002; Rajasthan PET 1999; Karnataka CET 2000]
 (a) 1 (b) 2 (c) $\pi/4$ (d) π
129. The value of $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$ is [SCRA 1992]
 (a) $\frac{2}{\log 3} \cdot (3^{\sqrt{2}} - 1)$ (b) 0 (c) $2 \cdot \frac{\sqrt{2}}{\log 3}$ (d) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$
130. $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is equal to [SCRA 1986; Karnataka CET 1999]
 (a) πab (b) $\pi^2 ab$ (c) $\frac{\pi}{ab}$ (d) $\frac{\pi}{2ab}$
131. If $I_1 = \int_0^x e^{zx} e^{-z^2} dz$ and $I_2 = \int_0^x e^{-z^2/4} dz$, then, [MP PET 1990]
 (a) $I_1 = e^x I_2$ (b) $I_1 = e^{x^2} I_2$ (c) $I_1 = e^{x^2/2} I_2$ (d) None of these
132. $\int_1^x \frac{\log(x^2)}{x} dx =$ [DCE 1999]
 (a) $(\log x)^2$ (b) $\frac{1}{2}(\log x)^2$ (c) $\frac{\log x^2}{2}$ (d) None of these
133. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is [SCRA 1980]
 (a) $\tan^{-1}\left(\frac{1-e}{1+e}\right)$ (b) $\tan^{-1}\left(\frac{e-1}{e+1}\right)$ (c) $\frac{\pi}{4}$ (d) $\tan^{-1} e - \frac{\pi}{4}$
134. $\int_0^{\pi/2} (\sin x - \cos x) \log(\sin x + \cos x) dx =$ [SCRA 1986]
 (a) -1 (b) 1 (c) 0 (d) None of these
135. $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3 \cos x + 2} dx$ is equal to [MNR 1981; Rajasthan PET 1990; MP PET 1990]
 (a) $\log\left(\frac{8}{9}\right)$ (b) $\log\left(\frac{9}{8}\right)$ (c) $\log(8, 9)$ (d) None of these
136. $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx =$ [UPSEAT 1999]
 (a) $\log \frac{4}{3}$ (b) $\log \frac{1}{3}$ (c) $\log \frac{3}{4}$ (d) None of these
137. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx =$ [Karnataka CET 1999]
 (a) $\frac{\pi}{2} - 2 \log \sqrt{2}$ (b) $\frac{\pi}{2} + 2 \log \sqrt{2}$ (c) $\frac{\pi}{4} - \log \sqrt{2}$ (d) $\frac{\pi}{4} + \log \sqrt{2}$
138. $\int_0^{\pi/4} \sec^7 \theta \sin^3 \theta d\theta =$
 (a) $\frac{1}{12}$ (b) $\frac{3}{12}$ (c) $\frac{5}{12}$ (d) None of these

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- 139.** The correct evaluation of $\int_0^{\pi/2} \sin x \sin 2x \, dx$ is [MP PET 1993, 2003]
 (a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$
- 140.** $\int_{\pi/4}^{\pi/2} \cos \theta \operatorname{cosec}^2 \theta \, d\theta =$ [Roorkee 1978]
 (a) $\sqrt{2} - 1$ (b) $1 - \sqrt{2}$ (c) $\sqrt{2} + 1$ (d) None of these
- 141.** $\int_1^2 \frac{\cos(\log x)}{x} \, dx =$ [MP PET 1990]
 (a) $\sin(\log 3)$ (b) $\sin(\log 2)$ (c) $\cos(\log 3)$ (d) None of these
- 142.** $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} \, dx =$ [Roorkee 1984]
 (a) $\frac{\pi}{4} + \frac{1}{2} \log 2$ (b) $\frac{\pi}{4} - \frac{1}{2} \log 2$ (c) $\frac{\pi}{2} + \log 2$ (d) $\frac{\pi}{2} - \log 2$
- 143.** $\int_0^{\pi/4} \frac{4 \sin 2\theta \, d\theta}{\sin^4 \theta + \cos^4 \theta} =$ [SCRA 1986]
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) None of these
- 144.** If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then $I_8 + I_6$ equals [Kurukshetra CEE 1996]
 (a) $1/4$ (b) $1/5$ (c) $1/6$ (d) $1/7$
- 145.** $\int_0^1 \sqrt{\frac{1-x}{1+x}} \, dx$ equals [Rajasthan PET 1997; Karnataka CET 1993; Bihar CEE 1998; AIEEE 2004]
 (a) $\left(\frac{\pi}{2} - 1\right)$ (b) $\left(\frac{\pi}{2} + 1\right)$ (c) $\pi/2$ (d) $(\pi + 1)$
- 146.** The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x \, dx$ is [IIT Screening; MP PET 2003]
 (a) $\frac{3}{2}$ (b) $-\frac{8}{3}$ (c) $\frac{3}{8}$ (d) $\frac{8}{3}$
- 147.** $\int_0^{\pi/2} \frac{dx}{2 + \cos x} =$ [BIT Ranchi 1992; Rajasthan PET 1993]
 (a) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\sqrt{3} \tan^{-1}(\sqrt{3})$ (c) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $2\sqrt{3} \tan^{-1}(\sqrt{3})$
- 148.** $\int_0^1 \tan^{-1} x \, dx =$ [Karnataka CET 1993; Rajasthan PET 1997]
 (a) $\frac{\pi}{4} - \frac{1}{2} \log 2$ (b) $\pi - \frac{1}{2} \log 2$ (c) $\frac{\pi}{4} - \log 2$ (d) $\pi - \log 2$
- 149.** $\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx$ [IIT 1984]
 (a) $\frac{1}{2} + \frac{\sqrt{3}\pi}{12}$ (b) $\frac{1}{2} - \frac{\sqrt{3}\pi}{12}$ (c) $\frac{1}{2} - \frac{\sqrt{3}\pi}{12}$ (d) None of these

150. $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx =$ [MNR 1984; CEE 1993]
 (a) $\pi + 2$ (b) $\pi + \frac{3}{2}$ (c) $\pi + 1$ (d) None of these
151. $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx =$ [Ranchi BIT 1984]
 (a) $\frac{\pi}{2} \log 2$ (b) $\pi \log 2$ (c) $-\frac{\pi}{2} \log 2$ (d) $-\pi \log 2$
152. $\int_0^{\pi/2} \frac{\sin x \cos x}{1 + \sin^4 x} dx =$ [AISSSE 1988]
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$
153. $\int_0^1 \frac{dx}{[ax + b(1-x)]^2} =$ [SCRA 1986]
 (a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) ab (d) $\frac{1}{ab}$
154. $\int_0^1 \log \sin\left(\frac{\pi}{2} x\right) dx$ is equal to [Rajasthan PET 1997]
 (a) $-\log 2$ (b) $\log 2$ (c) $\frac{\pi}{2} \log 2$ (d) $\frac{-\pi}{2} \log 2$
155. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\lim_{n \rightarrow \infty} n[I_n + I_{n-2}]$ equals [AIEEE 2002]
 (a) $1/2$ (b) 1 (c) ∞ (d) 0
156. The value of $\int_0^1 (x^3 + 3e^x + 4)(x^2 + e^x) dx$ is [Rajasthan PET 1987]
 (a) $(3e - 2)6$ (b) $(3e + 2)6$ (c) $(3e - 2)^2/36$ (d) None of these
157. $\int_{-\pi/2}^{\pi/2} \cos^3 x (1 + \sin x)^2 dx$ equals [EAMCET 1996]
 (a) $8/5$ (b) $5/8$ (c) $-8/5$ (d) $-5/8$
158. $\int_0^{\pi/2} \frac{dx}{1 + \sin x}$ equals [MNR 1983; Rajasthan PET 1990; Kurukshetra CEE 1997]
 (a) 0 (b) 1 (c) -1 (d) 2
159. $\int_0^{\pi/2} \frac{\sin 2x}{a \sin^2 x + b \cos^2 x} dx$ equals [Rajasthan PET 1991]
 (a) $\frac{1}{a-b} \log \frac{b}{a}$ (b) $\frac{1}{a-b} \log \frac{a}{b}$ (c) $\frac{1}{a-b} \log(ab)$ (d) $\frac{1}{a+b} \log \frac{a}{b}$
160. If $u_n = \int_0^{\pi/4} \tan^n x dx$, then $u_2 + u_4, u_3 + u_5, u_4 + u_6, \dots$ are in [MP PET 1990]
 (a) A.P. (b) G.P. (c) H.P. (d) None
161. $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is equal to
 (a) 1 (b) 2 (c) e (d) 37
162. $\int_0^1 \frac{dx}{x + \sqrt{x}}$ equals [Rajasthan PET 1993]
 (a) 0 (b) $\log 2$ (c) $\log 3$ (d) $\log 4$

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163. $\int_0^{\pi/4} \frac{\sin \sqrt{x}}{\sqrt{x}} dx$ [Rajasthan PET 1994]
 (a) 2 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi^2}{8}$
164. $\int_2^4 \frac{\sqrt{(x^2-4)}}{x} dx =$ [Rajasthan PET 1992]
 (a) $2(3\sqrt{3} - \pi)/3$ (b) π (c) $2(3\sqrt{3} - \pi)$ (d) None of these
165. The value of $\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx$ is
 (a) $\pi/4$ (b) $\pi/3$ (c) π (d) 2π
166. $\int_0^{\pi/2} \frac{dx}{1 + 2 \sin x + \cos x}$ equals [Rajasthan PET 1991]
 (a) $(1/2)\log 3$ (b) $\log 3$ (c) $(4/3)\log 3$ (d) None of these
167. $\int_0^1 \frac{dx}{(x^2 + 1)^{3/2}}$ is equal to [Kurukshetra CEE 1991]
 (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$
168. The value of $\int_0^1 \frac{x^3}{\sqrt{1-x^8}} dx$ is
 (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$
169. The value of $\int_0^1 \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ is
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 1 (d) 0
170. $\int_0^{\pi} \frac{dx}{a + b \cos x}$ equals [Karnataka CET 1993]
 (a) $\frac{\pi}{\sqrt{a^2 - b^2}}$ (b) $\frac{\pi}{ab}$ (c) $\frac{\pi}{\sqrt{a^2 + b^2}}$ (d) $\pi(a + b)$
171. $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ equals [Rajasthan PET 1987]
 (a) $\pi^2/4$ (b) $\pi/4$ (c) $\pi^2/8$ (d) $\pi/8$
172. If $\int_0^1 \frac{e^t dt}{t+1} = a$, then $\int_{b-1}^b \frac{e^{-t} dt}{t-b-1}$ is equal to [MP PET 1990]
 (a) ae^{-b} (b) $-ae^{-b}$ (c) $-be^{-a}$ (d) ae^b
173. The value of the integral $\int_{1/3}^1 \frac{(x-x^3)^{1/3}}{x^4} dx$ is [MP PET 1990]
 (a) 6 (b) 0 (c) 3 (d) 4
174. $\int_0^{\pi/2} \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx =$
 (a) 0 (b) $\pi/4$ (c) $\frac{1}{b^2} \log \left(\frac{a^2 + b^2}{a^2} \right)$ (d) $\frac{1}{b^2} \log(a^2 + b^2)$

175. $\int_0^{\pi/2} \frac{\sin x \cos x}{1 + \sin x} dx$ is
- (a) $\frac{3}{2} - \log 2$ (b) $1 - \log 2$ (c) $3 - \log 2$ (d) $3 + \log 2$
176. The value of the integral $\int_2^4 \frac{\sqrt{x^2 - 4}}{x^4} dx$ is [MP PET 1990]
- (a) $\sqrt{\frac{3}{32}}$ (b) $\frac{\sqrt{3}}{32}$ (c) $\frac{32}{\sqrt{3}}$ (d) $-\frac{\sqrt{3}}{32}$
177. If $\int_{\log 2}^x \frac{1}{\sqrt{e^x - 1}} dx = \frac{\pi}{6}$, then x is equal to [MP PET 1990]
- (a) e^2 (b) $1/e$ (c) $\log 4$ (d) none of these
178. $\int_0^1 \frac{x^7}{\sqrt{1 - x^4}} dx$ is equal to [AMU 2000]
- (a) 1 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{\pi}{3}$
179. $\int_0^{-1} \frac{(1 - x^2)}{1 + x^2 + x^4} dx$ equals
- (a) $(1/2) \log 2$ (b) $(1/2) \tan^{-1} 3$ (c) $(1/2) \log 3$ (d) none of these
180. If $\int_0^{\pi/2} \frac{d\theta}{13 - 4 \sin^2 \theta - 9 \cos^2 \theta} = \pi k$, then [MP PET 1990]
- (a) $k = \frac{1}{3}$ (b) $k = \frac{1}{6}$ (c) $k = \frac{1}{12}$ (d) $k = \frac{1}{13}$
181. The value of $\int_1^4 \frac{dx}{x^2 - 2x + 10}$ is
- (a) 0 (b) ∞ (c) $\pi/12$ (d) $\pi/6$
182. The value of $\int_0^{\pi/2} \cos x \cdot e^{\sin x} dx$ is [MP PET 1990]
- (a) 0 (b) 1 (c) -1 (d) $e - 1$
183. If $u_n = \int_0^{\pi/4} \tan^n x dx$, then $u_n + u_{n-2} =$ [UPSEAT 2002]
- (a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{1}{2n-1}$ (d) $\frac{1}{2n+1}$
184. $\int_{a+c}^{b+c} f(x-c) dx$ is equal to
- (a) $\int_c^b f(x-c) dx$ (b) $\int_a^b f(x) dx$ (c) $\int_a^b f(a+b+c+x) dx$ (d) none of these
185. $\int_0^{\pi/4} \frac{\cos x}{\sqrt{1 - 2 \sin^2 x}} dx$ is equal to
- (a) $\pi/\sqrt{2}$ (b) $\pi/2\sqrt{2}$ (c) π (d) $\pi/2$
186. The value of the integral $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is [MP PET 1990]
- (a) $\log 2$ (b) $\log 3$ (c) $\frac{1}{4} \log 3$ (d) $\frac{1}{8} \log 3$

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187. $I = \int_0^{\pi/2} \frac{\sin 2x}{1 + 4 \cos^2 x} dx$ is equal to
 (a) $\frac{1}{4} \log 2$ (b) $\frac{1}{4} \log 4$ (c) $\frac{1}{4} \log 3$ (d) $\frac{1}{4} \log 5$
188. $\int_0^{\pi} \frac{dx}{1 + 2 \sin^2 x} =$
 (a) $\pi/\sqrt{3}$ (b) $\pi/3\sqrt{3}$ (c) $\pi/3$ (d) None of these
189. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x , then $\int_{\sin \theta}^{\operatorname{cosec} \theta} f(x) dx$ equals
 (a) $\sin \theta + \operatorname{cosec} \theta$ (b) $\sin^2 \theta$ (c) $\operatorname{cosec}^2 \theta$ (d) 0
190. $\int_{\pi/6}^{\pi/4} \frac{1}{\sqrt{\cos x \sin^3 x}} dx$ is equal to
 (a) 1 (b) $\frac{1}{3}$ (c) -2 (d) none of these
191. The tangent to the graph of the function $y = f(x)$ at the point with abscissa $x = a$ makes with x -axis an angle of $\pi/3$ and at the abscissa $x = b$ an angle of $\frac{\pi}{4}$. The value of the integral $\int_a^b f'(x) f''(x) dx$ is
 (a) $\frac{1}{2}(1 - \sqrt{3})$ (b) $\frac{1}{2}(1 + \sqrt{3})$ (c) -1 (d) none
192. If $\frac{d[f(x)]}{dx} = g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)g(x)dx$ equals [CEE 1993]
 (a) $f(b) - f(a)$ (b) $g(b) - g(a)$ (c) $\frac{[f(b)]^2 - [f(a)]^2}{2}$ (d) $\frac{[g(b)]^2 - [g(a)]^2}{2}$
193. The value of $\int_0^{\pi/2} \frac{1}{9 \cos x + 12 \sin x} dx$ is
 (a) $\frac{1}{15} \log_{10} 6$ (b) $\frac{1}{15} \log_e 6$ (c) $\log\left(\frac{6}{15}\right)$ (d) none of these.
194. Let $I = \int_0^1 \frac{e^x}{x+1} dx$, then the value of the integral $\int_0^1 \frac{x e^{x^2}}{x^2 + 1} dx$ is
 (a) I^2 (b) $\frac{1}{2} I$ (c) $2 I$ (d) $\frac{1}{2} I^2$

Advance Level

195. Let $\frac{d}{dx} f(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = f(k) - f(1)$, then one of the possible values of k is
 (a) 15 (b) 16 (c) 63 (d) 64
196. $\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}} =$ [Rajasthan PET 1991; UPSEAT 2003]
 (a) $\frac{1}{2} \log \frac{5}{3}$ (b) $\frac{1}{3} \log \frac{5}{3}$ (c) $\frac{1}{2} \log \frac{3}{5}$ (d) $\frac{1}{5} \log \frac{3}{5}$

197. $\int_0^{\pi/2} \frac{1+2 \cos x}{(2+\cos x)^2} dx =$ [CEE 1993]
 (a) $\frac{\pi}{2}$ (b) π (c) $\frac{1}{2}$ (d) None of these
198. $\int_0^{\pi/4} \frac{\sin x + \cos x}{9+16 \sin 2x} dx =$ [IIT 1983]
 (a) $\frac{1}{20} \log 3$ (b) $\log 3$ (c) $\frac{1}{20} \log 5$ (d) None of these
199. The value of $\int_{-4}^{-5} e^{(x+5)^2} dx + 3 \int_{1/3}^{2/3} e^{9(x-2/3)^2} dx$ is [MP PET 1990]
 (a) e^5 (b) e^4 (c) $3 e^2$ (d) 0
200. $\int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}] dx$ equals
 (a) $\sqrt{2}\pi$ (b) $\pi/2$ (c) $\pi/\sqrt{2}$ (d) 2π
201. $\int_0^{2\pi} e^{x/2} \cdot \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx =$ [Roorkee 1982]
 (a) 1 (b) $2\sqrt{2}$ (c) 0 (d) None of these
202. $\int_0^{\pi/4} \frac{\sec x}{1+2 \sin^2 x}$ is equal to [MNR 1994]
 (a) $\frac{1}{3} \left[\log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}} \right]$ (b) $\frac{1}{3} \left[\log(\sqrt{2}+1) - \frac{\pi}{2\sqrt{2}} \right]$ (c) $3 \left[\log(\sqrt{2}+1) - \frac{\pi}{2\sqrt{2}} \right]$ (d) $3 \left[\log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}} \right]$
203. If $I(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$, then
 (a) $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ (b) $I(m, n) = \int_0^\infty \frac{x^m}{(1+x)^{m+n}} dx$ (c) $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ (d) Both (a) and (c)
204. If $I_n = \int_{\pi/4}^{\pi/2} (\tan x)^{-n} dx$ ($n > 1$), then $I_n + I_{n+2} =$ [MP PET 1990]
 (a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $-\frac{1}{n+1}$ (d) $\frac{1}{n} - 1$
205. The value of $\int_0^\pi \left(\sum_{r=0}^3 a_r \cos^{3-r} x \sin^r x \right) dx$ depends on [MP PET 1990]
 (a) a_0 and a_2 (b) a_1 and a_2 (c) a_0 and a_3 (d) a_1 and a_3
206. The value of the integral $\int_0^3 \frac{dx}{\sqrt{x+1} + \sqrt{5x+1}}$ is [MP PET 1990]
 (a) $\frac{11}{15}$ (b) $\frac{14}{15}$ (c) $\frac{2}{5}$ (d) None of these
207. $\int_0^{\pi/4} \cos^{3/2} 2\theta \cos \theta d\theta$ equal to
 (a) $3/8 \sqrt{2}$ (b) $3\pi/16 \sqrt{2}$ (c) $3\pi/16$ (d) None of these
208. The value of the integral $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)} dx$ is [MP PET 1990]

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- (a) $\frac{\pi}{4}(\beta - \alpha)^2$ (b) $\frac{\pi}{2}(\beta - \alpha)^2$ (c) $\frac{\pi}{8}(\beta - \alpha)^2$ (d) None of these.
209. Let $I_1 = \int_0^1 \frac{\tan^{-1} x}{x} dx$ and $I_2 = \frac{1}{2} \int_0^{\pi/2} \frac{t}{\sin t} dt$, then [MP PET 1990]
 (a) $I_1 = I_2$ (b) $I_1 < I_2$ (c) $I_1 > I_2$ (d) None of these.
210. $\int_0^1 \log(\sqrt{1+x} + \sqrt{1-x}) dx =$
 (a) $\frac{1}{2} \left(\log 2 - \frac{\pi}{2} + 1 \right)$ (b) $\frac{1}{2} \left(\log 2 + \frac{\pi}{2} + 1 \right)$ (c) $\frac{1}{2} \left(\log 2 + \frac{\pi}{2} - 1 \right)$ (d) None of these
211. Let $\frac{d}{dx} f(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = f(k) - f(1)$ then one of possible values of k is
 (a) 4 (b) 16 (c) 2 (d) None of these
212. $\int_0^1 \sin \left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx =$ [EAMCET 2003]
 (a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) π
213. $\int_0^\pi \frac{dx}{1 - 2a \cos x + a^2}$ equals [Haryana CEE 1993; MNR 1997]
 (a) $\frac{\pi}{2(1-a^2)}$ (b) $\pi(1-a^2)$ (c) $\frac{\pi}{1-a^2}$ (d) None of these

Properties of Definite Integration

Basic Level

214. $\int_0^1 f(1-x) dx$ has the same value as the integral [SCRA 1990]
 (a) $\int_0^1 f(x) dx$ (b) $\int_0^1 f(-x) dx$ (c) $\int_0^1 f(x-1) dx$ (d) $\int_{-1}^1 f(x) dx$
215. $\left[\sum_{n=1}^{10} \int_{-2n-1}^{2n} \sin^{27} x dx \right] + \left[\sum_{n=1}^{10} \int_{-2n}^{2n+1} \sin^{27} x dx \right]$ equals [MP PET 2002]
 (a) 27^2 (b) -54 (c) 36 (d) 0
216. Let a, b, c be non-zero real numbers such that $\int_0^3 (3ax^2 + 2bx + c) dx = \int_1^3 (3ax^2 + 2bx + c) dx$, then
 (a) $a + b + c = 3$ (b) $a + b + c = 1$ (c) $a + b + c = 0$ (d) $a + b + c = 2$
217. $\int_0^1 |\sin 2\pi x| dx$ is equal to
 (a) 0 (b) $-\frac{1}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{2}{\pi}$
218. $\int_0^2 |x-1| dx =$ [UPSEAT 2003]
 (a) 0 (b) 2 (c) $1/2$ (d) 1

219. $\int_{-1}^2 |x| dx =$ [DCE 1999]

(a) $5/2$

(b) $1/2$

(c) $3/2$

(d) $7/2$

220. $\int_0^3 |2-x| dx =$ [Rajasthan PET 1999]

(a) $2/7$

(b) $5/2$

(c) $3/2$

(d) $-3/2$

221. The value of $\int_1^5 (|x-3| + |1-x|) dx$ is [IIT Screening]

(a) 10

(b) $\frac{5}{6}$

(c) 21

(d) 12

222. $\int_{1/e}^e |\log x| dx =$ [UPSEAT 2001]
 (a) $1 - \frac{1}{e}$ (b) $2\left(1 - \frac{1}{e}\right)$ (c) $e^{-1} - 1$ (d) none of these
223. The correct evaluation of $\int_0^{\pi/2} \left| \sin\left(x - \frac{\pi}{4}\right) \right| dx$ is [MP PET 1993]
 (a) $2 + \sqrt{2}$ (b) $2 - \sqrt{2}$ (c) $-2 + \sqrt{2}$ (d) 0
224. The value of $\int_0^1 |3x^2 - 1| dx$ is [AMU 1999]
 (a) 0 (b) $4/3\sqrt{3}$ (c) $3/7$ (d) $5/6$
225. $\int_0^{\pi/2} |\sin x - \cos x| dx =$ [Roorkee 1990; MP PET 2001; UPSEAT 2001]
 (a) 0 (b) $2(\sqrt{2} - 1)$ (c) $\sqrt{2} - 1$ (d) $2(\sqrt{2} + 1)$
226. $\int_0^2 |(1-x)| dx =$ [SCRA 1990; Rajasthan PET 2001]
 (a) 1 (b) 2 (c) 3 (d) 0
227. $\int_{-4}^4 |x+2| dx =$
 (a) 50 (b) 24 (c) 20 (d) None of these
228. $\int_0^{2\pi} |\sin x| dx =$
 (a) 0 (b) 1 (c) 2 (d) 4
229. Let $f(x) = x - [x]$, for every real number x , where $[x]$ is the integer part of x , then $\int_{-1}^1 f(x) dx$ is
 (a) 1 (b) 2 (c) 0 (d) $1/2$
230. Find the value of $\int_0^9 [\sqrt{x} + 2] dx$, where $[.]$ is the greatest integer function [UPSEAT 2002]
 (a) 31 (b) 22 (c) 23 (d) None of these
231. If $[x]$ denotes the greatest integer less than or equal to x , then the value of the integral $\int_0^2 x^2 [x] dx$ is
 (a) $5/3$ (b) $7/3$ (c) $8/3$ (d) $4/3$
232. The value of $I = \int_0^1 x \left| x - \frac{1}{2} \right| dx$ is
 (a) $1/3$ (b) $1/4$ (c) $1/8$ (d) None of these
233. The value of $\int_0^{\sqrt{2}} [x^2] dx$ where $[.]$ is the greatest integer function
 (a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$ (c) $\sqrt{2} - 1$ (d) $\sqrt{2} - 2$
234. $\int_0^{2\pi} (\sin x + |\sin x|) dx =$ [Karnataka CET 2003]
 (a) 0 (b) 4 (c) 8 (d) 1
235. $\int_0^{\pi} |\sin x + \cos x| dx$ is equal to [WB JEE 1994]
 (a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) $1/\sqrt{2}$

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- 236.** The value of integral $\int_{-2}^4 x[x]dx$ is
 (a) $\frac{41}{2}$ (b) 20 (c) $\frac{21}{2}$ (d) None of these.
- 237.** The value of $\int_{-2}^3 |1-x^2| dx$ is
 (a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$
- 238.** If $a < 0 < b$, then $\int_a^b x|x| dx =$
 (a) $\frac{1}{2}(a^2 + b^2)$ (b) $\frac{1}{3}(b^2 - a^2)$ (c) $\frac{1}{3}(a^3 + b^3)$ (d) None of these
- 239.** The value of $\int_{-1}^3 (|x-2| + [x])dx$ is ($[x]$ stands for greatest integer less than or equal to x)
 (a) 7 (b) 5 (c) 4 (d) 3
- 240.** $\int_0^{\pi/2} \frac{d\theta}{1+\tan\theta}$ is equal to [Roorkee 1980; Karnataka CET 1993; MP PET 1996; DCE 1999]
 (a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$
- 241.** $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ is [DCE 2001]
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$
- 242.** $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx =$ [MNR 1985; BIT Ranchi 1986; UPSEAT 2002]
 (a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{3\pi^2}{2}$ (d) $\frac{\pi^2}{3}$
- 243.** The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ is [Karnataka CET 1999]
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π
- 244.** $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ is equal to [MNR 1984]
 (a) $\frac{\pi}{2} - 1$ (b) $\pi\left(\frac{\pi}{2} + 1\right)$ (c) $\frac{\pi}{2} + 1$ (d) $\pi\left(\frac{\pi}{2} - 1\right)$
- 245.** The value of $\int_0^1 \frac{dx}{x + \sqrt{1-x^2}}$ is
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$
- 246.** The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is [IIT 1993; DCE 2001]
 (a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$
- 247.** $\int_0^{\pi} x \sin x dx =$ [SCRA 1980, 1991]
 (a) π (b) 0 (c) 1 (d) π^2

248. $\int_0^{\pi/2} \frac{1}{1 + \sqrt{\tan x}} dx =$ [Rajasthan PET 1995; Kurukshetra CEE 1998]
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) 1
249. $\int_0^{\pi/2} (a \cos^2 x + b \sin^2 x) dx$ equals [Ranchi BIT 1994]
 (a) $(a+b)\pi/4$ (b) $(a+b)\pi/2$ (c) $(a+b)\pi/3$ (d) None of these
250. $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ equals [Rajasthan PET 1996; Kerala (Engg.) 2002]
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$
251. $\int_0^{2a} \frac{f(x)}{f(x) + f(2a-x)} dx =$ [Haryana CEE 1997; Assam JEE 1999; IIT 1999; Karnataka CET 2000]
 (a) a (b) $\frac{a}{2}$ (c) $2a$ (d) 0
252. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx =$
 (a) 2 (b) -2 (c) 0 (d) None of these
253. $\int_0^{\pi} |\cos x| dx =$ [MP PET 1998]
 (a) π (b) 0 (c) 2 (d) 1
254. $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx =$ [MNR 1989; UPSEAT 2002]
 (a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) None of these
255. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx =$ [MP PET 1990, 1995; IIT 1983; MNR 1990]
 (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$
256. $\int_0^{\pi/2} \log \sin x dx =$ [MP PET 1994; Rajasthan PET 1995, 96, 97]
 (a) $-\left(\frac{\pi}{2}\right) \log 2$ (b) $\pi \log \frac{1}{2}$ (c) $-\pi \log \frac{1}{2}$ (d) $\frac{\pi}{2} \log 2$
257. The value of $\int_0^{\pi/2} \frac{e^{x^2}}{e^{x^2} + e^{\left(\frac{\pi-x}{2}\right)^2}} dx$ is [AMU 1999]
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $e^{\pi^2/16}$ (d) $e^{\pi^2/4}$
258. The maximum and minimum value of the integral $\int_0^{\pi/2} \frac{dx}{(1 + \sin^2 x)}$ are
 (a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) Both (a) and (c)
259. $\int_0^{\pi/2} \log \tan x dx$ [MP PET 1999; Rajasthan PET 2001, 02; Karnataka CET 1999, 2000, 01, 02]
 (a) $\frac{\pi}{2} \log_e^2$ (b) $-\frac{\pi}{2} \log_e^2$ (c) $\pi \log_e^2$ (d) 0

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260. $\int_0^{\pi/2} \sin 2x \log \tan x \, dx =$ [Kerala (Engg.) 2002; AI CBSE 1990; Karnataka CET 1996, 98]
 (a) 1 (b) -1 (c) 0 (d) None of these
261. $\int_0^{\pi/4} \log(1 + \tan x) \, dx =$ [SCRA 1986; Karnataka CET 2000; IIT 1997]
 (a) $\frac{\pi}{4} \log 2$ (b) $\frac{\pi}{4} \log \frac{1}{2}$ (c) $\frac{\pi}{8} \log 2$ (d) $\frac{\pi}{8} \log \frac{1}{2}$
262. $\int_0^{2\pi} \frac{\sin 2\theta}{a - b \cos \theta} d\theta =$ [Roorkee 1988]
 (a) 1 (b) 2 (c) $\frac{\pi}{4}$ (d) 0
263. $\int_0^{\pi} x \sin^3 x \, dx =$ [CEE 1993]
 (a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) 0 (d) None of these
264. If f and g are continuous function on $[0, a]$ satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = 2$ then $\int_0^a f(x)g(x) \, dx =$ [IIT 1989]
 (a) $\int_0^a f(x) \, dx$ (b) $\int_a^0 f(x) \, dx$ (c) $2 \int_0^a f(x) \, dx$ (d) None of these
265. If $\int_0^{\pi} x f(\cos^2 x + \tan^2 x) \, dx = k \int_0^{\pi/2} f(\cos^2 x + \tan^4 x) \, dx$, then the value of k is
 (a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) None of these
266. The value of $\int_{\pi/4}^{3\pi/4} \frac{\phi}{1 + \sin \phi} d\phi$, is [AI CBSE 1990; IIT 1993]
 (a) $\pi \tan \frac{\pi}{8}$ (b) $\log \tan \frac{\pi}{8}$ (c) $\tan \frac{\pi}{8}$ (d) None of these
267. The value of $\int_0^{\pi} e^{\cos^2 x} \cos^5 3x \, dx$ is [Bihar CEE 1994]
 (a) 1 (b) -1 (c) 0 (d) None of these
268. If $\int_{-1}^1 f(x) \, dx = 0$, then [SCRA 1990]
 (a) $f(x) = f(-x)$ (b) $f(-x) = -f(x)$ (c) $f(x) = 2f(x)$ (d) None of these
269. $\int_{-\alpha}^{\alpha} f(x) \, dx =$ [MP PET 1994]
 (a) $2 \int_0^{\alpha} f(x) \, dx$ (b) $\int_{-\alpha}^{\alpha} f(-x) \, dx$ (c) 0 (d) None of these
270. The value $\int_{-2}^2 \left[p \ln \left(\frac{1+x}{1-x} \right) + q \ln \left(\frac{1-x}{1+x} \right)^{-2} + r \right] dx$ depends on [Orissa JEE 2003]
 (a) The value of p (b) The value of q (c) the value of r (d) The value of p and q
271. $\int_{-\pi/2}^{\pi/2} \log \left(\frac{2 - \sin x}{2 + \sin x} \right) dx =$
 (a) 0 (b) 1 (c) 2 (d) None of these
272. $\int_{-1}^1 \log \left(\frac{1+x}{1-x} \right) dx =$ [MP PET 1995]
 (a) 2 (b) 1 (c) 0 (d) π

273. To find the numerical value of $\int_{-2}^2 (px^2 + qx + s)dx$ it is necessary to know the values of constants [IIT 1992]
 (a) p (b) q (c) s (d) p and s
274. $\int_{-a}^a \sin x f(\cos x)dx =$ [Rajasthan PET 1997]
 (a) $2 \int_0^a \sin x f(\cos x)dx$ (b) 0 (c) 1 (d) None of these
275. $\int_{-\pi/2}^{\pi/2} \frac{\sin x}{1 + \cos^2 x} e^{-\cos^2 x} dx$ is equal to [AMU 1999]
 (a) $2e^{-1}$ (b) 1 (c) 0 (d) None of these
276. $\int_{-3}^3 \frac{x^2 \sin 2x}{x^2 + 1} dx =$
 (a) 0 (b) 1 (c) $2 \log_e 3$ (d) None of these
277. $\int_{\frac{1}{2}}^{\frac{1}{2}} \cos x \ln \frac{1+x}{1-x} dx$ is equal to [AMU 2000; MNR 1998]
 (a) 0 (b) 1 (c) 2 (d) $\ln 3$
278. $\int_{-1/2}^{1/2} (\cos x) \left[\log \left(\frac{1-x}{1+x} \right) \right] dx =$ [Karnataka CET 2002]
 (a) 0 (b) 1 (c) $e^{1/2}$ (d) $2e^{1/2}$
279. The value of $\int_{-\pi/2}^{\pi/2} (3 \sin x + \sin^3 x) dx$ is [MP PET 2003]
 (a) 3 (b) 2 (c) 0 (d) $\frac{10}{3}$
280. $\int_{-1}^1 \log \frac{2-x}{2+x} dx =$ [Roorkee 1986; Kurukshetra CEE 1998]
 (a) 2 (b) 1 (c) -1 (d) 0
281. $\int_{-1}^1 x |x| dx =$ [MP PET 1990]
 (a) 1 (b) 0 (c) 2 (d) -2
282. $\int_{-2}^2 |x| dx =$ [MP PET 2000]
 (a) 0 (b) 1 (c) 2 (d) 4
283. The value of $\int_{-1}^1 (\sqrt{1+x+x^2} - \sqrt{1-x+x^2}) dx$ is
 (a) 0 (b) 1 (c) -1 (d) None of these
284. $\int_{-1}^1 \sin^3 x \cos^2 x dx =$ [MNR 1991; UPSEAT 2000]
 (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2
285. $\int_{-1}^1 \sin^{11} x dx$ is equal to [MNR 1995]
 (a) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$ (b) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$ (c) 1 (d) 0
286. If $f:R \rightarrow R$ and $g:R \rightarrow R$ are one to one, real valued functions, then the value of the integral $\int_{-\pi}^{\pi} [f(x) + f(-x)][g(x) - g(-x)] dx$ is

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- (a) 0 (b) π (c) 1 (d) None of these [DCE 2001]
287. $\int_{-1}^1 x \tan^{-1} x \, dx$ equals [Rajasthan PET 1997]
 (a) $\left(\frac{\pi}{2} - 1\right)$ (b) $\left(\frac{\pi}{2} + 1\right)$ (c) $(\pi - 1)$ (d) 0
288. If $f(x)$ is an odd function of x , then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\cos x) \, dx$ is equal to [MP PET 1998]
 (a) 0 (b) $\int_0^{\frac{\pi}{2}} f(\cos x) \, dx$ (c) $2 \int_0^{\frac{\pi}{2}} f(\sin x) \, dx$ (d) $\int_0^{\pi} f(\cos x) \, dx$
289. $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 \, dx$ is equal to (where p and q are integers) [IIT 1992]
 (a) $-\pi$ (b) 0 (c) π (d) 2π
290. The value of $\int_{-1}^1 \frac{\sin x - x^2}{3 - |x|} \, dx$ [Roorkee 1995]
 (a) 0 (b) $2 \int_0^1 \frac{\sin x}{3 - |x|} \, dx$ (c) $2 \int_0^1 \frac{-x^2}{3 - |x|} \, dx$ (d) $2 \int_0^1 \frac{\sin x - x^2}{3 - |x|} \, dx$
291. $\int_{-\pi/2}^{\pi/2} \sqrt{\frac{1}{2}(1 - \cos 2x)} \, dx =$
 (a) 0 (b) 2 (c) $1/2$ (d) None of these
292. $\int_{-1}^1 x^{17} \cos^4 x \, dx =$ [MP PET 1990]
 (a) -2 (b) -1 (c) 0 (d) 2
293. If $f(x) = \frac{e^x}{1 + e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\} \, dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\} \, dx$, then the value of $\frac{I_2}{I_1}$ is [AIEEE 2004]
 (a) 1 (b) -3 (c) -1 (d) 2
294. $\int_{-\pi/2}^{\pi/2} \sqrt{(\cos x - \cos^3 x)} \, dx =$ [Rajasthan PET 1991]
 (a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 0
295. Let m be any integer. Then the integral $\int_0^{\pi} \frac{\sin 2m x}{\sin x} \, dx$ equals
 (a) 0 (b) π (c) 1 (d) None of these
296. $\int_0^{2a} f(x) \, dx =$ [Rajasthan PET 2002]
 (a) $2 \int_0^a f(x) \, dx$ (b) 0 (c) $\int_0^a f(x) \, dx + \int_0^a f(2a - x) \, dx$ (d) $\int_0^a f(x) \, dx + \int_0^{2a} f(2a - x) \, dx$
297. If $\int_0^{2a} f(x) \, dx = 2 \int_0^a f(x) \, dx$, then [SCRA 1986]
 (a) $f(2a - x) = -f(x)$ (b) $f(2a - x) = f(x)$ (c) $f(a - x) = -f(x)$ (d) $f(a - x) = f(x)$
298. $\int_0^{\pi} \frac{\cos x \, dx}{[\cos(x/2) + \sin(x/2)]^3}$ equals
 (a) 1 (b) -1 (c) 0 (d) 2
299. The value of $\int_0^{2\pi} \cos^{99} x \, dx$ is

- (a) 1 (b) -1 (c) 99 (d) 0
300. The value of $\int_0^{2\pi} |\sin^3 x| dx$ is
 (a) 0 (b) 3/8 (c) 8/3 (d) π
301. $\int_0^{\pi} \log \sin^2 x dx =$ [MP PET 1997]
 (a) $2\pi \log_e \left(\frac{1}{2}\right)$ (b) $\pi \log_e 2 + c$ (c) $\frac{\pi}{2} \log_e \left(\frac{1}{2}\right) + c$ (d) None of these
302. $\int_0^{2\pi} \sin^5 x dx$ equals
 (a) 0 (b) 16/15 (c) 32/15 (d) None of these
303. The value of $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$ is [Rajasthan PET 1988]
 (a) 1 (b) 0 (c) $\pi/4$ (d) None of these
304. $\int_0^{\pi/2} \frac{\sin 8x \log(\cot x)}{\cos 2x} dx$ equals
 (a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) None of these
305. $\int_0^{\pi} \sin 2x \sin^3 x dx$ is equal to [RPET 1993]
 (a) 0 (b) 1 (c) 2 (d) 4
306. If $f(x) = f(2 - x)$, then $\int_{0.5}^{1.5} x f(x) dx$ equals [AMU 1999]
 (a) $\int_0^1 f(x) dx$ (b) $\int_{0.5}^{1.5} f(x) dx$ (c) $2 \int_{0.5}^{1.5} f(x) dx$ (d) 0
307. The value of the integral $\int_{\frac{1}{n}}^{\frac{a n - 1}{n}} \frac{\sqrt{x}}{\sqrt{a - x} + \sqrt{x}} dx$ is [AMU 2002]
 (a) $\frac{a}{2}$ (b) $\frac{na + 2}{2n}$ (c) $\frac{na - 2}{2n}$ (d) None of these
308. Let $I_1 = \int_a^{\pi-a} x f(\sin x) dx$, $I_2 = \int_a^{\pi-a} f(\sin x) dx$, then I_2 is equal to [AMU 2000]
 (a) $\frac{\pi}{2} I_1$ (b) πI_1 (c) $\frac{2}{\pi} I_1$ (d) $2 I_1$
309. Let f be a positive function. Let $I_1 = \int_{1-k}^k x f\{x(1-x)\} dx$, $I_2 = \int_{1-k}^k f\{x(1-x)\} dx$ where $2k - 1 > 0$. Then I_1 / I_2 is [IIT 1997]
 (a) 2 (b) k (c) 1/2 (d) 1
310. The value of $\int_2^3 \frac{dx}{\sqrt{(x-2)(3-x)}}$ is
 (a) 1 (b) $\pi/2$ (c) π (d) 2π
311. If $I = \int_0^{100\pi} \sqrt{1 - \cos 2x} dx$, then the value of I is
 (a) $100\sqrt{2}$ (b) $200\sqrt{2}$ (c) $50\sqrt{2}$ (d) None of these
312. $\int_{\pi}^{10\pi} |\sin x| dx$ is [AIEEE 2002]
 (a) 20 (b) 8 (c) 10 (d) 18

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313. $\int_0^{1000} e^{x-[x]} dx$ is [AMU 2002]
 (a) $e^{1000} - 1$ (b) $\frac{e^{1000} - 1}{e - 1}$ (c) $1000(e - 1)$ (d) $\frac{e - 1}{1000}$
314. If n is a positive integer and $[x]$ is the greatest integer not exceeding x , then $\int_0^x \{x - [x]\} dx$ equals
 (a) $n^2 / 2$ (b) $n(n - 1)/2$ (c) $n/2$ (d) $\frac{n^2}{2} - n$
315. The value of $\int_{-\frac{\pi}{2}}^{\frac{199\pi}{2}} \sqrt{1 + \cos 2x} dx$ equals [AMU 1999]
 (a) $50\sqrt{2}$ (b) $100\sqrt{2}$ (c) $150\sqrt{2}$ (d) $200\sqrt{2}$
316. If $\int_0^{50\pi} (\sin^4 x + \cos^4 x) dx = k \int_0^{\pi/2} \left(\frac{3}{4} + \frac{1}{4} \cos 4x \right) dx$, then $k =$
 (a) 200 (b) 100 (c) 50 (d) 25
317. $\int_0^\pi x f(\sin x) dx =$ [IIT 1982; Kurukshetra CEE 1993]
 (a) $\pi \int_0^\pi f(\sin x) dx$ (b) $\frac{\pi}{2} \int_0^\pi f(\sin x) dx$ (c) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x) dx$ (d) None of these
318. If $\int_0^\pi x f(\sin x) dx = A \int_0^{\pi/2} f(\sin x) dx$, then A is [AIEEE 2004]
 (a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0
319. The value of the definite integral $\int_0^1 \frac{x dx}{x^3 + 16}$ lies in the interval $[a, b]$. The smallest such interval is
 (a) $\left[0, \frac{1}{17}\right]$ (b) $[0, 1]$ (c) $\left[0, \frac{1}{27}\right]$ (d) None of these
320. If $f(x)$ is a periodic function with period T , then
 (a) $\int_a^b f(x) dx = \int_a^{b+T} f(x) dx$ (b) $\int_a^b f(x) dx = \int_{a+T}^b f(x) dx$ (c) $\int_a^b f(x) dx = \int_{a+T}^{b+T} f(x) dx$ (d) $\int_a^b f(x) dx = \int_{a+T}^{b+2T} f(x) dx$
321. If $f(x)$ is an odd function defined on $[-T/2, T/2]$ and has period T , then $\phi(x) = \int_a^x f(t) dt$ is
 (a) A periodic function with period $T/2$ (b) A periodic function with period T
 (c) Not a periodic function (d) A periodic function with period $T/4$

Advance Level

322. If $[x]$ denotes the greatest integer less than or equal to x , then the value of $\int_1^5 [|x - 3|] dx$ is
 (a) 1 (b) 2 (c) 4 (d) 8
323. $\int_{-2}^2 |[x]| dx =$ [EAMCET 2003]
 (a) 1 (b) 2 (c) 3 (d) 4
324. If for a real number y , $[y]$ is the greatest integer less than or equal to y , then the value of the integral $\int_{\pi/2}^{3\pi/2} [2 \sin x] dx$ is [IIT 1999]

- (a) $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$
325. The value of $\int_{\pi}^{2\pi} [2 \sin x] dx$, where $[.]$ represents the greatest integer function is [IIT 1995]
- (a) $-\pi$ (b) -2π (c) $-\frac{5\pi}{3}$ (d) $\frac{5\pi}{3}$
326. If $f(x) = \min \{ |x-1|, |x|, |x+1| \}$, then $\int_{-1}^1 f(x) dx$ equals
- (a) 1 (b) 0 (c) 2 (d) None of these
327. The value of $\int_0^2 [x^2 - x + 1] dx$, (where $[x]$ denotes the greatest integer function) is given by
- (a) $\frac{5-\sqrt{5}}{2}$ (b) $\frac{6-\sqrt{5}}{2}$ (c) $\frac{7-\sqrt{5}}{2}$ (d) $\frac{8-\sqrt{5}}{2}$
328. $\int_{-2}^2 \min(x-[x], -x-[-x]) dx$ equals ($[x]$ represent greatest integer less than or equal to x)
- (a) 2 (b) 1 (c) 4 (d) 0
329. Let a, b, c be non-zero real numbers such that $\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c) dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c) dx$. Then the quadratic equation $ax^2 + bx + c = 0$ has
- (a) No root in $(0, 2)$ (b) At least one root in $(0, 2)$ (c) A double root in $(0, 2)$ (d) None of these
330. $\int_0^{\pi/2} f(\sin 2x) \sin x dx = \int_0^{\pi/2} f(\sin 2x) \cos x dx = \sqrt{2} \int_0^{\pi/4} f(\cos 2x) \cos x dx$ [IIT 1996]
- (a) Question is true (b) Question is false (c) Some data is missing (d) None of these
331. If $g(x) = \int_0^x \cos^4 t dt$ then $g(x+\pi)$ equals [IIT 1997 Re-Exam; DCE 2001; UPSEAT 2001]
- (a) $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$ (c) $g(x) \cdot g(\pi)$ (d) $g(x) / g(\pi)$
332. $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x} =$ [Karnataka CET 2003]
- (a) $\frac{\pi}{ab}$ (b) $\frac{\pi}{2ab}$ (c) $\frac{\pi^2}{ab}$ (d) $\frac{\pi^2}{2ab}$
333. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is [AIEEE 2003]
- (a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$ (c) $\frac{1}{n+1} - \frac{1}{n+2}$ (d) $\frac{1}{n+1} + \frac{1}{n+2}$
334. $\int_0^1 \tan^{-1} \left(\frac{1}{x^2 - x + 1} \right) dx$ is [Orissa JEE 2003]
- (a) $\ln 2$ (b) $-\ln 2$ (c) $\frac{\pi}{2} + \ln 2$ (d) $\frac{\pi}{2} - \ln 2$
335. $\int_0^1 \tan^{-1}(1-x+x^2) dx =$ [IIT 1998]
- (a) $\log 2$ (b) $\log \frac{1}{2}$ (c) $\pi \log 2$ (d) $\frac{\pi}{2} \log \frac{1}{2}$

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336. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is [AIEEE 2002]
 (a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$
337. The value of the integral $\int_{-\pi}^{\pi} \sin mx \sin nx dx$ for $m \neq n (m, n \in I)$, is
 (a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π
338. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is
 (a) π (b) $a\pi$ (c) $\frac{\pi}{2}$ (d) 2π

Summation of series by integration

Basic Level

339. $\lim_{n \rightarrow \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^5}$ [AIEEE 2003]
 (a) $\frac{1}{30}$ (b) Zero (c) $\frac{1}{4}$ (d) $\frac{1}{5}$
340. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{\sqrt{r}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2} =$
 (a) $\frac{1}{7}$ (b) $\frac{1}{10}$ (c) $\frac{1}{14}$ (d) None of these
341. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{(r^2 n - m)^{1/3}}{r n} =$
 (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 0
342. The value of $\lim_{n \rightarrow \infty} \left[\frac{(2n)!}{n! n^n} \right]^{1/n}$ is equal to
 (a) $4e$ (b) $e/4$ (c) $4/e$ (d) None of these
343. $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{n+n} \right]$ is equal to [Karnataka CET 1993]
 (a) $\log_e 3$ (b) 0 (c) $\log_e 2$ (d) 1
344. $\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n^2} \right) \left(1 + \frac{2^2}{n^2} \right) \dots \left(1 + \frac{n^2}{n^2} \right) \right]^{1/n} = \dots$ [WB JEE 1992, 93]
 (a) $2e^{(\pi+4)/2}$ (b) $2e^{\pi/4-1}$ (c) $2e^{(\pi-4)/2}$ (d) None of these
345. $\lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{r=1}^n r e^{r/n} =$ [EAMCET 1992]
 (a) 0 (b) 1 (c) e (d) $2e$
346. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals [IIT 1997]
 (a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

347. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} \log_e \left(1 + \frac{r}{n} \right)$ equals
- (a) $\log \left(\frac{27}{4e} \right)$ (b) $\log \left(\frac{27}{e^2} \right)$ (c) $\log \left(\frac{4}{e} \right)$ (d) None of these
348. If f is continuous then $\lim_{n \rightarrow \infty} \frac{1}{n} \left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + f\left(\frac{3}{n}\right) + \dots + f\left(\frac{n}{n}\right) \right]$ is nothing but
- (a) $\int_0^1 f\left(\frac{1}{x}\right) dx$ (b) $\int_0^1 x f(x) dx$ (c) $\int_0^1 \frac{1}{x} f\left(\frac{1}{x}\right) dx$ (d) $\int_0^1 f(x) dx$
349. $\lim_{n \rightarrow \infty} \left[\frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \dots + \frac{1}{nb} \right]$ is equal to
- (a) $\log \left(\frac{b}{a} \right)$ (b) $\log \left(\frac{a}{b} \right)$ (c) $\log a$ (d) $\log b$
350. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\tan \frac{\pi}{4n} + \tan \frac{2\pi}{4n} + \dots + \tan \frac{n\pi}{4n} \right] =$
- (a) $\frac{1}{\pi} \log 2$ (b) $\frac{2}{\pi} \log 2$ (c) $\frac{4}{\pi} \log 2$ (d) None of these
351. $\lim_{n \rightarrow \infty} \left[\frac{1}{1+n^3} + \frac{4}{8+n^3} + \dots + \frac{r^2}{r^3+n^3} + \dots + \frac{1}{2n} \right] =$
- (a) $\frac{1}{2} \log 2$ (b) $\frac{1}{3} \log 2$ (c) $\log \frac{1}{2}$ (d) None of these
352. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \sqrt{\frac{n+r}{n-r}} =$
- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} + 1$ (c) π (d) None of these
353. $\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{(n^2-1)}} + \frac{1}{\sqrt{(n^2-2^2)}} + \dots + \frac{1}{\sqrt{[n^2-(n-1)^2]}} \right] = \dots$
- (a) 0 (b) $\pi/2$ (c) π (d) None of these

Advance Level

354. If $na=1$ always and $n \rightarrow \infty$ then the value of $\Pi\{1+(ar)^2\}^{1/r}$ is
- (a) 1 (b) $e^{\pi^2/8}$ (c) $e^{\pi^2/24}$ (d) $e^{-\pi^2/12}$
355. The estimated value of $\frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \dots + \frac{1}{2000}$ is
- (a) 1 (b) $\log_e 3$ (c) $\log_e 2$ (d) None of these

Gamma function

Basic Level

356. $\int_0^{\pi/2} \sin^2 x \cos^3 x dx =$

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- (a) 0 (b) $\frac{2}{15}$ (c) $\frac{4}{15}$ (d) None of these
357. The value of $\int_0^{\pi/2} (\sqrt{\sin \theta} \cos \theta)^3 d\theta$ is [AMU 1999]
 (a) 2/9 (b) 2/15 (c) 8/45 (d) 5/2
358. $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx =$ [EAMCET 2002]
 (a) $\frac{3\pi}{64}$ (b) $\frac{3\pi}{572}$ (c) $\frac{3\pi}{256}$ (d) $\frac{3\pi}{128}$
359. $\int_0^a x^4 \sqrt{a^2 - x^2} dx =$
 (a) $\frac{\pi}{32}$ (b) $\frac{\pi}{32} a^6$ (c) $\frac{\pi}{16} a^6$ (d) $\frac{\pi}{8} a^6$
360. The value of $\int_0^{2\pi/3} \cos^4(3x/4) dx$ is
 (a) $\pi/8$ (b) $9\pi/64$ (c) $9\pi/128$ (d) $\pi/4$

Walli's formula

Basic Level

361. $\int_0^{\pi/2} \sin^5 x dx =$
 (a) $\frac{8}{15}$ (b) $\frac{4}{15}$ (c) $\frac{8\sqrt{\pi}}{15}$ (d) $\frac{8\pi}{15}$
362. Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$, then $\int_0^{\pi/2} f(x) dx =$ [IIT 1987]
 (a) $\frac{\pi}{4} + \frac{8}{15}$ (b) $\frac{\pi}{4} - \frac{8}{15}$ (c) $-\frac{\pi}{4} - \frac{8}{15}$ (d) $-\frac{\pi}{4} + \frac{8}{15}$
363. $\int_0^{\pi/2} \sin^{2m} x dx =$
 (a) $\frac{2m!}{(2^m \cdot m!)^2} \cdot \frac{\pi}{2}$ (b) $\frac{(2m)!}{(2^m \cdot m!)^2} \cdot \frac{\pi}{2}$ (c) $\frac{2m!}{2^m (m!)^2} \cdot \frac{\pi}{2}$ (d) None of these
364. $\int_0^{\pi} \cos^3 x dx =$ [Rajasthan 1995; MP PET 1996]
 (a) -1 (b) 0 (c) 1 (d) π
365. $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$ equals [Kurukshetra CEE 1996]
 (a) 16/15 (b) 32/15 (c) 8/15 (d) 5/6
366. The value of $\int_a^{a+(\pi/2)} (\sin^4 x + \cos^4 x) dx$ is

- (a) Independent of a (b) $a\left(\frac{\pi}{2}\right)^2$ (c) $\frac{3\pi}{8}$ (d) $\frac{3\pi a^2}{8}$

367. $\int_0^a x(2ax - x^2)^{3/2} dx =$

- (a) $a^5\left[\frac{3\pi}{16} - 1\right]$ (b) $a^5\left[\frac{3\pi}{16} + 1\right]$ (c) $a^5\left[\frac{3\pi}{16} - \frac{1}{5}\right]$ (d) None of these

Leibnitz's rule

Basic Level

368. The least value of the function $F(x) = \int_{5\pi/4}^x (3 \sin u + 4 \cos u) du$ on the interval $[5\pi/4, 4\pi/3]$, is

- (a) $\sqrt{3} + \frac{3}{2}$ (b) $-2\sqrt{3} + \frac{3}{2} + \frac{1}{\sqrt{2}}$ (c) $\frac{3}{2} + \frac{1}{\sqrt{2}}$ (d) None of these

369. The function $L(x) = \int_1^x \frac{dt}{t}$ satisfies the equation

[IIT 1996; DCE 2001]

- (a) $L(x+y) = L(x) + L(y)$ (b) $L\left(\frac{x}{y}\right) = L(x) + L(y)$ (c) $L(xy) = L(x) + L(y)$ (d) None of these

370. If $\int_{\pi/2}^x \sqrt{3-2\sin^2 u} du + \int_0^y \cos t dt = 0$, then $\frac{dy}{dx} =$

- (a) $\frac{\sqrt{4-3\sin^2 x}}{\cos y}$ (b) $-\frac{\sqrt{3-2\sin^2 x}}{\cos y}$ (c) $\sqrt{3-2\sin^2 x} + \cos y$ (d) None of these

371. If $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$, then the value of $f(1)$ is

[IIT 1998]

- (a) $1/2$ (b) 0 (c) 1 (d) $-1/2$

372. If $f(t) = \int_{-t}^t \frac{dx}{1+x^2}$, then $f'(1)$ is

[Roorkee 2000]

- (a) Zero (b) $\frac{2}{3}$ (c) -1 (d) 1

373. If $f(t) = \int_x^1 \frac{dt}{1+t^2}$ and $I_2 = \int_1^{1/x} \frac{dt}{1+t^2}$ for $x > 0$, then

- (a) $I_1 = I_2$ (b) $I_1 > I_2$ (c) $I_2 = \cot^{-1} x - \pi/4$ (d) Both (a) and (c)

374. If $\int_0^t \frac{bx \cos 4x - a \sin 4x}{x^2} dx = \frac{a \sin 4t}{t} - 1$ where $0 < t < \frac{\pi}{4}$, then the value of a, b are equal to

- (a) $\frac{1}{4}, 1$ (b) $-1, 4$ (c) $2, 2$ (d) $2, 4$

375. The equation of tangent to the curve $y = \int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}}$ at $x = 1$ is equal to

- (a) $x\sqrt{3} - y + (\sqrt{3} + 1) = 0$ (b) $x\sqrt{3} - y + 1 = 0$ (c) $x - y\sqrt{2} - 1 = 0$ (d) None of these

376. If $f(x) = \int_2^{x^2} \frac{(\sin^{-1} \sqrt{t})^2}{\sqrt{t}} dt$ then the value of $(1-x^2)\{f''(x)\}^2 - 2f'(x)$ at $x = \frac{1}{\sqrt{2}}$ is

- (a) $2 - \pi$ (b) $3 + \pi$ (c) $4 - \pi$ (d) None of these

370 Definite integral

377. If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x) =$ [MNR 1982; Karnataka CET 1999]
(a) $\cos x + x \sin x$ (b) $x \sin x$ (c) $x \cos x$ (d) None of these
378. Let $f(x) = \int_1^x \frac{\log_e t}{1+t} dt$, and $f(x) + f\left(\frac{1}{x}\right) = k(\log_e x)^2$, then $k =$
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$
379. The equation $\int_0^x (t^2 - 8t + 13) dt = x \sin\left(\frac{a}{x}\right)$ has a solution if $\sin\left(\frac{a}{6}\right)$ is
(a) Zero (b) -1 (c) 1 (d) None of these

Advance Level

380. The difference between the greatest and least values of the function $\phi(x) = \int_0^x (t+1) dt$ on $[2, 3]$ is
(a) 3 (b) 2 (c) $7/2$ (d) $11/2$
381. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in [IIT Screening 2003]
(a) $(2, 2)$ (b) No value of x (c) $(0, \infty)$ (d) $(-\infty, 0)$
382. The value of $\int_0^{n\pi+\pi} |\sin x| dx$ is
(a) $2n+1+\cos \pi$ (b) $2n+1-\cos \pi$ (c) $2n+1$ (d) $2n+\cos \pi$
383. The value of $\int_0^{\sin^4 x} \sin^{-1} \sqrt{t} \, dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} \, dt$ is [MP PET 2001]
(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi}{4}$ (d) None of these
384. The point of extreme of $\int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$ are [IIT Screening]
(a) $x = -2$ (b) $x = 1$ (c) $x = 0$ (d) All of these
385. Let $g(x) = \int_0^x f(t) dt$ where $\frac{1}{2} \leq f(t) \leq 1, t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in (1, 2]$, then [IIT Screening 2000]
(a) $-\frac{3}{2} \leq g(2) < \frac{1}{2}$ (b) $0 \leq g(2) < 2$ (c) $\frac{3}{2} < g(2) \leq \frac{5}{2}$ (d) $2 < g(2) < 4$
386. If $f(x) = \int_0^{\sin x} \cos^{-1} t \, dt + \int_0^{\cos x} \sin^{-1} t \, dt, 0 < x < \frac{\pi}{2}$, then $f\left(\frac{\pi}{4}\right) =$
(a) 0 (b) $\pi\sqrt{2}$ (c) 1 (d) $1 + \frac{\pi}{2\sqrt{2}}$
387. The function $f(x) = \int_0^x t(t-1)(t-2) dt$ is minimum, when
(a) $x = 0, 1$ (b) $x = 1, 2$ (c) $x = 0, 2$ (d) None of these

388. $\int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2} =$ [Rajasthan PET 2000, 2002]

- (a) $\pi \log 2$ (b) $-\pi \log 2$ (c) $\frac{\pi}{2} \log 2$ (d) $\frac{-\pi}{2} \log 2$

389. $\int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)} =$ [Karnataka CET 2003]

- (a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 1

390. Given that $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then the value of $\int_0^{\infty} \frac{x^2 dx}{(x^2+4)(x^2+9)}$ is [Karnataka CET 1993]

- (a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

391. $\int_0^{\infty} \frac{dx}{(x + \sqrt{x^2+1})^3} =$ [EAMCET 1992]

- (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) None of these

392. The value of the integral $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx$ is [Karnataka CET 1997; AMU 2000]

- (a) 0 (b) $\log 7$ (c) $5 \log 13$ (d) None of these

393. $\int_0^{\infty} \frac{1}{1+e^x} dx =$

- (a) $\log 2 - 1$ (b) $\log 2$ (c) $\log 4 - 1$ (d) $-\log 2$

394. $\int_0^{\infty} \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ equals [Rajasthan PET 1988]

- (a) 0 (b) π (c) 1 (d) $\pi/2$

395. $\int_0^{\infty} \frac{x}{1+x^4} dx$ equals [Rajasthan PET 1994]

- (a) $\pi/8$ (b) $\pi/4$ (c) $\pi/2$ (d) π

Advance Level

396. The value of the integral $I = \int_1^{\infty} \frac{x^2 - 2}{x^3 \sqrt{x^2 - 1}} dx$ is

- (a) 0 (b) $2/3$ (c) $4/3$ (d) None of these

Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	c	d	a	c	b	c	c	b	c	c	c	c	b	a	a	d	b	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	a	c	a	a	a	c	a	b	d	d	d	b	a	a	b	c	b	a	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	a	d	b	a	c	b	a	a	c	b	c	c	b	c	c	c	b	a
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
c	a	a	b	b	b	b	c	a	c	a	c	a	d	b	a	d	a	c	c
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
d	b	d	b	a	d	b	a	a	c	c	d	b	b	c	b	b	c	b	c
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
b	d	d	c	c	c	c	b	c	c	d	c	c	b	d	b	a	b	d	d
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
b	d	d	b	b	a	b	a	a	d	d	a	b	c	b	a	a	c	d	a
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
b	b	c	d	a	b	c	a	b	a	c	d	d	a	b	d	a	b	b	c
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
b	d	a	a	a	a	b	d	c	a	c	b	a	c	b	b	c	b	c	c
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
c	d	a	b	b	c	d	a	d	d	c	c	b	b	d	a	c	a	d	c
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
c	a	d	b	d	d	b	c	a	c	b	b	c	a	d	a	d	d	a	b
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
d	b	b	b	b	a	c	d	a	a	b	c	c	b	c	a	d	c	a	d
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
c	a	a	d	d	d	a	b	a	c	a	c	c	c	c	a	a	d	d	c
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
c	d	b	a	b	a	c	b	b	c	a	c	d	b	c	a	a	a	c	d
281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
b	d	a	a	d	a	a	c	d	c	b	c	d	c	a	c	b	c	d	c
301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320
a	a	b	a	a	b	c	c	c	c	b	d	c	c	d	b	b	b	a	c
321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340
b	b	d	c	c	d	c	b	b	a	a	d	c	d	a	b	a	c	d	c
341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360
b	c	c	c	b	b	a	d	a	b	b	b	b	c	c	b	c	c	b	d
361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380
a	c	b	b	a	c	c	b	c	b	a	d	d	a	c	d	b	d	c	c
381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396				

d	b	c	d	b	d	c	a	c	a	a	a	b	c	b	a
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