CBSE Test Paper 02

Chapter 13 Kinetic Theory

1. Calculate the change in internal energy of 3.00 mol of helium gas when its temperature is increased by 2.00 K. **1**
   a. 85.0 J
   b. 75.0 J
   c. 65.0 J
   d. 95.0 J
2. A spherical balloon with a volume of 4 000 $\mathrm{cm3}$ contains helium at an (inside) pressure of 1.20 $\times$ 10^5 Pa. How many moles of helium are in the balloon if each helium atom has an average kinetic energy of 3.60 $\times$ 10^{-22} J? **1**
   a. 3.82 mol
   b. 3.12 mol
   c. 3.32 mol
   d. 3.42 mol
3. 1 mole of a monoatomic gas is mixed with 3 moles of a diatomic gas. What is the molecular specific heat of the mixture at constant volume? **1**
   a. 18.7 J / mol K
   b. 15.2 J / mol K
   c. 12.5 J / mol K
   d. 22.6 J / mol K
4. Air at 20.0$^\circ\mathrm{C}$ in the cylinder of a diesel engine is compressed from an initial pressure of 1.00 atm and volume of 800.0 cm^3 to a volume of 60.0 cm^3. Assume that air behaves as an ideal gas with $\gamma = 1.4$ and that the compression is adiabatic. Find the final temperature of the air **1**
   a. 826 K
   b. 679 K
   c. 765 K
   d. 898 K
5. The molar specific heat at constant volume, $\mathrm{C_v}$ for diatomic gases is **1**
   a. $\frac{7}{2}$R

b. $\frac{5}{2}$ R
c. R
d. $\frac{3}{2}$ R

6. What is mean free path? **1**

7. What is the minimum possible temperature on the basis of Charles' law? **1**

8. Calculate the molecular kinetic energy of 1 g of helium (molecular weight 4) at 127°C. (Given, R = 8.31 $Jmol^{-1}K^{-1}$) **1**

9. What do you mean by degrees of freedom of a gas molecule? Write the number of degrees of freedom for a monatomic gas. **2**

10. Find out the ratio between most probable velocity, average velocity and root mean square velocity of gas molecules? **2**

11. Calculate the mean kinetic energy of one mole of hydrogen at 273 K. Take 8.3 J mol-1 K^{-1}. **2**

12. Establish the relation between $\gamma\left(=\frac{C_P}{C_V}\right)$ and degrees of freedom (n). **3**

13. Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find out the temperature of the mixture if the masses of molecules are m_1 and m_2 and number of molecules is n_1 and n_2? **3**

14. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C? **3**

15. Estimate the average thermal energy of a helium atom at **5**

    i. room temperature (27 °C),
    ii. the temperature on the surface of the Sun (6000 K),
    iii. the temperature of 10 million Kelvin (the typical core temperature in the case of a star).

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Answer

1. b. 75.0 J

   Explanation: Helium is a monoatomic gas.(C_V = 1.5R)

   change in internal energy

   $\Delta U = nC_V\Delta T = 3 \times 1.5 \times 8.31 \times 2 = 75J$

2. c. 3.32 mol

   Explanation: $P = \frac{2}{3}\frac{N}{V}\left(\bar{K}\right)$

   $N = \frac{3}{2}\frac{PV}{\left(\bar{K}\right)} = \frac{3}{2} \times \frac{\left(1.20\times10^5\right)\left(4.00\times10^{-3}\right)}{3.60\times10^{-22}} = 2 \times 10^{24}$

   $n = \frac{N}{N_A} = \frac{2\times10^{24}}{6.023\times10^{23}}$ = 3.32 mole

3. a. 18.7 J / mol K

   Explanation: for monoatomic gas

   $C_V = \frac{3}{2}R, C_P = \frac{5}{2}R$

   from conservation of energy

   $C_V = \frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2} = \frac{(1\times1.5R)+(3\times2.5R)}{1+3} = \frac{9}{4}R$

   $C_V = \frac{9}{4} \times 8.31$ = 18.7J/mol K

4. a. 826 K

   Explanation: $T_1{V_1}^{\gamma-1} = T_2{V_2}^{\gamma-1}$ $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 1 \times \left(\frac{800}{60}\right)^{1.4-1}$ = 826K

5. b. $\frac{5}{2}$R

   Explanation: $C_V = \frac{1}{2}Rf$

   for diatomic gases f = 5

   $C_V = \frac{5}{2}R$

6. Mean free path is defined as the average distance a molecule travels between collisions. It is represented by λ (lambda). Its Units is meters (m).

7. The minimum possible temperature on the basis of Charles' law is - 273.15°C at which all the gases become zero in volume.

8. Given, temperature(T) = 273 + 127 = 400 K
   Helium is a mono-atomic gas.
   Average kinetic energy per mole of helium gas = $\frac{3}{2}$ RT
   Average kinetic energy of 1 g of helium = $\frac{3\times 8.31\times 400}{2\times 4}$ = 1246.5 J (since 1 gm of Helium contains 1/4 mole)

9. Degrees of freedom of a gas molecule is the minimum number of coordinates (number of independent variables) required to completely specify the position (state of motion or energy) of it. For a monatomic gas e.g., He, Ne, etc., a molecule can have translational motion in any direction in the container and hence its speed v can be supposed to be consisting of three components v_x, v_y, and v_z along three principal axes and, thus, have three degrees of translational motion. A monatomic gas molecule does not possess rotational or vibrational motion. So, a total number of degrees of freedom per molecule of a monatomic gas is three only.

10. Since,
    Most Probable velocity, $V_{mp} = \sqrt{\frac{2KT}{m}}$
    Average velocity, $\overline{V} = \sqrt{\frac{8KT}{\pi m}}$
    Root Mean Square velocity: $V_{r.m.s.} = \sqrt{\frac{3KT}{m}}$
    So, $\mathrm{V_{mp}} : \overline{\mathrm{V}} \mathrm{Vrm.s} = \sqrt{\frac{2\mathrm{KT}}{\mathrm{m}}} : \sqrt{\frac{3\mathrm{KT}}{\pi \mathrm{m}}} : \sqrt{\frac{3\mathrm{KT}}{\mathrm{m}}}$
    $= \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$
    $V_{mp} : \overline{V}$ $V_{r.m.s.}$ = 1 :1.3 : 1.23

11. Standard temperature T = 273 K.
    ∴ Mean kinetic energy of one mole of hydrogen at STP
    $\overline{E} = \frac{3}{2} RT$
    = $\frac{3}{2} \times 8.3 \times 273$
    = 3403 J
    = 3.4×10^3J

12. Now $\gamma = \frac{C_P}{C_V}$
    Where C_P = specific heat at constant pressure

and C_V = Specific heat at constant volume.

and n = Degrees of freedom $\rightarrow$ which is the total number of co-ordinates or independent quantities required to describe completely the position and configuration of the system.

Suppose, a polyatomic gas molecule has 'n' degrees of freedom.

$\therefore$ Total energy associated with one mole molecule of the gas with

N_A = Total number of molecules (Avogadro's number)

R = Universal Gas Constant

$R = N_A K_B$

K_B = Boltzmann Constant

$E = n \times \frac{1}{2}\mathrm{K_B T} \times \mathrm{N_A} = \frac{n}{2}RT$ ....(i) (from law of equipartition of energy, we know that for each degree of freedom of a gas molecule, energy = $\frac{1}{2}K_BT$)

As,

Specific heat at constant volume,

$C_V = \frac{dE}{dT}$

$\Rightarrow C_V = \frac{d}{dT}\left(\frac{n}{2}RT\right)$ [putting the value of energy E from equation (i)]

$\therefore C_V = \frac{n}{2}R$

Now Specific heat at constant Pressure, $C_P = C_V + R$ (using known equation $C_P - C_V = R$)

$\therefore C_P = \frac{n}{2}R + R$

$\Rightarrow C_P = \left(\frac{n}{2} + 1\right)R$

As, $\gamma = \frac{C_P}{C_V}$

$\therefore \gamma = \frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}R}$

$\Rightarrow \gamma = \left(\frac{n}{2} + 1\right) \times \frac{2}{n}$

$\Rightarrow \gamma = \frac{2}{n} \times \left(\frac{n}{2} + 1\right)$

$\Rightarrow \gamma = \left(1 + \frac{2}{n}\right)$, This is the required relation γ and degrees of freedom, n.

13. In a perfect gas, there is no mutual interaction between the molecules.
    Now, kinetic energy of gas $= \frac{1}{2}mv^2$
    From the law of equi-partition of energy, kinetic energy of a single molecule with 3 degrees of freedom:
    $\frac{1}{2}mv^2 = \frac{3}{2}KT$

Kinetic energy of n_1 molecules of one gas at temperature $T_1 = n_1 \times \left(\frac{3}{2}KT_1\right) \rightarrow (1)$

Kinetic energy of n_2 molecules of other gas at temperature T_2

$= n_2 \times \left(\frac{3}{2}KT_2\right) \rightarrow (2)$

n_1, n_2 = Number of molecules in the two gases

K = Boltzmann's Constant

T_1, T_2 $\rightarrow$ Temperatures of the two gases

Total K.E. $= \frac{3}{2}K\left(n_1T_1 + n_2T_2\right)$ ....(3) (adding equation (1) & (2))

Let T be the absolute temperature of the mixture of gases

Then,

Total Kinetic energy of the mixture $= n_1 \times \left(\frac{3}{2}KT\right) + n_2 \times \left(\frac{3}{2}KT\right)$

Total Kinetic energy $= \frac{3}{2}KT\left(n_1 + n_2\right) \rightarrow 4)$

Since there is no loss of energy, hence on comparing equations (3) & (4) for total kinetic energy: $\rightarrow$

$\frac{3}{2}KT\left(n_1 + n_2\right) = \frac{3}{2}K\left(n_1T_1 + n_2T_2\right)$

$\Rightarrow$ T (n_1 + n_2) = (n_1T_1 + n_2T_2)

$\Rightarrow T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ This is required temperature of the mixture.

14. When the air bubble is at 40m depth, then

V_1 = 1.0cm^3

$= 1.0 \times 10^{-6}$m^3

T_1 = 12^{o}C

= 12 + 273 = 285K

P_1 = 1 atm + hpg

$= 1.01 \times 10^5 + 40 \times 10^3 \times 9.8$

= 4,93,000 Pa

When the air bubble reaches at the surface of lake,

then

V_2 =?

T_2 = 35^{o}C = 35 + 273K

= 308K

$P_2 = 1\text{atm} = 1.01 \times 10^5\text{Pa}$

Now $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

or $\mathrm{V}_2 = \frac{\mathrm{P_1V_1T_2}}{\mathrm{T_1P_2}}$

$\mathrm{V}_2 = \frac{493000 \times 1.01 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5}$

$= 5.328 \times 10^{-6}\mathrm{m}^3$.

15. i. According to law of equipartition of energy we know that the energy of a gas molecule per degree of freedom = $\frac{1}{2}kT$. For helium atom in the given question, there are 3 degrees of freedom with total thermal energy = $3 \times \frac{1}{2}kT = \frac{3}{2}kT$.

   Now at room temperature, T= 27°C = (273+27)K=300 K

   We know that, the average thermal energy per molecule $= \frac{3}{2}kT$

   Where k is the Boltzmann constant $= 1.38 \times 10^{-23}\mathrm{m}^2\mathrm{kg}s^{-2}\mathrm{K}^{-1}$

   $\therefore\ \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$ (k = 1.38 × 10^{-23}kg $m^2s^{-2}K^{-1}$, T = 300 K)

   $= 6.21 \times 10^{-21}J$

   Hence, the average thermal energy of a helium atom at room temperature (27°C) is $6.21 \times 10^{-21}J$.

   ii. On the surface of the sun, T= 6000 K

   Average thermal energy $= \frac{3}{2}kT$ with k = 1.38 × 10^{-23} kg $m^2s^{-2}K^{-1}$

   $= \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000$

   $= 1.241 \times 10^{-19}J$

   Hence, the average thermal energy of a helium atom on the surface of the sun having temperature 6000 K is $1.241 \times 10^{-19}J$.

   iii. At temperature, T= 10 million K = 100 lakhs K = 10^5 × 10^2 K = 10^7K

   Average thermal energy $= \frac{3}{2}kT$ with k = 1.38 × 10^{-23}kg $m^2s^{-2}K^{-1}$

   $= \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7$

   $= 2.07 \times 10^{-16}J$

   Hence, the average thermal energy of a helium atom at the core of a star is $2.07 \times 10^{-16}J$.